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An inverse electromagnetic scattering problem for periodic chiral structures

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Abstract. In this paper, we study the reconstruction of the chirality admittance of periodic chiral structures. The media is homogeneous and the structure is periodic in one direction and invariant in another direction. The electromagnetic fields inside the chiral medium are governed by Maxwell equations together with the Drude-Born-Fedorov equations. We simplify the problem to a two-dimensional inverse scattering problem and simply present the general recursive linearization procedure (continuous homotopy method) for solving the problem.

1. Introduction

The phenomenon of optical activity in special materials has been known since the beginning of the last century. Whereas optical activity has been considered in optics and in quantum mechanics for many years, its analysis within the framework of the classical electromagnetic field theory is much more recent. Recently, there has been a considerable interest in the study of scattering and diffraction by chiral media. In general, the electromagnetic fields inside the chiral medium are governed by Maxwell equations together with the Drude-Born-Fedorov equations in which the electric and magnetic fields are coupled. The chiral media is characterized by the electric permittivity ε, the magnetic permeability µ, and the chirality measure β. On the other hand, periodic structures (gratings) have received increasing attentions through the years because of importance applications in integrated optics, optical lenses, antireflective structures, et al.

Consider the reconstruction of the chirality admittance of periodic chiral structures. The media is homogeneous and the chiral structure is periodic in $x_1$-direction and invariant in $x_3$-direction. In this paper, we simplify the problem to a two-dimensional one and discuss the existence and uniqueness of the scattering problem by using the integral equation method. Then we formulate the inverse scattering problem to reconstruct the chirality admittance and simply present the general recursive linearization procedure (continuous homotopy method) for solving the problem. An important step here is to reduce problem into a two-dimensional domain which is bounded in $x_1$-direction. This is done by noticing the geometric properties of the structures, using the assumptions on the electromagnetic fields and decomposition of the electromagnetic fields. We emphasize that the integral equation method is very general.

For some interesting explanation and references of the model equations, we refer to Lakhtakia [10] and Lakhtakia, Varadan, Varadan [11] (non-periodic chiral structures), Ammi and Nédélec.
[2]. Results and references on closely related periodic structures may be found in Bao and Dobson [4], Dobson and Friedman [8], Chen and Friedman [5], Athanasiadis, Costakis and Stratis [3] and Gerlach [12].

The outline of this paper is as follows. In section 2, the Maxwell equations, the constitutive equations (the Drude-Born-Fedorov equations), and the scattering problem are presented. Section 3 is devoted to the reformulation of the problem; we discuss the existence and uniqueness of the scattering problem by using the integral equation method. In the last section, we formulate the inverse scattering problem and simply present the general recursive linearization procedure.

2. The scattering problem

The electromagnetic fields are governed by the time-harmonic Maxwell equations (time dependence $e^{-i\omega t}$):

\begin{align*}
\nabla \times E - i\omega B &= 0, \quad (2.1) \\
\nabla \times H + i\omega D &= 0, \quad (2.2)
\end{align*}

where $E, H, D$ and $B$ denote the electric field, the magnetic field, the electric and magnetic displacement vectors in $\mathbb{R}^3$, respectively.

In a homogeneous isotropic chiral medium, the following Drude-Born-Fedorov constitutive equations hold:

\begin{align*}
D &= \varepsilon(E + \beta \nabla \times E), \quad (2.3) \\
B &= \mu(H + \beta \nabla \times H), \quad (2.4)
\end{align*}

where $\varepsilon$ is the electric permittivity, $\mu$ is the magnetic permeability, and $\beta$ is the chirality admittance.

Eliminating $B$ and $D$, we get

\begin{align*}
\nabla \times E &= \gamma^2 \beta E + i\omega \mu \frac{\gamma^2}{k^2} H, \quad (2.5) \\
\nabla \times H &= \gamma^2 \beta H - i\omega \varepsilon \frac{\gamma^2}{k^2} E, \quad (2.6)
\end{align*}

where $k = \omega \sqrt{\varepsilon\mu}$, $\gamma^2 = \frac{k^2}{1 - k^2\beta^2}$. With respect to the physical parameters, we make the additional assumption that $|k| < 1$, $x \in \mathbb{R}^3$.

In this paper, we assume that the structure is periodic in $x_1$-direction with period $\Lambda$ and is infinitely long in $x_3$-direction. Assume additionally that all fields don’t depend on the $x_3$ variable.

Consider a plane wave

\begin{align*}
E^i = se^{iq \cdot x}, \quad H^i = pe^{iq \cdot x} \quad (2.7)
\end{align*}

incident on the structure, where $q = (\alpha, -\beta, 0) = \omega \sqrt{\varepsilon_1\mu_1}(\cos \theta, -\sin \theta, 0)$ is the incident wavevector whose direction is specified by $\theta$ with $0 < \theta < \pi$. The vectors $s$ and $p$ satisfy

\begin{align*}
s = \frac{1}{\omega \varepsilon_1}(p \times q), \quad q \cdot q = \omega^2 \varepsilon_1 \mu_1, \quad p \cdot q = 0. \quad (2.8)
\end{align*}

We look for bounded quasi-periodic solutions, i.e. bounded solutions $E, H$ such that $E_\alpha, H_\alpha$ defined by

\begin{align*}
E_\alpha(x_1, x_2) &= e^{-i\alpha x_1} E(x_1, x_2), \quad (2.9) \\
H_\alpha(x_1, x_2) &= e^{-i\alpha x_1} H(x_1, x_2)
\end{align*}

are periodic in $x_1$ direction of period $\Lambda$. 

3. Reformulation of the problem

In chiral medium $\Omega_c$, left-handed and right-handed waves can both propagate independently and with different phase speed. To see this, we consider the decomposition of $E, H$ into suitable Beltrami fields $Q_L, Q_R$

$$
E_c = Q_L + Q_R, \quad H_c = -i\eta_c^{-1}(Q_L - Q_R),
$$

where $\eta_c = \sqrt{\mu_c/\varepsilon_c}$ is the intrinsic impedance of the chiral medium, and

$$
\nabla \times Q_L = \gamma_L Q_L, \quad \nabla \times Q_R = -\gamma_R Q_R,
$$

with

$$
\gamma_L = k_c(1 - k_c\beta)^{-1}, \quad \gamma_R = k_c(1 + k_c\beta)^{-1},
$$

being the wave numbers of $Q_L, Q_R$, respectively. For details on the physical problem we refer to [10] (Lakhtakia et al., 1989) and [11] (Lakhtakia 1994).

As far as the physical parameters, we make the standard assumptions (Lakhtakia, 1994):

$$
\varepsilon, \mu, \beta \in \mathbb{C}, \quad \text{Re} \eta, \text{Re} \gamma_L, \text{Re} \gamma_R > 0, \quad \text{Im} \gamma_L, \text{Im} \gamma_R \geq 0 \quad \text{and} \quad |k\beta| < 1.
$$

Let $Q_L = (u_1, u_2, u)^T, Q_R = (v_1, v_2, v)^T$, then from (3.2), we have

$$
u_1 = \gamma_L^{-1}\partial_{x_2} u, \quad v_1 = -\gamma_R^{-1}\partial_{x_2} v,
$$

and

$$\Delta u + \gamma_L^2 u = 0, \quad \Delta v + \gamma_R^2 v = 0, \quad \text{in } \Omega_c.
$$

In achiral medium $\Omega$, let $E = (e_1, e_2, e)^T, H = (h_1, h_2, h)^T$. Then, we have

$$
e_1 = \frac{i}{\omega \varepsilon_0} \partial_{x_2} h, \quad h_1 = -\frac{i}{\omega \mu_0} \partial_{x_2} e, \quad e_2 = -\frac{i}{\omega \varepsilon_0} \partial_{x_1} h, \quad h_2 = \frac{i}{\omega \mu_0} \partial_{x_1} e,
$$

and

$$\Delta e + k_0^2 e = 0, \quad \Delta h + k_0^2 h = 0, \quad \text{in } \Omega.
$$

From the following transmission boundary conditions (Lakhtakia, 1994, p.466):

$$
n \times (E - E_c) = 0, \quad n \times (H - H_c) = 0,
$$

where $n$ is the unit normal to the boundary, we have the following transmission conditions for $e, h, u$ and $v$ on the boundary:

$$
u + v - e = 0, \quad u - v - i\eta_c h = 0,
$$

and

$$
\frac{1}{\gamma_L} \partial_n u + \frac{1}{\gamma_R} \partial_n v - \frac{\eta_c}{\omega \mu_0} \partial_n e = 0, \quad \frac{1}{\gamma_L} \partial_n u - \frac{1}{\gamma_R} \partial_n v - \frac{i}{\omega \varepsilon_0} \partial_n h = 0.
$$
As can be seen from the transmission conditions, a coupling of electric and magnetic fields occurs. To partly reduce the coupling, we introduce new fields \( a, b \) by

\[
a = \frac{1}{2} \left( \sqrt{\varepsilon_0 \mu_e} e + i \sqrt{\varepsilon_0 \mu_r} h \right),
\]

\[
b = \frac{1}{2} \left( \sqrt{\varepsilon_0 \mu_e} e - i \sqrt{\varepsilon_0 \mu_r} h \right).
\]

With the new variables we have the following equivalent formulation where the unknown functions satisfy the Helmholtz equation.

**Problem** For given \( f_1, f_2, g_1, g_2 \), find bounded quasi-periodic functions \( a, b \in C^2(\Omega) \cap \Theta(\Pi) \) and \( u, v \in C^2(\Omega_c) \cap \Theta(\Pi_c) \) which satisfy the differential equations

\[
\Delta a + k_0^2 a = 0, \quad \text{in } \Omega, \quad \Delta u + \gamma_L^2 u = 0, \quad \text{in } \Omega_c,
\]

\[
\Delta b + k_0^2 b = 0, \quad \text{in } \Omega, \quad \Delta v + \gamma_R^2 v = 0, \quad \text{in } \Omega_c,
\]

and the boundary conditions

\[
\begin{align*}
\frac{\partial u}{\partial n} - \frac{\gamma_L}{k_0} \frac{\partial a}{\partial n} &= g_1, & u - \xi_1 a - \xi_2 b &= f_1, \quad \text{on } L_1, \\
\frac{\partial v}{\partial n} - \frac{\gamma_R}{k_0} \frac{\partial b}{\partial n} &= g_2, & v - \xi_2 a - \xi_1 b &= f_2, \\
\frac{\partial u}{\partial n} - \frac{\gamma_L}{k_0} \frac{\partial a}{\partial n} &= 0, & u - \xi_1 a - \xi_2 b &= 0, \quad \text{on } L_2, \\
\frac{\partial v}{\partial n} - \frac{\gamma_R}{k_0} \frac{\partial b}{\partial n} &= 0, & v - \xi_2 a - \xi_1 b &= 0,
\end{align*}
\]

and the constants are given by

\[
\lambda = (\mu_0 \varepsilon_e)^{1/2} (\mu_e \varepsilon_0)^{-1/2}, \quad \xi_1 = \frac{1}{2} (\lambda + \lambda^{-1}), \quad \xi_2 = \frac{1}{2} (\lambda - \lambda^{-1}).
\]

4. Solvability of the scattering problem

From now on, we consider the problem in one period. Let

\[
\begin{align*}
\Omega_1 &= \{ x \in \mathbb{R}^2; 0 < x_1 < \Lambda, f_1(x_1) < x_2 \}, \\
\Omega_c &= \{ x \in \mathbb{R}^2; 0 < x_1 < \Lambda, f_2(x_1) < x_2 < f_1(x_1) \}, \\
\Omega_2 &= \{ x \in \mathbb{R}^2; 0 < x_1 < \Lambda, x_2 < f_2(x_1) \}, \\
L_1 &= \{ x \in \mathbb{R}^2; 0 < x_1 < \Lambda, x_2 = f_1(x_1) \}, \\
L_2 &= \{ x \in \mathbb{R}^2; 0 < x_1 < \Lambda, x_2 = f_2(x_1) \},
\end{align*}
\]

where functions \( f_1, f_2 \in C^2, f_1(x_1 + \Lambda) = f(x_1), f_2(x_1 + \Lambda) = f(x_1), f_2(x_1) < f_1(x_1), \forall x_1 \in \mathbb{R} \).

We denote the quasi-periodic fundamental solution of the Helmholtz equation \( \Delta u + k_0^2 u = 0 \) by

\[
\Psi(x) = \frac{1}{2\Lambda} \sum_{n \in \mathbb{Z}} \frac{1}{\beta_n} e^{i\beta_n |x_2| - i(\alpha_n - \alpha) x_1}, \quad x \in \mathbb{R}^2 \setminus \Theta,
\]

(4.1)
where

\[ k_0^2 \neq (\alpha_n - \alpha)^2, \quad \forall n \in \mathbb{Z}, \]
\[ \alpha_n = 2n\pi/\Lambda, \]
\[ \beta_n = e^{i\theta_n/2}|k_0^2 - (\alpha_n - \alpha)^2|^{1/2}, \quad n \in \mathbb{Z}, \]
\[ \theta_n = \arg\{k_0^2 - (\alpha_n - \alpha)^2\}, \]
\[ 0 \leq \theta_n < 2\pi, \]
\[ \Theta = \{ x \in \mathbb{R}^2; x = (n\Lambda, 0), n \in \mathbb{Z} \}, \]

and it is easy to see that

\[ \beta_n = \begin{cases} \sqrt{k_0^2 - |\alpha + \alpha_n|^2}, & k_0^2 > |\alpha + \alpha_n|^2; \\ i\sqrt{|\alpha + \alpha_n|^2 - k_0^2}, & k_0^2 < |\alpha + \alpha_n|^2. \end{cases} \]

By using Green formula and the penetrable boundary conditions, we get the generalized Lippmann-Schwinger equations:

\[ u(x) = \frac{\gamma_L}{k_0} a^i + (\gamma_L^2 - k_0^2) \int_{\Omega_c} u(y)\Psi(x, y)dy \]
\[ + \frac{\gamma_L}{k_0} \int_{L_1 \cup L_2} \{(\frac{k_0}{\gamma_L} - \xi_1)u(y) + \xi_2v(y)\}\partial_n\Psi(x, y)dy, \quad x \in \Omega_c, \quad (4.2) \]

\[ v(x) = \frac{\gamma_R}{k_0} b^i + (\gamma_R^2 - k_0^2) \int_{\Omega_c} v(y)\Psi(x, y)dy \]
\[ + \frac{\gamma_R}{k_0} \int_{L_1 \cup L_2} \{(\frac{k_0}{\gamma_R} - \xi_1)v(y) + \xi_2u(y)\}\partial_n\Psi(x, y)dy, \quad x \in \Omega_c, \quad (4.3) \]

\[ a_1(x) = a^i + \frac{k_0}{\gamma_L} (\gamma_L^2 - k_0^2) \int_{\Omega_c} u(y)\Psi(x, y)dy \]
\[ + \int_{L_1 \cup L_2} \{(\frac{k_0}{\gamma_L} - \xi_1)u(y) + \xi_2v(y)\}\partial_n\Psi(x, y)dy, \quad x \in \Omega_1 \cup \Omega_2, \quad (4.4) \]

\[ b_1(x) = b^i + \frac{k_0}{\gamma_R} (\gamma_R^2 - k_0^2) \int_{\Omega_c} v(y)\Psi(x, y)dy \]
\[ + \int_{L_1 \cup L_2} \{(\frac{k_0}{\gamma_R} - \xi_1)v(y) + \xi_2u(y)\}\partial_n\Psi(x, y)dy, \quad x \in \Omega_1 \cup \Omega_2, \quad (4.5) \]

For equations (4.2) and (4.3), let \( x \to \partial \Omega_c \); then from potential theory, we have the following second Fredholm integral equations

\[ (\mathcal{M} + \mathcal{K})w = w^i, \quad w \in (L^2(\Omega_c))^2 \times (L^2(L))^2 \]

(4.6)

where \( w = (u, v, \phi, \psi)^T, \phi = u|_{L}, \psi = v|_{L}, L = L_1 \cup L_2, \)

\[ \mathcal{M} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 + \frac{\gamma_L}{k_0} \xi_1 & -\frac{\gamma_L}{k_0} \xi_2 \\ 0 & 0 & -\frac{\gamma_R}{k_0} \xi_2 & 1 + \frac{\gamma_R}{k_0} \xi_1 \end{pmatrix}. \]
the operators in \( \overline{K} \) are defined by

\[
\begin{pmatrix}
(k_0^2 - \gamma_L^2) S & 0 & \frac{\gamma_L}{k_0} \xi_1 - 1) K & -\frac{\gamma_L}{k_0} \xi_2 K \\
0 & (k_0^2 - \gamma_R^2) S & -\frac{\gamma_R}{k_0} \xi_2 K & \frac{\gamma_R}{k_0} \xi_1 - 1) K \\
(k_0^2 - \gamma_L^2) \tilde{S} & 0 & \frac{\gamma_L}{k_0} \xi_1 - 1) \tilde{K} & -\frac{\gamma_L}{k_0} \xi_2 \tilde{K} \\
0 & (k_0^2 - \gamma_R^2) \tilde{S} & -\frac{\gamma_R}{k_0} \xi_2 \tilde{K} & \frac{\gamma_R}{k_0} \xi_1 - 1) \tilde{K}
\end{pmatrix},
\]

It is obvious that matrix \( \overline{M} \) is invertible iff \( 1 + \frac{\gamma_L \gamma_R}{k_0^2} + \xi_1 \frac{\gamma_L + \gamma_R}{k_0^2} \neq 0 \). The operator \( \overline{K} : (L^2(\Omega_c))^2 \times (L^2(S))^2 \to (L^2(\Omega_c))^2 \times (L^2(S))^2 \) is compact. Then from Fredholm alternative theorem, we have the following theorem.

**Theorem 1** For all but possibly a discrete set of wave numbers, there is a unique solution to the integral equation (4.6) in \( (L^2(\Omega_c))^2 \times (L^2(S))^2 \).

5. The inverse scattering problem

In this section, we study reconstruction of the chirality admittance \( \beta \) of periodic homogeneous chiral structures. The problem is: given the shape, the electric permittivity and magnetic permeability of the periodic homogeneous chiral medium, to reconstruct the chirality admittance \( \beta \) from the incident waves and the scattered fields.

Now we propose a general recursive linearization method. Let \( \Omega_1 = \{(x_1, x_2); 0 \leq x_1 \leq \Lambda, x_2 = f_1(0)\}, \Omega_2 = \{(x_1, x_2); 0 \leq x_1 \leq \Lambda, x_2 = f_2(0)\}, f_1^0 = f_1(0) \) and \( f_2^0 = f_2(0) \). We also let

\[
\begin{align*}
f(x_1, t) &= tf_1(x_1) + (1 - t)f_1^0, \\
g(x_1, t) &= tf_2(x_1) + (1 - t)f_2^0,
\end{align*}
\]

where \( 0 \leq x_1 \leq \Lambda \). Define

\[
L_1(t) = \{(x_1, x_2); 0 \leq x_1 \leq \Lambda, x_2 = f(x_1, t)\},
\]

\[
L_2(t) = \{(x_1, x_2); 0 \leq x_1 \leq \Lambda, x_2 = g(x_1, t)\},
\]

and

\[
\begin{align*}
\Omega_1(t) &= \{(x_1, x_2); 0 \leq x_1 \leq \Lambda, L_1(t) < x_2\}, \\
\Omega_2(t) &= \{(x_1, x_2); 0 \leq x_1 \leq \Lambda, L_2(t) < x_2 < L_1(t)\}
\end{align*}
\]

If \( t = 1 \), then \( \Omega_c(1) = \Omega \); if \( t = 0 \), we can get the analytic expression of the solution for the scattering problem.
Let
\[ F(\beta, t) = \begin{pmatrix} F_1(\beta, t) \\ F_2(\beta, t) \end{pmatrix}, \tag{5.1} \]
where
\[ F_1(\beta, t) = a^i + \frac{k_0}{\gamma_L} (\gamma_L^2 - k_0^2) \int_{\Omega_c(t)} u(y) \Psi(x, y) dy \]
\[ + \int_{L_1(t) \cup L_2(t)} \left\{ \left( \frac{k_0}{\gamma_L} - \xi_1 \right) u(y) + \xi_2 v(y) \right\} \partial_{n_y} \Psi(x, y) ds_y, \]
\[ F_2(\beta, t) = b^i + \frac{k_0}{\gamma_R} (\gamma_R^2 - k_0^2) \int_{\Omega_c(t)} v(y) \Psi(x, y) dy \]
\[ + \int_{L_1(t) \cup L_2(t)} \left\{ \left( \frac{k_0}{\gamma_R} - \xi_1 \right) v(y) + \xi_2 u(y) \right\} \partial_{n_y} \Psi(x, y) ds_y. \]

The inverse problem can be formulated as the following nonlinear operator equation:
\[ F(\beta, t) = y, \tag{5.2} \]
where \( F : (\beta, t) \in \mathbb{R} \times \mathbb{R} \mapsto y \in Y \), the parameter space \( \mathbb{R} \times \mathbb{R} \) and the observation space \( Y = L^2(\Gamma_j) \times L^2(\Gamma_j) \) are Hilbert spaces.

From equation (5.2), we have
\[ F_\beta(\beta, t) \beta'(t) + F_1(\beta, t) = 0, \tag{5.3} \]
where \( F_\beta(\beta, t) \) and \( F_1(\beta, t) \) are the Fréchet derivatives of \( F \) with respect to \( \beta \) and \( t \), respectively.

We simply present the general recursive linearization procedure (continuous homotopy method) for solving the problem (5.2).

Assume that \( \forall t \in [0, 1] \), there exists a unique solution \( \tilde{\beta}(t) \) of \( F(\beta, t) = y \) which depends on \( t \) continuously. Let
\[ \tilde{\beta}_0 = \tilde{\beta}(0), \tilde{\beta}_1 = \tilde{\beta}(t_1), \ldots, \tilde{\beta}_N = \tilde{\beta}(1) \]
be exact solutions of the equation (5.2) at \( t = t_0, t_1, \ldots, t_N \), respectively, where \( t_i = i \Delta t, i = 0, 1, \ldots, N, \Delta t = 1/N \). A recursive linearization method for finding an approximation of \( \tilde{\beta}(1) \) is as follows. Let \( \beta_0, \beta_1, \ldots, \beta_N \) be the approximation solutions for (5.2). Assume that \( \alpha > 0 \) is a regularization parameter and \( R_k, \alpha, k = 1, 2, \ldots, N, \) are the regularization schemes. Given \( \beta_0 = \beta(0) \), for \( k = 0, 1, \ldots, N, \beta_k \) is computed by
\[ \beta_k = \beta_{k-1} - R_k, \alpha (F(\beta_{k-1}, t_k) - y), \]
where \( R_k, \alpha = (\alpha I + A_k^* A_k)^{-1} A_k^* \) with \( A_k = F_\beta(\beta_{k-1}, t_k) \) is the Tikhonov regularization. Finally, \( \beta_N \) is taken to be an approximation of the solution \( \tilde{\beta}(1) \) of \( F(\beta, 1) = y \).

In the above method, we have to compute \( \partial_{\beta} u \) (denoted by \( u_\beta \)) and \( \partial_{\beta} v \) (denoted by \( v_\beta \)). From equations (4.2) and (4.3), we have
\[ u_\beta = u_0 + S_1(u, v) + T_1(u_\beta, v_\beta) \]
\[ v_\beta = v_0 + S_2(u, v) + T_2(u_\beta, v_\beta) \tag{5.4} \]
where \( u_0 = \frac{\gamma_L^2}{k_0} a^i, v_0 = -\frac{\gamma_R^2}{k_0} b^i \),
\[ S_1(u, v) = 2\gamma_L^2 \int_{\Omega_c} u(y) \Psi(x, y) dy \]
\[ + \frac{\gamma_L^2}{k_0} \int_{L_1(t) \cup L_2} \{ -\xi_1 u(y) + \xi_2 v(y) \} \partial_{n_y} \Psi(x, y) ds_y, \]
\[ S_2(u, v) = 2\gamma_R^2 \int_{\Omega_c} v(y) \Psi(x, y) dy \]
\[ + \frac{\gamma_R^2}{k_0} \int_{L_1(t) \cup L_2} \{ -\xi_1 v(y) + \xi_2 u(y) \} \partial_{n_y} \Psi(x, y) ds_y. \]
\[ S_2(u, v) = -2\gamma_3^2 \int_{\Omega} v(y) \Psi(x, y) dy \]
\[ + \frac{\gamma_3^2}{k_0} \int_{L_1 \cup L_2} \left\{ \xi_1 v(y) - \xi_2 u(y) \right\} \partial_{n_y} \Psi(x, y) ds_y. \]
\[ T_1(u, v) = (\gamma_L^2 - k_0^2) \int_{\Omega} u(y) \Psi(x, y) dy \]
\[ + \frac{\gamma_L}{k_0} \int_{L_1 \cup L_2} \left\{ \left( \frac{k_0}{\gamma_L} - \xi_1 \right) u(y) + \xi_2 v(y) \right\} \partial_{n_y} \Psi(x, y) ds_y, \]
\[ T_2(u, v) = (\gamma_R^2 - k_0^2) \int_{\Omega} v(y) \Psi(x, y) dy \]
\[ + \frac{\gamma_R}{k_0} \int_{L_1 \cup L_2} \left\{ \left( \frac{k_0}{\gamma_R} - \xi_1 \right) v(y) + \xi_2 u(y) \right\} \partial_{n_y} \Psi(x, y) ds_y. \]

We can first solve the equations (4.2)-(4.3), and then the equation (5.4). After that, we can get \( u_\beta \) and \( v_\beta \) and compute \( F_\beta(\beta, t) \).

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