Numerical analysis of binary fluid convection in extended systems

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Numerical analysis of binary fluid convection in extended systems

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Abstract.
Several aspects of the dynamics of convecting binary mixtures in large aspect ratio containers are analysed using high resolution numerical tools based on spectral methods. For two binary mixtures with separation ratio $S = -0.127$ and $S = -0.021$, we compute the Eckhaus stability boundaries of the spatially uniform travelling wave states arising in the primary subcritical bifurcation. The dynamics triggered by Eckhaus instability is discussed for the $S = -0.127$ mixture. Additionally, for the $S = -0.021$ mixture, we obtain the complex small-amplitude states observed in experiments for mixtures with a separation ratio very close to zero for the first time with DNS of the full convection equations.

1. Introduction
Thermal convection in a binary fluid layer heated from below is a system that exhibits a great variety of pattern forming phenomena when driven away from equilibrium [1]. Compared to convection in a pure fluid, the spatio-temporal properties of the flow patterns are more complex, due to the extra degree of freedom associated with the concentration field. In binary mixtures the temperature and concentration fields are coupled through the Soret effect. The separation ratio of the mixture, $S$, is the nondimensional parameter that describes the extent to which buoyancy is modified by the Soret effect. Of special interest is the dynamics arising in $S < 0$ binary mixtures. For such mixtures, the onset of convection is via a subcritical Hopf bifurcation that gives rise to oscillatory patterns. The final selected nonlinear state depends on the parameters of the mixture and can take the form of large-amplitude uniform travelling waves (TW), steady convection rolls (usually called stationary overturning convection, SOC) or several types of localized states and highly irregular states, which exhibit spatiotemporal chaotic motions.

In order to achieve translation invariant systems that can support uniform travelling waves, experiments are usually performed on long, narrow annular cells. For mixtures with negative values of the separation ratio, different behaviours have been observed depending on the actual strength of $S$. In the case of the experiment reported in [2] a $S = -0.257$ water-ethanol mixture is used. A final state consisting of large amplitude uniform travelling waves is observed when the threshold of stability of the conduction state is crossed. This TW branch is found to be stable until a saddle-node bifurcation is reached by decreasing the Rayleigh number. In contrast, for mixtures with a separation ratio value closer to zero, such as the $S = -0.021$ binary mixture used in experiment [3], uniform travelling waves are never observed, despite numerical
computations confirming their existence. These completely different behaviours may be attributed to the different stability properties of the travelling wave solutions when extended domains are considered, due to Eckhaus instabilities (instabilities that modify the periodicity of the basic solution). At least two experimental papers, using \( S = -0.26 \) and \( S = -0.127 \) mixtures, have been devoted to the determination of the Eckhaus stability boundaries of travelling waves in binary fluid convection [4, 5].

In addition, a series of experiments on mixtures with separation ratio of around \( S = -0.02 \) [3, 6] show that, near the onset of the primary instability, the weakly nonlinear oscillatory convection can be in the form of small-amplitude states and can exhibit dispersive chaos, which is a dynamical state characterised by an erratic growth and decay of the convection amplitude. These small-amplitude waves consist of the repetitive formation and sudden collapse of spatially localized pulses that lead to erratic dynamics with no stable saturated state. These states are never observed for mixtures with a larger value of \( |S| \).

The main difficulty in modelling this system numerically is the large size of the annular containers used in experiments. Most of the existing numerical works consider a single pair of rolls, so they model only the dynamics of uniform travelling waves and stationary states. This is the case of the extensive numerical work of Barten et al [7], which studies in detail the branches of travelling waves (TW) and steady states (SOC) for different values of the separation ratio. The system they consider is a two-dimensional cell such that only a single wavelength fits in the domain (the length of the roll is usually two times its height). In this way, the authors have obtained the bifurcation diagrams of the uniform TW and SOC states and have located the transition between them. However, this type of study cannot describe the non-uniform states and the dynamics arising from the Eckhaus instability.

The available numerical works considering the stability of extended patterns in binary miscible fluids, as far as the authors know, reduce to [8] and [9]. Huke et al [8] analyse the stability of steady rolls and square convection for positive separation ratio mixtures, including in their analysis the Eckhaus instability. In Buchel et al [9] transitions between different wavenumber travelling wave patterns are observed for negative separation ratios and the dynamics is explained as a manifestation of Eckhaus instabilities. Nevertheless, narrow rectangular containers are considered in their simulations, so the pattern selection dynamics is influenced by the presence of the lateral boundaries.

The purpose of our work is to analyse numerically binary convection in large aspect ratio two-dimensional periodic containers. To that aim, we have developed several accurate numerical tools based on spectral methods that enable us to compute the arising nonlinear patterns. We present results corresponding to different aspects of the dynamics of water-ethanol mixtures in periodic domains of aspect ratio around 80. In the first part of our study, we analyse the Eckhaus instability of the uniform travelling waves arising in the primary subcritical bifurcation for binary mixtures with two values of the separation ratio, \( S = -0.021 \) and \( S = -0.127 \), corresponding to the mixtures used in experiments [3, 4]. This analysis is an extension of the results presented in [10]. In the second part of the study, we obtain and analyse the complex low-amplitude convection observed in experiments [3, 6]. This rich dynamics is observed in binary mixtures with \( S > -0.04 \) in sufficiently long containers for Rayleigh numbers slightly above the threshold of convection. We present results for a \( S = -0.021 \) binary mixture.

2. The system
We consider Boussinesq binary-fluid convection in a narrow annular cell in the presence of a vertical gravitational field \( \mathbf{g} = -g \mathbf{e}_z \). A vertical temperature gradient is imposed by fixing a temperature difference \( \Delta T \) between the horizontal plates, with the temperature at the bottom being higher than at the top. We are interested in modelling experiments in cells with cross section width of the same order of the height \( d \) and mean circumference \( L \) much larger than
In such systems convection settles in the form of straight rolls with the axis in the radial direction, the dynamics being purely two-dimensional. Ignoring variations along the roll axes, we use a simplified geometry consisting of a two dimensional domain \((x, \hat{z}) \in [0, L] \times [0, d]\), with the aspect ratio \(\Gamma\) defined as \(\Gamma = L/d\) much greater than one. This system admits the following basic conductive state with constant gradients of temperature and concentration

\[
\begin{align*}
\mathbf{u}_c &= 0, \\
T_c &= T_0 - \Delta T (\hat{z} - \frac{1}{2}), \\
C_c &= C_0 + C_0 (1 - C_0) S_T \Delta T (\hat{z} - \frac{1}{2}),
\end{align*}
\]

where \(\mathbf{u} = (u, w)\) is the velocity field; \(T\) and \(C\) are the fields of temperature and concentration of the denser component, respectively; \(T_0\) and \(C_0\) are their mean values, and \(S_T\) is the Soret coefficient.

The dynamics of the system is governed by the continuity equation, the Navier-Stokes equations and the energy and mass conservation equations. In their nondimensional form, scaling length with the height of the layer \(d\), time with the vertical thermal diffusion time \(d^2/\kappa\), \(\kappa\) being the thermal diffusivity, and temperature with \(\Delta T\), the equations explicitly read

\[
\begin{align*}
\nabla \cdot \mathbf{u} &= 0, \\
\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} &= -\nabla p + \sigma \nabla^2 \mathbf{u} + R \sigma [(1 + S) \Theta + S \eta] \hat{e}_z, \\
\partial_t \Theta + (\mathbf{u} \cdot \nabla) \Theta &= w + \nabla^2 \Theta, \\
\partial_t \eta + (\mathbf{u} \cdot \nabla) \eta &= -\nabla^2 \Theta + \tau \nabla^2 \eta.
\end{align*}
\]

Here, \(\Theta\) denotes the departure of the temperature from its conduction profile, \(\Theta = (T - T_c)/\Delta T\), and \(\eta = -(C - C_c)/(C_0(1 - C_0)S_T \Delta T) - \Theta\). The dimensionless parameters in the above equations are the Rayleigh number \(R\), the Prandtl number \(\sigma\), the Lewis number \(\tau\) and the separation ratio \(S\), defined as

\[
R = \frac{\alpha \Delta T g d^3}{\kappa \nu}, \quad \sigma = \frac{\nu}{\kappa}, \quad \tau = \frac{D}{\kappa}, \quad S = \frac{C_0(1 - C_0)}{\beta} \cdot \frac{\beta}{\alpha} S_T,
\]

where \(\alpha\) and \(\beta\) are the thermal and concentration expansion coefficients, \(\nu\) is the kinematic viscosity and \(D\) is the mass diffusivity.

The boundary conditions are taken to be periodic in \(x\) with period \(\Gamma\). No-slip, fixed temperature and no mass flux at the top and bottom plates are considered

\[
\mathbf{u} = \Theta = \partial_z \eta = 0 \quad \text{on} \quad \hat{z} = 0, 1.
\]

As a measure of the heat transport by convection, we use the Nusselt number \(Nu\), defined as the ratio of heat flux through the top plate to that of the corresponding conductive solution. It has the following expression

\[
Nu = 1 - \Gamma^{-1} \int_{x=0}^{x=\Gamma} \partial_z \Theta(z = 1) \, dx.
\]

3. Numerical tools

3.1. Time evolution code

To integrate the equations in time, we have used the second order time-splitting algorithm proposed in [11], combined with a pseudo-spectral discretization in space (Galerkin-Fourier in \(x\) and Chebyshev-collocation in \(\hat{z}\)). The Helmholtz and the Poisson equations resulting from the time-splitting are solved by using a diagonalization technique [12]. The authors have successfully used this algorithm previously to study binary convection in large aspect ratio rectangular containers [13].
3.2. Computation of stationary solutions and travelling waves

Most of the calculations carried out in this paper are aimed at obtaining spatial periodic solutions (either SOC or TW states) that, fitting in the domain of periodicity $\Gamma$, contain many wavelengths $a$. To calculate these steady solutions in an efficient way we have adapted a first-order time-stepping formulation to carry out Newton’s method [14, 15]. In the preconditioned version of Newton’s iteration, the corresponding linear system is solved by an iterative technique using a GMRES package [16]. Travelling waves have been obtained in a similar way, by assuming time independent functions $g_n(z)$ in the following Fourier expansion of any variable

$$
\chi_{TW}(x, z, t) = \sum_{n=-N}^{N} g_n(z) e^{i n k(x - ct)},
$$

being $k = \frac{2\pi}{a}$ the basic wavenumber of the travelling wave. Letting $\tilde{x} = x - ct$, we solve a steady problem. The phase velocity $c$ is determined fixing the phase of the solution. Steady and travelling waves have also been calculated with a Newton-Raphson iterative scheme in a streamfunction formulation [17, 18, 19].

3.3. Stability of stationary solutions and travelling waves

To study the stability of the two-dimensional waves and stationary solutions in a periodic box that contains $M$ basic wavelengths $a$, we perform an Eckhaus stability analysis [17]. Since the basic solution has period $a$, the associated linear operator has the same periodicity and, according to Floquet theory, the set of perturbations splits as

$$
\{ \chi_m^*(\tilde{x}, z, t) = \chi_m(\tilde{x}, z)e^{id_m k \tilde{x}} e^{\lambda_m t} \}_{m=0\ldots M-1},
$$

where

$$
\chi_m(\tilde{x}, z) = \chi_m(\tilde{x} + a, z)
$$

with $d_m = m/M$ being the spatial Floquet parameter. Thus $\chi_m^*(\tilde{x}, z, t)$ admits the following expansion

$$
\chi_m^*(\tilde{x}, z, t) = \sum_{n=-N}^{N} g_n^*(z)e^{i(n + d_m)k \tilde{x}} e^{\lambda_m t}.
$$

The corresponding eigenvalue problem has to be solved for every value of $m$. If for any value of $m$, the real part of $\lambda_m$ is positive, the TW is unstable, otherwise it is stable. If $d = 0$, the perturbation has the same wavelength as the basic waves and the solution that bifurcates still contains $M$ wavelengths. In this stability analysis, we always obtain a zero eigenvalue corresponding to the trivial phase shift solution. In the case $d \neq 0$, subharmonic disturbances are considered. The basic periodicity $a$ is now broken and a new solution with a larger basic period emerges. As discussed in [17], the eigenfunctions for the problem with $d_{M-m}$, as well as the eigenvalues $\lambda_{M-m}$, can be obtained by conjugating those with $d_m$. Then it suffices to consider perturbations with $d_m \in (0, 1/2]$.

The linear stability analysis of the steady solutions (SOC) is carried out in a similar way. The details can be found in [18].

4. Results

4.1. Dynamics of uniform solutions with fixed spatial periodicity for $S < 0$ mixtures

For negative values of the separation ratio, the primary instability of the conduction state is oscillatory. Since the wavenumber $k$ of the dominant perturbation is nonzero, the translation invariance is broken and a pattern of wavelength $a = 2\pi/k$ arises. As expected in this
type of Hopf bifurcation with $O(2)$ symmetry, two branches of nontrivial solutions bifurcate simultaneously [20]. The instability evolves either to a pattern of standing waves (SW) or into waves that travel in either $x$-direction (TW). If the basic wavenumber $k$ is not allowed to vary, the TW branch typically bifurcates subcritically (see Fig. 1), acquiring stability at a secondary saddle-node bifurcation. When the Rayleigh number is increased from the saddle-node point, the TW branch disappears in a parity-breaking bifurcation of steady solutions usually called SOC states (stationary overturning convection), to which stability is transferred [21]. The phase velocity of the travelling waves increases as the Rayleigh number decreases, being maximum at the saddle-node point; from this point the velocity decreases continuously until vanishing at the bifurcation point that gives rise to the stable SOC states. The standing waves (SW) are unstable from the onset and usually disappear in a global bifurcation in which the SW connects with an unstable SOC state.

The precise location of the bifurcations presented in Fig. 1 for the arising $k = \pi$ patterns in the two mixtures we are analysing in this paper is indicated in Table 1. $R_c$ denotes the critical Rayleigh number at the onset of convection, $R_{TW_{SN}}$ the Rayleigh number at which the secondary stabilising saddle-node bifurcation in the TW branch takes place, $R^*$ the Rayleigh number of the parity breaking bifurcation in which the TW branch disappears and transfers stability to the SOC solution and $R_{SOC_{SN}}$ the Rayleigh number of the saddle-node bifurcation in the SOC branch.

<table>
<thead>
<tr>
<th>$S$</th>
<th>$R_c$</th>
<th>$R_{TW_{SN}}$</th>
<th>$R_{SOC_{SN}}$</th>
<th>$R^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.021</td>
<td>1760.81</td>
<td>1743.69</td>
<td>1743.35</td>
<td>1746.96</td>
</tr>
<tr>
<td>-0.127</td>
<td>1960.5</td>
<td>1863.7</td>
<td>1795.8</td>
<td>1941.5</td>
</tr>
</tbody>
</table>

Table 1. Critical values of the Rayleigh number of the type of bifurcations showed in Fig. 1 for the two cases considered in the paper, $S = -0.127$ and $S = -0.021$. $R_c$ indicates the primary Hopf bifurcation of the conductive state, $R_{TW_{SN}}$ and $R_{SOC_{SN}}$ the saddle-node bifurcations in the branches of TW and SOC solutions and $R^*$ the parity-breaking bifurcation of the SOC solutions, where the TW branch disappears.

4.2. Eckhaus instability analysis for a $S = -0.127$ mixture

We present results obtained for a binary mixture with parameters $S = -0.127$, $\sigma = 6.86$ and $\tau = 0.0083$, corresponding to the mixture used in experiment [5], which is performed on an annular cell of aspect ratio $\Gamma = 84$. In annular containers the basic wavenumber of the arising
pattern \( k \) is a discrete quantity, since the number of pairs of rolls of the solution gets adapted to the size of the cell. The critical wavenumber for a \( \Gamma = 84 \) cell is \( k = \pi \), which corresponds to a travelling wave of wavelength \( a = 2 \) consisting of \( n = 42 \) pairs of rolls. Nevertheless, in large aspect ratio annular cells, stable uniform travelling waves with a slightly different value of the wavenumber can coexist with the critical one. These solutions, which represent travelling waves with a different number of roll pairs, i.e. \( n = \ldots, 40, 41, 43, 44, \ldots \) in a \( \Gamma = 84 \) cell, bifurcate from the conduction state for a value of the Rayleigh number very close to the critical one. Therefore, it is of interest to analyse not only the dynamics of the \( k = \pi \) TW, but also of its neighbouring solutions.

To begin, we consider uniform travelling waves with basic wavenumber \( k = \pi \) (\( n = 42 \) pairs of rolls). The bifurcation diagram of this solution is shown in Fig. 1. In order to know whether this diagram is modified or not when the periodicity of the basic solution is allowed to change, we have computed the Eckhaus instability in the TW branch, both in the lower part of the branch, before the saddle-node bifurcation point, where travelling waves are unstable, and in the upper part, after the saddle-node point. Our computations show that, although several subharmonic bifurcations in the lower part of the TW branch have been identified, the stability of the \( k = \pi \) TW branch is only very slightly modified when the Eckhaus instability is taken into account. The first destabilising Eckhaus bifurcation in the TW branch has a Floquet parameter \( d_1 = 1/42 \) and takes place very close to the onset of convection. Successive destabilisations against perturbations of different Floquet parameter occur, but the TW solution regains stability against most of these perturbations before the saddle-node point is reached. The last stabilising Eckhaus bifurcation takes place in the upper part of the branch for a Rayleigh number \( R = 1864.26 \), which is slightly superior to that of the saddle-node, \( R_{SN}^{TW} = 1863.66 \). No more Eckhaus bifurcations have been identified in the upper part of the TW branch, so the travelling waves remain stable until their connection with the SOC branch.

For solutions with different wavenumber, the region of stability of the uniform TW depends very much on whether the number of rolls is increased or decreased with respect to the critical number of rolls. For the \( n = 43 \) TW, the last Eckhaus bifurcation that stabilises the TW takes place at a Rayleigh number superior to the one corresponding to the saddle-node point, but the solution goes on being Eckhaus stable in a significant region. When there is a decrease in one roll pair, \( n = 41 \) TW, the last Eckhaus bifurcation is shifted upwards in the branch. As a result, the \( n = 41 \) TW is stable in a considerably smaller region than the \( n = 42 \) and \( n = 43 \) TW. This result seems to indicate that there is a trend to favour the stability of solutions with wavenumber larger than the critical one (solutions with a larger number of roll pairs).

The Eckhaus instability band can be clearly visualised in Fig. 2. In this figure we plot the Rayleigh number at the onset of convection (squares), the Rayleigh number at which the TW-SOC

---

**Figure 2.** Stability boundaries in a wavenumber \((n)\)-Rayleigh number diagram corresponding to solutions for the parameters \( S = -0.127, \sigma = 6.86, \tau = 0.0083, \Gamma = 84 \). The bifurcation points for travelling waves containing different number \( n \) of roll pairs are indicated by using: squares for the bifurcation of the conductive state, triangles for the saddle-node bifurcation of TW solutions, diamonds for the parity breaking bifurcation TW-SOC and circles for the Eckhaus instability which stabilises the TW branch.
SOC transition takes place (diamonds), the position of the last stabilising Eckhaus bifurcation (circles) and the saddle-node point in the TW branch (triangles) as a function of the number of roll pairs in the cell. This defines the four corresponding stability curves. As can be inferred from the plot, travelling wave solutions containing \( n = 38, \ n = 39 \) or \( n = 46 \) pairs of rolls, are always unstable. As a consequence, for these cases the SOC solution is also unstable after the TW-SOC bifurcation. The Eckhaus bifurcation that stabilises the solutions with these wavenumbers now takes place in the SOC branch.

Once the Eckhaus boundaries are determined it is of interest to analyse the features of the dynamics that this instability triggers. To do this, we run the time evolution code for a Rayleigh number outside the stability band, giving as an initial solution uniform TW states with different number of roll pairs.

If the number of rolls of the solution is higher than the critical value \( n = 42 \), two different behaviours are observed. For the \( n = 43 \) TW, the system makes a transition back to the conduction state for all the trials we have made with several values of the Rayleigh number. During the transient, the dominant mode decreases in amplitude and the rest of modes begin to grow, as if a phase slip leading to a change in the wavenumber was going to take place, but finally the system is unable to select a stable state different from the conductive one. In the case of the \( n = 44 \) TW, when the Eckhaus stability limit is approached from the stable side, a transition to a \( n = 42 \) TW state takes place (see Fig. 3). The phase modulations induced by the Eckhaus instability, which travel in the same sense as the TW and are highly localized, result in the annihilation of two pairs of rolls. In this case, the relaxation back to the stable wavenumber band is relatively simple, and few different states are approached by the system during the transient.

If the number of rolls of the TW is decreased in a unity with respect to the critical state, the uniform \( n = 41 \) TW undergoes a longer transition (see Fig. 4), which ends up again in a

![Figure 3](image-url)
n = 42 TW state. In this case, as can be appreciated in the space-time plot in Fig. 4, while the
TW travels to the right, the group velocity of the phase modulations has the opposite sense.
Besides, the wavenumber modulation affects a wider region of the cell than in the previous case
(transition of the n = 44 TW to the n = 42 TW). We do not observe the tendency of the system
to execute damped oscillations in mean wavenumber before settling down to a stable state inside
the Eckhaus band that is reported in experiment [4].

Therefore, our numerical results agree with the experimental observations about the nature
of the Eckhaus instability [4, 5], which is subcritical and consists of phase modulations that
trigger the creation or annihilation of roll pairs, although the transients observed in experiments
are far more complex.

Finally, our results for the n = 40 TW do not allow us to understand completely the dynamics
in the neighbourhood of the Eckhaus instability. In some runs, the n = 40 TW state is quenched
outside the Eckhaus boundary and the long transients lead to a n = 42 stationary state, which
is still stable for the Rayleigh we consider. This result is consistent with the Eckhaus instability
being subcritical. However, near the Eckhaus bifurcation of the n = 40 TW, we have also
obtained a branch of weakly modulated travelling waves, although its dynamical behaviour does
not fit well the expected behaviour if the Eckhaus bifurcation were supercritical. Nevertheless,
it is important to notice that this region of the parameter space (Ra ≈ 1950) is complicated
from the dynamical point of view, since the TW-SOC bifurcation point is extremely close to the
Eckhaus bifurcation. The parameters we are considering are very close to a codimension-two
point.

4.3. Eckhaus instability analysis and small amplitude convection for a S = −0.021 mixture
We present now results corresponding to the binary mixture with parameters S = −0.021,
σ = 6.22 and τ = 0.009 used in experiment [3], performed on an annular cell of aspect ratio
Γ = 80. For this values of the aspect ratio, the wavenumber of the critical TW is again k = π,
but it corresponds to a solution with n = 40 pairs of rolls.

Figure 4. (Left) Temporal series showing the variation of the Nusselt number and of the n = 40,
n = 41, n = 42 and n = 43 horizontal mode of the vertical velocity and (right) space-time plot of
the temperature during a transient that brings the solution back inside the Eckhaus instability
band. The initial solution is a n = 41 TW state, which is Eckhaus unstable for the Rayleigh
number we consider, R = 1930. The d = 1/41 Eckhaus instability triggers the growth of the
n = 40 and n = 42 adjacent modes. The system finally selects the n = 42 TW state. We are
considering a S = −0.127, σ = 6.86 and τ = 0.0083 binary mixture.
The Eckhaus bifurcations that have been identified for the \( k = \pi \) TW are included in Fig. 5. The open circles show the location of the destabilising bifurcations, while the solid circles correspond to the stabilising ones. The first Eckhaus instability, with Floquet parameter \( d_1 = 1/40 \), occurs extremely soon, at \( R = 1760.67 \), which is nearly at the critical point \( (R_c = 1760.81) \). There are two more destabilising bifurcations with Floquet parameters \( d_2 = 2/40 \) and \( d_3 = 3/40 \). In this case, the Eckhaus stability is not retrieved before the saddle-node point. As can be clearly seen in the figure, the \( k = \pi \) TW is Eckhaus unstable against disturbances of Floquet parameter \( d_1 \) until \( R = 1746.40 \), which is practically the position of the transition from TW to SOC \( (R^* = 1746.96) \). So in this case, the region of stability of the uniform travelling wave is extremely small. This result agrees with the experimental observations reported in [3], where nonlinear saturated TW states have not been observed.

The behaviour does not change when the number of rolls of the TW is increased or decreased in one. The the \( n = 39 \) and \( n = 41 \) travelling waves behave like the \( n = 40 \) TW. They are Eckhaus unstable in most of the region between the saddle-node and the TW-SOC transition; indeed they are only stable in a tiny interval of Rayleigh numbers.

A completely different convective regime consistent with experimental observations is identified for slightly supercritical Rayleigh numbers \( (R \geq R_c = 1760.8) \). Depending on the initial conditions, the system remains in long-lived very small amplitude states instead of making a transition to a stable nonlinear state, which for the \( S = -0.021 \) mixture in a \( \Gamma = 80 \) container is a uniform \( n = 40 \) SOC state (see Fig. 5).

As can be seen in Fig. 6 these states are far from being periodical. On the left part of the figure, we are plotting the real part of the dominant \( n = 40 \) temperature mode in a point located at a height \( 2d/3 \) as a function of time for different values of the Rayleigh number \( (R = 1761 - 1766) \). On the right part of the figure, the time dependence of the Nusselt number is represented for the same values of the control parameter. The Nusselt number is a global variable, so it gives an indication of the state of convection in the whole annulus. The most outstanding feature of these states is that they remain very small in amplitude for a long time until quite suddenly a burst of amplitude takes place. After most of these growths of amplitude, the system goes back to the small amplitude original state, but sometimes the system makes a transition to the extended pattern of steady, spatially uniform convective rolls. These sequence of small amplitude convection followed by a burst and subsequent collapse of amplitude repeats irregularly.

To illustrate the spatiotemporal nature of the solution, Fig. 7 shows the contour plots of the concentration field during a burst of amplitude observed for a Rayleigh number \( R = 1762 \). The weakly nonlinear convective regimes consist of several types of wave packets of small amplitude.

![Figure 5. Bifurcation diagram (Nusselt-1 versus the Rayleigh number) for a uniform train of travelling waves of wavenumber \( k = \pi \) contained in a periodic domain of aspect ratio \( \Gamma = 80 \). The remaining parameters are \( S = -0.021, \sigma = 6.22, \tau = 0.009 \) (the parameters considered in the diagram at the top of Fig. 2). The points where subharmonic instabilities take place are indicated in the figure together with the corresponding Floquet parameter \( d_{m/M} \). Following the curve of TW waves from the onset of convection, open and solid circles are used to indicate loss and gain of stability, respectively.](image-url)
Figure 6. Temporal series showing the variation of the $n = 40$ temperature mode in one point (left) and of the Nusselt number (right) for different values of the Rayleigh number in a mixture with parameters $S = -0.021$, $\sigma = 6.22$, $\tau = 0.009$.

travelling waves which propagate around the cell. Previously to a burst of amplitude, a fast travelling, nearly uniform TW of small amplitude develops ($t = 150$ in the plot). When the amplitude of the TW begins to grow, a spatially localized pulse appears in the pattern and the propagation velocity of the TW is reduced significantly ($t = 195$). Two different convective regimes coexist: a region of slowly travelling finite amplitude convection and a localized travelling pulse ($t = 210 - t = 220$). As the pulse collides with the extended structure, an abrupt collapse of the structure takes place ($t = 220 - t = 230$), leaving the system in a slightly perturbed conduction state at the end of the process ($t = 280$).

Increasing the Rayleigh number affects the frequency and intensity of the bursts. The variations of the structure in time are more rapid and the dynamics becomes more erratic. A reduction in the amplitude of the bursts can be appreciated. The resulting spatiotemporal chaotic state is known as dispersive chaos, because these behaviour is associated to strong nonlinear dispersion [3].

5. Summary
In this paper, we presented numerical results on convection of binary fluids in large aspect ratio annular geometry. We have focused on binary mixtures with negative values of the separation ratio, for which the primary bifurcation is subcritical and oscillatory. Several aspects of the non uniform dynamics arising in extended domains have been considered.

For one hand, the Eckhaus instability, which is known to play an important role influencing
the stability of the uniform patterns, has been analysed for TW states. We have obtained the Eckhaus stability band for $=-0.127$ and $S=-0.021$ binary mixtures in $\Gamma = 84$ and $\Gamma = 80$ aspect ratio cells, respectively. In the $S = -0.127$ case, as expected, the region of stability of the uniform travelling wave becomes smaller as the difference in the number of roll pairs with the critical solution increases. Nevertheless, a tendency to favour the stability of solutions with a larger number of rolls compared with those with a smaller number has been observed. The numerical Eckhaus curve we have obtained does not fully compare to the experimental one reported in [4]. The main difference is the location of the minimum of the Eckhaus curve, which is shifted in the experimental curve. The minimum is observed to be in the $n = 40$ TW, instead of being in the $n = 42$ TW state, which is the critical solution. The cause of these discrepancies might be the finite width of the experimental cell, which is neglected in our computations. In the $S = -0.021$ case, our results fully agree with the experimental observations [3]. In this experiment no stable nonlinear travelling waves are observed. This observation agrees with our Eckhaus stability analysis, which determines that all the uniform travelling waves are Eckhaus unstable until the TW-SOC bifurcation point is nearly reached.

The dynamics triggered by the Eckhaus instability has been studied for the $=-0.127$ mixture. Long transients, which bring the solution back within the stability band or to the conductive state, have been analysed. This is consistent with the subcritical nature of the instability reported in experiments [4, 5].

On the other hand, we have obtained the complex small-amplitude states observed in experiments on sufficiently long cells for mixtures with a separation ratio very close to zero for the first time by means of DNS of the full convection equations. These states are characterized by the repetitive, chaotic bursting and collapse of the wave amplitude. The dynamics is dominated by the nonlinear dispersion of the system, which gets stronger by reducing the value of $|S|$. We
think that the branches of solutions that arise from the Eckhaus bifurcations in the lower part of the unstable TW branch modify the phase space allowing these long-lived, small amplitude states to exist.

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References