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# Destroying black holes with test bodies

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## **Abstract.**

If a black hole can accrete a body whose spin or charge would send the black hole parameters over the extremal limit, then a naked singularity would presumably form, in violation of the cosmic censorship conjecture. We review some previous results on testing cosmic censorship in this way using the test body approximation, focusing mostly on the case of neutral black holes. Under certain conditions a black hole can indeed be over-spun or over-charged in this approximation, hence radiative and self-force effects must be taken into account to further test cosmic censorship.

## **1. Black holes and naked singularities**

Penrose and Hawking have shown that singularities are inevitable in gravitational collapse [1, 2]. These results do not rely on specific solutions of Einstein's equations or special symmetry requirements. Therefore, spacetime singularities are physically relevant and not simply mathematical peculiarities of special solutions. While the results do not say anything about the precise nature of the singularities, they do indicate the breakdown of general relativity. Einstein's theory is unable to predict the outcome of events in the vicinity of the singularity, and unusual phenomena could take place there. Visible singularities could provide exciting access to the so far unobserved physics of quantum gravity.

In the case of a black hole, however, the event horizon hides the singularity. Although this is a lost opportunity for quantum gravity phenomenology, it is also a blessing, if all we wish to understand is what can be seen from outside without having to grapple with the physics of a singularity. But does an event horizon always hide any singularities that form from a collapsing object? As Roger Penrose first put it in 1969 [3], "does there exist a cosmic censor who forbids the appearance of naked singularities, clothing each one in an absolute event horizon?" The conjecture that such a censor indeed exists is called the "cosmic censorship conjecture".

The physics of event horizons, unlike that of singularities, should be well described by classical general relativity, so cosmic censorship can be tested and possibly proven without going beyond known physics. Evidence from several directions suggests that any singularity arising to the future of generic (not infinitely finely tuned) non-singular initial conditions, may indeed always be hidden behind a black hole event horizon [4, 5]. However, we are far from having a definitive proof.

Much of the evidence in favor of cosmic censorship comes from the failure of attempts to violate it in thought experiments. Given the difficulty of a direct attack on the general question,

this strategy continues to offer an attractive approach to the problem. Even in the context of simplifying approximations, such thought experiments can uncover mechanisms tending to uphold censorship, and they can produce scenarios where censorship would be violated if the approximations were valid. In the latter case, they focus our attention on those “dangerous” scenarios and on the limits of validity of the approximation scheme. This type of approach is what will be discussed here<sup>1</sup>.

## 2. “Destroying” a black hole

In general relativity, the spacetime in the vicinity of the black hole is described by the Kerr–Newman (K-N) metric which contains three parameters: the mass  $M$ , the spin angular momentum  $J$ , and the electric charge  $Q$ . The K-N metric describes a black hole as long as the mass is sufficiently large compared to a combination of the charge and angular momentum,  $M^2 \geq a^2 + Q^2$ , where  $a = J/M$ . (We adopt units with Newton’s constant  $G$  and the speed of light  $c$  both set equal to unity.) The case where  $M^2 = a^2 + Q^2$  is called an extremal black hole, while for  $M^2 < a^2 + Q^2$  there is no event horizon and the K-N metric actually describes a naked singularity. Therefore, it would naively seem that to create a naked singularity all one need do is to start with a black hole and toss in matter with enough angular momentum or charge so as to drive its parameters beyond the extremal limit, leaving it no option but to expose the singularity.

On further thought there are some subtleties involved in such a scenario:

- (i) The K-N metric describes stationary configurations, so the proposed strategy can work as stated only if, after having absorbed the matter, the system settles down to a stationary configuration containing all the mass, angular momentum and charge, i.e. without having shed the excess angular momentum or charge in the settling down process. Such an outcome is by no means guaranteed. Indeed, it seems rather unlikely, given the evidence that the trans-extremal K-N metric is unstable [8, 9]. If such instability occurs (the uncertainty lies in the proper boundary conditions at the singularity), the system may well shed sufficient angular momentum and/or charge, or not settle down to such a metric, and at present nobody knows what it would do. What this means is that to demonstrate the creation of a naked singularity one would have to follow the evolution further than the initial “absorption” of the extra matter. So the possibility of initially overspinning or overcharging an initial black hole configuration can only be taken as an indication that cosmic censorship *might* fail.
- (ii) The notion of “exposing” the singularity may be inappropriate, since the singularity inside a perturbed charged or rotating stationary black hole cannot send signals to any point, even those interior to the horizon. (Here we assume that the Cauchy horizon inside the black hole is indeed unstable, as evidence indicates [10].) That is, it has no nonsingular future. Hence it is not so clear that, if a horizon could be “destroyed”, the result would be to expose the singularity that would have been there had the horizon *not* been destroyed. It might produce a wholly different singularity. Nevertheless, either way the process would violate cosmic censorship.

Setting these issues aside for the present, we focus here just on the question of whether a black hole can initially absorb sufficient angular momentum or charge to send its parameters over the extremal limit.

Given the non-linear nature of Einstein’s equations, following the evolution of a black hole exactly is a very difficult problem that presumably requires numerical solution of the Einstein

<sup>1</sup> This paper is based partially on a talk given by T.P.S. at the 1st Mediterranean conference on Classical and Quantum gravity following the lines of Ref. [6] and partially on an essay prepared for the “FQXi essay contest: What is Ultimately Possible in Physics?” [7].

equation. Therefore, most studies of cosmic censorship to date either imposed symmetry conditions or have been carried out in the simple framework of a test-body moving on the black hole spacetime. The test-body approximation imposes the conditions

$$\delta E \ll M, \quad \delta J \ll M^2, \quad \delta Q \ll M, \quad (1)$$

where  $\delta E$ ,  $\delta J$ , and  $\delta Q$  denote the energy, angular momentum and charge of the body. These conditions seem sufficient to ensure that the influence of the test body can be treated as a small perturbation on the background spacetime. (In particular, we need not assume that  $\delta J \ll J$  or  $\delta Q \ll Q$ , since addition of angular momentum to a nonspinning black hole, or charge to a neutral black hole, can perfectly well be a small perturbation.) However, they certainly do not guarantee that the effects due to the gravity of the body, such as gravitational radiation and self-force, can be neglected when studying the motion of the body. Issues that arise when including these effects are discussed briefly at the end of this paper.

Provided the body can be tossed into the black hole, the final composite object would have mass  $M + \delta E$ , angular momentum  $J + \delta J$  and charge  $Q + \delta Q$ . In order for the K-N metric with these parameters to be a naked singularity they would have to satisfy the inequality

$$(M + \delta E)^2 < \left( \frac{J + \delta J}{M + \delta E} \right)^2 + (Q + \delta Q)^2. \quad (2)$$

Various special cases of such test body experiments have been considered in the literature. Wald focused on an exactly extremal black hole and showed that it cannot be overcharged or over-spun using a particle with charge and/or orbital angular momentum. He also showed that an extremal neutral rotating black hole cannot be overspun using a particle with spin angular momentum falling along the spin axis [11]. However, de Felice and Yu have shown that an extremal charged black hole *can* be sent over the extremal limit by accretion of a neutral spinning test body [14]. If one starts with a non-extremal black hole, the results differ from what Wald found for the extremal case. In particular, Hubeny showed that one can overcharge a *near*-extremal Reisser–Nördstom black hole by tossing in a test body [12], and Hod showed that a near-extremal rotating black hole can be overspun in the limiting case where a particle carrying angular momentum is lowered all the way to the horizon of a black hole and dropped from there [13]. We will return to these results later on and comment on them more extensively. A case we will not consider further is that of dyonic black holes which carry both electric and magnetic charge [15, 16]. We will also not consider here test field experiments via wave scattering, classical or quantum, as discussed for example in [17] and references therein.

Astrophysical black holes tend to increase their angular momentum by accreting matter, whereas they tend to decrease their charge by attracting opposite and repelling same charges. For a point of principle, one may ignore these facts, but nevertheless it would be much more provocative and promising if arguments showed that a naked singularity could be created using only angular momentum, since that might then actually occur in nature. We therefore focus mostly on the case where the both the black hole and the test body have only angular momentum and no charge.

We shall demonstrate that in the test body approximation a trans-extremal condition can be attained, even when taking into account constraints on the size and structure of the body. The limitations of the test-body approximation will then be addressed.

### 2.1. Over-spinning a neutral black hole

With  $Q = \delta Q = 0$ , the inequality in eq. (2) takes the form

$$J + \delta J > (M + \delta E)^2. \quad (3)$$

This yields a lower bound on the required angular momentum carried by the body, for a given energy  $\delta E$ :

$$\delta J > \delta J_{min} = (M^2 - J) + 2M\delta E + \delta E^2. \quad (4)$$

Since we are assuming  $\delta E \ll M$ , it might seem that the  $\delta E^2$  term may as well just be neglected at this stage. However, as we will see shortly, the presence of that term imposes an upper bound on  $\delta E$  and  $\delta J$  and, therefore, should not be neglected.

We can already extract a useful piece of information from eq. (4). Dividing both sides by  $\delta E^2$ , and observing that each term on the right hand side is positive and therefore should by itself be smaller than the left hand side, we get

$$\delta J / \delta E^2 > 2M / \delta E \gg 1 \quad (5)$$

(where the last inequality follows from (1). If  $\delta E$  is comparable to the rest mass of the body (it can be much less if the body is deeply bound by the gravitational field of the black hole), and if  $\delta J$  comes from spin (rather than orbital angular momentum), this would imply that the body has angular momentum far over the extremal ratio. In that case the body could not be a black hole. This does not mean that it would have to be a naked singularity itself, as there is no a priori upper limit to this ratio for bodies other than black holes. Stars for instance can easily have ratios much larger than 1.

The requirement that the composite object be a naked singularity gave us the lower bound (4) on the angular momentum of the body. An upper bound is obtained from the requirement that the body does indeed cross the horizon. One can use the equations of motion for the body in order to derive the bound [11]. These are the Papapetrou equation if the body's angular momentum includes spin, and the geodesic equation if it is purely orbital angular momentum. But a simpler and more transparent method is to just consider the flux of energy and angular momentum into the black hole when the body falls across the horizon. The requirement that the energy momentum tensor satisfies the null energy condition (which follows for example if the energy density is positive in all local reference frames) yields (see for example [6])

$$\delta E \geq \Omega_H \delta J, \quad (6)$$

where  $\Omega_H = a/2Mr_+$  is the angular velocity of the horizon and  $r_+ = M + (M^2 - a^2)^{1/2}$  is the horizon radius in Boyer-Lindquist coordinates. This condition can be written as

$$\delta J \leq \delta J_{max} = \frac{2Mr_+}{a} \delta E. \quad (7)$$

It guarantees that the body can fall across the horizon starting from *some* point outside, although in general the body is in a bound orbit that does not come from spatial infinity.

We now have both an upper and a lower bound for the angular momentum of the body, for a given energy. As long as  $\delta J_{min} < \delta J_{max}$  for some  $\delta E$ , there will be values of  $\delta J$  and  $\delta E$  satisfying both inequalities (4) and (7).

First let us suppose the black hole starts exactly extremal, i.e.  $J = M^2$ . Then  $a = M = r_+$ , so one has  $\delta J_{min} = 2M\delta E + \delta E^2$  and  $\delta J_{max} = 2M\delta E$ . This implies that  $\delta J_{min} > \delta J_{max}$  so it is impossible to over-spin the black hole. Thus we recover the result of Wald [11] mentioned earlier. (The analysis here is significantly simpler than that of Ref. [11].)

The physical interpretation of this result is the following: In the case of the spinning body, the spin-spin interaction with the spin of the black hole is sufficiently repulsive to prevent the body from falling in if it would have overspun the black hole. In the orbital angular momentum case, If the body has the angular momentum required to overspin then the impact parameter of the body is too large for it to hit the horizon.

In the sub-extremal case, however, the inequalities *can* be satisfied, as was shown in [6]. As mentioned earlier, the limiting case where the body is dropped from a point on the horizon had been considered previously by Hod [13] (it corresponds to  $\delta J = \delta J_{max}$ ). To understand the range of overspinning parameters, it is helpful to visualize the inequalities graphically. If  $\delta J_{max}$  and  $\delta J_{min}$  are plotted vs.  $\delta E$ , the former is a straight line through the origin, with slope  $2Mr_+/a$ , while the latter is a parabola with positive intercept, slope  $2M$  at the intercept, and curved upwards. For a sub-extremal black hole we have  $r_+ > M > a$ , so the slope of the  $\delta J_{max}$  line is greater than the initial slope of the  $\delta J_{min}$  parabola. Some algebra reveals that the parabola always intersects the straight line in two points. The allowed values of  $\delta E$  and  $\delta J$  are those in the compact region above the parabola and on or below the straight line. Note that if the  $\delta E^2$  term is neglected in (4), the parabola is replaced by a straight line, and the allowed region becomes an infinite wedge, with no upper bound.

To determine whether the overspinning can be accomplished with a small perturbation satisfying (1) we can expand in the dimensionless quantity  $\epsilon$  defined by

$$J/M^2 = a/M = 1 - 2\epsilon^2. \quad (8)$$

(Hubeny [12] used the same parameter to analyze the charged case, see below.) The parameter  $\epsilon$  measures how close to extremality the black hole is to begin with, and we must have  $\epsilon \ll 1$  if the overspinning perturbation is to be small. It is now useful to adopt units with  $M = 1$ , to keep the expressions simpler. Then we have, from (4) and (7),

$$\delta J_{min} = 2\epsilon^2 + 2\delta E + \delta E^2 \quad (9)$$

$$\delta J_{max} = (2 + 4\epsilon)\delta E, \quad (10)$$

where terms of order  $O(\epsilon^2\delta E)$  have been dropped in (10). The allowed range of  $\delta E$  lies where the difference

$$\delta J_{max} - \delta J_{min} = -2\epsilon^2 + 4\epsilon\delta E - \delta E^2 \quad (11)$$

is positive, i.e.

$$(2 - \sqrt{2})\epsilon < \delta E < (2 + \sqrt{2})\epsilon. \quad (12)$$

In particular,  $\delta E$  must be of order  $\epsilon$ . For a given  $\delta E$ , the allowed values of  $\delta J$  are near  $2\delta E$ , so we must have

$$\delta J \sim \delta E. \quad (13)$$

We conclude that if  $\epsilon \ll 1$  the overspinning values of  $\delta E$  and  $\delta J$  can indeed be consistent with the perturbative requirement.

Note that the *width* (11) of the allowed range of  $\delta J$  is only of order  $\epsilon^2 \ll \epsilon$ . Note also that  $a - 1 = 2\epsilon^2$  is parametrically smaller than  $\epsilon$ . For example, if  $\epsilon = 10^{-2}$ , then the initial black hole must have  $a = 0.9998$ . For a thought experiment, we can imagine even smaller values of  $\epsilon$ .

## 2.2. Over-charging or over-spinning a charged black hole

Let us return to similar conclusions reached in the cases mentioned earlier, where different assumptions for the quantities characterizing the black hole were made. Hubeny considered the case of adding charge to a charged black hole ( $J = \delta J = 0$ ) [12]. In analogy to the spinning case, in the charged case the two constraints are

$$\delta Q > M - Q + \delta E, \quad (14)$$

$$\delta Q \leq \frac{r_+}{Q}\delta E, \quad (15)$$

where now  $r_+ = M + \sqrt{M^2 - Q^2}$ . If the black hole starts out extremal,  $M = Q = r_+$ , then overcharging is impossible. However if  $M > Q$ , then  $r_+ > Q$ , and one easily sees by visualizing the inequalities graphically (now they are both straight lines) that there is an infinite range of solutions, once  $\delta E$  is greater than a certain minimum value.

de Felice and Yu considered the case of adding angular momentum to an extremally charged black hole ( $Q = M$ ,  $J = \delta Q = 0$ ) [14]. In this case the minimum  $\delta J$  to overspin is given by

$$\delta J > (M + \delta E)\sqrt{2M\delta E + \delta E^2}, \quad (16)$$

and there is no maximum  $\delta J$ , since the only requirement for the body to fall across the horizon is  $\delta E \geq 0$ , which does not involve  $\delta J$ . Note that to lowest order in  $\delta E/M$  the minimum overspinning  $\delta J$  is  $\delta J_{\min} = M\sqrt{2M\delta E}$ .

### 3. Size and structure requirements

So far we have characterized the body only by its energy  $\delta E$ , angular momentum  $\delta J$  and charge  $\delta Q$ . We did not consider restrictions placed on its size and structure: It should be sufficiently small to justify use of a test particle approximation, and it should be composed of matter having positive energy density and no unphysically large stresses. The next step is to take into account these requirements.

#### 3.1. Adding spin angular momentum to a neutral black hole

We begin with the case of a spinning test body. For simplicity we assume that the body is dropped along the rotational axis of the black hole. We first consider the case where the body has  $\delta E \sim m$ , and is not spinning relativistically, so its spin angular momentum is given by  $\delta J \sim mvR \sim \delta E vR$ , where  $v$  is the surface velocity and  $R$  is the equatorial radius. The condition  $v < 1$  then implies  $R > \delta J/\delta E$ . We saw above that the ratio  $\delta J/\delta E$  must be of order unity (13), that is of order  $M$ . In this case the body must be larger than the black hole, so it simply will not “fit” in the transverse direction, and in any case treating it as a point particle with spin would be unjustified, since that rests on the smallness of the size of the body compared to the ambient radius of curvature. Moreover, one can show [6] that the radial tidal stress required to hold the body together would be larger than the energy density, violating energy conditions. It cannot help to allow ultra-relativistic tangential velocity: as a simple Newtonian estimate shows, that would require unphysical stresses holding the body together. The conclusion is that it is impossible to over-spin the black hole if the body’s energy is close to its rest mass,  $\delta E \sim m$ .

Since the angular momentum involves the rest mass  $m$ , not the energy  $\delta E$ , it might be possible to achieve a large enough  $\delta J$  with a small enough size  $R$ , without requiring unphysical matter, by dropping the body from a position where it is deeply bound,  $\delta E \ll m$ . This might be achieved by slowly lowering the body on a tether, down to the near the black hole horizon, before dropping it in. Now we reconsider whether the size restrictions can be met in this setting.

We begin with the restrictions on the rest mass  $m$ . If  $m$  is much greater than  $\delta E$ , then the test body approximation requires that we impose not only  $\delta E \ll 1 (= M)$  (1), but also  $m \ll 1$ . There is also a lower bound on  $m$ , coming from an upper bound on  $R$ : the angular momentum is  $\delta J \sim mvR$ , hence (restricting to nonrelativistic spin  $v < 1$  as required by the previously mentioned result)  $R > \delta J/m \simeq 4\epsilon/m$ . The requirement  $R \ll 1$  then yields  $m \gg \epsilon$ . The mass and size must therefore fall within the ranges

$$\epsilon \ll m \ll 1, \quad 4\epsilon/m \lesssim R \ll 1. \quad (17)$$

To these conditions we must add the requirement  $R \gtrsim m$  that the body is not a black hole, as explained above.

The inequality (6) guarantees that the body can cross the horizon with the chosen values of energy and angular momentum, but since the deeply bound drop point lies at a finite distance from the horizon it is necessary to check that (a) the spinning body would actually fall into the black hole rather than being repelled, and (b) it is possible to choose the polar radius of the body  $R_{\text{polar}}$  to be smaller than the proper distance  $d$  from the horizon to the drop point

$$R_{\text{polar}} < d, \quad (18)$$

so that it can fully “fit” outside the black hole and be localized at the drop point. Now it turns out [6] that, in order to fall in, the maximum value that  $d$  can have, given the allowed values of  $\delta E$  and  $\delta J$ , is

$$d_{\text{max}} \simeq \epsilon/m. \quad (19)$$

Thus we arrive at the bound

$$R_{\text{polar}} < \epsilon/m. \quad (20)$$

Together with (17), this means that the body must be at least somewhat oblate,  $R_{\text{polar}} \lesssim R/4$ .<sup>2</sup> We conclude that the body can be large enough to possess the requisite angular momentum and also have a physically acceptable stress and fit outside the black hole at the drop point<sup>3</sup>.

### 3.2. Adding orbital angular momentum to a neutral black hole

We turn now to the case of orbital angular momentum in the equatorial plane. Here the issue is that in order to have the required values of  $\delta E$  and  $\delta J$ , the body might have to be in a bound orbit, which would have a turning point at a maximum radius. In that case we would need to require that the body be small enough to fit outside the horizon at this radius. Since the body can be no smaller than a black hole with the same rest mass, it is not clear in advance whether this requirement could be met. However, as has been shown in [6] this size constraint is not an issue, since in fact there are suitable orbits that come in from infinity with no turning point. This can be shown numerically, but also analytically by the use of the effective potential governing the motion of a test particle in a Kerr spacetime (Kerr-Newman with no charge).

### 3.3. Cases involving a charged black hole

Let us briefly consider the size and structure requirements when attempting to overcharge or overspin a charged black hole. In the case with no angular momentum Hubeny [12] showed that the body can have the required charge and mass, with low internal stresses and size much smaller than the black hole. Also, she demonstrated that there are charged test particle trajectories that fall from infinity into the black hole. Therefore, much like the orbital angular momentum case, size constraints are not an issue. On the contrary, for spinning particles dropped radially with radial spin into an extremal charged black hole, de Felice and Yu [14] found that the test body must be bound very close to the horizon. The same is true for a particle carrying orbital angular momentum but no spin, as we now show. The radial motion is governed by the equation  $\dot{r}^2 + (1 - 1/r)^2(1 + \tilde{L}^2/r^2) = \tilde{E}^2$ . Here  $\dot{r}$  is the derivative of the Reissner-Nordstrom radial coordinate with respect to the particle proper time, and  $\tilde{E} = (\delta E)/m$  and  $\tilde{L} = (\delta J)/m$  are the energy and angular momentum divided by the particle rest mass  $m$ , and we have again set  $M = 1$ . As mentioned after (16), the overspinning requirement is  $\delta J/\delta E \gtrsim \sqrt{2/\delta E} \gg 1$ , hence

<sup>2</sup> In [6] the possibility that  $R_{\text{polar}} \neq R_{\text{equator}}$  was overlooked, so it was erroneously concluded that no value of  $R$  could meet all requirements.

<sup>3</sup> de Felice and Yu made a similar analysis for the case of dropping a spinning body into an extremal charged black hole, but they computed the coordinate radius corresponding to  $d_{\text{max}}$ , rather than the proper distance. In the extremal case, the proper distance to the horizon is infinite in the direction orthogonal to the Killing vector, so there is apparently no requirement that the body be disk-shaped, contrary to what was stated in [14].



$\tilde{L}/\tilde{E} \gg 1$ . For an unbound orbit  $\tilde{E} \geq 1$ , so we infer that  $\tilde{L} \gg 1$ . There are therefore two turning points where  $\dot{r} = 0$ . To fall into the hole the particle must lie inside the inner turning point, which lies at a radius  $r_{\text{inner}}$  very close to the horizon, where  $r_{\text{inner}} - 1 \simeq \tilde{E}/\tilde{L} \lesssim \sqrt{\delta E/2} \ll 1$ . However, although the radial coordinate must be very close to that of the horizon, the proper distance to the horizon, measured in the static frame, is infinite for an exactly extremal black hole. Hence, in both the spin and orbital cases, no further size constraints appear to be imposed by the location of the turning point (see also footnote 3).

#### 4. Beyond the test body approximation

Our considerations thus far have been based on an approximation which neglects loss of energy and angular momentum in gravitational radiation and does not take into account self-force effects. Recall that our purely kinematic considerations above yielded a finely tuned relation between the energy and angular momentum of the dropped body for over-spinning to occur. Both quantities have to be of order  $\epsilon$  in magnitude, but the allowed window for angular momentum, given the energy, is only of order  $\epsilon^2$ . In the light of this delicate balance, it is certainly possible that, although small, gravitational radiation and/or self-force effects may always manage to preclude the over-spinning.

Given that the inequalities (4) and (7) need only hold on the horizon, one could imagine that the loss of energy and angular momentum in gravitational radiation might be compensated by adjusting the initial conditions. In the case of an axially symmetric spinning body falling along the black hole spin axis there is no radiation of angular momentum, so it should be possible to simply compensate for the energy radiated. To determine whether compensation is actually possible in the orbital case requires further investigation. Perhaps more worrisome than radiation are the self force effects. Indeed Hubeny found strong indications that for the charged case the electromagnetic self-force might prevent the overcharging, although her calculations were not conclusive [12].

Another distinct effect that might prevent the creation of a naked singularity is the tides raised on the black hole horizon by the falling body. These would be irrelevant for the orbital angular momentum case since the body falls in from spatial infinity. In the spinning body case, however, in which the body is lowered to the horizon and then dropped, the tidal bulge of the horizon might perhaps make it impossible for the body to start out in the exterior while still satisfying the size constraints.

Given the existing evidence for cosmic censorship, it seems indeed likely that neglected gravitational effects will come to its rescue. The examples discussed here suggest dynamical regimes in which it may be interesting to study these neglected effects.

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