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A multi-level strategy for regularized parameter identification using nonlinear reparameterization with sample application for reservoir characterization

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Abstract. Level-set methods are popular for identifying piecewise constant structures. We propose an approach inspired by the level-set idea to identify coarse scale features of a scalar field, where the transitions between different regions can be both sharp and smooth. The nonlinear and coarse reparameterization structure provides regularization on the inverse problem, which is important if the quality of the available information is poor.

In our identification strategy, the resolution of the representation of the parameter function is refined gradually; for several inverse problems, the nonlinearity of the problem is correspondingly carefully increased. Hence, when using a gradient-based optimization routine, the approach may reduce the risk of getting trapped in a local minimum of the objective function.

We demonstrate the identification strategy for estimation of fluid conductivity in a porous medium based on sparsely distributed, transient data. The methodology is well suited for determining channels and barriers as well as smooth structures based on limited information.

1. Introduction
The need to solve inverse problems related to the identification of a spatially distributed parameter function arises in a wide range of practical applications. Examples are characterization of organs in medical inverse problems, constructional inverse problems, identification of electrical conductivity in electrical resistance tomography, and characterization of oil or ground-water reservoirs. These inverse problems are, in general, ill-posed due to limited information content in the data. High nonlinearity in the relation between the data and the model parameters causes additional difficulties.

To meet these problems, and avoid instabilities in the solution of the inverse-problem, the inverse problem must be regularized. We consider regularization by reparameterization, and approach the task of determining a reparameterization structure that is in agreement with the available information. Typically, this is a challenge since the resolving power of the available data with respect to the chosen forward model is not known prior to the estimation.

Our approach is inspired by the level-set idea for implicit representation of a two-zone segmentation structure: The zero level-set of a function, which is called the level-set function,
determines the geometry of the segmentation. The level-set idea was first introduced by Osher and Sethian [10], and has become popular for solving inverse problems related to shape and topology optimization [5].

In the level-set approach, discontinuous Heaviside functions are applied to determine the relationship between the level-set function and the parameter function so that the result is a segmentation. In this work, we enable identification of smooth as well as sharp transitions by replacing the Heaviside functions with related, smooth, nonlinear functions. This gives a smooth representation determined by the function that take the role of the level-set function; this function is called a structure function since it determines the structure of the representation. In the same manner as for the level-set approach, we can enhance the complexity of the parametrization by increasing the number of structure functions.

As we treat the parameter identification as a pure numerical optimization problem [7], a discretization of the structure functions must be chosen. Using the level-set approach, the resolution of the structure functions is normally defined by the grid for the forward computations. In previous work [3, 4, 9] using the level-set idea, we have proposed to reparametrize the level-set functions to reduce the number of values to optimize (see also [7, 12]). In this work, we follow the approach in [3] and use a discretization of the structure functions by a linear expansion with continuous basis functions.

The identification algorithm is based on a multilevel strategy [3, 4, 9], where the main idea is to gradually increase the resolution of the representation. There are two possibilities for achieving this:

- Adding a new structure function to the representation of the parameter function, providing increased flexibility with respect to representing multiple regions where the parameter value is close to constant.
- Refinement of the representations of the structure functions, providing increased flexibility in geometry of the transition zones between the regions of constant parameter value (see also [7]).

A combination of the two strategies enables identification of a parameter function based on the available information. A new estimation is started with the result from the previous estimation as initial state each time the representation is refined. This strategy makes it possible to obtain a representation where the number of estimated coefficients is in correspondence with the available information. For nonlinear inverse problems, the estimation strategy can avoid local minima of the objective function in a gradient-based optimization since the nonlinearity often is lower for coarse-scale representations.

The rest of the paper is organized as follows; in Section 2, we motivate and describe our solution approach based on parametrization, before we, in Section 3, apply the proposed methodology for solving an inverse problem related to identification of the fluid conductivity in a porous media based on sparsely distributed, transient data. This problem is a prime example of an inverse problem where the available information is poor and the relationship between the parameter function and the data is highly nonlinear.

2. Parameter Identification

2.1. Inverse Problem

The inverse problem at consideration consists in identifying a spatially distributed parameter function based on information given by a vector of data \( d \). Since the number of data points is finite, the distributed parameter function must be approximated by a function \( p(x) \) with a finite number of degrees of freedom (see Section 2.2).

The forward operator \( m \) maps \( p \) to the space of data. The relationship is given by

\[ m(p) + r = d, \]  

(1)
where the residual $r$ is a result of error in the forward operator, error in the representation of the parameter function, and data error. In general, the forward operator $m$ is the result of solving one or more equations that governs the mathematical model for the forward problem. Typically, one must rely on numerical solution methods due to the complexity of the models. In the application part of this paper, we investigate reservoir characterization problems, where the mathematical model consists of a system of nonlinear partial differential equations modeling two-phase porous-media flow.

The inverse problem is normally solved by minimizing an objective function of the type

$$J(p) = [m(p) - d]^T C^{-1} [m(p) - d] + R(p),$$

where $m(p)$ denote the calculated model output given a specific parameter function $p$, $C$ denotes the covariance of the data and model errors, and $R(p)$ is a penalizing term based on prior assumptions of the solution properties.

### 2.2. Representation of the parameter function

We consider a representation of the parameter function that enables both dominating regions of constant parameter value and more or less smooth transitions between these regions. As stated in the introduction, our approach is inspired by a level-set approach for identifying piecewise constant parameter functions; hence, we start by revealing the level-set idea.

Consider a partitioning of a domain $\Omega$ into two nonoverlapping (and possibly disconnected) zones $\Omega_1$ and $\Omega_2$, and define a function $\phi$ with the following properties:

$$\begin{align*}
\phi(x) &> 0 \quad \text{for } x \in \Omega_1; \\
\phi(x) &< 0 \quad \text{for } x \in \Omega_2; \\
\phi(x) &= 0 \quad \text{for } x \in \partial \Omega_1 \cap \partial \Omega_2.
\end{align*}$$

(3)

If we now define

$$\Psi_1(\phi) = H(\phi), \quad \Psi_2(\phi) = 1 - H(\phi),$$

(4)

where $H$ is the Heaviside function, we can represent the parameter function as

$$p(x) = c_1 \Psi_1(\phi(x)) + c_2 \Psi_2(\phi(x)),$$

(5)

where the coefficients $c_1$ and $c_2$ give the parameter value for each zones. The zero level-set of $\phi(x)$ now corresponds to the zonation boundaries, and we denote it the level-set function. Observe also that $\phi$ gives rise to a parametrized nonlinearity in the representation.

In this work, we suggest to replace the Heaviside function in (4) by a smooth function to obtain a smooth representation that still can describe dominating regions of nearly constant parameter value. To be more specific, we choose

$$\tilde{H}(\phi) = \frac{1}{\pi} \tan^{-1}(\phi(x)) + 1/2.$$ 

(6)

Hence, it is no longer only the zero isocontour of $\phi$ that determines the structure of the representation, but, for simplicity, we do not change the notation. As $\phi$ determines the structure of the parametrization, we call it a structure function.

Using the representation (4)–(6), the degree of smoothness in the parameter function $p(x)$ is determined by $\phi(x)$. For steep gradients in $\phi(x)$ around the zero point (or zero isocontour in two spatial dimensions) the transitions between regions of approximately constant parameter value are sharp. For more slowly varying $\phi(x)$ the parameter function exhibits smoother transitions.
Hence, applying $\tilde{H}(\phi)$ in the basis representation of $p(x)$ enables optimization with respect to the smoothness of the parameter function.

The representation (4)–(6) is inspired by the level-set idea, and, as described above, the approaches are strongly related. In the same manner as the level-set approach can be expanded on for representation of more than two zones, the smooth nonlinear expansion we consider can be expanded to enable more complex structures. Following Vese and Chan [11] we can increase the complexity of the representation allowing for more structure functions. The representation by means of structure functions is then

$$p(x) = \sum_{k=1}^{2^n} c_k \Psi_k(\phi_1, \ldots, \phi_n),$$

where $\{c_k\}$ denotes the set of coefficients in the discretization and $\{\Psi_k\}$ denotes a set of linearly independent basis functions given by

$$\Psi_k(x) = \prod_{r=1}^{n} R_k(\phi_r(x)), \quad R_k(\phi_r) = \begin{cases} \tilde{H}(\phi_r) & \text{for } b^k_r = 0, \\ 1 - \tilde{H}(\phi_r) & \text{for } b^k_r = 1, \end{cases}$$

where $b^k_r$ is the $r$th element in the binary expansion of $(k-1)$ written on the form $(b^1_k, b^2_k, \ldots, b^n_k)$. The difference from the level-set approach is just that we have replaced the Heaviside function, $H$, by the smooth function $\tilde{H}$.

### 2.3. Parametrization of structure functions

To treat the parameter function identification problem as a numerical optimization problem, we need a discrete representation of $\phi(x)$. Whereas the common strategy in the conventional level-set approach is to use the same resolution in the representation of the level-set function as for the grid that is used for the forward simulations, we consider a coarse-scale reparameterization, where each structure function is represented by a few parameters only [4, 7, 9, 12].

In this work, we use a continuous representation of $\phi(x)$ [3]. We consider a linear expansion given by

$$\phi(x) = \sum_{j=1}^{s} a_j \theta_j(x),$$

where $\{\theta_j\}_{j=1}^{s}$ denotes a set of continuous basis functions. As the set of coefficients $\{a_j\}_{j=1}^{s}$ in (9) determines the structure of the representation, we call these coefficients the shape parameters. Looking at the representation (7)–(9), we see that whereas the coefficients $\{c_k\}_{k=1}^{2^n}$ are linearly related to $p(x)$, the shape parameters are nonlinearly related to $p(x)$. Hence, we have a parametrized nonlinearity in the representation. In the same manner as the level-set representation, our representation provides a type of “topological” regularization [8] of the inverse problem.

The low number of degrees of freedom in the representation (9) results in a structure function that is restricted with respect to which shapes it can take; this provides additional regularization on the inverse problem. When it comes to the choice of continuous basis functions $\{\theta_j\}_{j=1}^{s}$, various alternatives are possible. However, once a set of basis functions is chosen, the reparameterized structure functions are restricted to a certain function space.

### 2.4. Multilevel estimation algorithm

In identifying the parameter function, we suggest a multi-level strategy. The aim is to determine the parametrization of the parameter function during estimation such that the level of detail in
the representation corresponds to the information content of the data with respect to the chosen forward model.

Assuming that we have no prior knowledge of the parameter function, we start by optimizing an approximation to the parameter function by representing it by a single parameter value in the entire domain. The reparameterization of the parameter function is then altered by adding a structure function (discretized by some basis function expansion), and the new representation is optimized. At this point there are two possibilities: Either the basis representation of the current structure function could be refined, thus allowing for more detail in the structure of the representation; or a new structure function could be added to increase the number of coefficients in the representation.

Each time the reparameterization of the parameter function is altered, either by adding a new structure function or refining the resolution of the representation of the given structure functions, a new estimation is started with the result from the previous estimation as initial state. This construction allows for increasing detail in the representation, which may improve the convergence speed and, for nonlinear problems, also avoid local minima of the objective function.

How the two strategies should be combined must depend on the considered application. In some applications, information about coarse-scale variation is known prior to the estimation; hence, the number of structure functions can be set fixed, and we can focus on finding the resolution of the structure functions.

A problem in solving ill-posed parameter identification problem is that the results may depend on the initial guess for the structure function. To evaluate different choices, a possibility is to use some kind of performance measure that approximates the ability of a refined parameterization structure to reduce the objective-function value. Examples are: the magnitude of the objective-function gradient and a prediction of the decreased objective-function value based on a linearization of the model response; see, for example [2].

3. Application to Reservoir Characterization
An oil or ground water reservoir consists of various porous rocks with different properties. A crucial factor for the flow is the single-phase fluid conductivity, or permeability, of the porous medium. This property will vary on different scales within the reservoir. For good reservoir management, reliable permeability estimates are important. In the following, we identify coarse-scale variations in permeability based on pressure data from a water-flooding scenario in an oil reservoir.

3.1. Forward model
We consider two-phase, incompressible, and immiscible flow, assuming that gravity can be neglected and that the permeability is isotropic. Given two fluid phases, \( w \) (water) and \( o \) (oil), occupying the pores, the model equations describing this problem are

\[
\varphi(x) \frac{\partial S_o}{\partial t} - \nabla \cdot (k(x)k_{ro}(S_o)\mu_o^{-1}\nabla P_o) = q_o,
\]

\[
\varphi(x) \frac{\partial S_w}{\partial t} - \nabla \cdot (k(x)k_{rw}(S_w)\mu_w^{-1}\nabla P_w) = q_w.
\]

Here, \( S_i \) denotes the fluid saturation, \( P_i \) is the fluid pressure, \( \mu_i \) the fluid viscosity and \( q_i \) the external volumetric flow rates, all with respect to the \( i \)th fluid phase. Furthermore, \( \varphi(x) \) denotes porosity. The parameter function \( k(x) \) denotes the permeability; and \( k_{ri} \) denotes relative permeability, which is the factor the fluid conductivity of the \( i \)th fluid phase is reduced due to the presence of the other phase.
The equations are defined for \((x, t) \in \Omega \times [0, T]\), where \(\Omega\) is a bounded two-dimensional reservoir domain and \(T\) is finite. For our test case, we assume that the reservoir is completely saturated, \(S_o + S_w = 1\), and that the capillary pressure, \(P_c = P_o - P_w\), is zero.

The following specifications of the model setup are presented dimensionless. We consider a computational domain with extent equal to 1 for both horizontal directions, and thickness equal to 0.1. The fluid conductivity is assumed to be heterogeneous in the horizontal directions only. The initial saturations are \(S_w = 0.1\) and \(S_o = 0.9\), respectively; furthermore, \(\mu_w = 1.0\), \(\mu_o = 1.3\), \(\varphi = 0.3\), \(k_{rw}(S_w) = (S_w - 0.1)^2/0.04\) and \(k_{ro}(S_w) = (0.8 - S_w)^2/0.04\).

To generate synthetic observation data, \(d\), for pressure in the injection wells, we added Gaussian random noise to the pressure values obtained from a run of the simulator with a reference permeability field. For the test example we have investigated, the standard deviation, \(\sigma\), for a pressure observation corresponds to 1% of the pressure difference between the observation and the constant production pressure.

The forward flow simulations are done on a 33 \(\times\) 33 grid, where the permeability is approximated as constant on each grid cell. The injection wells are shown as white dots in the figure and the production wells as black dots. The wells form a five-spot pattern and there are 9 injection wells and 4 production wells. If the rate in the injection well located in the center of the domain is \(I\), then the rate for the wells in the corners are set to \(I/4\), and the rates for the other wells on the boundary of the domain are set to \(I/2\). For the test case presented in the following, we have chosen \(I = 0.010\). For each injection well, we have 200 pressure observations evenly distributed in time, where the final observation is taken at \(\frac{4}{3}\) PVI (pore volumes injected).

### 3.2. Permeability identification

We consider solely inversion of transient pressure observations in wells. To illustrate the regularizing effect of the coarse level-set representation, we minimize a weighted least squares objective function (2) with no additional regularizing term, that is

\[
J(p) = (m(p) - d)^T C^{-1} [m(p) - d].
\]

Here \(d\) denotes the available data, and \(m\) denotes the corresponding pressures calculated using the forward simulator.

We consider a two-dimensional rectangular computational domain, and to represent each structure function, we choose a bilinear basis on rectangular elements. The number of elements is increased according to the estimation algorithm, starting with a regular grid of four elements, and dividing each element into four each time the representation of the structure function is refined.

Our chosen estimation strategy is to first search for the appropriate number of structure functions, by adding one structure function at a time until the relative reduction in the objective function after including a new structure function is less than some threshold \(L\). For our test case, we have chosen \(L = 20\%\).

When the number of structure functions is determined, we gradually refine their resolution by refining the bilinear representation in each direction.

The initial guess for a structure function is set to either \(\phi^0_1(x, y) = -1 + 2x\) or \(\phi^0_2(x, y) = -1 + 2y\), where the choice between the two alternatives are determined by a performance measure based on a linearization of the model response; see, for example [2, 6].

We have based the optimization on the Levenberg-Marquardt (L-M) algorithm. For calculating the sensitivity matrix, we apply a direct method [1], where the different types of derivatives are found by utilizing a chain rule.
3.3. Results
In the example we show here, we identify a channel-like structure, both with the level-set representation and with the related, continuous representation. Results are shown in Figure 1.

While the algorithm chooses to add structure functions when using the level-set representation; refinement of the structure function is selected in the first stages when using the continuous representation. This effect is not surprising due to the increased flexibility of the continuous representation. For comparison, we have also shown the result of the estimation using the discontinuous representation with a single structure function.

When we altered the threshold that determines refinement or addition of structure functions, the choice between adding structure functions and refinement of the representation of the structure functions could change, but the results were still in favour of the continuous representation. For this representation, the results were also visually similar.

4. Concluding remarks
In this paper, we have proposed an approach for parameter identification that focuses on recovering coarse-scale trends of the parameter function. The nonlinear expansion we have utilized for representing the parameter function is inspired by the level-set idea, but it is able to describe both smooth and sharp transitions as well as dominating regions of constant parameter function
value.

To identify a representation that has a resolution corresponding to the information content of the data, we have suggested a multilevel strategy, where the representation can be refined either by adding structure functions to the representation, or allowing for a higher resolution in the representation of these structure functions. Hence, the risk of over-parametrization, which results in unstable estimates, is diminished. Searching for coarse-scale estimates reduce the nonlinearity of several nonlinear inverse problems, which helps to avoid getting trapped in a local minimum during estimation.

We have demonstrated the methodology for estimation of permeability in an oil reservoir based on scarce data. The result show that the main coarse-scale features of the permeability field are recovered with a low number of estimated coefficients and shape parameters in the representation.

References