

OPEN ACCESS

Hydrodynamics of the electroweak phase transition in an extension of the Standard Model with dimension-6 interactions

To cite this article: M Sopena and SJ Huber 2010 *J. Phys.: Conf. Ser.* **259** 012048

View the [article online](#) for updates and enhancements.

You may also like

- [Quantum transport and electroweak baryogenesis](#)
T Konstandin
- [Andrei Sakharov's research work and modern physics](#)
B L Altshuler
- [Scalar triplet leptogenesis in the presence of right-handed neutrinos with \$S_3\$ symmetry](#)
Subhasmita Mishra and Anjan Giri



ECS
The
Electrochemical
Society
Advancing solid state &
electrochemical science & technology

DISCOVER
how sustainability
intersects with
electrochemistry & solid
state science research

Hydrodynamics of the electroweak phase transition in an extension of the Standard Model with dimension-6 interactions

M Sopena¹, SJ Huber^{1, 2}

¹ Department of Physics and Astronomy, Sussex University, Brighton, East Sussex BN1 9QE, UK

E-mail: m.sopena@sussex.ac.uk

Abstract. Extensions of the Standard Model are being considered as viable settings for a first-order electroweak phase transition which would satisfy Sakharov's three conditions for the generation of the baryon asymmetry of the Universe. These extensions would provide a sufficiently strong phase transition and remove the main obstacles which appear in the context of the Standard Model: A far-too-high lower bound on the Higgs mass, immediate wipeout of the newly-created baryon asymmetry and insufficient CP violation. We apply standard semiclassical treatments of the hydrodynamics of a first-order phase transition to the case of a recently-introduced dimension-6 extension of the Standard Model which (within the present bounds on the Higgs mass) could produce the observed baryon asymmetry of the Universe. We express the friction term in the hydrodynamic equations in terms of the particle content of the model and produce predictions for the velocity of the expanding bubble wall in the stationary regime.

1. Introduction

A 1st-order electroweak phase transition at the ~ 100 GeV scale could satisfy Sakharov's three conditions for electroweak baryogenesis: Deviation from thermal equilibrium, CP violation, and baryon number violation. In the Standard Model, B-violation proceeds through finite-temperature 'sphaleron' transitions in the presence of external W fields. These are Boltzmann-suppressed and become relevant at $T \gtrsim 100$ GeV, with $E_{Sph}/T \sim v/T$, where v is the Higgs VEV. In a first-order transition, bubbles of the new phase nucleate and grow, with B-violation taking place *outside* the growing bubbles, in the symmetric phase, and the baryon asymmetry then being transported across the bubble wall into the new phase where it must survive washout. However we now know that within the Standard Model the washout-preventing condition $v/T > 1$ is extremely hard to satisfy, not to mention that the electroweak phase transition is not even first-order for $m_h \gtrsim 75$ GeV [1]. This is the origin of the current interest in extended settings (including supersymmetry) in which a first-order electroweak phase transition which avoided washout of the newly-created baryon asymmetry could take place within existing experimental bounds. Such a setting could even be constrained through signatures detectable in the near future, for instance at the LHC or through measurements of an electric dipole moment for the neutron.

² DPhil supervisor

We present the results of applying the usual WKB-approximation hydrodynamic analysis to one such (non-supersymmetric) extension of the Standard Model analysed in [2]. The model features dimension-6 interactions regulated by a cut-off scale M . The model is fully parametrised by M and the Higgs mass m_h and provides a broad enough region in parameter space where a baryon-asymmetry-preserving, first-order phase transition may take place.

2. The dimension-6 potential

The finite-temperature effective potential for the dimension-6 model is written as [2]

$$\begin{aligned}
 V(\phi, T) = & \frac{1}{2}(-\mu^2 + (\frac{1}{2}\lambda + \frac{3}{16}g_2^2 + \frac{1}{16}g_1^2 + \frac{1}{4}y_t^2)T^2)\phi^2 \\
 & - \frac{g_2^3}{16\pi}T\phi^3 + \frac{\lambda}{4}\phi^4 + \frac{3}{64\pi^2}y_t^4\phi^4 \ln(\frac{Q^2}{c_F T^2}) \\
 & + \frac{1}{8M^2}(\phi^6 + 2\phi^4 T^2 + \phi^2 T^4)
 \end{aligned} \tag{1}$$

where $Q \equiv m_{top} = 178 \text{ GeV}$, $c_F \approx 13.94$, and μ and λ are determined via the conditions

$$\left. \frac{\partial V(\phi, 0)}{\partial \phi} \right|_{\phi=v_0} = 0, \quad \left. \frac{\partial^2 V(\phi, 0)}{\partial \phi^2} \right|_{\phi=v_0} = m_h^2 \tag{2}$$

where $v_0 = 246 \text{ GeV}$. Dimension-6 non-renormalisable operators can parametrise the effects of new physics beyond the cut-off M . In order for these to be relevant at the weak scale we require $M \lesssim 1 \text{ TeV}$.

3. Nucleation temperature. The hydrodynamic equations.

For a first-order transition to take place, the expansion of the effective potential at the relevant temperature scale, which has a global minimum at zero Higgs VEV, must develop a *second* local minimum at a nonzero VEV as T decreases (see eg [3]). As T decreases further the value of V at the second minimum approaches the value at zero until both become degenerate at the so-called *critical temperature*. For $T < T_c$ nucleation of bubbles of the broken symmetry phase becomes possible. As T falls further below T_c , production of bubbles of the broken symmetry phase accelerates and existing bubbles grow, eventually filling all space. The *nucleation temperature* $T_n < T_c$ is that at which the integrated probability of bubble nucleation in the horizon volume reaches unity,

$$P(T = T_n) = \int_{T_n}^{T_c} dP = \int_{T_n}^{T_c} (\Gamma / \text{Vol}) \cdot V_H \cdot dt = \int_{T_n}^{T_c} \frac{T^4}{H^4} e^{-F_c/T} \frac{dT}{T} = 1 \tag{3}$$

where we have taken $\Gamma / \text{Vol} = \Lambda^4(T) e^{-F_c/T} \approx T^4 e^{-F_c/T}$, F_c being the free energy of the *critical bubble* (large enough to spontaneously grow) at the temperature T .

The phase transition ends when the fraction of the horizon volume taken over by the new phase becomes one.

The usual treatment of the hydrodynamical problem models the early Universe plasma as a perfect relativistic fluid with a conserved energy-momentum tensor,

$$\begin{aligned}
\partial_\mu T^{\mu\nu} &= \partial_\mu (T_{field}^{\mu\nu} + T_{fluid}^{\mu\nu}) = \\
&= \partial_\mu \left(\partial^\mu \phi \partial^\nu \phi - g^{\mu\nu} \left(\frac{1}{2} \partial_\alpha \phi \partial^\alpha \phi \right) + \right. \\
&\quad \left. + (\rho + P) u^\mu u^\nu - P g^{\mu\nu} \right) = 0
\end{aligned} \tag{4}$$

which, applying the relevant thermodynamic relations and introducing a *friction term* [4], results, in the rest frame of the advancing bubble wall, working in a single spatial dimension (perpendicular to the planar wall, along the direction of advance), and assuming a stationary situation, in the system

$$\frac{d^2 \phi(x)}{dx^2} = \frac{\partial V(\phi, T)}{\partial \phi} + \eta \frac{\phi^2}{T_{s1}} v \gamma \frac{d\phi(x)}{dx} \tag{5}$$

$$(4aT^4 - T \frac{\partial V(\phi, T)}{\partial T}) \gamma^2 v = C_1 \tag{6}$$

$$(4aT^4 - T \frac{\partial V(\phi, T)}{\partial T}) \gamma^2 v^2 + P_r - V(\phi, T) + \frac{1}{2} \left(\frac{d\phi}{dx} \right)^2 = C_2 \tag{7}$$

where v (the fluid velocity), T and the Higgs VEV ϕ depend exclusively on the spatial coordinate and γ is the usual relativistic factor $(1 - v^2)^{-1/2}$. η is our friction parameter and T_{s1} the plasma temperature in the symmetric phase ahead of the wall. We solve this system with vanishing ϕ derivative at both extremes of the integration interval and vanishing Higgs VEV at the extreme in the symmetric phase and we obtain the shape of the wall (fig 1). Solutions to the hydrodynamic equations are usually described as either *subsonic* (deflagration) or *supersonic* (detonation) relative to the velocity of sound in the plasma [4] [5] [6]. Subsonic solutions are preceded by a shock front which heats up and accelerates the plasma at rest. A supersonic wall is followed by a rarefaction wave which brings the plasma back to rest. Stationary solutions of the hydrodynamic equations are similarity solutions, maintaining their shape as the bubble grows. For a deflagration we calculate the whole bubble profile by solving the conservation equation in spherical coordinates in the region between the bubble wall and the shock front [6] and the leap in v , T across the shock front [4]. The results of the calculation for a choice of model parameters is shown in figure 2.

4. The friction parameter

The friction term introduced in a phenomenological way above can be calculated explicitly from the particle populations in the plasma [7]. The equation of motion for the Higgs field can be written covariantly as

$$\square \phi + \frac{\partial V_{eff}(\phi, T)}{\partial \phi} + \Sigma \frac{dm^2}{d\phi} \int \frac{d^3 p}{(2\pi)^3 2E} \delta f(p, x) = 0 \tag{8}$$

with the mass dependence on the Higgs VEV $m^2 = \frac{y^2 \phi^2}{2}$ for fermions, $m^2 = \frac{g^2 \phi^2}{4}$ for bosons, and δf expressing the *deviation from equilibrium* of particle populations responsible for the friction (the sum is over all relevant species). In the WKB approximation distributions evolve following a Boltzmann equation

$$\frac{df}{dt} = \partial_t f + \dot{\vec{x}} \cdot \partial_{\vec{x}} f + \dot{\vec{p}} \cdot \partial_{\vec{p}} f = -C[f] \tag{9}$$

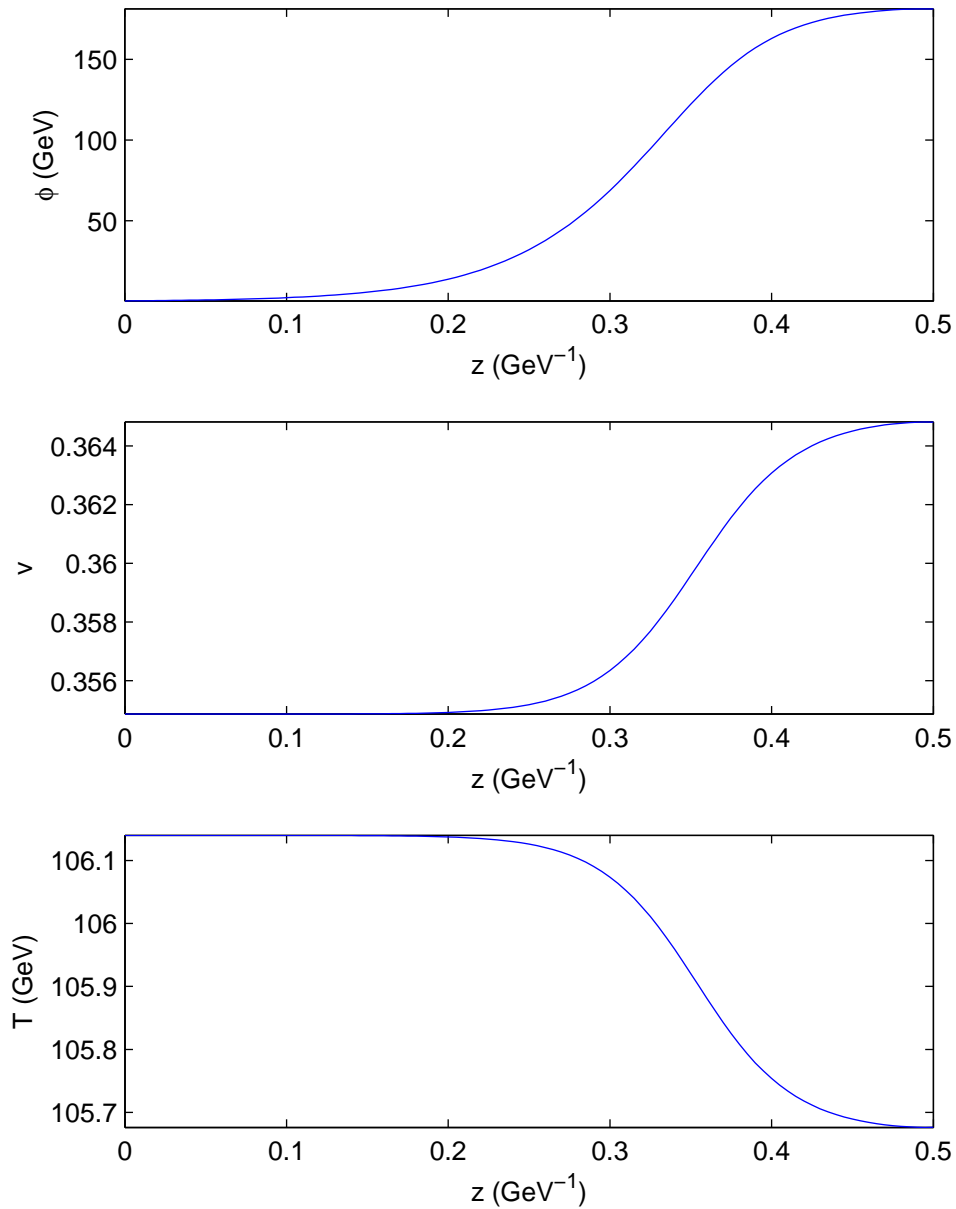


Figure 1. Higgs VEV, fluid velocity and temperature profiles across a subsonic bubble wall in the case $M = 800$, $m_h = 115$, $\eta = 0.398$. Quantities are in the rest frame of the wall. The symmetric phase is to the left [9].

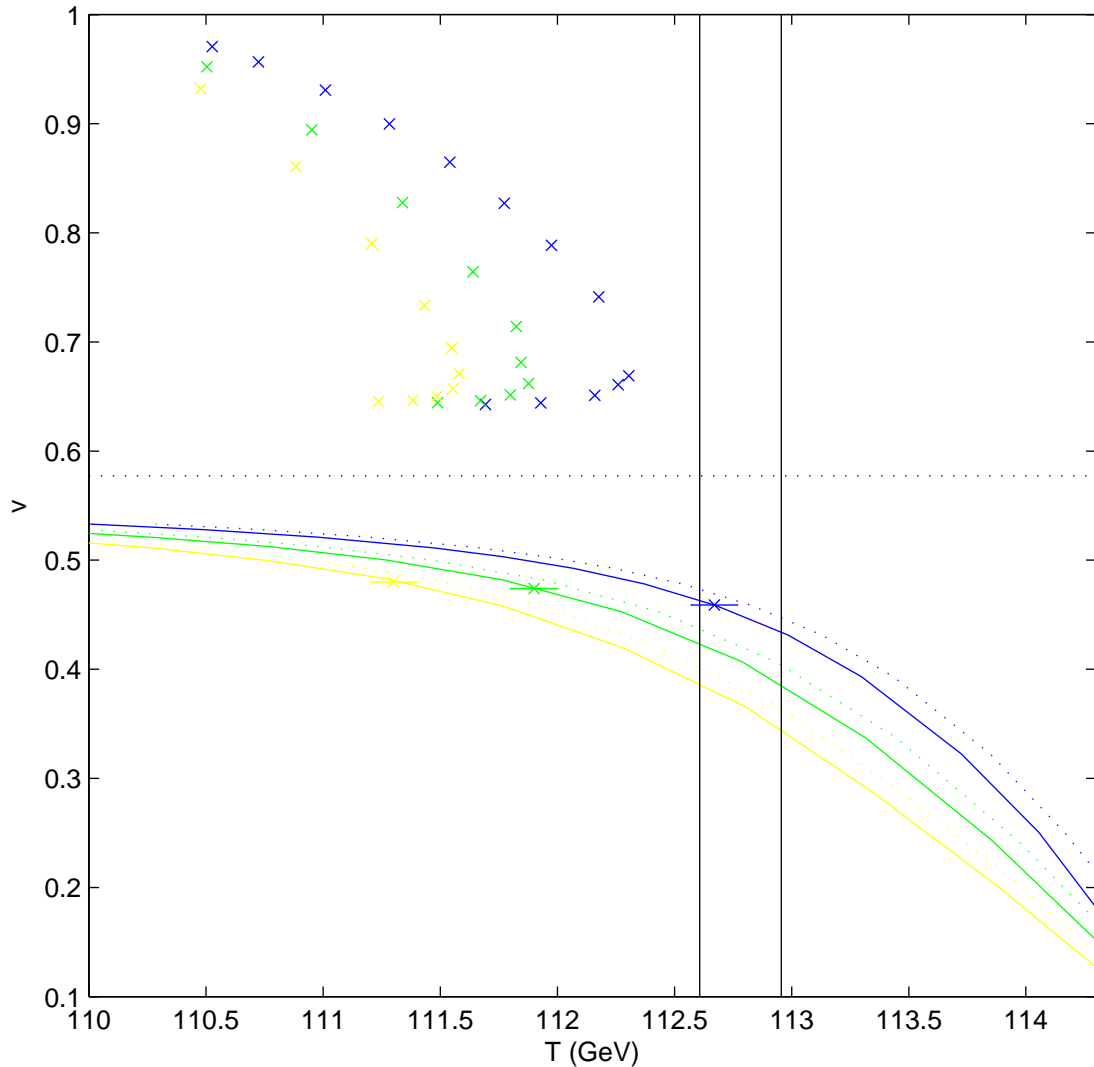


Figure 2. Wall velocity vs temperature of the universe for values of the friction coefficient η 0.3 (blue), 0.4 (green) and 0.5 (yellow), $M = 800$ GeV, $m_h = 120$ GeV. The horizontal dotted line marks the speed of sound in the plasma. The continuous lines below are subsonic solutions. For comparison, dotted lines to the right of them mark solutions found when neglecting the sphericity of the bubble. The horizontal error bars mark the point where the stability criterium for subsonic solutions [8] changes sign (the stable region lies to the right of the error bars). The crosses above the velocity of sound mark two branches of supersonic solutions, the lower one being unphysical [5]. The two vertical lines mark the nucleation and finalisation temperatures for the phase transition for this choice of parameters. In this example and for these values of η stationary supersonic solutions would be excluded for the duration of the phase transition [9].

Table 1. Fitted η s and velocities for different values of M for two slices at $m_h = 115, 150$ GeV [9].

m_h	M	T_n	$\xi = \phi_0/T_n$	η	v_w
115	900	115.92	1.26	0.477	0.34
	800	105.49	1.74	0.398	0.38
	700	88.86	2.47	0.305	0.45
	650	75.10	3.14	0.240	0.53
	630	67.00	3.56	0.207	?
	610	54.70	4.44	0.153	0.74
	600	45.65	5.35	0.110	0.83
150	700	144.64	0.92	0.539	0.35
	650	136.62	1.19	0.490	0.36
	600	126.46	1.48	0.438	0.39
	550	112.71	1.87	0.380	0.43
	500	91.61	2.53	0.298	0.50

Following [7] we adopt the *relaxation time approximation* to the collision integral $C[f] = -\frac{\delta f}{\tau}$. In this way we can reexpress δf and write the friction term (for one particle species) as

$$\phi^2 \phi' \tau \beta \gamma v \int \frac{d^3 p}{(2\pi)^3 4E^2} \frac{e^{\beta \gamma (E - v p_z)}}{(e^{\beta \gamma (E - v p_z)} \pm 1)^2} \quad (10)$$

The sum of these contributions is equivalent to $\frac{\eta}{T} \phi^2 \phi' \gamma v$ (the equilibrium distribution for fermions/bosons boosted to a frame moving with velocity v in the z direction is $f_0 = \frac{1}{e^{\beta \gamma (E - v p_z)} \pm 1}$ with $\beta = \frac{1}{T}$, $\gamma = \frac{1}{\sqrt{1-v^2}}$). In this way we can calculate the friction parameter (and thus the wall velocity) for any Standard Model-like situation as a function of the strength of the phase transition given by $\xi_n \equiv \frac{\phi_0}{T_n}$ (here ϕ_0 is the Higgs VEV in the broken symmetry phase). The relevant prefactors to the momentum integral can be calibrated by reproducing the Standard Model wall velocities in [7].

Proceeding in an iterative fashion for each choice of model parameters we arrive at the results in table 1. Hydrodynamic equations become increasingly hard to solve numerically for wall velocities close to the speed of sound, as in the case $m_h = 115$ GeV, $M = 630$ GeV. The slice $m_h = 115$ features a jump for a strength of the phase transition $\xi \sim 3.5$ from subsonic solutions (usually assumed in standard baryogenesis scenarios) to supersonic ones (the standard assumption for production of gravitational waves).

In summary, we apply standard hydrodynamic treatments to the electroweak phase transition in an extension of the Standard Model with dimension-6 interactions. We calculate the friction coefficient in the hydrodynamic equations as a function of the particle content of the cosmic fluid and produce predictions for the bubble wall velocity for a range of model parameters.

References

- [1] Kajantie K, Laine M, Rummukainen K and Shaposhnikov ME 1996 *Phys. Rev. Lett.* **77** 2887-90
- [2] Bodeker D, Fromme L, Huber SJ and Seniuch M 2005 *JHEP* **0502** 026 [arXiv:hep-ph/0412366].
- [3] Anderson GW and Hall LJ 1992 *Phys. Rev. D* **45** 2685.
- [4] Ignatius J, Kajantie K, Kurki-Suonio H and Laine M 1994 *Phys. Rev. D* **49** 3854 [arXiv:astro-ph/9309059].
- [5] Megevand A and Sanchez AD 2009 *Nucl. Phys. B* **820** 47 [arXiv:0904.1753 [hep-ph]].
- [6] Espinosa JR, Konstandin T, No JM and Servant G 2010 *JCAP* **1006** 028 [arXiv:1004.4187 [hep-ph]].
- [7] Moore GD and Prokopec T 1995 *Phys. Rev. D* **52** 7182 [arXiv:hep-ph/9506475].
- [8] Huet PY, Kajantie K, Leigh RG, Liu BH and McLerran LD 1993 *Phys. Rev. D* **48** 2477 [arXiv:hep-ph/9212224].
- [9] Huber SJ and Sopena M, to appear 2010