Global nature of zonal flow due to the finite band width

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Global nature of zonal flow due to the finite band width

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Abstract. The spectral effects of both micro-scale electron temperature gradient 
driven turbulence and zonal flow on the zonal flow generation are investigated 
theoretically and computationally based on the Hasegawa-Mima turbulence equation. 
As the minimum model describing the most primary spectral feature, we have developed 
an eight-wave interaction model which includes two sets of the conventional four-wave 
coupling systems with adjacent radial spectral difference of $\delta k_r$. It is found that the 
zonal flow is characterized by a global nature with an enhancement of growth rate $\gamma_q$, 
namely, global zonal flow eigen-mode, which is attributed to the appearance of new cross 
mode couplings due to the finite band width of zonal flow. This is qualitatively different 
from the monochromatic character of zonal flow described in the conventional four-wave 
coupling model. Direct numerical simulations of the Hasegawa-Mima equation with a 
zonal flow spectral structure have clearly proven the analytical results by employing a 
rigorous spectral code. The calculations further show that the global zonal mode is not 
only governed by the spectral structure of pump waves, but also their phase relation.

1. Introduction
The importance of secondary large scale structures nonlinearly generated by the 
ambient turbulence is now being widely recognized[1], such as zonal/mean flows, 
streamers, geodesic acoustic modes (GAMs), and secondary low-frequency long wavelength 
fluctuations. Among them, toroidally and poloidally symmetric low frequency zonal flow 
with $k_\theta = k_\phi = 0$, but $k_r \neq 0$ is found to play an important role leading to a suppression of 
turbulent transport[2, 3, 4]. The generation mechanisms of zonal flow have been intensively 
studied. The modulational instability has been discussed as one of the plausible candidates of 
the zonal flow generation. In studying the modulation process, two approaches have 
been developed based on the Hasegawa-Mima (HM) turbulence model[5] which describes 
the most primary nonlinear interaction process of inviscid drift wave turbulence. One is 
the method using the coherent mode coupling (CMC)[6, 7, 8, 9, 10] and the other is 
that using the wave-kinetic (WK) equation[11, 12, 13]. In the CMC model, the four-wave 
model which samples the turbulent fluctuation as a monochromatic pump wave with the 
excitation of two sideband components through nonlinear coupling with a zonal seed has 
been proposed as the minimum model. In the WK model, the action invariant of the 
turbulence and associated evolution are represented in terms of the eikonal equation of 
the drift wave turbulence based on the assumption of scale separation between ambient
turbulence and zonal flow[14]. It is noted that the dispersion relation obtained from the WK model is described in terms of the spectral density of turbulence.

Recently, these two different methods have been compared for the generation of zonal flow from the ion temperature gradient (ITG) mode for monochromatic pump wave and shown that two models give qualitatively the same results even though the assumptions such as scale separation between turbulence and zonal flow are different[13]. Meanwhile, the effects of the turbulent spectra on the zonal flow generation were investigated employing the advantage of the WK model. A turbulent spectrum with a Gaussian shape has been applied in the analysis of the WK model[12]. We have also investigated the zonal flow generation by representing the turbulent spectrum with two monochromatic pump waves by keeping the total pump energy constant. From these analyses, however, no essential difference for the zonal flow instability was obtained. It is noticed that these studies still assume monochromatic wave with respect to the zonal flow. Namely, no theoretical works that take into account the effect of zonal flow spectra have been attempted.

In the present paper, we have extended the previous modulational instability analysis of the zonal flow generation originated from electron temperature gradient (ETG) driven turbulence[7] by including the effect of the zonal flow spectra in the CMC model. We propose eight waves which involves two sets of the four-wave coupling system with adjacent radial spectral difference of dk_r. Namely, we represent the effect of the zonal flow spectra by means of two monochromatic waves with the radial wave number given by k_\eta and k_\eta + dk_r, which couple with two monochromatic pump waves and also four sidebands. We then derive the dispersion relation and found from the analyses that the zonal flow generation is qualitatively different in two cases with and without the spectral structure of the zonal flow. Namely, due to a new cross coupling between two sets of the modulational loop, two zonal flow components are no longer independent, but couple with each other, leading to a global zonal flow eigen-mode. The growth rate of the global mode is found to be enhanced compared with that estimated from the four-wave model.

In order to justify the validity of the model, we have performed direct numerical simulations of the HM equation. The formation of the global zonal flow eigen-mode and associated enhancement of the instability are clearly proved. We have also performed simulations with a quasi-continuous Gaussian-type spectrum for both zonal flow and pump wave. It is noticed that the radial structure of the pump waves can be different depending on the phase relation of each Fourier harmonics even if the shape of the power spectrum is identical. Due to this fact, it is found that the radial structure of the zonal flow and associated enhancement nature of the instability are influenced not only by the spectrum shape but also by their phases. Namely, the zonal flow is preferentially excited in the region where the intensity of the pump wave is higher. From these analyses, we found that the zonal flow spectra and resultant global nature of zonal flow play an important role for zonal flow generation whereas previous works only deal with the spectrum of turbulence.

The paper is organized as follows. In section 2, we present our theoretical model of zonal flow generation including the effect of spectral of both zonal flow and turbulence. The ETG mode is considered as an example. Based on the theoretical model, we derived a dispersion relation and present the numerical results. In section 3, we present numerical simulations of the HM equation to justify our theoretical model. We also examine the effect of initial phase of turbulence on the zonal flow instability. Finally, we summarize the results and give concluding remarks in section 4.
2. Theoretical model and numerical results

2.1. Theoretical model

As a model to investigate secondary instability in the zonal flow-drift wave turbulence, the well-known HM equation is conveniently employed to analyze the modulation process due to nonlinear coupling between the zonal flow and the pump wave through the production of the sidebands. The final normalized evolution equation reads as

\[ (1 - \nabla^2) \frac{\partial \phi}{\partial t} = \frac{\partial \phi}{\partial y} + [\phi, \nabla^2 \phi], \]

where \( \phi \) and \( \nabla^2 \) denote the electrostatic potentials of the drift-wave turbulence and two-dimensional Laplacian perpendicular to the magnetic field, respectively. Here, we assume the turbulence due to the ETG mode, so that the adiabatic ion response is employed. The adiabatic ion response is a direct consequence of short wavelength limit of ion scale perturbation obtained from the gyrokinetic formalism. In equation (1), space coordinate, time, and potential are normalized as follows

\[ (x, y, t, \phi) \rightarrow \left( \frac{x}{\rho_c}, \frac{y}{\rho_c}, \frac{v_{te}}{L_{ne}}, \frac{L_{ne}}{\rho_c T_e} \right), \]

where \( \rho_c = v_{te}/\omega_{ce}, v_{te} = \sqrt{T_e/m_e}, L_{ne} = (d\ln n_e/dx)^{-1} \). This equation is the same in form as the reduced version of the three-field nonlinear evolution equation of ETG driven turbulence under the lowest order and electrostatic limits in a slab geometry. The Poisson bracket presents the dominant nonlinear term which is originated from the convective nonlinearity.

As explained in section 1, two approaches, i.e., the CMC method and the WK one are employed. In these methods, a same assumption has been taken in which monochromatic wave of zonal flow is sampled. However, the generation of the zonal flow is purely a nonlinear modulation process, which essentially involves the complex coupling along whole spectral distribution under the wave number and frequency match conditions. In the description of the zonal flow generation, we should not only consider the spectral structure of the turbulent pump fluctuations, but also need to incorporate the spectral characteristics of the zonal flow. In this case, each spectral component of zonal flow may be linked successively due to the nonlinear couplings with pumps through the modulation excitation of sidebands. As a result, the zonal flow instability is then characterized as a global mode rather than a monochromatic wave.

In this section, we propose a minimal model to describe the primary nonlinear interaction involved among the ambient turbulence and zonal flow in order to capture the essential feature of zonal flow spectral effects arising in the zonal flow generation processes, i.e., the effect of finite band width. Hence, as a minimal model to characterize the spectral effects of both zonal flow and the ambient pump turbulence, we represent those spectrum by means of two monochromatic waves with a same spectral difference \( dk_x \). The corresponding four sidebands can be produced through the nonlinear interaction similar to those appeared in the conventional four-wave coupling model, as illustrated in figure 1. This is the proposed our minimal eight-wave coupling model. It looks to be equivalent to a system involving two sets of four-wave coupling loops with a spectral shift \( dk_x \). However, this system is not a simple superposition of two sets, a new cross coupling loop appears between these two sets through the sideband, which links two modulation processes as explained in the following. This may change the characteristics of zonal flow instability. Based on this idea, we will derive a dispersion relation of zonal flow instability in the frame of coherent mode coupling method as follows.
The pump wave packets are represented as

$$
\tilde{\phi}_p(\mathbf{r}, t) = \sum_{j=0}^{1} \phi_{k_{xj}, k_y} \exp \{ i(k_{xj}x + k_yy - \omega_{0j}t) \} + c.c.,
$$

where \((k_{xj}, k_y) = (k_x + jdk_x, k_y)\) are the radial and poloidal wave numbers, and the frequencies \(\omega_{0j}\) satisfy the dispersion relation of the drift wave under equation (1), i.e., \(\omega_{0j} = -k_y/\left(1 + k_x^2 + k_y^2\right)\). Arbitrarily small perturbation is chosen as a seed of the zonal flow with two wave components

$$
\tilde{\phi}_q(\mathbf{r}, t) = \sum_{j=0}^{1} \phi_{k_{qj}, 0} \exp \{ i(k_{qj}x - \omega_{qj}t) \} + c.c.,
$$

where \(k_{qj} = k_q + jdk_x\) are the radial wave numbers. Corresponding side bands can be produced through the nonlinear coupling between the zonal components and pump waves as expressed as follows

$$
\tilde{\phi}_{\pm}(\mathbf{r}, t) = \sum_{j=0}^{1} \phi_{k_{x\pm j}, k_y} \exp \{ i(k_{x\pm j}x + k_yy - \omega_{\pm j}t) \} + c.c.
$$

with denotations \(k_{x\pm j} = k_{xj} \pm k_q\) and \(\omega_{\pm j} = \omega_{0j} \pm \omega_{qj}\). The frequencies of all sidebands are also subject to the dispersion relation of drift wave. Considering all possible nonlinear coupling among these eight waves involved in the model under equation (1), the perturbed equations of the zonal flow components and corresponding sidebands can be given as follows

$$
\begin{align*}
- \omega_y(1 + k_q^2)\phi_{k_q, 0} \\
= k_qk_y(2k_xk_q + k_y^2)\phi_{k_q+k_xk_y} - k_x - k_y + k_qk_y(k_xk_q - k_q^2)\phi_{k_q+k_xk_y} \\
+ k_y^2[(k_x + k_q + dk_x)^2 - (k_x + dk_x)^2]\phi_{-k_x - dk_x - k_y \phi_{k_x + k_y}} \\
+ k_y^2[-(k_x + k_q - dk_x)^2 + (k_x + dk_x)^2]\phi_{k_x + k_y - k_x - dk_x - k_y}
\end{align*}
$$

\[\tag{6}\]
\[ -i[(\omega_0 + \omega_q)(1 + k_x^2 + k_y^2 + 2k_x k_y) + k_y] \phi_{k_x + k_y} + k_y \phi_{k_x, k_y} = k_y \phi_y (k^2_0 - k_y^2) \phi_{k_x, k_y} \phi_{k_x, 0} \]  
\[ (7) \]

\[ -i[-(\omega_0 + \omega_q)(1 + k_x^2 + k_y^2 - 2k_x k_y) - k_y] \phi_{-k_x + k_y, -k_y} \]
\[ = k_x k_y (k^2_0 - k_y^2) \phi_{-k_x, -k_y} + (k_y + dik_x k_y)[k^2_0 - k_y^2 + 2(k_x - k_y)d k_x] \phi_{-k_x - dik_x, -k_y} \phi_{k_x + dik_x, 0} \]  
\[ (8) \]

\[ -i[(\omega_0 + \omega_q)(1 + (k_x + k_y + dik_x)^2 + k_y^2) + k_y] \phi_{k_x + dik_x, k_y + dik_y} \]
\[ = k_x k_y [(k^2_0 - k_y^2) + 2k_x d k_x + dik_y] \phi_{k_x + dik_x, k_y + dik_y} \phi_{k_x, 0} \]  
\[ + (k_y + dik_x) k_y (k^2_0 - k_y^2) + 2k_x d k_x + dik_y \phi_{k_x + dik_x, k_y + dik_y} \phi_{k_x + dik_x, 0} \]  
\[ (9) \]

\[ -i[(-\omega_0 + \omega_q)(1 + (-k_x + k_y - dik_x)^2 + k_y^2) - k_y] \phi_{-k_x + dik_x, -k_y} \]
\[ = -k_x k_y (k^2_0 - k_y^2) + 2k_x d k_x + dik_y \phi_{-k_x - dik_x, -k_y} \phi_{k_x + dik_x, 0} \]  
\[ (10) \]

\[ -i[\omega_q(1 + (k_y + dik_y)^2)] \phi_{k_x + dik_x, 0} \]
\[ = (k_y + dik_y) k_y [(k_x + k_y + dik_y)^2 + k_y^2 - k^2_0] \phi_{-k_x, -k_y} + (k_x + dik_y) k_y (k_x + dik_y)^2 - (k_x + k_y)^2 + dik_y \phi_{k_x + dik_x, k_y + dik_y} \phi_{-k_x + dik_x, -k_y} \]  
\[ (11) \]

Note that we have assumed that two complex frequencies of zonal flow components are the same, i.e., \( \omega_{q1} = \omega_{q2} = \omega_{q1} \). From equation (6) to equation (11), we can derive a dispersion relation of the zonal flow:

\[ \omega_q = \omega_q(k_x, k_y, dik_x, dik_y, |\phi_{k_x, k_y}|^2), \]  
\[ (12) \]

which is expressed by the eleventh order algebraic equation with respect to the complex zonal flow frequency \( \omega_q = \Omega_q + i\gamma_q \). The maximum growth rate is chosen from the eleven complex roots. Note that in the zonal flow equations, two components are linked through the cross coupling overlapping on the side band components with wave numbers \( (k_{x\pm1}, k_y) \), which depend on the finite band width of the zonal flow. This new cross coupling may enhance or reduce the zonal flow generation due to the complex modulation processes. In the following we will calculate the dispersion relation to investigate the effect of spectral band width.

### 2.2. Numerical results of dispersion relation

First, the growth rate of the zonal flow in this newly developed eight-wave coupling model is compared with that in the conventional four-wave model. Note that the generation of secondary structures is related to the spectral isotropy of ambient turbulence, namely, a radially longer wavelength pump wave \( k_x \ll k_y \) tends to excite zonal flow[15]. Here, we choose radial wave number \( k_x \) from 0.1 to 0.5 and poloidal wave number \( k_y = 0.98 \) for pump fluctuation in order to excite zonal flow efficiently. The spectral difference between two turbulent wave packets or two zonal flow components is chosen to be \( dik_x = 0.0125 \). Note that the generation of the zonal flow depends on the pump amplitude or pump energy, it is assumed that the pump energy is the same in both models to directly compare the growth rates. Hence the pump amplitude of the two components in the eight-wave coupling model is reduced by a factor \( 1/\sqrt{2} \). Figure 2 plots the growth rate of the zonal flow \( \gamma_q \) with respect to the radial wave number of pump modes \( k_x \) in both models. It shows that the growth rate of zonal flow in the eight-wave model is larger than that in the four-wave model. It is understood that this enhancement of zonal flow instability originates from a new modulation loop due to the spectral band width of zonal flow as explained
Figure 2. The growth rate $\gamma_q$ of zonal flow as a function of the wave number of turbulence $k_x$. A solid line shows the growth rate from the eight-wave model and a dotted line is that from the four-wave model. Other parameters are $(k_y, |\phi_0|^2) = (0.98, 4.0)$ for pump wave. We choose wave numbers of the zonal flow $k_q = 0.75$ which provides the maximum growth rate. The distance between two radial wave numbers of turbulence $dk_x$ is chosen to 0.0125.

above. It has been examined that if the modulation contribution of the new cross coupling between the pump ($k_{x1}, k_y$) and the zonal component ($k_{x0}, 0$) to the sideband component ($k_{x1} + k_{q0}, k_y$), which is the same as ($k_{x0} + k_{q1}, k_y$), is artificially removed, the growth rate of each zonal flow component in the eight-wave model become the same as that in the four-wave model as $dk_x \to 0$. On the other hand, if monochromatic zonal flow wave is considered in the coupling system with spectral band width of the pumps but keeping the same potential energy, the result is completely the same as that in four-wave coupling model. These analyses indicate that the new cross coupling due to the spectral band width of zonal flow plays an essential role in the enhancement of zonal flow instability. This new cross coupling path can not only link the two sets of four-wave coupling loops, but also enhance the modulation process through the contribution to the sidebands.

Next, we examine the dependence of the zonal flow growth rate on the spectral difference $dk_x$. Figure 3 plots the growth rate of the zonal flow $\gamma_q$ with respect to $dk_x$. The spectral difference is chosen from $dk_x = 0$ to 0.1. It is shown that the growth rate is approximately 0.594 for $dk_x < 0.1$ where two turbulent components are adjacent to each other, and the $dk_x$ dependence of the zonal flow growth rate is found to be weak in this region. At the limit of $dk_x = 0$ which corresponds to the four-wave model, the growth rate is approximately 0.516. Note that the growth rate is discontinuous at $dk_x = 0$. The result shows that there is a qualitative difference between the models with and without zonal flow spectrum, suggesting that this global nature of the zonal flow should be taken into account in theoretical analyses of the zonal flow generation.

3. Numerical simulation
As shown in theoretical analysis in the minimal modeling, the zonal flow instability is enhanced due to the spectral effects of the zonal flow. In order to justify the validity of the analysis and to evaluate the role of the zonal flow spectrum in realistic turbulence, a spectral code is advanced to solve 2D HM equation. Replacing differential operators in
Figure 3. The growth rate of the zonal flow $\gamma_q$ with respect to the difference between two radial wave numbers of turbulence $dk_x$. Other parameters are $(k_x, k_y) = (0.16, 0.98)$ and $|\phi_0|^2 = 4.0$ for pump wave. We choose wave numbers of the zonal flow $k_q = 0.75$ which provides the maximum growth rate.

space coordinates by radial and poloidal wave numbers, we solve the ordinary differential equations by using the 6-th order Runge-Kutta method with respect to the complex amplitude of each component $\phi(r,t) = \phi_{k_x, k_y}(t) \exp(i(k_x r + k_y y + \theta_{k_x}) + c.c.)$, where $\theta_{k_x}$ indicates initial phase for different $k_x$ in the Fourier space. The grid size $\delta k_x$ and simulation box size $k_{x_{max}}$ in the radial wave number space are chosen to be $\delta k_x = 0.0125$, $k_{x_{max}} = 4.0$, respectively. Here, single component $k_y = 1.0$ is chosen along $y$-direction. Along the $x$-direction, we assume the Gaussian-type spectral distribution in $k_x$ space for the turbulence as $E_{k_x} = E_{k_x}^{0} \exp[-(k_x - k_{xc})^2/2(\Delta k_x)^2]/\Delta k_x$, where $k_{xc}$ and $\Delta k_x$ represent the center value of the spectrum and spectral width, respectively. In performing numerical calculation, we choose 27 discrete pump waves by keeping total pump energy constant, i.e., $E = \sum_{k_x} E_{k_x}^{0} \exp[-(k_x - k_{xc})^2/2(\Delta k_x)^2]/\Delta k_x$, which is the same as that in the four-wave model. Here, $dk_x = 0.0125$, $k_{xc} = 0.1$ and $\Delta k_x = 0.5$, respectively.

Many zonal flow components satisfying $(k_x \neq 0, k_y = 0)$ can be generated through the nonlinear interaction due to the pump spectrum. Firstly, we examine the effect of the zonal flow spectrum on the zonal flow instability by performing two simulations with and without a zonal flow spectrum. The former is a multi-mode case, where we allow all zonal flow components in the numerical calculation. In the latter case, only one zonal flow component is chosen with wave number $k_q = 0.75$ which corresponds to the maximum growth rate in the conventional four-wave model. Figure 4 plots the temporal evolution of total zonal flow energy for all cases. Here, we assume a coherent phase for all initial turbulent components i.e., $\theta_{k_x} = 0$ for simplicity. The growth rate in the case with single zonal flow component is observed to be identical to that in the four-wave model. It can be also seen that the growth rate in the multi-mode case is approximately 1.7, which is more than three times larger than that in the single mode case. Figure 5 shows the growth rate of zonal flow as a function of number of zonal flow $n$. The zonal flow component with wave numbers $(k_y \pm (n - 1)dk_x, 0)$, $n = 1, 2, 3, \cdots$ is included in each numerical calculation one by one. Note that $n = 1$ corresponds to the single mode case as shown in figure 4, i.e., $\gamma_q = 0.52$, and the growth rate finally converges to $\gamma_q \simeq 1.7$, which corresponds to the
Figure 4. The temporal evolution of total zonal flow energy for all cases. We choose \((k_x, k_y) = (0.1, 1.0)\) and \(E(0) = 4.0\) for pump wave. Wave number of the zonal flow seed is chosen as \(k_q = 0.75\) which provides the maximum growth rate. \(dk_x\) is chosen to 0.0125. The grid size \(\delta k_x\) and simulation box size \(k_{x_{\text{max}}}\) for radial wave number space are chosen to \(\delta k_x = 0.0125\), \(k_{x_{\text{max}}} = 4.0\), respectively.

multi-mode case. It is found that the growth rate is enhanced with the number of zonal flow, and two zonal flow components \((n = 2)\) are sufficient to enhance the growth rate. These results support the theoretical analysis above, showing that the eight-wave model can be reasonably proposed as a minimal model including the spectral effect of the zonal flow.

Figure 5. The growth rate of zonal flow as a function of number of zonal flow components \(n\). Parameters are same as shown in figure 4.

Often observed in most experimental and numerical results, the realistic turbulence shows a strongly anisotropic structure. In order to represent this feature, we set a finite but non-zero random phase \(\theta_{k_x}\) for the initial turbulent components in the numerical
calculation. In figure 6, the temporal evolution of zonal flow energy for two different cases of $\theta_{k_x}$ is illustrated. When $\theta_{k_x} \neq 0$, growth rate is approximately 0.85, showing that the enhancement of growth rate is weakened compared to the $\theta_{k_x} = 0$ case. This result reflects the relation of the spatial structure between turbulence and the zonal flow. Figure 7 shows potential structure of both turbulence and zonal flow in $x$-direction at the saturation state $t = 20$ in the cases of (a) $\theta_{k_x} = 0$ and (b) $\theta_{k_x} \neq 0$. The blue and red curves represent potential of turbulence and zonal flow, respectively. When $\theta_{k_x} = 0$, the turbulence and the zonal flow are spatially localized around $-50 < x < 50$. On the other hand, they are delocalized and spreading to the whole region when $\theta_{k_x} \neq 0$. If the initial spatial structure of turbulence is localized, the zonal flow nonlinearly generated via the Reynolds stress of the ambient turbulence is also localized. As a result, the locally large amplitude turbulence may enhance the growth rate of zonal flow. It is also noted that in the case with $\theta_{k_x} \neq 0$, the zonal flow evolves gradually before it reaches the final saturation level.
4. Summary and discussion

In this work, we have theoretically and numerically investigated the spectral effects of both the turbulence and the zonal flow on zonal flow instability in micro-scale ETG turbulence based on the HM equation. Here, a coherent mode coupling method is employed for the modulational analysis of zonal flow generation. To capture the essential feature of the spectral effects of zonal flow, we have extended the conventional four-wave coupling model to a minimal eight-wave model, which includes two sets of the conventional four-wave coupling systems with a spectral shift \( dk_x \). Assuming turbulence as a sum of two sinusoidal waves, a dispersion relation of the zonal flow instability which is represented by the eleventh order algebraic equation with respect to a complex frequency \( \omega_q = \Omega_q + i\gamma_q \) is derived. Numerically solving the dispersion relation, we have shown that the spectral effect of zonal flow can increase its growth rate. The result shows that there is a qualitative difference between models with or without zonal flow spectrum. It is found that a new cross coupling loop through the overlapping of sidebands can enhance the modulation instability of zonal flow generation. The zonal components are coupled each other so that the zonal flow is characterized by a global nature of the zonal flow should be taken into account in some theoretical analyses on the zonal flow generation.

In order to justify the validity of the eight-wave model and to investigate the dependence of the initial phase of the turbulence on the zonal flow instability, a spectral code to solve 2D HM equation is advanced. As an example, we choose a sum of 27 discrete modes for the pump wave to approximate the Gaussian-type turbulent structure in the Fourier space. Numerical simulation shows that the more initial phase becomes coherent, the more the zonal flow instability increases. This result is understandable because the turbulence and the zonal flow are spatially localized with coherent phase relation, the interaction becomes strong locally. In the case with single zonal flow, the growth rate is the same as that from the conventional four-wave model. We conclude that the zonal flow spectrum is a more essential characteristic of the zonal flow instability whereas only the turbulence is characterized by a spectrum in some previous works. Here, we have discussed the zonal flow structure based on the micro-scale ETG driven turbulence-zonal flow system, but the present result can be applied to that of ITG driven turbulence-zonal flow system.

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