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Black holes, quantum theory and cosmology

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Abstract. Some reasons are given for believing that the rules of quantum (field) theory must be changed when general relativity becomes seriously involved. If full quantum mechanical respect is paid to the principle of equivalence, we find that a superposition of gravitational fields leads to an illegal superposition of different vacua, giving support to a proposal for spontaneous quantum state reduction made earlier by Diósi, and then independently by the author. A different line of attack involves the over-riding role of black holes in the total entropy content of the universe, and in the operation of the 2^{nd} Law of thermodynamics. The author's proposal of conformal cyclic cosmology is reviewed in order to highlight a seeming paradox, according to which the entropy of the universe of the remote future seems to return to the small kind of value that it had at the big bang. The paradox is resolved when we take into account the information loss that, from this perspective, necessarily occurs in Hawking's black-hole evaporation, with the accompanying loss of unitarity.

1. Quantum state reduction as a gravitational effect

In various places elsewhere I have expressed some of my reasons for believing that, when Einstein's theory of gravity is brought into the picture, the standard rules of quantum theory must change – with the hope that when the appropriate changes are made, the quantum *measurement paradox* will be resolved. For an account of most of these arguments, see Chapter 30 of my book *The Road to Reality.* [1] It is suggested that, in accordance with a proposal put forward initially by Diósi [2] and later, but with different motivations, by myself, [3] a quantum superposition of two states, where each on its own would be stationary, should spontaneously decay to one or the other in a time scale of the order $\sim \hbar/E_G$, where E_G is the gravitational self-energy of the mass distribution which is the *difference* between the mass distributions of the two states.

The most compelling motivation, [4] in my opinion, comes from an argument that regards Einstein's principle of equivalence as an essential ingredient in the description of quantum processes taking place in a gravitational field. Two ways of treating a uniform gravitational field may be envisaged. One is the "Newtonian" way, in which gravitation is considered as just another force, and so it merely provides a term in the Hamiltonian in addition to those describing all the other forces of relevance to the problem at hand. The other is the "Einsteinian", in which we work in a frame in free fall, so that the gravitational force disappears from the Hamiltonian, and we can then transform back for comparison with the Newtonian frame that we used before. It is found that the Newtonian wavefunction Ψ_N is simply a *phase* multiple of the Einsteinian one Ψ_E. In a sense this justifies a viewpoint that quantum theory is consistent with the principle of equivalence, a fact that has found experimental confirmation in the well-known "COW"

experiment. [5] However, the phase factor (where $\mathbf{a}$ is the 3-vector of the acceleration, $\bar{\mathbf{x}}$ the position 3-vector of the mass centre, m the total mass, and t the time) appearing in the formula

$$\Psi_E = \mathrm{e}^{i(\frac{1}{6}t^3\mathbf{a}\cdot\mathbf{a}+t\bar{\mathbf{x}}\cdot\mathbf{a})m/\hbar}\Psi_N$$

relating these two wavefunctions is *non-linear in the time*, because of the term $\frac{i}{6}t^3\mathbf{a}\cdot\mathbf{a}$ in the exponent, which has the effect that the two wavefunctions belong to *different vacua.* This is actually the Galilean (i.e. $c \to \infty$) limit of the *Unruh* effect, [6] where an accelerating observer (i.e. not in Einsteinian free fall) experiences a different vacuum – in fact a *thermal* vacuum – from a freely falling one at the same location. In the fully relativistic Unruh effect, the acceleration of the observer gives rise to a temperature $\mathbf{T}$ proportional to a/c, experienced by the observer. But this gives $\mathbf{T}= 0$ in the Galilean limit $c \to \infty$. Nevertheless, the two vacua are still different because of the above phase factor, which survives in the limit. [7]

I am adopting the viewpoint that the Einsteinian treatment of the gravitational field is the more fundamental one, in the quantum (as well as the classical) context. This has no real consequence if we just doing quantum mechanics within a single classically treatable gravitational field. But it does have an important implication that if we try to consider doing quantum physics in the presence of a quantum *superposition* between two gravitational fields, where the two free-fall frames differ in the neighbourhood of some point P. For then we have two distinct vacua around P involved in the superposition, so that, strictly speaking, quantum superpositions between states that are entangled with the gravitational field become "illegal" – since scalar products between states belonging to the different vacua become divergent. Since these scalar products involve integrations out to infinity, I take the view that these infinities only begin to make their mark after a finite time-scale T, and that T can be estimated by use of Heisenberg's time-energy uncertainty relation, where the energy uncertainty E_G is estimated from the coefficient of the t^3 term in the phase factor relating the two vacua, integrated over all space. Taking T to represent the rough maximum time that our quantum superposition can persist, before "decaying" to one or the other of the two gravity states under superposition, we obtain the above-mentioned ("Diósi-Penrose") proposal for the lifetime of a quantum superposition. Although current experiments have not reached the degree of sensitivity needed to test this proposal, there are projected schemes under development that might eventually do so. [8]

2. Space-time singularities and the 2nd Law

Among the other reasons for believing that the rules of quantum (field theory) must become seriously modified in the context of the curved space-time of (Einstein's) general relativity, are those relating to black holes and cosmology. Most particularly, it is normally anticipated that there ought to be a physically correct theory of "quantum gravity" that should account for the actual physical behaviour that takes place at those space-time locations where, when we attempt to describe them according to classical Einstein theory, we encounter *space-time singularities.* Yet, what we find when we try to understand the structure of the classical space-time in the neighbourhood of such singularities is something that is grossly asymmetrical in time, a feature which is hard to understand if we insist on playing according to the rules of standard quantum field theory. This time-asymmetry is deeply related to the second law of thermodynamics (which I here refer to simply as the "2nd Law"). This is not a time-asymmetry that can be explained *by* the 2nd Law; rather it is the 2nd Law (in the particular form that we find in our universe) that arises *because* of this gross time-asymmetry in space-time-singularity structure.

This point needs illumination. The 2nd Law roughly tells us that the entropy S of an isolated system should increase with time (or at least should not decrease, apart from, perhaps, occasional fluctuations). Boltzmann's definition of entropy is

$$S = k \log V,$$

k being Boltzmann's constant, where V is the volume of the coarse-graining region $\mathcal{V}$, in phase space $\mathcal{P}$, of all the possible micro-states of the system which could underlie the given macro-state under consideration. These coarse-graining volumes differ enormously from one another, a fact that is somewhat disguised by the logarithm in Boltzmann's formula (and by the smallness of Boltzmann's constant k in ordinary terms), so that a tiny entropy increase signals a vast increase of the volume of the relevant coarse-graining region. Thus, the huge majority of time-evolution trajectories within $\mathcal{P}$, which pass through a given coarse-graining region $\mathcal{V}$, will *next* enter a vastly larger region, and this corresponds to a significant entropy increase in the future time direction, as is indeed the behaviour that comes about according to the 2$^{\text{nd}}$ Law. However, this reasoning does not agree with observed physical behaviour in the *past* time direction, because the vast majority of time-evolution trajectories entering $\mathcal{V}$ will also have *come from* a vastly larger region, seeming to suggest that a larger entropy in the past is also to be expected. We cannot explain why the 2$^{\text{nd}}$ Law held in the past, simply from general considerations of this kind. Instead, we need an understanding of why it was that the entropy of the *initial* state of our universe was so extraordinarily small. If we accept this as an initial constraint, and then allow ourselves to consider *only* those trajectories originating within that utterly tiny coarse-graining region $\mathcal{B}$ relevant to our *actual* Big Bang, then we do expect to find that the vast majority of time evolutions constrained in this way will encounter increasing entropies – which is the central content of the 2$^{\text{nd}}$ Law.

3. The extraordinary specialness of the Big Bang

The fact that $\mathcal{B}$ does indeed have to be an extremely tiny region within $\mathcal{P}$, is a manifest implication of the 2$^{\text{nd}}$ Law, yet there is a seeming paradox here, in view of the most direct piece of observational evidence for an actual "primordial fireball" encompassing our entire universe some 1.4×10^{10} years ago. This evidence is provided by the *cosmic microwave background* (CMB), coming to us from all directions in space. Its frequency spectrum accords extremely closely with the Planck formula, for a temperature of 2.725K, so what we appear to be observing is an adiabatically expanding universe in a very closely thermal state, where the expansion of the universe has cooled the radiation from an extremely high temperature (of around 3000K) when the universe was a mere 3.8×10^5 years old (about 1/37000 of its present age) – this being the time of "decoupling", when the universe became fully transparent to electromagnetic radiation. This has the appearance of paradox, because the Planck spectrum is indicative of a state of *thermal equilibrium*, in other words, of *maximum* rather than minimum of entropy. The resolution of this paradox has nothing to do with the universe in its early stages being "small". There would have been as many degrees of freedom potentially available to it in its early highly condensed stages as in its later expanded stages, for the simple reason that the phase space $\mathcal{P}$, is just a *given* thing which does not itself "grow" with time (time evolution being described by trajectories *within* $\mathcal{P}$).

An issue arises, with a spatially infinite universe, that the total volumes of $\mathcal{P}$ and $\mathcal{B}$ would normally be infinite. This is not really problematic, however, in the context of standard cosmologies because, adequately for the present purposes, we can consider that our descriptions apply to a very large *co-moving* spatial volume, rather than to the entire universe. It will be sufficient to take this co-moving volume to be that originally lying within the *particle horizon* of our present space-time location, and we then find that this volume has a baryon number of around 10^{80}. I shall consider our phase space $\mathcal{P}$ and its Big-Bang-subspace $\mathcal{B}$ to refer to this co-moving space-time volume only, and then we find a finite value for the volume P of $\mathcal{P}$ and therefore also for the smaller volume B of $\mathcal{B}$.

In order understand how the 2$^{\text{nd}}$ Law is set on its course, we need to appreciate how B can be enormously smaller than P, despite the appearance of thermal equilibrium of the primordial fireball. A clue lies in another striking aspect of the CMB, namely its extremely closely isotropic

nature, with variations of temperature in different directions in the sky being only at the level of around 10^{-5}, once the Doppler correction is made for the proper motion of the Earth. This provides a strong indication that the initial Big-Bang singularity must itself have been extremely closely isotropic, and if we assume that there is nothing significant in our particular spatial location [9] in the universe ("Copernican principle") then we conclude that it must have been closely homogeneous also. This is all in accordance with the picture that our overall universe, with its Big-Bang singularity, is rather closely approximated by a standard Friedmann-Lemaître-Robertson-Walker (FLRW) cosmological model, as is standard practice in modern cosmology.

However, it is this very closeness to an FLRW-symmetrical structure, that we apparently find in the Big Bang, that characterizes it as having an extraordinarily low entropy. The FLRW models are all very special in that there can be no gravitational contribution to the entropy, this arising only when departures from complete homogeneity or isotropy occur. In general terms, it is *gravitational clumping* that represents entropy increase. We see this most particularly, from the perspective of life here on Earth, in the formation of the Sun by gravitational clumping from a previously rather uniform distribution of gas, this being the ultimate reason for the manifest departure from thermal equilibrium that the contrast between the hot sun and the dark sky provides, and which our continued well-being on Earth depends upon. The most extreme gravitational clumping is the formation of *black holes*, this resulting in an enormous increase in entropy. In fact, the totality of large black holes residing in galactic centres provides easily the largest (known) contribution to the entropy of the universe. This is despite the fact that the contribution from the CMB itself constitutes a larger entropy per baryon than all known physical processes other than these.

Now, if we want to get some feeling for how much larger the entropy of the universe *might have been* had the Big Bang been of a completely generic nature, we can first consider the time-reversed situation of a *collapsing* universe, which acts in accordance with the 2$^{\text{nd}}$ Law, and in which there are some initially fairly small deviations from FLRW symmetry that represent a certain amount of gravitational clumping. As the entropy of the universe increases with this universe's collapse, matter accumulates into black holes, and as the collapse continues, the deviations from FLRW increase relentlessly. Many black holes merge together, swallowing each other up at an increasing rate, forming a great mess of a final space-time singularity, with little left of the symmetry or uniformity of the FLRW model that it initially resembled. From the perspective of strong cosmic censorship, it is not unreasonable to assume that this final ("Big-Crunch") singularity has an essentially *spacelike* character, [10] so that with a suitable choice of time parameter, it may be regarded as a simultaneous event. The entropy of the universe – or at least of the co-moving part which lies within the particle horizon of our present space-time location – may be fairly well estimated by use of the Bekenstein-Hawking formula for the entropy of the (effective) black hole which encompasses all the matter in the (relevant co-moving) universe. Taking the baryonic part of this matter to consist of 10^{80} baryons, this formula gives us an entropy of around 10^{123} in absolute units (i.e. taking $c = \hbar = k = 1$, although this choice makes little difference to this figure). In fact, baryonic matter appears to account for only $\sim 15\%$ of the dynamical material content of the universe, the remaining $\sim 85\%$ being some unknown substance referred to as "dark matter". Taking this into account, and assuming that this dark material could also contribute to the total possible black-hole entropy, we get a figure for the largest possible entropy value of perhaps 10^{125}, but owing to the uncertainties about the nature of this material, I shall take the figure 10^{123} as representing a reasonable lower limit to the possible entropy value. Inverting Boltzmann's entropy formula, we find the value $e^{10^{123}}$ as a reasonable lower limit for the volume P of the phase space $\mathcal{P}$:

$$\mathrm{e}^{10^{123}} \leq P.$$

With regard to the volume B, there is more uncertainty, since all we directly see is the CMB,

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and the primordial fireball which created it may well have been involved in some significant entropy-raising processes in its $\sim 3.8\times10^5$ preceding years, following the Big Bang singularity. Figures of 10^8 or 10^9 have been quoted for the entropy per baryon of this radiation, so we may take 10^{89} as a rough upper bound for the entropy in the CMB, and therefore as an upper bound for the entropy in the Big Bang itself (which might actually have been significantly lower):

$$B \;\leq\; \mathrm{e}^{10^{89}}.$$

This tells us that the ratio of the volume of B to that of the whole phase space P is no greater than

$$\mathrm{e}^{10^{89}} \;\div\; \mathrm{e}^{10^{123}} \;=\; \mathrm{e}^{10^{89}-10^{123}} \;=\; \mathrm{e}^{-10^{123}}$$

to within the accuracy of the upper exponents, so we see that the $\mathrm{e}^{10^{89}}$ makes no significant impression on the far more enormous $\mathrm{e}^{10^{123}}$. The thermal nature, or otherwise, of the primordial fireball, would only affect the $\mathrm{e}^{10^{89}}$ and is thus essentially irrelevant to the far greater problem of the utter minuteness of $\mathrm{e}^{-10^{123}}$. On the other hand, the great uniformity of the CMB temperature across the sky is a reflection of the uniformity of the universe itself – a powerful confirmation of the fact that that the initial singularity *was* pretty closely of FLRW type, with gravitational degrees of freedom essentially inactivated and utterly *un*like the time-reverse of the maximum-entropy collapsing universe full of congealing black holes that was considered in the previous paragraph. That time reverse of a generic "Big Crunch" final singularity would have been a Big Bang of stupendous entropy ($\geq \mathrm{e}^{10^{123}}$) riddled with white holes (the time reverses of black holes) and it was the complete absence of such white-hole monstrosities that provides the way in which our universe was so enormously special.

4. Cosmic inflation

The foregoing discussion must be compared with the alternative viewpoint of *cosmic inflation*, [11] central to most modern discussions of the very early universe, and which is commonly claimed to provide a quite different perspective, and even an "explanation" of the uniformity of the early universe. It is argued that during a period roughly between the times 10^{-35}s and 10^{-32}s following the initial Big Bang singularity, the universe exponentially expanded by a factor of perhaps between 10^{30} and 10^{60}. Without such cosmic inflation, two fairly widely separated points on the CMB sky could never have been in causal contact with each other, at any time since the Big Bang. The view is taken that such causal contact would have been *necessary* in order that the primordial fireball could have reached its global state of thermal equilibrium. It is true that the presumed inflationary phase of the very early universe indeed serves the purpose of bringing all the points on our observed CMB sky into causal contact with one another. Accordingly, it is argued that the resulting inflationary thermalization of the primordial fireball would explain the very close equality of CMB temperatures over the whole sky.

However this argument goes no way towards explaining the very tiny entropy that, as we have seen, has to be a feature of the primordial fireball. [12] Indeed, the thermalization argument – *any* thermalization argument – acts in the opposite direction. We note that the very idea of bringing distant points on the CMB sky into causal contact is in order that initially differing temperatures at such places would become equal through the *thermalization* that is enabled by this causal contact. The problem with this kind of argument, if we are trying to explain the very special nature of the early universe, is that all thermalization processes are *entropy raising* effects, so that the situation before thermalization would have to have been even more special (i.e. even lower entropy) than after thermalization. The problem of the specialness of the Big Bang is simply made *worse* by arguments of this kind. We recall, from the previous section, that the nature of this specialness could be understood if we can find some reason for there being a closely FLRW structure for the Big Bang singularity. If we are already accepting that this singularity

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had this structure very closely, then the close uniformity of the CMB temperature over the sky is a *consequence*. (The initial FLRW symmetry implies FLRW symmetry at all later times, if we assume deterministic evolution equations.) This uniformity of temperature is not helped by a thermalization argument. Indeed, it should be borne in mind that explicit calculations for deriving implications of inflation are almost invariably carried out within a FLRW background, which completely begs the question of *why* the initial singularity should be closely of this form.

A similar kind of objection can be raised against the common argument that inflation serves to iron out spatial irregularities that might have been present in the early universe. To see why inflation by itself cannot iron out generic irregularities in a messy Big Bang, we need to take into account the fact that the dynamical equations that are to govern inflation (these involving a special "inflaton field" φ that is introduced) are all *time symmetrical*, the only time asymmetry arising in the dynamical behaviour (e.g. symmetry breaking) coming about through *consequences* of the 2^{nd} Law. Suppose that it is actually true that, with a generic starting point, the inflationary processes will almost inevitably lead to a smoothed-out expanding universe after inflation has finished. Clearly there will be many states that, if simply conjured up at that later time, would *not* be smooth (and if that were not the case, there would have been no need for the inflation to get rid of them). Let us reverse the direction of time for such a state and let the reversed-time dynamical evolution (with equations still allowing for the possibility of inflation, with φ-field, etc.) take over. This must lead us somewhere. Where we are led would generally be some very complicated state which is not at all like a FLRW model, but of the black-hole congealing variety that we considered in the collapsing irregular universe of the previous section. Reversing time back again we find an initial (far from FLRW) white-hole ridden initial singularity that inflation would have been powerless to smooth.

Despite the inadequacy of the arguments from cosmic inflation to explain the uniformity and smoothness of the universe (and hence the particular way in which the Big Bang was of extraordinarily low entropy), there have been observational aspects of the CMB which inflation accounts for in a fairly natural way, and which appear to be difficult to explain without invoking inflation. One of these is the closely *scale-invariant* (Harrison-Zel'dovich) nature of the deviations from uniformity in the primordial matter distribution and also in the temperature distribution in the CMB sky. [13] Inflationary cosmology accounts for this scale invariance essentially for the reason that inflation involves an *exponential* expansion, which has a self-similar character. Another observational fact is that there are correlations between the temperatures at points on the CMB sky which, according to the standard cosmological (Friedmann-Tolman) expansions rates, would never have been in causal contact. This appears to indicate that inflation is required in order for these correlations to be explained.

5. Conformal space-time geometry

Among the alternatives to inflation for accounting for such correlations are "pre-Big-Bang" models for which the correlations in the CMB sky arise because of a kind of "causal contact" which could have occurred in the previous phase of the universe, prior to the Big Bang. Likewise this previous phase could have undergone some kind of exponential behaviour, leading to a scale-invariant input into the structure of the Big Bang. [14,15] The usual idea of a pre-Big-Bang model had been to involve a previous universe phase in which *collapse* to a singularity occurred, followed by a quantum-gravity "bounce" that became our own Big Bang. [16] But a suggestion that I have myself been developing for more than three years now is that of *Conformal Cyclic Cosmology* (CCC) [17] which involves a quite different idea.

The driving geometrical notion underlying CCC is that of *conformal* space-time geometry. This geometry takes the null-cone structure, together with temporal orientation, as being more fundamental than the scale factor that provides an actual measure of time (and space) intervals. Standard general relativity is based on the full *metric tensor* **g** but, in conformal geometry, **g**

is determined only up to a local scale factor, so that two metrics $\mathbf{g}$ and $\hat{\mathbf{g}}$ are deemed to be equivalent to each other if there is a smooth scalar field $\mathbf{\Omega}$ for which

$$\hat{\mathbf{g}} = \mathbf{\Omega}^2\mathbf{g}.$$

It is normally taken that $\mathbf{\Omega}$ is positive, but I shall also allow $\mathbf{\Omega}$ to become zero or negative in some regions, or even to become infinite (where $\mathbf{\Omega}^{-1}$ is smooth). Basically, conformal geometry is the geometry where infinitesimal shapes (or angles) have meaning, but not the infinitesimal notion of *size* that is fundamental to Riemannian geometry. Here we are concerned with *space-time*, so our $\mathbf{g}$ (or, equivalently $\hat{\mathbf{g}}$) has a Lorentzian signature, which I take in the form $(+ - - -)$. The conformal structure is equivalent to the *null-cone* structure, but I also fix a time orientation, so that the future and past null cones are distinguished. This time-oriented Lorentzian conformal structure is equivalent, locally, to the *causal* structure [18] of space-time.

It should be noted that this conformal geometry, in 4 space-time dimensions is defined by 9 numbers per point, these being the 9 independent ratios of the 10 components of $\mathbf{g}$ so, in a sense, the conformal structure is *most* of the metric. Now, there are certain aspects of physics that depend only on the conformal structure of space-time and do not need the scale factor that determines the full metric. Maxwellian electromagnetism is the most important example, the source-free Maxwell equations being invariant [19] under conformal rescalings of the form $\mathbf{g} \to \hat{\mathbf{g}} = \mathbf{\Omega}^2\mathbf{g}$, and this invariance holds just as well for a curved space-time metric as for flat space. Conformal invariance extends also to situations when there are (given) charged sources present, so the electromagnetic *interaction* is conformally invariant as well as are the free-field equations. Moreover, this conformal invariance extends to the wave equation governing the quantum description of a massless particle [20] of any spin $\frac{n}{2}$, although when $n \geq 3$ there are matters of consistency, in curved space-time, that complicate things. The case of the gravitational field ($n = 4$) raises particular issues, but we can meaningfully say that the free field is conformally invariant whereas the interactions are not. In rough terms, the key point seems to be that it is *mass* and the interactions that are specifically to do with mass that break conformal invariance.

Another way of addressing the relation between mass and conformal structure is to think of a classical particle picture. A massive particle has a timelike world-line along which the metric $\mathbf{g}$ determines a "length" notion, which is actually the measure of the passage of time for that particle. If we borrow from quantum mechanics Planck's formula $E = h\nu$ and combine it with Einstein's $E = mc^2$ we see that any individual particle is a kind of clock, with frequency $\nu = m(c^2/h)$, or $2\pi\nu = m$ in absolute units, showing us that it is *mass* that determines clock rates, and therefore the scale that fixes a particular metric $\mathbf{g}$ consistent with the given conformal structure. Of course, to make a reliable clock (such as an atomic or nuclear clock – which, indeed, can now be extraordinarily accurate) we need a system involving a great many particles, but the central issue is the same: we need *mass* in order to build a clock, and therefore to pick out a particular $\mathbf{g}$ consistent with the conformal structure.

If we work our way back in time, approaching the physical situation close to the Big Bang, we find that temperatures get so great that the masses of individual particles become irrelevant by comparison (or because we find energies far in excess of the Higgs mass) so it is a not unreasonable assumption that we can treat all the particles in the very early universe as being *massless*. It seems, also, that it is a plausible assumption that all relevant interactions can be regarded as being conformally invariant. Accordingly, we may take the view that, as far as the material contents of the universe were concerned, at the vicinity of the Big Bang, the real geometry of relevance there was *conformal* geometry, rather than Einstein's slightly more restrictive metric geometry. [21] In fact, if we adopt an FLRW model for the initial state of the universe (particularly if we use Tolman's radiation for the energy-tensor source – or even Friedmann's pressure-free "dust"), then we find that the conformal structure extends smoothly

to a region *prior* to what we refer to as the Big Bang, where the space-time singularity that was the Big Bang is now stretched out to a smooth hypersurface $\mathscr{Z}$. One might even say that the material contents of the universe could not even clearly "notice" this singular event – for they could have been dynamically propagated across $\mathscr{Z}$ in a consistent way, owing to this conformal invariance.

The basic exception is *gravity*, which interacts with mass-energy in a non-conformally invariant way. But if we wish to express the particular condition on the Big Bang which appears actually to have held pretty closely – and which ensures a very tiny initial entropy for the universe, compared with what it might have been, whence the 2^{nd} Law was set on its course, then we need to ensure that gravitational degrees of freedom were not initially activated. For many years, I have tried to formulate this condition as what I have referred to as the "Weyl curvature hypothesis". This asserts, in rough terms, that the *Weyl curvature* – which is the entirely trace-free part of the Riemann curvature, and which measures the *conformal* space-time curvature (and the free gravitational field) – must be zero at initial-type space-time singularities, though it is unconstrained at final-type singularities (inside black holes).

However, there are various difficulties and ambiguities about the precise mathematical statement of the needed condition if it is formulated in this particular way, so it is fortunate that Paul Tod [22] has developed an alternative approach which is geometrically clear and unambiguous. According to this procedure, the required condition is that the conformal geometry can be extended smoothly to a "pre-Big-Bang" Lorentzian manifold. This is seen through the introduction of a local metric $\hat{\mathbf{g}}$, with $\hat{\mathbf{g}} = \mathbf{\Omega}^2\mathbf{g}$, where $\mathbf{g}$ is the conventional Einstein metric, but where $\hat{\mathbf{g}}$ is finite and smooth at the Big Bang (where $\mathbf{\Omega} \to \infty$, with $\mathbf{\Omega}^{-1}$ smooth) so the singularity is expanded out infinitely, providing us with a *smooth* past boundary $\mathscr{Z}$, which is a spacelike hypersurface instead of a singularity. Just as with the standard FLRW models, the metric $\hat{\mathbf{g}}$ extends smoothly to the other side of $\mathscr{Z}$, so we can say that the *conformal* metric of space-time extends across $\mathscr{Z}$.

The very possibility of performing this operation provides us with a powerful restriction on the conformal geometry at the Big Bang, this restriction being of an appropriate character for a good formulation of the "Weyl curvature hypothesis". Yet, it should be noted that Tod's procedure was introduced just as a kind of "mathematical trick", providing merely a plausible and mathematically elegant way of characterizing the sort of restriction on the Big Bang that could precisely characterize the starting region $\mathcal{B}$, within the phase space $\mathcal{P}$ of possible universes. No *physical* reality had been intended for this earlier phase of the universe.

6. Conformal cyclic cosmology

Here is where CCC adopts a fundamentally different attitude, and the proposal is indeed to assign an actual reality to the postulated space-time prior to the Big Bang. This would appear to provide a situation resembling earlier ideas [23] in which the Big Bang would be replaced by a kind of "bounce" from a previous phase of the universe which was in *collapse* – but in which the 2^{nd} Law had been presumably severely violated, in order for the entropy to get down to the tiny value that it had when emerging from the bounce. The idea behind CCC is more subtle than this, however, and to understand what is involved, we must pass to the other end of the scale of time for our universe and consider, instead, what is to be expected in the very remote *future*.

I assume that the so-called "dark energy" of our universe is simply a *cosmological constant* Λ (as introduced by Einstein in 1927), where $\Lambda > 0$. Then the accelerated expansion that appears to be going on now will continue indefinitely. Galaxies and galactic clusters may be expected to hang together, although much larger conglomerations such as superclusters are likely to disperse. Large black holes in galactic centres will swallow great numbers of stars and grow larger. Many stars will escape from their galaxies, but will remain within the cluster to which its galaxy

belongs, eventually to be swallowed by the huge black hole that results from the congealing of those in the various galaxies in the cluster. Other "rogue" stars will, however, escape altogether from the cluster, and will evolve to dead black dwarfs or to individual black holes of a few solar masses. Presumably, most of the dust and accompanying dark matter in the galaxies will eventually be swallowed by the black holes. Ultimately – and here we are thinking in terms of time scales of the order of 10^{100} years, or the like – even the largest of the black holes will evaporate away through mass loss due to Hawking radiation, where almost the entire black holes' mass-energy would ultimately be converted to electromagnetic radiation. A considerable fraction of the black holes' mass-energy would, however, have been converted to *gravitational* radiation during violent encounters between different black holes residing in galaxies of the cluster.

A picture is thereby presented of a final configuration of an expanding universe inhabited largely by massless radiation, and the issue arises as to whether one can adequately treat this ultimate "asymptotic" state of the universe as something which is inhabited only by massless entities. For the purposes of argument, I am going to assume this to be the case, although the status of rogue material such as the aforementioned black dwarfs, or stray individual particles, such as protons, electrons, and neutrinos, must be considered. The strict philosophy of CCC does indeed require that nothing massive is left in the future asymptotic limit, for then we can consider that conformal geometry again becomes the relevant space-time structure. For the purposes of the present discussion, it will be necessary merely to assume that the major part of the universe's contents can be modelled accurately in this way, though a plausible case can be made that the stronger assumption that, in the very far limit, mass will itself die away so that the *strict* CCC philosophy may well ultimately hold true.

A mathematical device for treating the future asymptotic behaviour of an expanding universe, with conformally invariant massless contents, has been around for some 45 years. [24] The idea is to introduce a conformal factor $\mathbf{\Omega}$ which appropriately goes to zero in the future asymptotic limit, so that the metric $\hat{\mathbf{g}} = \mathbf{\Omega}^2\mathbf{g}$ (where $\mathbf{g}$ is the conventional Einstein metric) remains finite and smooth in the asymptotic limit (where now $\mathbf{\Omega} \to 0$). The conformal space-time then acquires a future boundary $\mathscr{I}$ which represents the future "infinity" of the Einsteinian space-time. Here we have a positive cosmological constant, whence this future hypersurface boundary $\mathscr{I}$ turns out to be *spacelike*. (This is advantageous, from the strict mathematical point of view, [25] since problematic issues that arise when $\Lambda = 0$ with $\mathscr{I}$ null, are here avoided.) Both the electromagnetic and the gravitational radiation, escaping ultimately to infinity, will make their significant marks on $\mathscr{I}$, but they do not spoil the smoothness of this bounding hypersurface. Moreover, the conformal geometry of the space-time, in its asymptotic future, extends smoothly to a conformal space-time manifold on the other side of $\mathscr{I}$. As with Tod's formulation of the Weyl curvature hypothesis, no physical reality is intended to be attached to this region of "conformal space-time" on the other side of $\mathscr{I}$, but it is introduced simply as a mathematical convenience.

The central idea of CCC, however, is to take seriously both the space-time to the future of $\mathscr{I}$ obtained here, and that to the past of the Big-bang hypersurface $\mathscr{B}$ that occurs in Tod's proposal. According to CCC, the history of the universe consists of a (perhaps infinite) succession of *aeons*, where the indefinitely expanding remote future of each aeon can be joined smoothly as a conformal manifold to a big bang for the succeeding one. Thus, the "$\mathscr{I}$" of each aeon is identified with the "$\mathscr{B}$" of the next. The conformal geometry is taken to be smooth across the join. The entire succession of aeons taken together provides a conformal manifold which is non-singular in *past* directions, although there will be black-hole space-time regions of divergent Weyl curvature that are singularities in the conformal geometry, and which can be encountered as *future* end-points of particles' timelike world-lines.

7. The 2^{nd} Law and black-hole information loss

Finally, let us come to an important issue which is made particularly manifest in relation to CCC, but it also arises as a fundamental issue for the universe as we know it, quite independently of CCC. This is the question of how the 2^{nd} Law can remain true when we compare the situation close to the Big Bang with that in the very remote future. In CCC we require a very strong correspondence indeed, between the two, since as we pass across the $\mathcal{I}/\mathcal{Z}$ hypersurface – which I shall henceforth call the *crossover surface* – via which each aeon joins to the next, all the relevant fields and conformal geometry of space-time must match exactly, as we pass from $\mathcal{I}$ to $\mathcal{Z}$. We must ask how the entropy at $\mathcal{Z}$ can be so enormously different from the entropy at $\mathcal{I}$, while the two are physically almost identical, so long as we remain in the realm of conformally invariant physics.

It should first be realized that a conformal rescaling of the metric, according to $\hat{\mathbf{g}} = \mathbf{\Omega}^2\mathbf{g}$, does *not* affect the volume measure of phase space $\mathcal{P}$. The position coordinates scale in the opposite way from the momentum coordinates, so that the 2-form surface elements $\mathrm{d}x \wedge \mathrm{d}p$, of which the full (Liouville) volume measure is composed, are *invariant* under the rescaling. Thus, entropy measures are not affected by the conformal rescalings.

On the other hand, the temperature does undergo an enormous change, approaching zero just before crossover and infinity just following it. This is because energy certainly undergoes rescaling, whereas the degrees of freedom do not. Photon goes to photon at the crossover, but each individual photon's energy (and therefore frequency) is completely rescaled at crossover. If we think in terms of a local metric $\hat{\mathbf{g}}$ which is taken so as to be smooth over crossover, then the photon's energy (frequency), with respect to the metric $\hat{\mathbf{g}}$, remains unchanged as it passes through the crossover surface. But in the scaling back to the Einsteinian metric $\mathbf{g}$, we have an enormous stretching just prior to crossover (so we get physical photons which have very low energy) and an enormous squashing just following crossover (so we get physical photons of very high energy).

The situation with gravity is technically a little different, on account of the fact that there is a difference between the conformal scaling factors for the "graviton field" and for the Weyl conformal tensor, [26] the latter passing smoothly through zero at crossover. However, the graviton information also carries through the crossover surface without loss of information (and without change in entropy), where in CCC the information is converted into that carried by a *scalar field* which must be introduced at crossover. This scalar field arises initially simply from the conformal factor $\mathbf{\Omega}$, but it is then interpreted as the origin of new *dark matter* for the succeeding aeon (compensating for dark matter swallowed by black holes in the previous aeon).

If we do not adopt CCC, there is still a huge issue to be faced, since although we need not expect such a close correspondence between degrees of freedom at $\mathcal{I}$ and $\mathcal{Z}$ as we do in CCC, it is still very difficult to see how we can account for the enormous entropy difference at these two places. This difference would be far more than 10^{101} which is a very rough estimate of the entropy in black holes that are within our particle horizon now, and we must expect this discrepancy to be greatly increased in the future. The conformal geometry and physical content of the universe at $\mathcal{I}$ and $\mathcal{Z}$ are still rather similar at these two places, irrespective of CCC, and it would appear that there can be very little difference in the entropies at $\mathcal{I}$ and $\mathcal{Z}$ and certainly nothing like 10^{101} or more.

One issue that needs to be brought up at this stage, however, is the huge entropy that is often attributed to the presence of the *cosmological event horizons* that come about from a positive Λ. If we try to think of such a cosmological horizon as being similar to a black-hole horizon (and in certain respects they are indeed similar), then we might want to attribute an entropy to such a horizon in accordance with the Bekenstein-Hawking formula. This would provide us with a value of about 3×10^{122}, if the argument for this entropy is accepted. However, as we can see from the considerations of conformal rescaling just given (telling us that the entropy at

crossover is not affected by the conformal scale), this "entropy" does not actually correspond to any actual degrees of freedom – certainly if we take Λ to be a *constant*, as it should be in standard Einstein general relativity. Some cosmologists prefer to take the view that we should take Λ to represent some kind of "dark-energy field" which could be varying in time, and accordingly subject to spatial variation also. This presents us with various theoretical difficulties, however – especially in connection with the "weak energy condition", which is only marginally satisfied by a cosmological constant and would be likely to be violated as soon as $\Lambda\mathbf{g}$ is replaced by a varying field.

There are additional difficulties involved in attempting to attribute a Bekenstein-Hawking entropy to the whole universe on the basis of a particle-horizon area. Although this horizon does indeed have a finite cross-sectional area $12\pi\Lambda^{-1}$ (suggesting a Bekenstein-Hawking entropy of $3\pi\Lambda^{-1}$ in absolute units), it is not at all clear how this "entropy" could actually be interpreted. If it indeed applies to the *whole* universe, and if the universe is spatially infinite, then the single value "3×10^{122}" would provide only an *infinitesimal* contribution to the *finite* part of the universe that has been relevant to our discussion.

However, we need to take into account the fact that cosmological horizons (unlike black-hole horizons) are highly dependent on an observer's world-line, so if we consider that it is "our" world-line that is the relevant one, we might hope to find an entropy value that is relevant just to the ($\sim 10^{80}$-baryon) matter content that lies within our particle horizon. In fact we would need to extend our world-line to infinity, in order to arrive at the cosmological horizon, and this would lead (basing calculations on a constant Λ) to a matter content M within our ultimate particle horizon that would be somewhat over 3 times the volume that is within our present particle horizon. We might expect a cosmological entropy that would roughly correspond to the collapse of M to a black hole. This would be about an order of magnitude greater than the maximum entropy figure of 10^{124}, or so (i.e. $\sim 10^{125}$), for the matter (including dark matter) within our present particle horizon.

This figure is perhaps tantalizingly close (given various uncertainties) to the entropy figure of 3×10^{122} that we arrive at from the cosmological horizon size, although appearing to be somewhat *larger*. A first impression might be that an *equality* between these figures should somehow be a logical necessity. The limiting past light cone of points moving progressively up our world-line is indeed our cosmological event horizon, and it is tempting to think that this is actually a black-hole horizon for all the material M. However, this is not a correct identification, as the null cones along this horizon point in the wrong direction. A more accurate identification would be that this is the black-hole horizon for all the material in the universe which does *not* lie within our ultimate particle horizon, which would be far far larger, so this identification seems to make little sense. Accordingly, my own position (for these and other reasons which I hope to explain elsewhere) is to take the cosmological event-horizon entropy as spurious, or at least as having no dynamical content. The remainder of the discussion given here is in accordance with this standpoint.

In that case, we must look elsewhere for the seeming enormous discrepancy between, on the one hand, what the 2$^{\text{nd}}$ Law tells us coupled with the vastness of the entropy increase engendered by the formation of black holes and, on the other, the remarkably close (conformal) similarity between $\mathcal{Z}$ and $\mathcal{I}$. The clue indeed lies with the black holes. Why is this? We might say that the 2$^{\text{nd}}$ Law is beautifully exemplified by the entire history of a black hole's evolution, from its initial creation in gravitational collapse – the epitome of an irreversible process – through to its final (presumed) disappearance through Hawking evaporation. The relentless increase of the hole's surface area is seen as a close reflection, within classical general relativity, of the 2$^{\text{nd}}$ Law itself, once we have appreciated the Bekenstein-Hawking attribution of entropy to the hole's surface area, this increase being reversed only when the ambient temperature reduces below the Hawking temperature of the hole, so that it becomes thermodynamically favourable for the hole

to lose (mass-)energy to the outside world.

Yet all this refers to activity that is proceeding according to the well-understood physics of the world *outside* the black holes' horizons. If we ask what takes place deep within the holes' interiors, we get a different story. What we find is a space-time singularity with none of the simplicity of structure that we found at $\mathscr{Z}$, where it had been not unreasonable to suppose that the conformal structure of space-time could be extended beyond it. Instead, we find a singularity of such a violent nature that its conformal geometry cannot be extended beyond it and, as far as we can tell, all information is destroyed at this singularity.

Although not a great deal is known for sure about the structure of the space-time singularity within a generic collapse to a black hole, there are three things which we must take note of, all of which are certainly plausible. The first is that, on the basis of the conjecture of *strong cosmic censorship*, [10] the singularity should generally have a spacelike character, and all timelike world-lines originating within the horizon and extended as far as possible into the future, will encounter this singularity. The second is that, in accordance with the proposal of Belinskii, Khalatnikov, and Lifshitz, [27] the Weyl curvature will oscillate in an extremely wild and complicated manner, whilst diverging to infinity. Finally, in his original analysis, Hawking [28] deduced the temperature and entropy of a black hole using quantum field theory in a background describing a collapse to a black hole, where he explicitly assumed that all information falling into the hole was destroyed (at the singularity). This information loss is what gives rise to the thermal state of the black hole.

It must be mentioned that in more recent discussions, this information loss has now become a matter of considerable controversy, one of the main reasons being that it implies a breakdown of unitarity. However, I find this argument unconvincing, particularly since the very process of quantum measurement, in standard quantum mechanics, violates unitarity, and since my own view (as expressed in the first section of this article) is that the physical process of quantum state reduction is a gravitationally induced phenomenon, it is reasonable to expect violations of unitarity in other quantum processes involving gravity. Other arguments in favour of information loss [29] have also been put forward in recent years.

In fact, this information loss is, in my view, the crucial ingredient which resolves the conundrum that I am addressing in this section, whereby the 2$^{\text{nd}}$ Law is, in effect, *not* violated, even though the physical situation can return to a low-entropy state following the process of black-hole evaporation. To understand how this can be, we must return to Boltzmann's formula $S = k\log V$, and ask what volume this "V" is supposed to be measuring. When a black hole is present, there will be some degrees of freedom internal to the black hole that, sooner or later, will get destroyed by the hole's singularity, but *exactly* when this destruction is considered to have taken place will have no real concern for us. We must certainly consider that it is gone by the time that the black hole finally evaporates away, via Hawking's runaway process at the end of the hole's life. But we should probably not think of this as a "sudden" process which "takes place" at a particular time. It is just that this information gets lost at some stage, and has become irrelevant during the course of the black hole's existence. It has finally gone by the time the hole disappears. The volume "V" (occurring in Boltzmann's formula) that is relevant to physical processes that are *not* caught behind a black hole's horizon may be calculated either by taking into account this interior information or not. If we do, then the volume measure V_{ext}, of relevance to the external processes that concern us and which we would normally use for calculating the entropy of those processes, must be multiplied by the total volume V_{bh} of the phase space describing all the degrees of freedom in possible information caught in the black hole (and due, eventually, for destruction), giving a phase-space volume $V_{\text{ext}} \times V_{\text{bh}}$. But if we do *not* take these degrees of freedom lost inside the black hole into consideration, then we would just use V_{ext}. The difference between these choices merely amounts to whether or not we add the quantity $k\log V_{\text{bh}}$ into our measure of entropy for the physics external to the black hole.

I would guess that a rough estimate for the internal entropy $k\log V_{\mathrm{bh}}$ that disappears, in effect, when the black hole is gone, would generally be not hugely more than the Bekenstein-Hawking entropy of the black hole when it was at its maximum size, but it would be somewhat more than this value, by an amount that would depend upon the hole's thermal history. (A good estimate of the internal entropy, it seems to me, would be to take that of the hypothetical black hole that would have resulted, had there been no loss through Hawking evaporation.) The sum of all these contributions $k\log V_{\mathrm{bh}}$, arising from all separate black holes, could easily account the discrepancy that we are looking for. If we add these contributions in, then we get an entropy value consistent with the 2^{nd} Law holding for any particular aeon (or for the history of the entire universe, if we do not accept CCC). In CCC, these contributions must be removed – an effective "renormalization" of entropy – by the time all black holes have evaporated, so we can consider the entropy at the crossover surface, joining one aeon to the next, to be completely continuous.

My final comment is that however we look at this, we have another powerful reason for believing that the unitary rules of standard quantum mechanics (or quantum field theory) must be transcended when general relativity is involved.

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Notes

1. Penrose (2004).
2. Diósi (1984, 1987, 1989).
3. Penrose (1993, 1996).
4. See Penrose (2004), Ex.[21.6] and note 30.37.
5. Colella, Overhauser, and Werner (1975), Bonse and Wroblewski (1983).
6. Unruh (1976).
7. B.S.Kay (private communication).
8. Marshall *et al.* (2003).
9. Compare the discussion given in Ellis (2009), Section 5, for an alternative viewpoint.
10. Penrose (1978).
11. Guth (1981, 1997).
12. Penrose (1990).
13. See, for example, Kolb and Turner (1994).
14. Veneziano (1998).
15. Steinhardt and Turok (2007).
16. See, for example, Ashtekar, Pawlowski, and Singh (2006), Bojowald (2007).
17. Penrose (2006, 2008, 2009).
18. Kronheimer and Penrose (1967).
19. MacLennan (1956).
20. Penrose (1965), Penrose and Rindler (1984).
21. Rugh and Zinkernagel (2007).
22. Tod (2003).
23. See, for example, notes 14, 15, and 16.
24. Penrose (1965), Penrose and Rindler (1986).
25. Friedrich (1998).
26. Penrose (1965), Penrose and Rindler (1984, 1986).
27. Belinskii, Lifshitz, and Khalatnikov (1970, 1972).
28. Hawking (1975, 1976).
29. Braunstein and Pati (2007).

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