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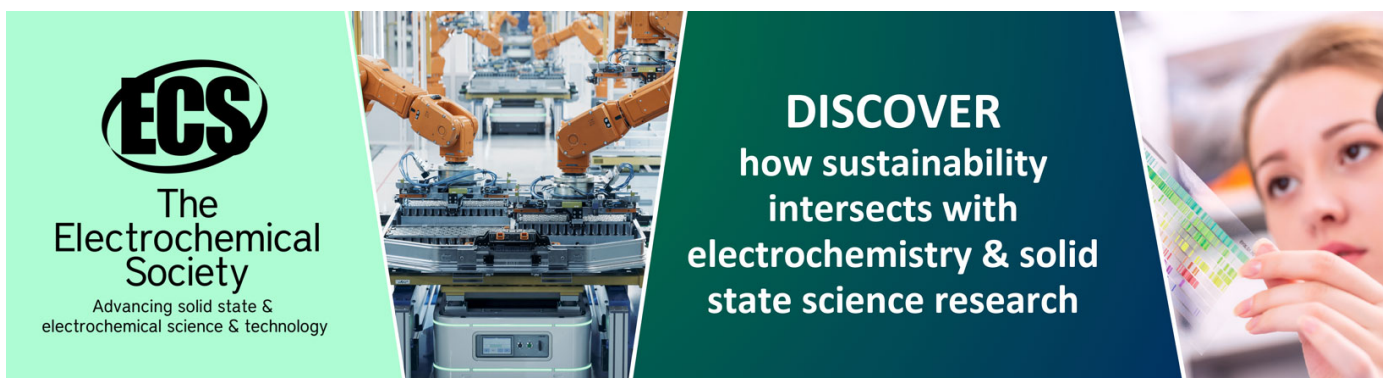
Measuring Resistance with an Uncalibrated Voltmeter

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Measuring Resistance with an Uncalibrated Voltmeter

Andrej Cvetkovski

Associate Professor, Mother Teresa University, Skopje, North Macedonia

e-mail: acvetk@gmail.com

Abstract. In this paper we examine the possibilities of measuring electrical resistance with an uncalibrated voltmeter. A digital voltmeter typically has high resolution and linearity regardless of how well calibrated it is. Based on this observation, we propose a two-step measurement method for performing resistance measurements by using a digital voltmeter. We show that the precision of the measurement so performed does not depend on the calibratedness of the used digital instrument. For the proposed scheme, we also provide guidelines on minimizing the measurement error while taking into account the some of the well-known properties of digital voltmeters.

1. Introduction

The Wheatstone bridge (Fig. 1) is a resistance measurement circuit that dates back as early as 1833 [1, 2]. The unknown resistance to be measured R_x is to be connected to form a bridge with three other previously known resistances, one of which is continuously variable (R_2) and the remaining two are fixed (R_1 , R_3). The input of the bridge is connected to a power source, whereas the output is abridged with a galvanometer. The measurement consists of balancing the bridge by varying R_2 until the galvanometer shows no current across, at which point $R_x/R_3 = R_2/R_1$, whence one simply obtains

$$R_x = R_2 \cdot R_3 / R_1. \quad (1)$$

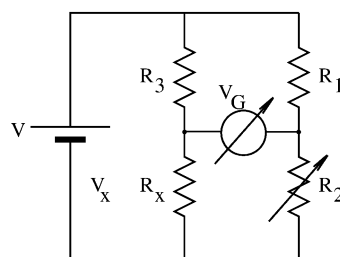


Figure 1. The Wheatstone bridge.

The obvious advantages of the Wheatstone bridge are that (i) the voltage of the source, as well as the resistance of the galvanometer need not be known precisely, nor do these values affect the measurement results or their repeatability; (ii) the galvanometer need be neither highly linear over a range, nor previously calibrated against a higher standard; the only requirements posed on it are having sufficient sensitivity relative to the source voltage V (which in turn can be raised arbitrarily),

and an ability to show deviations in both directions from an easily discernible zero state. The only prior knowledge needed to measure R_x , as in Eq. (1), comprises then the values of R_1 , R_2 and R_3 . And since (i) balancing of such a bridge can be done with very high precision even with a galvanometer of very simple construction, and (ii) the known resistors are expected to maintain relatively constant resistance over long periods of time, it turns out that, in this way, measurement of resistance can be completed with great precision using rudimentary and uncalibrated equipment, and the measurement precision is bounded, in essence, only by the precision of the prior knowledge of three resistances.

The essential design principle behind measurement circuits providing advantages such as those described above, is then: the measured quantity should depend solely, or to the extent possible, on the values of simple passive components as opposed to more complex mechanical, electrochemical, or electronic devices, whose predictability, linearity, consistency and permanence of characteristics, are generally lesser and harder to verify and maintain, compared to those of the passive components.

In the Wheatstone bridge, one ought to notice however, that by design, the complexity of the device as a whole is relocated from the construction of the galvanometer to the construction of the variable resistor R_2 . The precision of the measurement is then largely governed by the ability to read off precisely the value of R_2 from a mechanical scale after the bridge had been balanced. This, in turn, requires either complex mechanical construction of a potentiometer, or else will likely be the limiting factor in the overall precision of the procedure. Still, it seems that better linearity can be achieved with simple variable resistors than with galvanometers or mechanical voltmeters, and this observation itself is the justification of the usability of the measurement bridges even nowadays.

The advent of digital voltmeters however, changes the constellation significantly. Two notable properties of digital readout instruments compared to their analog counterparts are that (i) the parallax and reading errors typical for analog scales are eliminated and (ii) the resolution and the linearity of the readout over the measurement range are greatly increased [3]. To what extent can these two distinguishing properties of digital voltmeters be utilized to arrive at a resistance measurement procedure whose precision does not depend on how well the voltmeter is calibrated, is the subject of the rest of this consideration.

Results: We first lay down the circuit-theoretical elements of performing two-step measurements of electrical resistance by taking DC voltage measurements (Section 2). Based on an assumption that a digital voltmeter has linear characteristic regardless of how well calibrated it is, we show that the precision of this two-step measurement scheme does not depend on the calibratedness of the used instrument. Section 3 presents experimental results on four low-end digital voltmeters in support of the practical applicability of the proposed scheme. Section 4 further discusses the accuracy properties of digital voltmeters and their applicability to measurement of resistance using the principles described in Section 2. For our proposed measurement scheme, we also provide guidelines on minimizing the measurement error while taking into account some of the properties of digital voltmeters.

A brief survey of the contemporary literature in electrical measurements [4-10] shows that the two-step measurement scheme proposed herein, albeit elementary, is not explicitly represented in the classical corpus of elementary measurement methods, in spite of its applicability. Therefore, we believe that the described method represents an interesting addition to the resistance measurement toolkit, and a contemporary counterpart of the Wheatstone bridge, reflecting the present-day shift from analog to digital readout measurement instruments.

2. Measuring resistance with a voltmeter

Consider the simple-minded circuit shown in Fig. 2 (left). Denoting the true values of V and V_x as V_r and V_{xr} respectively, we have

$$R_x = R_0 \frac{V_{xr}}{V_r - V_{xr}}. \quad (2)$$

Hence, knowing the values of R_0 , and the true values V_r and V_{xr} , one could calculate R_x .

Let us assume now that we have at our disposal a voltmeter whose readout includes a constant multiplicative and additive error. Then the measured value V of a true value V_r using this voltmeter, can be modeled as

$$V = k_0 V_r + V_{k0} \quad (3)$$

whence $V_r = V / k_0 - V_{k0} / k_0 = kV + V_k$, with k_0 , k , V_{k0} , and V_k constant. Then Eq. (2) becomes

$$R_x = R_0 \frac{kV_x + V_k}{kV - kV_x}, \quad (4)$$

where V and V_x are the measured values by our voltmeter. By varying the source voltage V_r , for two different values V_{ar} , V_{br} , we can measure the values of V_r as V_a , V_b , and the corresponding values of V_{xr} as V_{xa} , V_{xb} . From Eq. (4) we obtain a system of equations in the unknowns R_x , k and V_k . Solving by R_x , we have

$$\begin{aligned} R_x &= R_0 \frac{V_{xa} - V_{xb}}{V_a - V_b - (V_{xa} - V_{xb})} \\ &= R_0 \frac{\Delta V_x}{\Delta V - \Delta V_x}, \end{aligned}$$

which could have also been obtained by inspection from (2) due to circuit linearity. Finally,

$$R_x = R_0 / \left(\frac{\Delta V}{\Delta V_x} - 1 \right). \quad (5)$$

Eq. (5) establishes that, as long as the model of Eq. (3) applies, the differential scheme of taking measurements at two points eliminates both the multiplicative and the additive errors of the voltmeter.

The above analysis assumed infinite resistance of the voltmeter R_v . In case it is to be taken into account, assuming that the power source is stabilized, the measured source voltages are unaffected, and the obtained value R'_x from Eq. (5) is the parallel connection of R_v and R_x (Fig. 2 right). Therefrom, the actual value of R_x can be obtained as

$$R_x = \frac{R_v R'_x}{R_v - R'_x}.$$

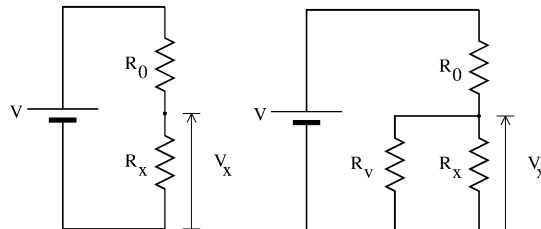


Figure 2. The measurement circuit

Compared to the Wheatstone bridge approach reviewed in Section 1, the advantages of the described differential measurement procedure on the circuit of Fig. 2 are that (i) it requires the prior

knowledge of only one resistor, (ii) there is no need of balancing, which may be time consuming, (iii) there is no need of variable resistors, nor reading values from a mechanical scale.

3. Experimental evaluation

In our experimental considerations we investigated the errors and the linearity of four cheap $3\frac{1}{2}$ -digit digital voltmeters acquired from the electronics hobbyist market. We compared the voltage readouts of each of those voltmeters in the same range ($0\div 1.999\text{V}$) against a mid-range, well documented voltmeter having accuracy of $\pm(0.15\%+2)$ and a resolution of 0.001V . The absolute errors with respect to our baseline etalon are depicted in Fig. 3.

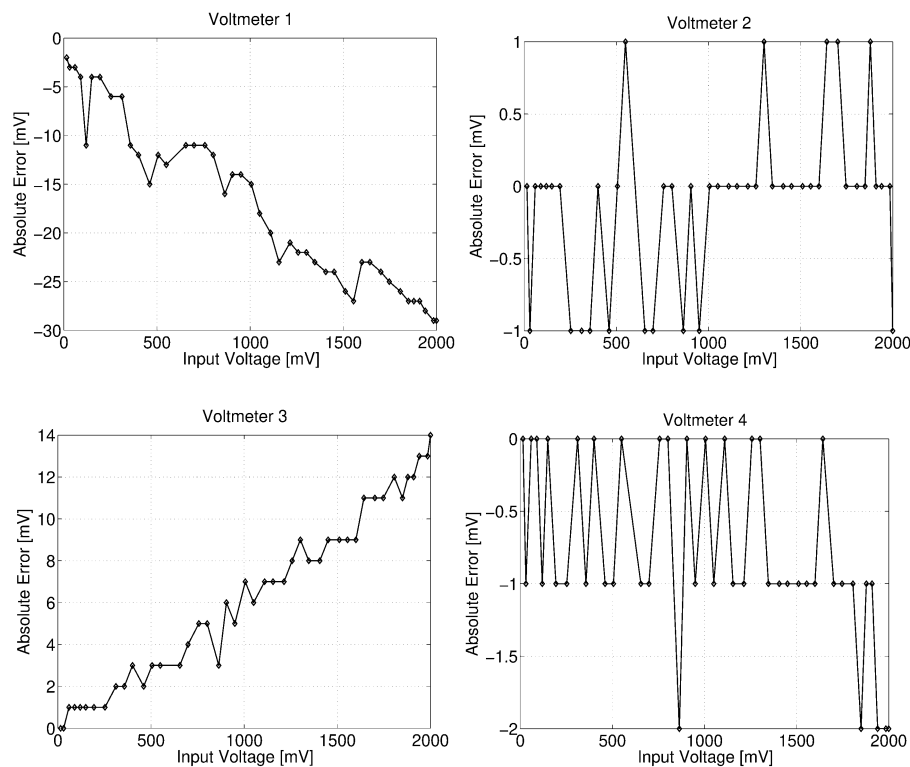


Figure 3. Absolute errors of four low-end voltmeters in the range $0\div 1.999\text{V}$

As in Fig. 3, all tested voltmeters showed very good adherence to the multiplicative and additive error model (3). Two of the voltmeters (V1 and V3) had measurable problem with the additive and multiplicative error. The other two voltmeters (V2 and V4) were surprisingly precise and exhibited only one or two counts of least-significant-digit error. From this quick estimate of our low-end voltmeters, we conclude that our assumption of linearity is satisfactory over the entire range and that our proposed differential measurement scheme is expected to provide good results in practice.

4. Discussion

Commercially manufactured voltmeters come with a specification of their accuracy. Analog voltmeters normally have the additional restriction that the published accuracy is applicable only if the reading is in the upper $\frac{2}{3}$ of the full scale.

Digital voltmeters specify their accuracy as the worst-case percent of the reading plus the count of erroneous least-significant digits. On the example of a $3\frac{1}{2}$ -digit instrument, in the 2V range, a complete accuracy specification of $\pm(0.15\% \text{ of readout} + 2)$ would mean $\pm(0.15\% \text{ of readout} + 2\text{mV})$ whereas in the 20V range, the same specification would mean $\pm(0.15\% \text{ of readout} + 20\text{mV})$. Therefore,

one should choose the smallest applicable range for a particular measurement in order to minimize the effect of the LSD errors.

In connection with the measurement scheme proposed herein, one should additionally be careful to perform both measurements needed for the differential calculation using the same range, which, in turn is not hard to achieve given that the choice of the two voltage points V_a and V_b is arbitrary.

Finally, as in the case of analog voltmeters, the upper portion of the chosen range is preferred to minimize the estimated error of the measurement.

5. Conclusion

The inherent linearity of resistive circuits enables the application of *the static differential approach* in measurements of resistance which, as we have clarified in this paper, eliminates both the multiplicative and the additive errors of digital voltmeters. Consequently, we have shown that it is possible to arrive at quite accurate resistance measurements using readily available, low-end, uncalibrated digital voltmeters. The accuracy of such measurements is supported by the fact that a digital voltmeter inherently has high resolution, good linearity, and eliminates the reading parallax error regardless of how well calibrated it is. We have also provided guidelines on minimizing the measurement error in differential measurements while taking into account the some of the properties of the digital voltmeters.

The differential approach presented shall be easily extendable to a number of measurement setups for measuring not only resistance, but other parameters of passive linear circuit elements as well. The application of differential measurement techniques largely eliminates the need of procurement of expensive specialized instrumentation together with the need for its periodic calibration.

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