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Theory of few body reaction frameworks: Application to Halo nuclei

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Abstract. The theory of Faddev/AGS few body reaction frameworks is presented. A comparison with other reaction formalisms (DWBA, CDCC) is made. Sucesses and shortcomings of the scattering approaches for breakup are analysed. Some set of calculated reaction observales for resonant and nonresonant breakup are presented to obtain insight into the physics incorporated on the scattering approaches.

1. Introduction

The use of beams of radioactive nuclei, allowing the exploration of the nuclear chart away from the stability line, has open a new era for the field of Nuclear Physics. From the fundamental point of view the study of extreme states of matter near the drip lines will allow a deeper understanding of the nuclear matter.

Away from the stability line the many-body concept may not be useful and cluster structure models are frequently used especially for light systems [1, 2, 3]. From the point of view of scattering, the study of these exotic nuclei is both exciting and challenging. The need to take into account the halo few-body nature ab initio into the reaction formalism must be addressed. This calls for novel developments in the reaction theory, in order to handle in a convenient way at least four body problems. In addition, a very tight control on the reaction theory must be performed on the reaction formalisms, in order to extract reliable information from increasingly high precision data.

In this contribution we make a detailed review of the available theories of fewbody reaction frameworks and its application for breakup.

2. The Faddeev/AGS multiple scattering framework

The Faddeev/Alt, Grassberger, and Sandhas(AGS) formalism [4, 5, 6] is a three-body multiple scattering reaction framework that treats all open channels (elastic, breakup and transfer in equal footing). In addition all resonant and nonresonant states of each pair are automatically included. Let us consider 3 particles (1,2,3) interacting by means of two-body potentials. We shall be using in this section the odd man out notation (appropriate for 3-body problems) which means for example that the interaction between the pair (2,3) is denoted as v_1. We assume that the system is non-relativistic and we write its total Hamiltonian as

$$H = H_0 + \sum_{\gamma} v_\gamma \ , \tag{1}$$

with H_0 the kinetic energy operator of the system and v_γ the interaction for the pair γ. The Hamiltonian can be rewritten as

$$H = H_\alpha + V^\alpha \ , \tag{2}$$

where H_α is the hamiltonian for channel α

$$H_\alpha = H_0 + v_\alpha \ , \tag{3}$$

and V^α represents the sum of interactions external to partition α

$$V^\alpha = \sum_{\gamma \neq \alpha} v_\gamma \ . \tag{4}$$

The total transition amplitude which describes the scattering from the initial state α to the final state β is given in the post form by (see Refs. [6, 7] for a derivation)

$$T_+^{\beta\alpha}(z) = V^\beta + \sum_{\gamma \neq \alpha} T_+^{\beta\gamma}(z) G_0 t_\gamma \ . \tag{5}$$

with the transition amplitude

$$t_\gamma = v_\gamma + v_\gamma G_0 t_\gamma \tag{6}$$

and the free resolvent

$$G_0(z) = (z - H_0)^{-1}. \tag{7}$$

The transition amplitude t_γ, althought containing the two-body potential v_γ, is a many body operator due to its dependence on the kinetic energy of the three particles. Equivalently in the prior form we get

$$T_-^{\beta\alpha}(z) = V^\alpha + \sum_{\gamma \neq \beta} t_\gamma G_0 T_-^{\gamma\alpha}(z) \ . \tag{8}$$

One then defines the operators $U^{\beta\alpha}$ which are equivalent to the transition amplitudes on the energy shell, such that

$$\begin{aligned} U^{\beta\alpha} &= \bar{\delta}_{\beta\alpha} G_\alpha^{-1} + T_+^{\beta\alpha} \\ &= \bar{\delta}_{\beta\alpha} G_\beta^{-1} + T_-^{\beta\alpha} \ , \end{aligned} \tag{9}$$

where $\bar{\delta}_{\beta\alpha} = 1 - \delta_{\beta\alpha}$. From Eqs. (5-8)

$$U^{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_{\gamma} U^{\beta\gamma} G_0 t_\gamma \bar{\delta}_{\gamma\alpha} \ , \tag{10}$$

or

$$U^{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_{\gamma} \bar{\delta}_{\beta\gamma} t_\gamma G_0 U^{\gamma\alpha} \ . \tag{11}$$

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The solution of the Faddeev/AGS equations can be found by iteration

$$
\begin{aligned}
U^{\beta\alpha} &= \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_{\gamma} \bar{\delta}_{\beta\gamma} t_\gamma \bar{\delta}_{\gamma\alpha} \\
&+ \sum_{\gamma} \bar{\delta}_{\beta\gamma} t_\gamma \sum_{\xi} G_0 \bar{\delta}_{\gamma\xi} t_\xi \bar{\delta}_{\xi\alpha} \\
&+ \sum_{\gamma} \bar{\delta}_{\beta\gamma} t_\gamma \sum_{\xi} G_0 \bar{\delta}_{\gamma\xi} t_\xi \sum_{\eta} G_0 \bar{\delta}_{\xi\eta} t_\eta \bar{\delta}_{\eta\alpha} \\
&+ \cdots .
\end{aligned}
\tag{12}
$$

The successive terms of this series can be considered as terms of zero order (which contribute only for rearrangment transitions), first order (single scattering), second order (double scattering) and so on in the transition operators t_α.

The solution of the Faddeev/AGS equations are in standard approches solved in momentum space and discretization of all momentum variables. The Coulomb interaction can be included as described in [9, 10].

We consider here the breakup series which is represented diagramatically up to third order in Figs. 1-2 where the upper particle is taken as particle one scattering from the bound state.

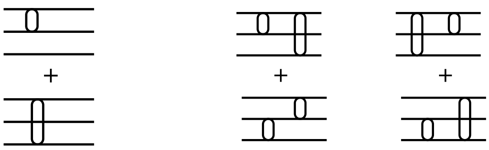

Figure 1. Multiple scattering diagrams (first to second order) for breakup in the Faddeev multiple scattering framework.

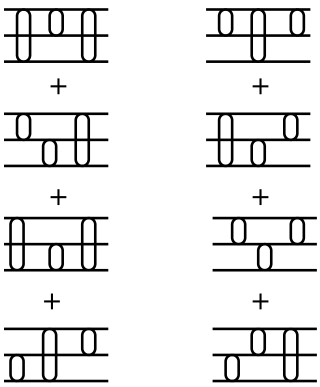

Figure 2. Multiple scattering diagrams (third order) for breakup in the Faddeev multiple scattering framework.

In order to solve the Faddeev/AGS equations we need to provide all the pair interactions. In the case examples we are presenting here envolving the scattering of a halo nucleus (well described by a core C and a valence particle n) on a proton target, p, we need then to provide the three pair interactions: $p-n$, $p-C$ and $n-C$ as described for example in the work of [11] for the case of p-^{11}Be scattering.

3. The DWIA/PWIA

Let us consider the reaction $A(a, ab)B$ where an incident particle a knocks out a nucleon or a bound cluster b in the target nucleus A resulting in three particles (a, b, B) in the final state.

This reaction has been analysed within the approximated reaction formalism, the distorted-wave impulse-approximation (DWIA) as described for example in the work of Chant and Roos [12]. This reaction framework relies mainly in two basic assumptions: (i) that the projectile strucks the ejected particle freely and (ii) that the 3-body wave function $\eta^{(-)}_{Bab}$ can be written as a factorized product of two wave functions that describe the $a + B$ and $b + B$ scattering. The transition amplitude is then written as

$$T_{AB} = \langle \eta^{(-)}_{aB} \eta^{(-)}_{bB} | t_{ab} | \phi_{Bb} \eta^{(+)}_{aA} \rangle, \tag{13}$$

where $\eta^{(-)}_{aB}$, $\eta^{(-)}_{bB}$ and $\eta^{(+)}_{aA}$ describe the relative motion of the particles in the entrance and exit channels: for the entrance channel one has

$$(T_{aA} + V_{aA} - V_{ab} - \epsilon_{aA}) \eta^{(+)}_{aA} = 0, \tag{14}$$

where ϵ_{aA} is the relative kinetic energy and T_{aA} the relative kinetic energy operator. For the exit channel, the relative wave functions satisfy:

$$(T_{aB} + V_{aB} - \epsilon_{aB}) \eta^{(+)}_{aB} = 0, \tag{15}$$

and

$$(T_{bB} + V_{bB} - \epsilon_{bB}) \eta^{(+)}_{bB} = 0. \tag{16}$$

The potentials V_{aB} and V_{bB} are taken to be the optical potentials which describe the $a + B$ and $b + B$ scattering at energies ϵ_{aB} and ϵ_{bB} respectively. Introducing plane waves in the exit channel and assuming

$$\eta^{(+)}_{aA} \sim (1 + G_0 t_{aB}) \chi^{(+)}_{aA} \ , \tag{17}$$

we then obtain

$$T_{AB} \sim \langle \chi^{(-)}_{aB} \chi^{(-)}_{bB} | T^{\rm exit} + T^{\rm entrance} | \phi_{Bb} \chi^{(+)}_{aA} \rangle, \tag{18}$$

where $T^{\rm exit}$ is given by

$$T^{\rm exit} = t_{ab} + t_{aB} G_0 t_{ab} + t_{bB} G_0 t_{ab} + t_{bB} G_0 t_{aB} G_0 t_{ab}. \tag{19}$$

The first order term is the single scattering and contains the scattering from the struck particle only. This is the only term retained in plane-wave impulse approximation calculations. The second and third order terms in Eq. (19) represented in Fig. 4 are due to distortion in the exit channel. The component $T^{\rm entrance}$ is

$$T^{\rm entrance} = t_{ab} G_0 t_{aB} + t_{aB} G_0 t_{ab} G_0 t_{aB} + t_{bB} G_0 t_{ab} G_0 t_{aB} + \mathcal{O}(t^4) \ , \tag{20}$$

and the corresponding terms are represented in Fig. 3.

We refer as DWIA-full the *truncated* multiple scattering expansion $T^{\rm exit} + T^{\rm entrance}$ given respectively by the terms in Eqs. (19) and (20). We note that further approximations are usually made in standard applications of the DWIA when evaluating the transition amplitude, namely (i) The factorization approximation, which is only exact in PWIA (ii) On-shell approximation of the transition amplitude. We refer to this as DWIA-standard.

The Faddeev/AGS calculations do not make use of such approximations in the summation of the multiple scattering expansion.

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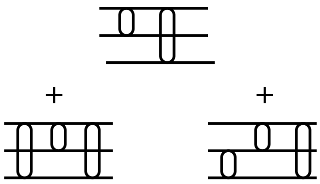

Figure 3. Diagrams due to the distortion in the income channel in the distorted-wave impulse approximation scattering framework.

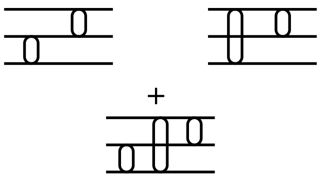

Figure 4. Diagrams due to the distortion in the exit channel in the distorted-wave impulse approximation scattering framework.

4. The CDCC scattering formalism

The continuum discretized coupled chanels (CDCC) [13, 14] reaction framework consists in solving the Schrödinger equation in a model space in which the three-body wavefunction is expanded in the internal states (bound and continuum resonant and nonresonant states) of the two-body projectile. The exact three-body wave function satisfies the Schrödinger equation

$$(H - E)\Psi^{+}_{\mathbf{K}_0}(\mathbf{R}, \mathbf{r}) = 0 \; , \tag{21}$$

where H is the three-body Hamiltonian, E is the total c.m. energy of the system, $\mathbf{R}$ the relative distance between the c.m. of the projectile and the target and $\mathbf{r}$ the relative distance between the valence particle and the core. Finally $\mathbf{K}_0$ is the incident wave number of the projectile in the c.m. frame.

The Hamiltonian H for this system can be written as a sum of three terms

$$H = T_R + U_{pt}(\mathbf{R}, \mathbf{r}) + H_p \; , \tag{22}$$

where T_R is the c.m. kinetic energy, U_{pt} the potential between the projectile constituents and the target which for a two-body target described by a core C and a valence particle v, is given by

$$U_{pt}(\mathbf{R}, \mathbf{r}) = \sum_{j=C,v} V_{jt}(\mathbf{R}, \mathbf{r}) \; , \tag{23}$$

and H_p internal Hamiltonian of the projectile. In the CDCC approach the exact wave function is expressed as an expansion in terms of the states (bound and bin) $\phi_\alpha(\mathbf{r})$ of the two-body Hamiltonian H_p as following

$$\Psi^{\mathrm{CDCC}}_{\mathbf{K}_0}(\mathbf{R}, \mathbf{r}) = \sum_{\alpha=0}^{N} \phi_\alpha(\mathbf{r})\omega_\alpha(\mathbf{R}) \tag{24}$$

where α=0 refers to the projectile ground state. The bin states includes both the resonant and the nonresonant part of the continuum. The wave functions $\omega_\alpha(\mathbf{R})$ of the projecticle-target relative motion are solutions of the coupled-channels equations

$$\left(E_\alpha - T_R - V_{\alpha\alpha}(\mathbf{R})\right)\omega_\alpha(\mathbf{R}) = \sum_{\beta\neq\alpha} V_{\alpha\beta}(\mathbf{R})\omega_\beta(\mathbf{R}) \; , \tag{25}$$

where $E_\alpha = E - \varepsilon_\alpha$ and the coupling potentials are

$$V_{\alpha\beta}(\mathbf{R}) = \langle\phi_\alpha|U_{pt}(\mathbf{R}, \mathbf{r})|\phi_\beta\rangle \; . \tag{26}$$

5. Bridging scattering approaches for breakup

In this section we show several calculated observables for breakup scattering making use of the previously discussed fewbody reaction frameworks. As an working example we have considered the case of the scattering of a ^{11}Be on a proton target at intermediate energies. We aim to pin down the relevant phyisics content of each fewbody scattering framework and to explore the validity of standard approximations in the calculated scattering observables.

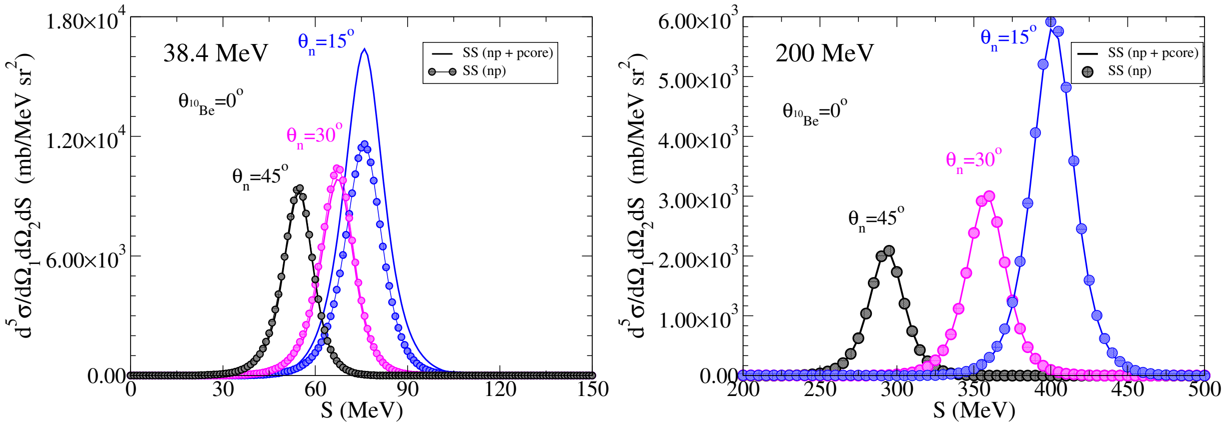

Figure 5. (Color online) Cross section for the breakup ^{11}Be(p,pn) at 38.4 MeV and 200 MeV including only the single scattering terms.

In Figs. 5 and 6 it is shown the Faddeev-type calculations performed for ^{11}Be breakup on proton target at 38.4 and 200 MeV/u incident energies at quasi free scattering conditions as described in the work of [15].

The curves represented in the figures, correspond to the calculated fivefold differential breakup cross section $d^5\sigma/d\Omega_1 d\Omega_2 dS$ versus the arclength S taking into account the single scattering term only. The kinematical configurations are characterized by the polar and azimuthal angles (θ_i, ϕ_i) of the two detected particles labeled 1 and 2 in Fig.10 of [15] . We assume those particles to be the ^{10}Be core (1) and the neutron (2). We take three quasi free scattering configurations: (conf 1) with $(\theta_1, \phi_1) = (0°, 0°)$ and $(\theta_2, \phi_2) = (45°, 180°)$; (conf 2) with $(\theta_1, \phi_1) = (0°, 0°)$ and $(\theta_2, \phi_2) = (30°, 180°)$; (conf 3) with $(\theta_1, \phi_1) = (0°, 0°)$ and $(\theta_2, \phi_2) = (15°, 180°)$.

Fig. 5 shows the breakup cross section calculated taking into account only the single scattering contribution of the Faddeev/AGS series. The case where both the proton-neutron and proton-core contributions is represented by the solid line. The circles correspond to the PWIA, i.e., without the proton-core scattering contribution. At 38.4 MeV/u this contribution is very small in the configurations 1 and 2, but is sizable when the neutron is emmited in the forward region. Therefore, even at the single scattering level, the DWIA calculations become innacurate in some configurations at low energies. At high energies the proton-core scattering contribution is negligible in all configurations.

Fig. 6 shows the Faddeev/AGS results taking multiple scattering terms up to second order (solid line) (shown in Fig 1) since the series is converged to this order.

The DWIA-full assumes that the contribution of the scattering between the proton and the neutron is dominant and therefore the single scattering contribution due to the scattering of the proton from the ^{10}Be core is neglected. The circles include the first and second order diagrams originated from the distortions in the exit channel represented in Fig. 4, and the dashed curve includes, in addition, the second order diagrams originated from the distortion in the incoming channel represented in Fig. 3. The DWIA-full results to second order (dashed line) provide a poor approximation to the Faddeev/AGS at 38.4 MeV in any configuration. As the energy increases

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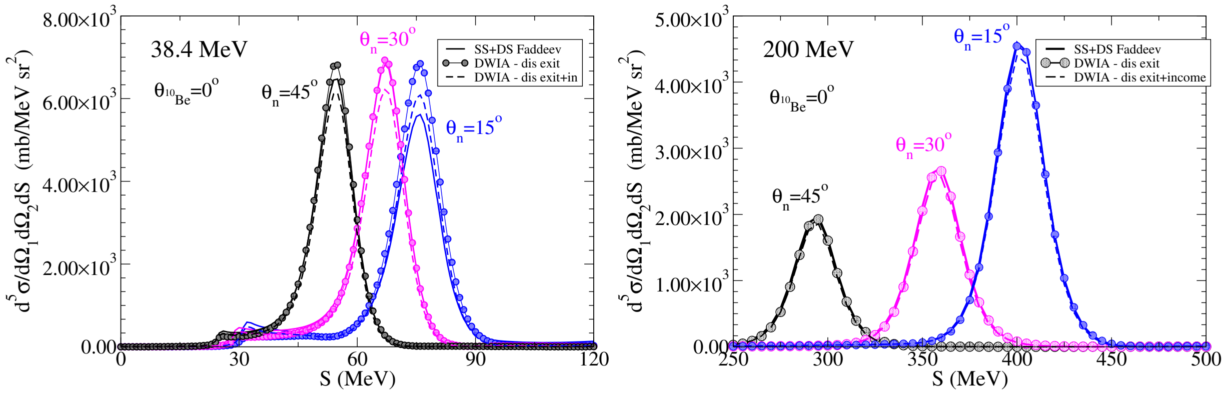

Figure 6. (Color online) Cross section for the breakup ^{11}Be(p,pn) at 38.4 MeV and 200 MeV including single and double scattering term.

DWIA-full approaches the exact result. However one should keep in mind that DWIA-standard calculations assume further approximations

As a final example Fig. 7 shows the breakup angular distribution for ^{11}Be breakup on a proton target @ 63.7 MeV/nucleon. The figure shows the inclusive angular distribution $d\sigma/d\Omega_{\mathrm{c.m.}}$. Very small angles $\theta_{\mathrm{c.m.}} < 5°$ (where there is no data) were not included because the convergence of the results with respect to the Coulomb screening radius is slow.

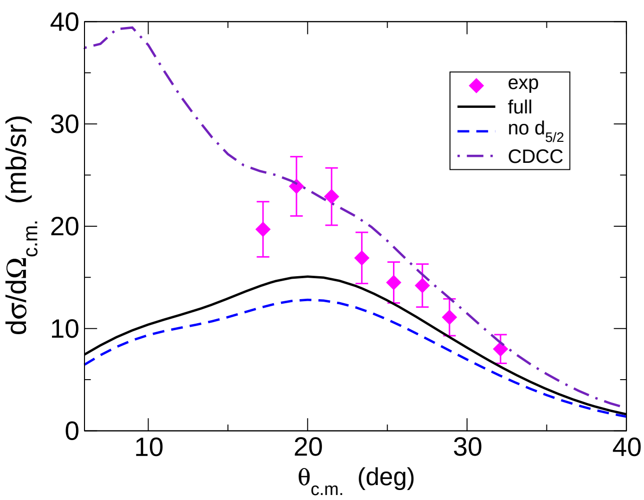

Figure 7. (Color online) Angular distribution for the breakup p(^{11}Be,p)^{10}Be n at 63.7 MeV/u integrated over the energy range $E_{\mathrm{rel}} = 0 - 2.5$ MeV. The solid line was obtained by solving the Faddeev/AGS equations with all partial waves. The dashed line represents the calculated observable excluding the $d_{\frac{5}{2}}$ partial wave contribution.

Due to the energy resolution of the experimental setup, the relative core-neutron energy was integrated around the resonance $E_{\mathrm{r}} = 1.275$ MeV in the energy range $E_{\mathrm{rel}} = 0 - 2.5$ MeV. The solid curve which includes both the resonant and nonresonant contributions reproduces fairly well the experimental data except for $\theta_{\mathrm{c.m.}} \leq 20°$. The dashed line represents the results excluding the $d_{\frac{5}{2}}$ partial wave. The dashed-dotted lines were obtained using the CDCC scattering framework with similar inputs as those used in the work of ref. [16]. A large disagreement between the two calculations for $\theta_{\mathrm{c.m.}} \leq 20°$ is evident and the source of discrepency needs to be investigated.

6. Conclusions

We have reviewed the theory of the Faddeev/AGS few body reaction framework along with other standard formalisms (DWIA and CDCC). In the Faddeev/AGS framework the sum of the series can be included. Practical applications of the DWIA approach envolves a truncated multiple scattering terms which may lead to significant contributions. A strong disagreement between calculated breakup observables was found between the Faddeev and the CDCC framework that needs to be understood.

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