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# Andreev Reflection and the Kondo Effect in Side-Coupled Double Quantum Dots

Yoichi Tanaka<sup>1</sup>, Norio Kawakami<sup>2</sup> and Akira Oguri<sup>3</sup>

<sup>1</sup> Condensed Matter Theory Laboratory, RIKEN, Saitama 351-0198, Japan

<sup>2</sup> Department of Physics, Kyoto University, Kyoto 606-8502, Japan

<sup>3</sup> Department of Material Science, Osaka City University, Osaka 558-8585, Japan

E-mail: tanaka-y@riken.jp

**Abstract.** We study the transport due to the Andreev reflection through side-coupled double quantum dots, using the numerical renormalization group (NRG). We find that superconducting interference via the side dot is suppressed as increasing the Coulomb interaction, and the conductance in the Kondo regime is enhanced. This behavior stands in stark contrast to the normal conductance, in which a wide valley is seen in the Kondo regime. It is further elucidated that two separate Fano structures appear in the gate-voltage dependence of the Andreev transport, where a single plateau appears in the normal transport.

## 1. Introduction

Recent remarkable progress in nanofabrication has stimulated extensive investigations on transport properties of a mesoscopic system with hybrid normal(metal)-superconductor junction, in which the Andreev reflection plays a key role in controlling characteristic transport properties. Furthermore, the interplay between the Andreev scattering and the Kondo effect has been studied intensively for a quantum dot (QD) coupled to normal (N) and superconducting (SC) leads, theoretically [1–8] and experimentally [9]. So far, however, the Andreev-Kondo physics has been discussed mainly for a single dot.

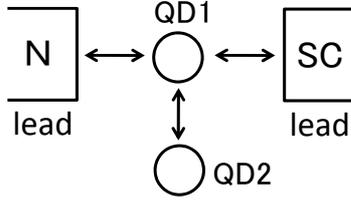
In this paper, we study the Andreev transport through a double quantum dot (DQD) with a T-shape geometry (see Fig. 1), which is a typical system sensitive to an interference effect due to multiple paths. We calculate the conductance and discuss the interplay of superconductivity, the Kondo effect and the interference effect.

## 2. Model

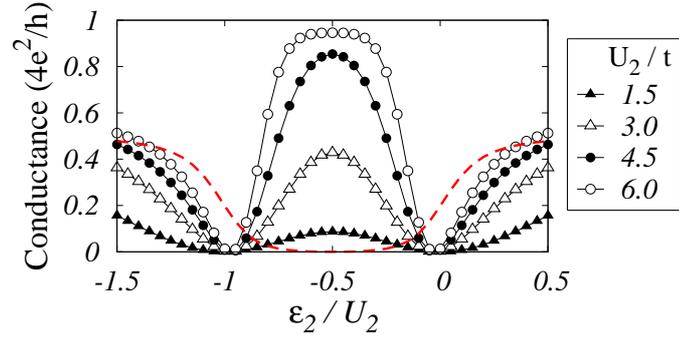
The Hamiltonian of the side-coupled DQD connected to SC and normal leads is given by

$$H = H_{DQD} + H_S + H_N + H_{T,S} + H_{T,N}. \quad (1)$$

$H_{DQD} = \sum_{i=1,2} (\sum_{\sigma} \varepsilon_i n_{i\sigma} + U_i n_{i\uparrow} n_{i\downarrow}) + t \sum_{\sigma} (d_{1\sigma}^{\dagger} d_{2\sigma} + \text{H.c.})$  is the Hamiltonian of the QD1 ( $i = 1$ ) and QD2 ( $i = 2$ ) in Fig. 1. Here  $\varepsilon_i$  is the energy level,  $U_i$  the Coulomb interaction,  $n_{i\sigma} = d_{i\sigma}^{\dagger} d_{i\sigma}$ , and  $t$  the inter-dot hopping matrix element.  $H_S = \sum_{k,\sigma} \varepsilon_k c_{S,k\sigma}^{\dagger} c_{S,k\sigma} - \sum_k (\Delta c_{S,k\uparrow}^{\dagger} c_{S,-k\downarrow}^{\dagger} + \text{H.c.})$  and  $H_N = \sum_{k,\sigma} \varepsilon_k c_{N,k\sigma}^{\dagger} c_{N,k\sigma}$  describe the SC and normal leads,



**Figure 1.** Double dots coupled to normal (N) and superconducting (SC) leads with T-shape configuration.



**Figure 2.** Conductance vs  $\varepsilon_2/U_2$  for several side-dot Coulomb interactions  $U_2$ , where we set  $\Gamma_S = 1.0t$ ,  $\Gamma_N = 0.5t$  and  $\varepsilon_1 = U_1 = 0$ . The dashed line is the normal conductance without the SC gap ( $\Delta = 0$ ) for  $U_2 = 6t$ .

respectively.  $H_{T,\nu} = \sum_{k,\sigma} V_\nu (c_{\nu,k\sigma}^\dagger d_{1\sigma} + \text{H.c.})$  denotes the tunneling part between the QD1 and the leads. Since we are interested in the transport due to the Andreev reflection, which occurs inside the SC gap, we assume that the amplitude of the SC gap  $|\Delta|$  is sufficiently large ( $|\Delta| \rightarrow \infty$ ). In this case, the role of the SC lead is replaced simply with an extra onsite SC gap in the QD1,  $H_S^{\text{eff}} = -\Delta_1 (d_{1\uparrow}^\dagger d_{1\downarrow}^\dagger + \text{H.c.})$  [8]. Here,  $\Delta_1 \equiv \Gamma_S$ , and  $\Gamma_{S(N)} (= \pi V_{S(N)}^2 \sum_k \delta(\varepsilon - \varepsilon_k))$  is the resonance strength between the QD1 and the SC (normal) lead. We calculate the linear conductance at zero temperature, using the combination of the Kubo formula and the numerical renormalization group (NRG) method.

### 3. Results

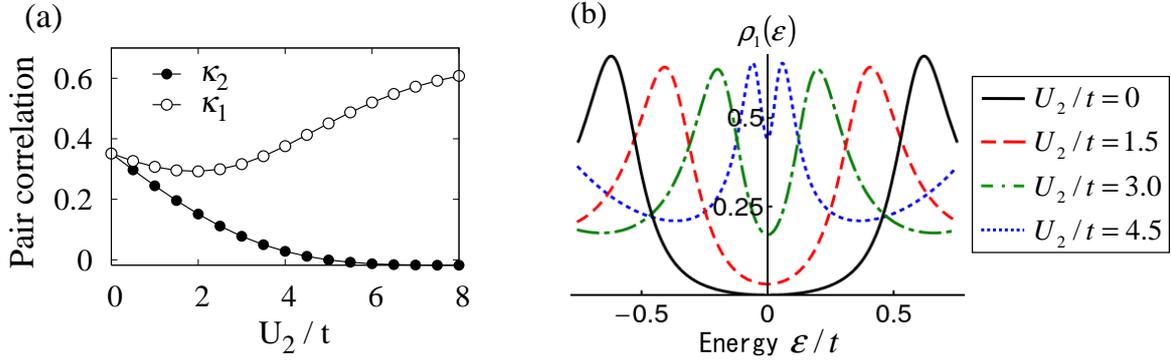
In this section, we show the numerical results and discuss the transport properties. For simplicity, we assume that  $U_1 = 0$  and concentrate on the correlation effect due to  $U_2$ .

#### 3.1. $\varepsilon_2$ -dependence of the conductance

Figure 2 shows the conductance as a function of the energy level  $\varepsilon_2$  of the side dot QD2 for several values of  $U_2$ , where the energy level of the QD1 is chosen to be  $\varepsilon_1 = 0$ . In the Kondo regime  $-U_2 < \varepsilon_2 < 0$ , the conductance is enhanced with increasing  $U_2$ . This behavior is quite different from that seen in the conductance through the side-coupled DQD connected to two normal leads [10–14]. In the latter case, the normal conductance in the Kondo regime is suppressed, and shows a wide minimum called a Kondo valley (see the dashed line in Fig. 2), because of the interference effect by the Kondo resonance in the side dot. In the following, we elucidate the reason why the conductance in the Kondo regime is enhanced in our system, focusing on how the Coulomb interaction  $U_2$  affects the Andreev transport.

Figure 3(a) shows the pair correlation  $\kappa_i \equiv \langle d_{i\uparrow}^\dagger d_{i\downarrow}^\dagger + d_{i\downarrow} d_{i\uparrow} \rangle$  at the QD- $i$  for  $\varepsilon_2/U_2 = -0.5$ , at which the correlation effect due to  $U_2$  appears most strongly. Note that the pair correlation  $\kappa_i$  gives a measure of SC proximity into the dots. As shown in Fig. 3(a),  $\kappa_2$  decreases monotonically and approaches zero with increasing  $U_2$ , while  $\kappa_1$  increases. The decrease of  $\kappa_2$  indicates the suppression of the SC proximity into the QD2, and the interference effect caused by the Andreev tunneling via the QD2 is also suppressed for large  $U_2$  ( $\gtrsim 4t$ ). This behavior is different from the normal transport, for which the interference effect still occurs even in the presence of  $U_2$ . Therefore, we see that the conductance in the Kondo regime is enhanced by the suppression of the destructive interference.

We can also see the suppression of the interference effect from the density of states (DOS)



**Figure 3.** (a) Pair correlation  $\kappa_i \equiv \langle d_{i\uparrow}^\dagger d_{i\downarrow}^\dagger + d_{i\downarrow} d_{i\uparrow} \rangle$  at the QD- $i$  and (b) DOS of the QD1,  $\rho_1(\epsilon)$ , around the Fermi energy ( $\epsilon = 0$ ). We set  $\epsilon_2/U_2 = -0.5$  and the other parameters are the same as Fig. 2.

in the dots. For  $\epsilon_2/U_2 = -0.5$ , the DOS of the QD1,  $\rho_1(\epsilon)$ , around the Fermi energy can be expressed in the form

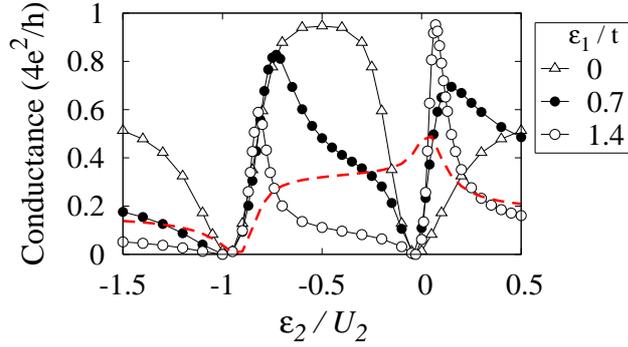
$$\rho_1(\epsilon) \simeq \frac{1}{\pi} \sum_{\alpha=\pm 1} \left[ \frac{\Gamma_N(\epsilon - \alpha\tilde{\Delta}_2)^2}{\{(\epsilon - \alpha\Gamma_S)(\epsilon - \alpha\tilde{\Delta}_2) - \tilde{t}^2\}^2 + \{\Gamma_N(\epsilon - \alpha\tilde{\Delta}_2)\}^2} \right]. \quad (2)$$

Here,  $\tilde{t}$  corresponds to a renormalized inter-dot hopping matrix element, and  $\tilde{\Delta}_2$  is an effective local SC gap induced in the QD2 by the Coulomb interaction  $U_2$ . These parameters can be deduced from the fixed-point of NRG [15]. Figure 3(b) shows the DOS  $\rho_1(\epsilon)$  around the Fermi energy. For the noninteracting case ( $U_2 = 0$ ), the DOS has a valley around the Fermi energy ( $\epsilon = 0$ ), and the value of the DOS at  $\epsilon = 0$  becomes zero. This is caused by the interference effect, and is also seen in the case of the two normal leads [10–14]. However, with the increase of  $U_2$ , the DOS around  $\epsilon = 0$  is gradually enhanced. This enhancement indicates that the interference effect is suppressed by the Coulomb interaction  $U_2$ , which also explains the enhancement of the conductance in the Kondo regime in Fig. 2.

We here comment on the Kondo effect for large  $U_2 (\gtrsim 4t)$  in our system. As discussed above, the penetration of the SC correlation into the QD2 is suppressed by the Coulomb interaction  $U_2$ , which induces the local spin moment at the QD2. In this case, conduction electrons in the normal lead screen this moment to form the Kondo singlet state. However, the electrons of the interfacial dot QD1 can not contribute to the Kondo screening, because the SC correlation at the QD1 tends to stabilize the local SC singlet state which consists of a linear combination of the empty and doubly occupied states. Thus, the conduction electrons of the normal lead tunnel into the QD2 virtually via the QD1 to screen the local moment. This process is analogous to a superexchange mechanism, which makes the singlet bond of the Kondo screening long compared to the case of two normal leads.

### 3.2. Fano structure for $\epsilon_1 \neq 0$

Next we discuss the change of the conductance due to the shift of the energy level  $\epsilon_1$ . Figure 4 shows the  $\epsilon_2$ -dependence of the conductance for some different values of  $\epsilon_1$ . We see for  $\epsilon_1 \neq 0$  that two asymmetric Fano structures, each of which consists of a pair of peak and dip, emerge at  $\epsilon_2 \simeq 0$  and  $\epsilon_2 \simeq -U_2$ , as the Fermi level crosses the energy corresponding to the upper and lower levels of the atomic limit. As discussed above, the Coulomb interaction  $U_2$  suppresses the interference effect due to the Andreev tunneling in the Kondo regime around  $\epsilon_2/U_2 = -0.5$ . Away from the symmetric point  $\epsilon_2/U_2 = -0.5$ , however, the correlation effect



**Figure 4.** Conductance vs  $\varepsilon_2/U_2$  for several values of  $\varepsilon_1$ , where  $U_1 = 0$  and  $U_2 = 6t$ . The dashed line is the normal conductance without the SC gap ( $\Delta = 0$ ) for  $\varepsilon_1 = 2.1t$ . The other parameters are the same as Fig. 2.

due to  $U_2$  becomes weak, and the interference effect begins to appear. Then, two Fano structures individually emerge around  $\varepsilon_2 \simeq 0$  and  $\varepsilon_2 \simeq -U_2$ , as shown in Fig. 4. In contrast, Maruyama et al. [14] showed that at low temperatures the normal conductance has only one Fano structure with a pair of the peak and dip, which are separated widely by a Fano-Kondo plateau [16] at  $-U_2 \lesssim \varepsilon_2 \lesssim 0$ , as shown in the dashed line in Fig. 4. Note that this separation is caused by the interference effect due to the Kondo scattering for large  $U_2$ . Therefore, the Andreev and normal conductance have the clearly distinct Fano structures, depending on whether the interference effect in the Kondo regime is suppressed by the Coulomb interaction in the side dot.

#### 4. Summary

We have studied the Andreev transport through the side-coupled DQD system using the NRG method. We have clarified that the Coulomb interaction in the side dot suppresses the destructive interference effect typical of the T-shape geometry, which in turn enhances the conductance. Furthermore, it has been elucidated that the two Fano structures appear in the gate-voltage dependence of the Andreev conductance, which is quite different from the normal conductance exhibiting a Fano-Kondo structure with a single plateau.

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