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Entrainment matrix for BSk energy-density functionals

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Abstract. The entrainment matrix is an important parameter of superfluid hydrodynamics and is extensively used to model, e.g., neutron-star oscillations and glitches. Here we present a detailed step-by-step algorithm to calculate this matrix for a series of modern BSk energy-density functionals. Using it, both the entrainment matrix and equation of state can be determined self-consistently, within the same microscopic approach.

1. Introduction

After its birth, neutron star (NS) rapidly cools down. At stellar temperatures $T \lesssim 10^8 - 10^{10}$ K baryons in NS interiors undergo transition to superfluid/superconducting state; after that NS should be described as a superfluid mixture. To model dynamics of such mixture one needs to calculate the so-called entrainment matrix Y_{ik} (indices i, k run over superfluid baryon species) [1, 2, 3, 4, 5], which is a generalization of the superfluid density concept (e.g., [6]) to relativistic mixtures. Since the number of baryons in the condensate depends on temperature, the entrainment-matrix is also temperature-dependent.

Here we present a step-by-step algorithm to calculate the elements of the matrix Y_{ik} for the NS core composed of neutrons, protons, electrons, and muons, assuming one of the modern BSk energy-density functionals [7, 8]. For illustration, we then apply this algorithm and calculate Y_{ik} for BSk24 energy-density functional.

2. Description of the algorithm

To calculate the relativistic entrainment matrix Y_{ik} (i, k run over superfluid neutrons, n, and superconducting protons, p), we propose to use general formulas derived in [3, 4, 5]. Namely, Y_{ik} is given by the following equation

$$Y_{ik} = n_i \gamma_{ik} \left(1 - \Phi_i\right). \tag{1}$$

Here n_i is the number density of particle species i; Φ_i is the temperature-dependent function to be specified below; and the elements of the matrix γ_{ik} equal

$$\gamma_{ii} = \frac{(n_i + G_{ii}\, m_i^*)\,(n_k + G_{kk}\, m_k^*\, \Phi_k) - G_{ik}^2\, m_i^*\, m_k^*\, \Phi_k}{m_i^* c^2\, S}, \tag{2}$$

$$\gamma_{ik} = \frac{G_{ik}\, n_k\,(1 - \Phi_k)}{c^2\, S}, \tag{3}$$

where

$$S = (n_i + G_{ii}\, m_i^*\, \Phi_i)\,(n_k + G_{kk}\, m_k^*\, \Phi_k) - G_{ik}^2\, m_i^*\, m_k^*\, \Phi_i\, \Phi_k. \tag{4}$$

In Eqs. (2)–(4) indices i and k refer to different particle species, $i \neq k$; m_i^* is the effective mass of particle species i, defined by

$$\frac{\mu_i}{m_i^*\, c^2} = 1 - \sum_k \frac{\mu_k G_{ik}}{n_i\, c^2}, \tag{5}$$

where μ_i is the chemical potential of particle species i and c is the speed of light. Further,

$$G_{ik} \equiv \frac{1}{9\pi^4\hbar^6}\, p_{\mathrm{F}i}^2 p_{\mathrm{F}k}^2\, f_1^{ik}, \tag{6}$$

where $p_{\mathrm{F}i} = (3\pi^2\hbar^3 n_i)^{1/3}$ is the Fermi momentum for particle species i, $\hbar$ is the Planck constant, f_1^{ik} are the Landau parameters. The latter can be calculated as it is described in [9], see their formulas (33)–(35):

$$f_1^{nn} = 2(C_0^j + C_1^j)\frac{p_{\mathrm{F}n}^2}{\hbar^2}, \tag{7}$$

$$f_1^{pp} = 2(C_0^j + C_1^j)\frac{p_{\mathrm{F}p}^2}{\hbar^2}, \tag{8}$$

$$f_1^{np} = 2(C_0^j - C_1^j)\frac{p_{\mathrm{F}n}p_{\mathrm{F}p}}{\hbar^2}, \tag{9}$$

In [9] the parameters C_0^j and C_1^j are presented for SLy4 energy-density functional (see the formulas A1, A4, and A5 in that reference). For a set of BSk energy-density functionals the parameters C_0^j and C_1^j depend on the baryon number density n_b, and equal (see equations A30e, A30f of [10], and equation A1 of [9])

$$C_0^j = -\frac{3}{16}t_1 - \frac{1}{4}t_2\left(\frac{5}{4} + x_2\right) - \frac{3}{16}t_4 n_b^\beta - \frac{1}{4}t_5\left(\frac{5}{4} + x_5\right) n_b^\gamma, \tag{10}$$

$$C_1^j = \frac{1}{8}t_1\left(\frac{1}{2} + x_1\right) - \frac{1}{8}t_2\left(\frac{1}{2} + x_2\right) + \frac{1}{8}t_4\left(\frac{1}{2} + x_4\right) n_b^\beta - \frac{1}{8}t_5\left(\frac{1}{2} + x_5\right) n_b^\gamma, \tag{11}$$

where the constants $t_1, t_2, t_4, t_5, x_1, x_2, x_4, x_5, \beta, \gamma$ are presented in Table II of [7] for BSk21, BSk22, BSk23, BSk24, BSk25, BSk26.

The thermodynamic functions n_i and μ_i, entering the above equations, can be determined directly for a particular energy-density functional. However, it can be easier to calculate them using the fits representing these quantities as analytic functions of n_b (in Ref. [8] the corresponding fits are presented for BSk22, BSk24, BSk25, BSk26, see sections C3.2 and C4.2 of that reference; [11] gives the fits for BSk19, BSk20, BSk21, but for n_i only, see section 4.1 of that reference).

The temperature-dependent function Φ_i was calculated and fitted (under assumption that the superfluid energy gap does not depend on the particle momentum) in [12] (see also [13]). The corresponding fit reads

$$\Phi_i = \left[0.9443 + \sqrt{0.0557^2 + (0.1886\, v_i)^2}\right]^{1/2} \exp\left(1.753 - \sqrt{1.753^2 + v_i^2}\right), \tag{12}$$

where v_i is the superfluid temperature-dependent energy gap (in the case of triplet-state pairing it is the effective energy gap) for particle species i, normalized to temperature, T. For the singlet-state pairing, relevant to protons, it can be fitted as [14, 13]

$$v_i = \sqrt{1 - \tau_i}\left(0.7893 + \frac{1.188}{\tau_i}\right). \tag{13}$$

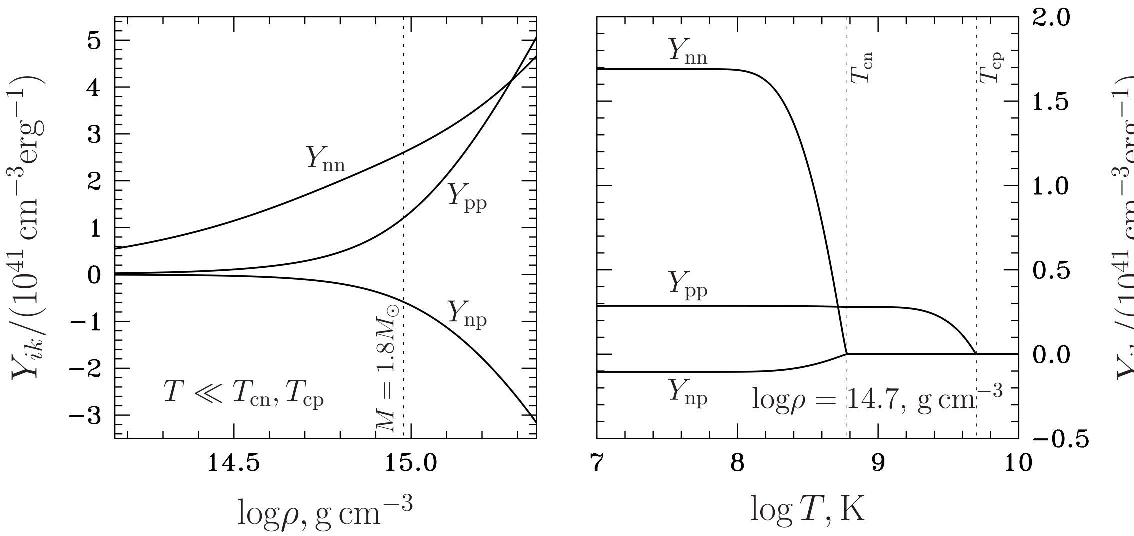

Figure 1. Entrainment matrix elements Y_{ik} for the BSk24 functional versus density ρ in the zero-temperature limit, $T \ll T_{cn}, T_{cp}$ (left panel) and versus temperature (right panel). Vertical dots in the left panel correspond to the central density of an NS with $M = 1.8M_\odot$. The right panel is plotted for $\log\rho = 14.7\,\mathrm{g\,cm^{-3}}$, and for neutron and proton critical temperatures $T_{\rm cn} = 6\times10^8$ K and $T_{\rm cp} = 5\times10^9$ K, respectively (shown by vertical dots).

In turn, for neutrons in the core, which are believed to be paired in the triplet state, the corresponding fit takes the form [15, 13]

$$v_i = \sqrt{1-\tau_i}\left(1.456 - \frac{0.157}{\sqrt{\tau_i}} + \frac{1.764}{\tau_i}\right), \tag{14}$$

where $\tau_i = T/T_{ci}$ and T_{ci} is the critical temperature for particle species $i = n$, p.

To determine Y_{ik} one also needs to specify the critical temperatures for neutrons and protons. While BSk energy-density functionals allow to calculate various thermodynamic quantities, they do not provide the values of T_{cn} and T_{cp}. Thus, T_{cn} and T_{cp} should be treated as external input parameters. (Note that there are many microscopic superfluidity models on the market, which predict very different baryon critical temperatures in the NS cores.) Once T_{ci} are specified, Y_{ik} can be easily calculated using the above formulas.

Fig. 1 illustrates the behavior of the entrainment matrix elements showing them as functions of density ρ (left panel) and temperature (right panel) for BSk24 energy-density functional. The left panel is plotted in the limit of vanishing temperature, $T \ll T_{cn}, T_{cp}$. Density in the left panel ranges from the core-crust interface to the central density for the limiting configuration of an NS with the maximum mass. For the reader's convenience, by vertical dots we show also the central density for an NS with the mass $M = 1.8M_\odot$ ($M_\odot$ is the solar mass). The right panel is plotted for $\log\rho = 14.7\,\mathrm{g\,cm^{-3}}$. Vertical dots correspond to the critical temperatures for (from left to right) neutrons and protons.

To summarize, in this note we have presented a step-by-step algorithm to calculate the entrainment matrix Y_{ik} for a set of BSk energy-density functionals. Using it, both Y_{ik} and equation of state can be determined self-consistently, within the same microscopic approach, which is important for realistic modeling of dynamical processes in superfluid NSs.

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