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Astrophysical reaction rates for ^{6}He and ^{9}Be production by electromagnetic radiative capture and four-body recombination

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Abstract. The triple alpha reaction $\alpha+\alpha+\alpha \rightarrow {}^{12}\text{C}+\gamma$ is considered to be the most relevant one in stars at the helium burning stage, since it permits to bridge the A=5 and A=8 instability gaps. However, at higher temperatures and neutron rich environments, other reactions can also play a relevant role. In this work we investigate the astrophysical reaction and production rates for the two-particle radiative capture processes $\alpha+n+n \rightarrow {}^6\text{He}+\gamma$ and $\alpha+\alpha+n \rightarrow {}^9\text{Be}+\gamma$. The hyperspherical adiabatic expansion method is used. With this method no assumption is made about the capture mechanism. The four-body recombination reactions $\alpha+\alpha+n+n \rightarrow {}^6\text{He}+\alpha$, $\alpha+n+n+n \rightarrow {}^6\text{He}+n$, $\alpha+\alpha+n+n \rightarrow {}^9\text{Be}+n$ and $\alpha+\alpha+\alpha+n \rightarrow {}^9\text{Be}+\alpha$ are also investigated as a possible alternative as a source of ^{6}He and ^{9}Be.

1. Introduction

Due to the lack of neutrons in the helium burning red giants, the triple alpha reaction $\alpha+\alpha+\alpha \rightarrow^{12} \text{C}+\gamma$ is considered to be the most relevant one at this stage of the star life. This reaction is the way such that the A=5 and A=8 instability gaps are bridged.

Nevertheless, other different scenarios are also possible. Stars more massive than about ten times the mass of the sun are expected to end its life as a Type-II supernova explosion, leading to a neutron star as a residue. The gravitational collapse of the star produces neutrinos and antineutrinos heating the external layers of the star, which are expelled in the form of neutrino wind. This rapidly expanding high-temperature and low density region is the so called "hot bubble", which is in fact a rapidly expanding matter with a significant neutron excess where temperatures of about (7$\sim$10) GK and nuclear statistical equilibrium favors also the presence of α-particles. This is the most likely environment where the rapid neutron capture (r-process) nucleosynthesis can happen [1].

Among these processes, two relevant ones are $\alpha+\alpha+n \rightarrow^9 \text{Be}+\gamma$ and $\alpha+n+n \rightarrow^6 \text{He}+\gamma$. They are an alternative way to overtake the A=5 and A=8 gaps. The triple α reaction is too slow at the temperature-density conditions in the hot bubble, and the reactions mentioned above play a crucial role. Determination of their reaction rates is essential for the initial condition of the r-process, namely, the amounts of seed elements and remaining neutrons. Calculation of these reaction and production rates are usually made assuming that the reactions take place in a two-step process (sequential capture) [2, 3].

The purpose of this work is two-fold. First, we investigate how simultaneous inclusion of sequential and direct capture modifies previous results already available for the radiative capture reactions $\alpha + n + n \rightarrow^6$ He $+ \gamma$ and $\alpha + \alpha + n \rightarrow^9$ Be $+ \gamma$. This will be done by use of the hyperspherical adiabatic expansion method [4]. Second, we shall estimate the production and reaction rates for the four-body recombination reactions $\alpha+\alpha+n+n \rightarrow^6$ He$+\alpha$, $\alpha + n + n + n \rightarrow^6$ He $+ n$, $\alpha + \alpha + n + n \rightarrow^9$ Be $+ n$ and $\alpha + \alpha + \alpha + n \rightarrow^9$ Be $+ \alpha$ which in principle compete with the radiative capture as a source of ^{6}He and ^{9}Be. Under the hot bubble conditions these reactions could become the dominating ones and contribute to the bridging of the A=5 and A=8 gaps.

2. The method

The production rate in some stellar environment for a reaction leading from an N-particle system to a final M-particle state ($A_1 + A_2 + \cdots + A_N \rightarrow B_1 + B_2 + \cdots + B_M$) gives the number of reactions taking place per unit time and unit volume in that star, and it can be written as

$$P^T = \int dEB(E,T)P(E), \tag{1}$$

where $P(E)$ is the production rate at a given kinetic energy E in the N-body center of mass, and $B(E,T)$ is the Maxwell-Boltzmann distribution giving the probability for finding the N particles with that precise relative kinetic energy [5]. This distribution takes the form:

$$B(E,T) = \frac{1}{\Gamma(\frac{3N-3}{2})} \frac{1}{K_BT} \left(\frac{E}{K_BT} \right)^{\frac{3N-5}{2}} e^{-\frac{E}{K_BT}}, \tag{2}$$

where K_B is the Boltzmann constant and T is the temperature of the star.

The presence of the exponential in the previous expression implies that, for a given temperature T, the only relevant energies are the ones smaller than no more that a few times K_BT. Typical temperatures in the stars (i.e., in the core of the sun) are of the order of 10^7 K, which leads to $K_BT \approx 0.001$ MeV. Even more, for the temperatures estimated in the hot bubble (7$\sim$10 GK) one has that $K_BT < 1$ MeV. Therefore, in the stellar medium only very low relative kinetic energies are relevant.

The total production rate at a given energy ($P(E)$) is the product of the so called *reaction rate* and the densities n_i of the N nuclei ($i = 1, \cdots, N$) involved in the initial state. These densities are usually written as

$$n_i = \rho N_A \frac{X_i}{A_i}, \tag{3}$$

where N_A is the Avogrado number, A_i and X_i are the mass number and mass abundance of the nucleus i, and ρ is the density of the star [5]. The density is, together with the temperature, the crucial property of the star determining the production rate. In fact, $P(E)$ is proportional to ρ^N, meaning that, for a sufficiently large ρ, processes involving more particles could play a role.

Finally, the reaction rate ($R(E)$) is given by the Fermi's golden rule integrated over all the possible momenta for the final products of the reaction. Assuming M particles in the final state, $R(E)$ is written as:

$$R(E) = \int \frac{2\pi}{\hbar} \left| \langle \Psi_i(E) | W | \Psi_f(E_f) \rangle \right|^2 \delta(E - E_f) \frac{d^3p_1}{(2\pi)^3} \cdots \frac{d^3p_M}{(2\pi)^3}, \tag{4}$$

where Ψ_i and Ψ_f are the initial and final wave functions, $\vec{p}_1, \cdots, \vec{p}_M$ are the momenta of the final nuclei, and W represents the interaction. When only two particles are involved in the initial state, the reaction rate is the cross section of the process times the relative velocity between the two particles.

Obviously, the matrix element contained in the integrand of Eq.(4) is the same for a given reaction and for the inverse process. It is then possible to relate the reaction rates (and therefore the production rates) corresponding to both processes. This means that the production rate for a reaction leading to two particles in the final state can be written in terms of the cross section of the inverse process.

2.1. Three-body radiative capture

Let us consider here the particular case of the radiative capture reaction $\alpha + n + n \rightarrow^6$ He $+ \gamma$. This a pure electromagnetic process where only the bound ^{6}He nucleus and a photon are found in the final state. Following the discussion above, the corresponding production rate can be written in terms of the photo-dissociation cross section (σ_γ) of ^{6}He. To be precise, this production rate takes the form:

$$P_{\alpha,2n}(\rho,T) = n_\alpha n_n^2 \frac{\hbar^3}{c^2}\left(\frac{m_\alpha + 2m_n}{m_\alpha m_n^2}\right)^{\frac{3}{2}} \frac{2\pi}{(K_BT)^3} e^{-\frac{Q}{K_BT}} \int_{|Q|}^{\infty} E^2 \sigma_\gamma(E) e^{-\frac{E}{K_BT}} dE \tag{5}$$

where $Q = m_{^6\mathrm{He}} - m_\alpha - 2m_n$, and $m_{^6\mathrm{He}}$, m_α, and m_n are the masses of ^{6}He, the α particle, and the neutron, respectively. A similar expression could be written for the reaction $\alpha+\alpha+n \rightarrow^9$ Be$+\gamma$.

The photodisintegration cross section $\sigma_\gamma(E)$ is usually expanded in terms of electric and magnetic multipoles, each of them given by a well known expression in terms of the strength function of the reaction [6]. For the particular case of the reactions $\alpha + n + n \rightarrow^6$ He $+ \gamma$ and $\alpha + \alpha + n \rightarrow^9$ Be $+ \gamma$, the electric dipole term ($E1$) dominates. For three identical charged bosons, like for instance in the $\alpha + \alpha + \alpha \rightarrow^{12}$ C $+ \gamma$ reaction, it can be seen that the electric quadrupole term is the leading contribution [7].

2.2. Four-body recombination

Similarly, for a the reaction like $\alpha + n + n + n \rightarrow^6$ He $+ n$, where one neutron takes the excess of energy released when the remaining particles combine into a bound state, it is possible to express the corresponding production rate in terms of the cross section $\sigma_n(E)$ for the inverse process (cross section for neutron breakup of ^{6}He). In particular, the production rate takes the form:

$$P_{\alpha,3n}(\rho,T) = n_\alpha n_n^3 \mu_{n^6\mathrm{He}} \left(\frac{m_\alpha + 3m_n}{m_\alpha m_n^3}\right)^{\frac{3}{2}} \frac{\hbar^6 (2\pi)^{\frac{5}{2}}}{(K_BT)^{\frac{9}{2}}} e^{-\frac{Q}{K_BT}} \int_{|Q|}^{\infty} E\sigma_n(E) e^{-\frac{E}{K_BT}} dE. \tag{6}$$

Calculation of $\sigma_n(E)$ requires a proper description of the four-body initial and final states. In this work we describe them as a three-body state ($\alpha+n+n$), whose wave function is accurately computed, plus a free neutron relative to the center of mass of the previous three-body system. In this way, the transition amplitude can be written as the sum of three terms, each of them containing the interaction between the spectator neutron and each of the constituents of the $\alpha + n + n$ system. More precisely, each of these three terms takes the form [8]:

$$T^{(i=1,2,3)} = \langle \Psi_c(\boldsymbol{r}_{jk}, \boldsymbol{r}_{jk,i}) | e^{i\boldsymbol{q}\cdot\boldsymbol{r}_{jk,i}} | \Psi_b(\boldsymbol{r}_{jk}, \boldsymbol{r}_{jk,i}) \rangle \langle e^{i\boldsymbol{P}'\cdot\boldsymbol{r}_{ni}} | V_{ni} | e^{i\boldsymbol{P}\cdot\boldsymbol{r}_{ni}} \rangle, \tag{7}$$

where Ψ_c and Ψ_b are the continuum and bound three-body wave functions of the $\alpha + n + n$ system, $\boldsymbol{q}$ is the momentum transfer to the three-body center of mass, and $\langle e^{i\boldsymbol{P}'\cdot\boldsymbol{r}_{ni}} | V_{ni} | e^{i\boldsymbol{P}\cdot\boldsymbol{r}_{ni}} \rangle$ is the Fourier transform of the two-body interaction between the neutron and constituent i in the $\alpha + n + n$ system.

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After squaring the sum of three terms in Eq.(7), one can then estimate the value of $|\langle\Psi_i(E)|W|\Psi_f(E_f)\rangle|^2$, which after insertion in Eq.(4) permits to obtain the reaction rate.

Expressions equivalent to the ones above could also be obtained for the other four-body recombination reactions considered in this work: $\alpha+\alpha+n+n \rightarrow^6$ He$+\alpha$, $\alpha+\alpha+n+n \rightarrow^9$ Be$+n$ and $\alpha+\alpha+\alpha+n \rightarrow^9$ Be $+\,\alpha$.

2.3. Three-body wave functions

As described above, calculation of the three-body radiative capture and four-body recombination reaction rates require knowledge of continuum and bound three-body wave functions. In this work they are obtained by solving the Faddeev equations in coordinate space. This is done by using the hyperspherical adiabatic expansion method, as described in [4].

For the particular case of the three-body continuum wave functions, they have been computed by imposing a box boundary condition. This procedure is automatically discretizing the continuum spectrum. Inclusion of all the discrete continuum states amounts to include all the possible capture mechanisms: direct capture, where intermediate two-body states are not populated, sequential capture, where intermediate two-body states are populated before capturing the third particle, and resonant capture, where a three-body resonance is populated. This point can be visualized by means of the complex scaling method, which, after the scaling transformation, permits to see how each of the discrete continuum states is associated to one of the possible mechanisms [9].

In all the cases the only nuclear interactions involved are the neutron-neutron, neutron-α, and α-α interactions. For the first two potentials we have used the ones given in Eqs.(57) and (58) of [10]. For the neutron-neutron interaction, they are simple gaussian potentials adjusted to reproduce the experimental s-wave and p-wave scattering lengths and effective ranges. For the neutron-α potential gaussian central and spin-orbit terms are adjusted to reproduce the phase shifts for s, p, and d waves up to 20 MeV. For the α-α we use the modified Ali-Bodmer potential [11] specified in [12]. The Coulomb repulsion has been introduced assuming a gaussian charge distribution reproducing the experimental charge radius of the α-particle.

3. Production of ^{6}He and ^{9}Be by radiative capture

The reaction rates (5) for these reactions are obtained from the photodissociation cross section of the inverse process. Calculation of this cross section requires knowledge of $d\mathcal{B}/dE$, where $\mathcal{B}$ is the transition probability from the ground state to each of the box discretized continuum states. For the particular case of the two reactions considered here, the dominant contribution to $d\mathcal{B}/dE$ comes from the electric dipole term.

Once the photodissociation cross sections are obtained, we can compute the reaction rates according to Eq.(5). In Fig.2 we show as a function of the temperature the (energy weighted) reaction rate for the $\alpha+n+n \rightarrow {}^6He+\gamma$ reaction. The thick solid lines are the results obtained in this work. The higher and lower curves correspond to the electric dipole and electric quadrupole contributions, respectively. As seen in the figure the dipole term dominates by roughly four orders of magnitude. This procedure includes all the possible capture mechanisms. As seen in the figure, the quadrupole result agrees with previous estimates by Görres et al.[2] (dot-dashed) and Fowler et al. (dotted). For the dipole contribution our reaction rate is about one order of magnitude higher than Görres et al., Efros et al., and Barlett et al [3]. The reason is that in these calculations a fully sequential capture process is assumed. In fact, in [3] they also estimated the dipole reaction rate including the contribution from dineutron capture. This estimate (dotted line) is above our calculation.

For the $\alpha+\alpha+n \rightarrow {}^9Be+\gamma$ reaction, the dominant contribution to the electric dipole reaction rate comes from the $1/2^+$ states, which produce a peak below 1 GK related to the low-lying resonant state seen in the photodissociation cross section [13]. Only for temperatures beyond 5

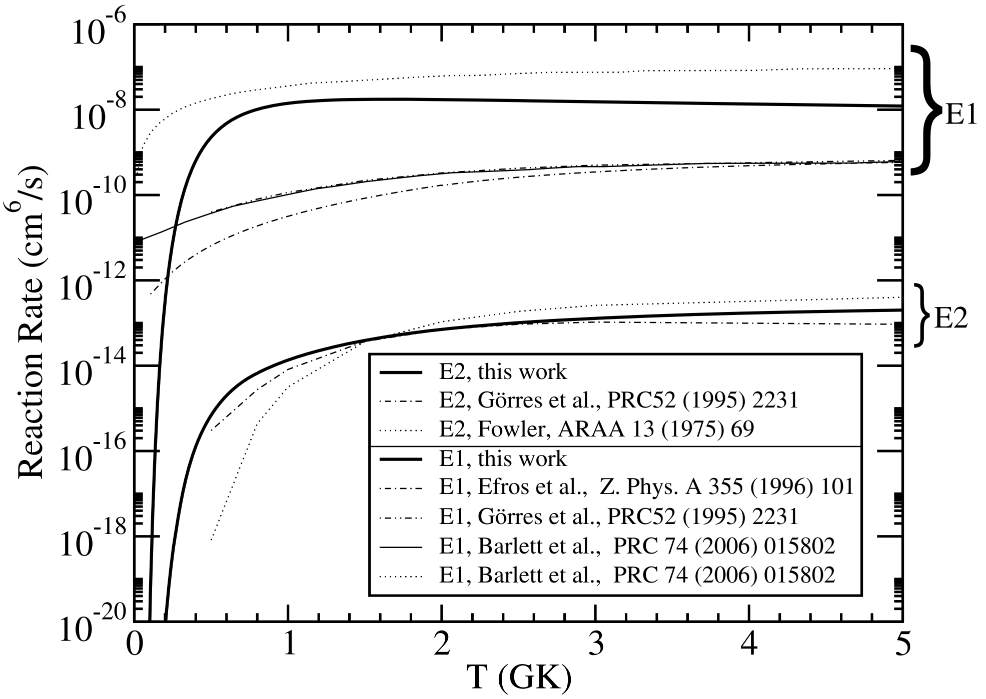

Figure 1. Dipole and quadrupole reaction rates for $\alpha+n+n \rightarrow {}^6\mathrm{He}+\gamma$. The thick solid lines are the calculations corresponding to this work. The dipole curve is about four orders of magnitude above the quadrupole curve. The different thin curves are the results found in the references given in the figure legend.

GK the $5/2^+$ contributions begins to dominate. This is due to the presence of a $5/2^+$ resonance in ^{9}Be at about 1.5 MeV above threshold. The $3/2^+$ contributions always plays a minor role.

4. Four-body recombination reactions and production rates

The production rate gives the velocity at which a certain nucleus (^{6}He or ^{9}Be in our case) is created in a given environment (number of reactions per unit time and unit volume). Direct comparison between different sources of a given reaction product is then possible. In general this can not be done from the reaction rates, since when the processes involve a different number of particles the reaction rates have different units.

Once the reaction rates have been computed, the corresponding production rates can be easily obtained by multiplying them by the densities of the nuclei involved in the reaction. These densities are given as in Eq.(3). This implies that the production rates depend on the mass density ρ, the abundances of the different elements, and of course on the temperature. Furthermore, it is clear that while the production rate for the two-particle radiative capture processes goes like ρ^3, the one for the four-body recombination reactions goes like ρ^4. Environments with large density favors then the production of ^{6}He or ^{9}Be through four-body recombination processes.

In Fig.2 we show the production rates for the different radiative capture and four-body recombination processes considered in this work. To permit an easier comparison between them we have normalized them to the production rate for the $\alpha + n + n \rightarrow {}^6\mathrm{He} + \gamma$ process. The left, central, and right panels correspond to mass abundances of the α particles of X_α=0.1, 0.5, and 0.9, respectively. They are plotted as a function of the temperature. In the calculation we have used a mass density of ρ=15000 g/cm^3, which amounts to a neutron density of the order of $n_n \sim 10^{27}$ neutrons/cm^3. Larger values of ρ would favor the production rates through the

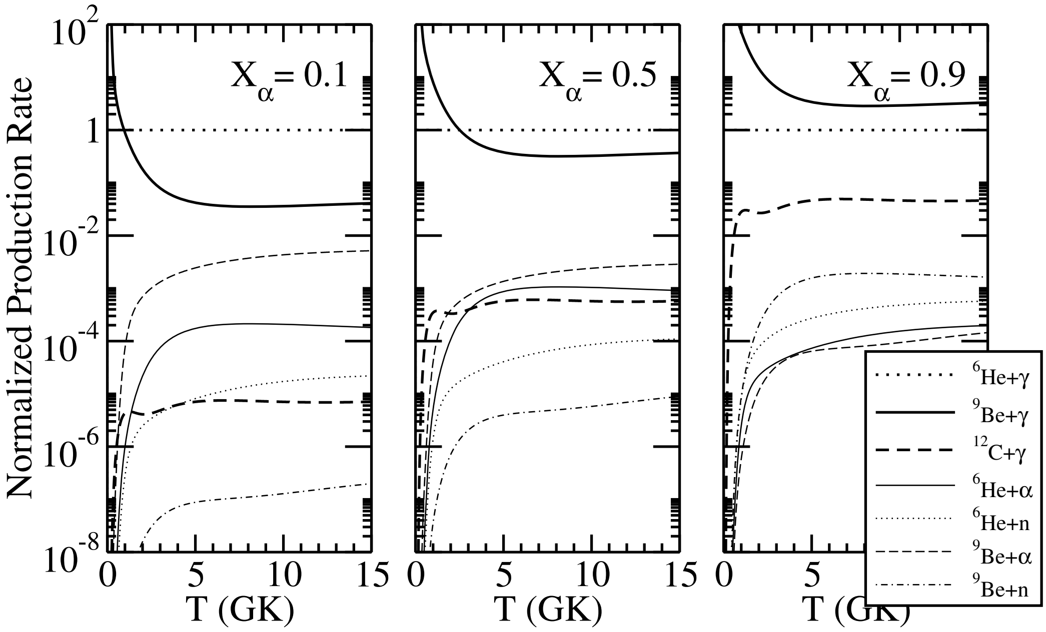

Figure 2. Normalized production rates for electromagnetic and four-body processes as a function of the temperature for α-abundances X_α=0.1, (left), 0.5 (middle), and 0.9 (right). The thick dashed line is the production rate for the triple α process computed as in Eq.(5). The production rate for the radiative capture process α+n+n $\rightarrow$^{6}He+γ is taken as reference.

four-body recombination reactions.

In the figure, the thick curves correspond to the radiative processes. Obviously, the horizontal dotted line at 1 refers to the $\alpha + n + n \rightarrow {}^6\text{He} + \gamma$ reaction. The thick solid line represents the $\alpha + \alpha + n \rightarrow {}^9\text{Be} + \gamma$ process, and the thick dashed line is the production rate for the triple α process computed as in Eq.(5). The production of ^{9}Be is dominating in all the cases for low temperatures. This is consistent with the fact that the reaction rate for the process leading to $^9\text{Be}+\gamma$ is, at low temperatures, orders of magnitude bigger than the one leading to $^6\text{He}+\gamma$ (this agrees with the result quoted in [2], where for a temperature of 0.5 GK the reaction rate for the $\alpha + \alpha + n \rightarrow {}^9\text{Be} + \gamma$ process is computed to be four orders of magnitude bigger than the one for the $\alpha + n + n \rightarrow {}^6\text{He} + \gamma$ reaction). For higher temperatures, as seen in Fig.2, the reaction rate for production of ^{9}Be is comparable to the one obtained for the production of ^{6}He, and the dominance of one process or another strongly depends on the abundance of neutrons and alphas. In fact, as seen in the figure, a small presence of α's makes the $\alpha + n + n \rightarrow {}^6\text{He} + \gamma$ reaction to be the dominant electromagnetic reaction, while when the abundance of α-particles increases the production rate for the $\alpha + \alpha + n \rightarrow {}^9\text{Be} + \gamma$ process progressively increases as well, and eventually become the dominant electromagnetic process in the whole range of temperatures. As also seen in the figure, a neutron density of about 10^{27} neutrons/cm^3 (as used in the three panels of Fig.2) is enough to make the production of ^{9}Be (of $E1$ character) dominant over the triple α reaction (of $E2$ character). For an environment with a neutron density orders of magnitude smaller (as in the core of the red giants) the triple α process appears however as the dominant one, and therefore as the responsible for the bridging of the A=5 and A=8 instability gaps.

The thin lines in the figure are the production rates corresponding the four-body recombination processes. The dominance of one four-body process over another strongly depends on the abundance of α's and neutrons. It is important to remember that an increase in the mass density by a certain factor, enhances by the same factor the relative four-body recombination production rate. In a hot bubble the neutron density is estimated to range between 10^{20} and 10^{30} neutrons/cm^3. The calculation shown in the figure has used $n_n \sim 10^{27}$ neutrons/cm^3. Therefore, for neutron densities in the higher limit of the estimated values, and for temperatures as the ones estimated for a hot bubble (units of GK), the four-body recombination reactions could easily be very relevant.

5. Summary and conclusions

The main goal when investigating nuclear reactions with astrophysical interest is to estimate their production rate, which gives the velocity (number of reactions per unit time and unit volume) at which the products of the reaction are created. Typical temperatures in the stellar medium are at most of a few GK. This fact implies that the reactions taking place in the interior of a star occur at very low energies (at the nuclear scale). Relative kinetic energies are often clearly smaller than 1 MeV.

The production rates depend critically on the star temperature and mass density. More precisely, the density appears as ρ^N, where N is the number of particles involved in the reaction. This means, that for a sufficiently high mass density, the factor ρ^N could compensate the smaller probability for the N particles to react simultaneously, and therefore become a relevant process.

In this work we have found that four-body recombination reactions can compete with three-body radiative processes provided that the temperature and density conditions are the appropriate. In particular they can play a role as a source of ^{6}He and ^{9}Be, and therefore be relevant in the process of bridging the A=5 and A=8 instability gaps.

In this work we have investigated the reaction and production rates for the radiative processes $\alpha + n + n \rightarrow^6$ He $+ \gamma$ and $\alpha + \alpha + n \rightarrow^9$ Be $+ \gamma$, as well as for the four-body recombination reactions $\alpha + \alpha + n + n \rightarrow^6$ He $+ \alpha$, $\alpha + n + n + n \rightarrow^6$ He $+ n$, $\alpha + \alpha + n + n \rightarrow^9$ Be $+ n$ and $\alpha + \alpha + \alpha + n \rightarrow^9$ Be $+ \alpha$. No particular assumptions about the reaction mechanism has been made. Discretization of the continuum spectrum by using a box boundary condition permits simultaneous inclusion of sequential, direct, and resonant processes. Contrary to what very often is assumed, we have found that the assumption of a sequential process (through intermediate two-body states) can lead to inaccurate results.

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