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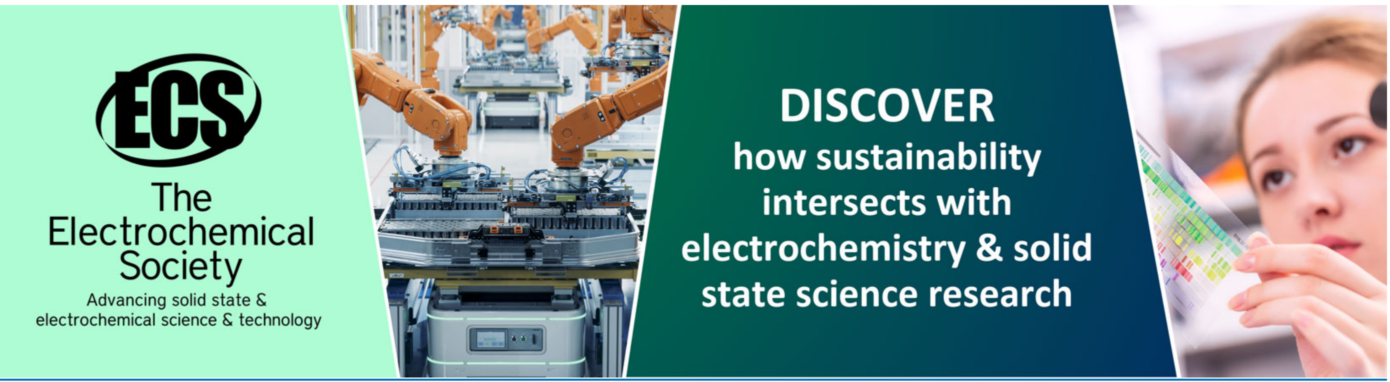

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Generating black strings

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Abstract. It is generated a five dimensional solution of vacuum Einstein equations, with three free parameters, by means of equipping the Zipoy-Voorhees solution with a scalar field then lifting it to a fifth dimension. This solution presents as particular cases the black string with arbitrary tension and the Gregory-Laflamme black string.

The trapped surfaces, mass, tension and the condition of spherical symmetry of the solution are studied.

1. Introduction

There has been interest in study higher dimensional solutions to Einstein field equations associated whit string theory and the proposed more than dimensional universes. One example are the static vacuum solutions of black holes in five dimensions [1]. Other authors have studied the construction of higher dimensional vacuum solutions, considering the scalar fields that appear naturally in the dimensional reduction in string theory [2].

2. 5D Solution Black String

In this section we will use the Zipoy-Voorhees solution and introducing a scalar field, we will generate a 5D solution, with the black string as special case.

Let us consider the Zipoy-Voorhees (ZV) solution [3] in Weyl coordinates:

$$ds^2 = -e^{2U} dt^2 + e^{-2U+2\sigma}(d\rho^2 + dz^2) + \rho^2 e^{-2U} d\phi^2$$

$$U = \frac{1}{2} \ln \left[\frac{r_+ + r_- - 2m}{r_+ + r_- + 2m} \right]^{\frac{q}{2}} \qquad \sigma = \frac{1}{2} \ln \left[\frac{(r_+ + r_-)^2 - 4m^2}{4r_+ r_-} \right]^{\frac{q^2}{4}} \tag{1}$$

where $r_\pm^2 = \rho^2 + (z \pm m)^2$. For $q = 2$ we have Schwarzschild solution that describes a black hole, while if $q = 4$ it represents, the Darmois solution.

Adding the scalar field

$$\varphi = \frac{A}{2} \ln \left[\frac{r_+ + r_- - 2m}{r_+ + r_- + 2m} \right], \tag{2}$$

that is a solution of Laplace's equation $\varphi_{,\rho\rho} + \frac{1}{\rho}\varphi_{,\rho} + \varphi_{,zz} = 0$, it is obtained the spacetime with a scalar field φ, that is expressed as;

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$$ds_{sc}^2 = -e^{2U}dt^2 + e^{-2U+2(\sigma+\sigma_{sc})}(d\rho^2 + dz^2) + \rho^2 e^{-2U} d\phi^2 \tag{3}$$

with

$$\sigma_{sc} = \frac{A^2}{2} \ln \left[\frac{(r_+ + r_-)^2 - 4m^2}{4r_+ r_-} \right] \tag{4}$$

Using the method described in [4], and considering the metric (3) as seed metric, we lift the solution to five dimensions:

$$dS_5^2 = e^{-\frac{2\varphi}{\sqrt{3}}}(ds_{sc}^2) + e^{\frac{4\varphi}{\sqrt{3}}} d\omega^2 \tag{5}$$

or explicitly

$$\begin{aligned} dS_5^2 \;\; = \;\; & -H^{\frac{q}{2}-\frac{A}{\sqrt{3}}}dt^2 + H^{-\frac{q}{2}-\frac{A}{\sqrt{3}}}\rho^2 d\phi^2 + H^{-\frac{q}{2}-\frac{A}{\sqrt{3}}} + H^{2A/\sqrt{3}}d\omega^2 \\ & + H^{-\frac{q}{2}-\frac{A}{\sqrt{3}}} \left[\frac{(r_+ + r_-)^2 - 4m^2}{4r_+ r_-} \right]^{\frac{q^2}{4}+A^2} (d\rho^2 + dz^2). \end{aligned} \tag{6}$$

where $H = \frac{r_+ + r_- - 2m}{r_+ + r_- + 2m}$.

Transforming the metric (6)to the Schwarzschild-like coordinates using;

$$\rho^2 = \left(1 - \frac{2m}{r}\right) r^2 \sin^2\theta, \qquad z = r\left(1 - \frac{m}{r}\right)\cos\theta, \tag{7}$$

we get:

$$\begin{aligned} dS_5^2 \;\; = \;\; & -G^{\frac{q}{2}-\frac{A}{\sqrt{3}}}dt^2 + G^{-\frac{q}{2}-\frac{A}{\sqrt{3}}+1} r^2 \sin^2\theta d\phi^2 + G^{\frac{2A}{\sqrt{3}}} d\omega^2 \\ & + \frac{(r-2m)^P r^Q}{(r^2 - 2mr + m^2\sin^2\theta)^{A^2+\frac{q^2}{4}-1}} \left(\frac{dr^2}{r^2 - 2mr} + d\theta^2 \right) \end{aligned} \tag{8}$$

Where $G = (1 - \frac{2m}{r})$, $P = A^2 + \frac{q^2}{4} - \frac{A}{\sqrt{3}} - \frac{q}{2}$ and $Q = A^2 + \frac{q^2}{4} + \frac{A}{\sqrt{3}} + \frac{q}{2}$.

We note that if we want to impose spherical symmetry, we must choose $A^2 + \frac{q^2}{4} = 1$ in (8). Doing so the term with $\sin^2\theta$ does not appear in the denominator of g_{rr} and $g_{\theta\theta}$, obtaining an element proportional to $r^2 d\Omega$.

For the moment keeping free A and q, (8) is a 5D solution with three free parameters A, q and m.

3. Trapped Surfaces and Horizons

One of the most interesting properties to study in the spacetimes is the trapping surfaces and located horizons. In this section we will study the trapped surfaces and horizons of the generated 5D solution (8).

If we want to study the trapped surfaces S_{X^a} and located horizons corresponding to the spacetime (8), we need to calculate the scalar κ (see [5]).

Fixing the coordinates $x^a = \{r, t\}$ and denoting the local coordinates $x^A = \{\theta, \phi, \omega\}$ on the surface S_{X^a} the value $G \equiv \sqrt{\det g_{AB}} \equiv e^{\check{U}}$ gives the canonical volume element of the surfaces S_{X^a}.

Introducing $\mathbf{g}_a \equiv g_{aA}dx^a$ and $H_\mu = \delta^a_\mu(\check{U}_{,a} - div\overrightarrow{g_a})$

$$\kappa_{x^a} = -g^{ab}H_bH_c|_{S_{x^a}}, \tag{9}$$

the invariant κ amounts to

$$\kappa = -g^{rr}\check{U}^2_{,r}, \tag{10}$$

where

$$\check{U} = \frac{1}{2}\ln\left[\frac{r^{2(A^2+\frac{q^2}{4}+1)}\sin^2\theta(1-\frac{2m}{r})^{1-q+A^2+\frac{q^2}{4}}}{(r^2-2mr+m^2\sin^2\theta)^{A^2+\frac{q^2}{4}-1}}\right] \tag{11}$$

and therefore, κ is obtained as

$$\kappa = -\frac{\{[r-m(1+\cos\theta)][r-m(1-\cos\theta)]\}^{A^2+\frac{q^2}{4}-3}[\mathrm{Num}\check{U}_{,r}]^2}{r^{A^2+\frac{q^2}{4}+\frac{A}{\sqrt{3}}+\frac{q}{2}+1}(r-2m)^{A^2+\frac{q^2}{4}-\frac{A}{\sqrt{3}}-\frac{q}{2}+1}} \tag{12}$$

with

$$\mathrm{Num}\check{U}_{,r} = (r-m)[2r(r-2m)+m^2\sin^2\theta(A^2+\frac{q^2}{4}+1)] - mq[r^2-2mr+m^2\sin^2\theta] \tag{13}$$

Let us analyze the trapped surfaces using κ (κ=0) in (12) and the singularities. If we assume that $A>0$ and $q>0$, then there exists a singularity at $r=0$ since $A^2+\frac{q^2}{4}+\frac{A}{\sqrt{3}}+\frac{q}{2}+1>0$.

Also there are two trapped surfaces in $r_\pm = m(1\pm\cos\theta)$ nevertheless they are wrapped by the naked singularity located at $r=2m$. Depending of the values that A and q have, $\mathrm{Num}\check{U}_{,r}$ in (12) can give a horizon that coincide with the naked singularity $r=2m$. In other cases $\mathrm{Num}\check{\mathrm{U}}_{,\mathrm{r}}$ gives surfaces that do not cover completely the naked singularity.

4. Mass and Tension

The expansion of the metrical functions g_{tt} and $g_{\omega\omega}$ allow us to determinate the mass and tension of the 5D black string.

We restrict our attention to the metric (8) with $r\to\infty$. Asymptotically to order $O(1/r)$ the elements of the metric (8) are compared to the asymptotic form of metric around a stationary matter source given in [6]. Then the mass and tension of the 5D black string (8) are given by.

$$M = \frac{2qm}{4G_5}, \qquad Ma = \tau = \frac{m}{4G_5}\left(\frac{6A}{\sqrt{3}}+q\right) \tag{14}$$

Therefore when lifted to 5D the solution (1) by means of scalar field $\varphi(A,\rho,m)$, the mass and tension are function of q and A, but the tension augments in $\frac{6A}{\sqrt{3}}$.

5. Solutions with spherical symmetry

In this section we consider the condition of spherical symmetry in the 5D solution (8). Obtaining two 5D solutions of the vacuum Einstein equations that are contained in the 5D generalized black string.

Let us impose the spherical symmetry condition $A^2+\frac{q^2}{4}=1$ and perform in (8) the coordinate transformation

$$r = \varrho \left(1 + \frac{k}{\varrho}\right)^2, \quad \text{with} \quad \mathrm{m} = 2\kappa \tag{15}$$

in such a form (8) is transformed into:

$$\begin{aligned} dS_5^2 &= -D^{-s}dt^2 + (1 - k/\varrho)^{2-\frac{1+a}{2-a}s}(1 + k/\varrho)^{2+\frac{1+a}{2-a}s}(d\varrho^2 + \varrho^2 d\theta^2 + \varrho^2 sen^2\theta d\phi^2) \\ &\quad + D^{\frac{1-2a}{2-a}s}d\omega^2 \end{aligned} \tag{16}$$

Where $D = \frac{1+k/\varrho}{1-k/\varrho}$ and A, q, a and s are related by

$$A = -\frac{1-2a}{2\sqrt{1-a+a^2}}, \quad q = \frac{3}{\sqrt{3(1-a+a^2)}}, \quad \frac{2}{\sqrt{3}}A + q = s \tag{17}$$

this represents black string solution with arbitrary tension, obtained by Chul H. Lee [7].

While the invariant κ (12) with the condition of spherical symmetry becomes,

$$\kappa = \frac{1}{r^4}\left(1 - \frac{2m}{r}\right)^{\frac{A}{\sqrt{3}}+\frac{q}{2}-2}(2r - 2m - mq)^2 \tag{18}$$

We obtain the horizon $r = 2m$ or $r = 4k$ with $2 > a$, since in this case the highest possible value of q is 2. (16) has two free parameters k and s. Let us consider the values $A = 0$ and $q = 2$, then we recover the black string solution [8]. In this case the solution has a horizon located at $r = 2m$, the mass and tension take the values $\frac{m}{G_5}$ and $\frac{m}{2G_5}$ respectively.

6. Conclusions

In this paper we lifted the solution ZV to five dimensions, using an scalar field. So we have generated a 5D generalized black string (8). Then we studied the case of spherical symmetry (black string and Chul H. Lee's solution) for this solution, obtaining that the horizons coincide with the naked singularity $r = 2m$.

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