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The spectral functions method for ultrasonic plane wave diffraction by a soft wedge

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Abstract. NDE examination of industrial structures requires the modelling of specimen geometry echoes generated by the surfaces (entry, backwall...) of inspected blocks. For that purpose, the study of plane elastic wave diffraction by a wedge is of great interest since surfaces of complex industrial specimen often include dihedral corners. There exist various approaches for modelling the plane elastic wave diffraction by a wedge but for the moment, the theoretical and numerical aspects of these methods have only been developed for wedge angles lower than π. Croisille and Lebeau [1] have introduced a resolution method called the Spectral Functions method in the case of an immersed elastic wedge of angle less than π. Kamotski and Lebeau [2] have then proven existence and uniqueness of the solution derived from this method to the diffraction problem of stress-free wedges embedded in an elastic medium. The advantages of this method are its validity for wedge angles greater than π and its adaptability to more complex cases. The methodology of Croisille and Lebeau [1] has been first extended by the authors of the current communication to the simpler case of an immersed soft wedge [3]. The outline of their methodology is presented here and an application to the case of longitudinal incident and scattered waves in the case of the acoustic limit of the elastic code is presented.

1. Introduction

Ultrasonic inspection of a specimen generates echoes from the entry and backwall surfaces of this specimen. If these surfaces contain wedges, it is then necessary to provide a correct model of the interaction between the ultrasonic beam and these wedges. These interactions may be linked to two different phenomena : reflection from the wedge faces and diffraction of the incident rays by the wedge edge. Both must be correctly taken into account by the model.

Abrahams has studied the problem of elastic wave diffraction by a half-space [4] and by a crack [5], but this work does not include the specific problem of wedge diffraction. A study of the existing models for the problem of wedge diffraction shows that the specular model (a ray-tracing method based on geometrical optics) developed by CEA/LIST and partners in the NDT simulation platform CIVA [6] is much faster than other numerical models (finite elements or finite differences for example). However, it computes reflection but not diffraction. Based on the Physical Theory of Diffraction (PTD) introduced by Ufmitsev [7], an ultrasonic system model has been developed for a half-plane by Zernov et al. [8] and extended to mimic ultrasonics with some head waves [9, 10]. Nevertheless, this ultrasonic PTD model can be time consuming for large specimen surfaces. A second solution to this problem, called the Uniform Theory of Diffraction (UTD) was proposed in elastodynamics by Kamta Djakou et al. [11] and developed for a half-plane scatterer. It combines the specular model with a diffraction model. Our future idea is to extend UTD to the wedge case, which needs the development of a robust wedge diffraction model.

To apply the aforementioned UTD method, a generic and trustworthy wedge diffraction model is necessary. Such models, such as the Laplace Transform (LT) method or the Sommerfeld Integral (SI) method, have been developed. The LT method was originally developed by Gautesen for elastic quarter-spaces [12, 13] and later extended to the case of a scattered Rayleigh wave [14, 15] for wedge angles smaller and greater than π. However the range of the wedge angle was restricted to the range $[63^o, 180^o]$ for angles smaller than π and to $[189^o, 327^o]$ for angles greater than π in order to avoid numerical instabilities. The SI method was first introduced for the case of an incident Rayleigh wave by Budaev and Bogy [16] and clarified by Kamotski et al. [17]. Both these methods have been extended by Gautesen and Fradkin [18] to the case of an elastic incident wave (not necessarily a Rayleigh wave) on a stress-free wedge, but only for wedge angles lower than π. In addition, they are not valid for 3D configurations, for which the incident wave vector is not necessarily in the plane normal to the edge, and have only been developed for stress-free wedges.

The spectral functions method for diffraction by a wedge presents numerous advantages. First, it works for all wedge angles (and notably for angles higher than π). Secondly, it requires a very short computation time. It is a generic method adaptable to much more complex cases such as 3D configurations or various types of media (impedant wedges for instance). It was originally developed by Croisille and Lebeau [1] and applied to an immersed elastic wedge. The methodology was extended to the case of a stress-free wedge embedded in an elastic solid by Kamotski and Lebeau [2] but the numerical aspects and the final solution were not dealt with. The study of further configurations is to come and an article concerning the case of a stress-free wedge in acoustics has been submitted [3].

The following paper begins by presenting the problem and defining the diffraction coefficient. Section 3 deals with defining the outgoing solution of the diffraction problem and finding an integral formulation of this solution. This integral formulation is expressed in terms of two unknown functions called the spectral functions. In section 4, a system of functional equations solved by the spectral functions is determined. This system will be solved semi-analytically. Section 5 presents some numerical results and section 6 gives the conclusions and perspectives of this work.

2. Problem statement

Let us consider the diffraction problem of a plane longitudinal elastic wave $\mathbf{u}^{inc}$ incident on a wedge delimited by the stress-free infinite plane faces $\mathcal{S}_1$ and $\mathcal{S}_2$. the inside of the wedge is defined by :

$$\Omega = \{(r\cos\theta, r\sin\theta)\backslash\theta \in]0, \varphi[\}$$

And the incident plane wave is of the form

$$\mathbf{u}^{inc}(\mathbf{x}, t) = \mathbf{A}_L e^{i(\mathbf{p}_L^{inc}\cdot\mathbf{x} - \omega t)}$$

$\mathbf{A}_L$ is the amplitude vector and $\mathbf{p}_L^{inc}$ is the incident wave vector. The Cartesian coordinate system $(O; \mathbf{e}_{x_1}, \mathbf{e}_{y_1})$ is linked to the face $\mathcal{S}_1$ of the wedge and $(O; \mathbf{e}_{x_2}, \mathbf{e}_{y_2})$ is linked to the face

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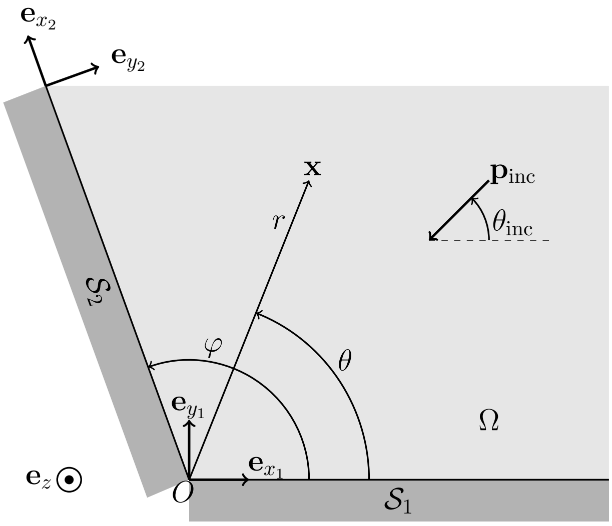

Figure 1: Plane wave incident on a stress-free wedge of angle φ

$\mathcal{S}_2$, as shown in figure 1. These Cartesian coordinate systems have the same origin located on the wedge edge which coincides with the z-axis. Let $\mathbf{x} = (x_1, y_1)_{(\mathbf{e}_{x_1},\mathbf{e}_{y_1})} = (x_2, y_2)_{(\mathbf{e}_{x_2},\mathbf{e}_{y_2})}$ be a position vector $\mathbf{x} = (r, \theta)$ in a local basis of polar coordinates associated to the Cartesian coordinates (x_1, y_1).

In the coordinate system $(\mathbf{e}_{x_1}, \mathbf{e}_{y_1})$, the incident wave vector is given by :

$$\mathbf{p}_L^{inc} = \frac{\omega}{c_L}\begin{pmatrix}\cos\theta_{inc}\\ \sin\theta_{inc}\end{pmatrix} \tag{1}$$

where $c_L = \sqrt{(\underline{\lambda} + 2\underline{\mu})/\rho}$ is the velocity of the longitudinal waves and $\underline{\lambda}, \underline{\mu}$ are the Lamé coefficients.

In the following, vectors are expressed in the coordinate system $(O; \mathbf{e}_{x_1}, \mathbf{e}_{y_1})$, except when explicitly stated otherwise. The displacement field $\mathbf{u}$ is then solution to the linear elasticity problem for an isotropic homogeneous material and verifies stress-free boundary conditions on $\mathcal{S}_1$ and $\mathcal{S}_2$. Bold letters will hereafter be reserved for matrices in order to simplify notations and the harmonic time-factor $e^{-i\omega t}$ is omitted.

Let us suppose that the total field is the sum of the incident field and of an edge diffracted field :

$$u = u^{inc} + u_0 \tag{2}$$

The dimensionless problem is obtained by applying the following change in variables :

$$u_0(x, y) = v\left(\frac{\omega}{c_L}x, \frac{\omega}{c_L}y\right) \tag{3}$$

The problem we wish to solve is now

$$(\mathcal{P}^*) \qquad \begin{cases} (E+1)v = 0 & (\Omega) \\ Bv = -Bv_L^{inc} & (\mathcal{S}_1 \cup \mathcal{S}_2) \end{cases} \tag{4}$$

where E is the dimensionless elasticity operator and B is the normal stress operator (n being in equation (5) the inward normal to each face of the wedge).

$$\begin{aligned} Ev &= \mu\Delta v + (\lambda+\mu)\nabla\,\nabla v \\ Bv &= (\lambda\nabla v.\mathbb{I}_{\mathbf{2}} + 2\mu\varepsilon(v)).n \end{aligned} \tag{5}$$

The dimensionless Lamé coefficients are given by :

$$\lambda = \frac{\lambda}{\rho c_L^2}, \quad \mu = \frac{\mu}{\rho c_L^2}$$

and we have :

$$\varepsilon_{ij} = \frac{1}{2}(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i})$$

The dimensionless incident longitudinal wave is given by :

$$v_L^{inc}(r,\theta) = \begin{pmatrix}\cos\theta_{inc} \\ \sin\theta_{inc}\end{pmatrix} e^{ir\cos(\theta-\theta_{inc})} \quad , \tag{6}$$

In the far field approximation ($\frac{\omega r}{c_L} >> 1$, r being the distance of propagation, see figure 1), the edge-diffracted longitudinal wave can be expressed as a cylindrical wave, proportional to the incident wave, with $\frac{1}{\sqrt{r}}$ spreading factor and weighted by a coefficient D_L^L called the diffraction coefficient which depends only on the direction of observation θ. The diffraction coefficient is therefore defined by :

$$v(r\cos\theta, r\sin\theta) = D_L^L(\theta)\frac{e^{-ir}}{\sqrt{r}} v_L^{inc}(r\cos\theta, r\sin\theta) \tag{7}$$

In order to obtain a far-field approximation of the longitudinal wave diffracted by a wedge impinged by a longitudinal plane wave, it is sufficient to compute the diffraction coefficient, defined by equation (7). The aim of the spectral functions method is to compute this coefficient.

3. Integral formulation of the solutions

Kamotski and Lebeau [2] have applied the spectral functions method to the case of a stress-free elastic wedge. They have used the method to prove the existence and uniqueness of the outgoing solution to the diffraction problem but have not computed or developed such a solution. In the following, the main steps of their method and its application to the computation of the solution are presented.

We begin by defining the double Fourier transform of a tempered distribution f and its inverse:

$$\hat{f}(\xi,\eta) = \int\int_{\mathbb{R}^2} f(x,y)e^{-i(x\xi+y\eta)}\,dx\,dy \tag{8a}$$

$$f(x,y) = \frac{1}{4\pi^2}\int\int_{\mathbb{R}^2} \hat{f}(\xi,\eta)e^{i(x\xi+y\eta)}\,d\xi\,d\eta \tag{8b}$$

Following the formalism of Kamotski and Lebeau [2], the outgoing solution of the problem ($\mathcal{P}^*$), meaning the one which corresponds to the physical reality of the wedge diffraction problem is the sum of two terms, v_1 is the contribution due to face $\mathcal{S}_1$ and v_2 is the contribution due to face $\mathcal{S}_2$.

$$v = v_1 + v_2 \tag{9}$$

with

$$v_j(x_j, y_j) = \frac{1}{4\pi^2} \lim_{\epsilon \to 0} \int_{-\infty}^{+\infty} e^{ix_j\xi} \left(\int_{-\infty}^{+\infty} e^{iy_j\eta} (\mathbf{M}(\xi,\eta) - e^{-2i\epsilon}\mathbb{I}_{\mathbf{2}})^{-1} \, d\eta \right) \Sigma_j(\xi) \, d\xi \tag{10}$$

where Σ_j are unknown functions called the spectral functions

$$\Sigma_j(\xi) = \begin{pmatrix} \hat{\alpha_j}(\xi) \\ \hat{\beta_j}(\xi) \end{pmatrix} \tag{11}$$

and the operator $\mathbf{M}$ is the Fourier transform of the operator E. The computation of the unknown functions $\alpha_j, \beta_j \in L^2(\mathcal{S})$ (more accurately of their Fourier transforms) will be treated in section 4.

The poles of $(\mathbf{M}(\xi,\eta) - e^{-2i\epsilon}\mathbb{I}_{\mathbf{2}})$ are $\eta = \pm\zeta_*^\epsilon(\xi)$, where

$$\zeta_*^\epsilon(\xi) = \sqrt{e^{-2i\epsilon}\nu_*^2 - \xi^2}, \tag{12}$$

and

$$\nu_L = 1, \qquad \nu_T = \frac{c_L}{c_T}, \tag{13}$$

where $c_T = \sqrt{\mu/\rho}$ is the velocity of the transversal waves. The square root is defined by choosing the branch cut which is continuous on $\mathbb{R}$ and whose imaginary part is positive along the real axis. The inner integral can be computed using Cauchy's residue theorem.

This integral can be approached in the far-field approximation by using the stationary phase method. By identifying the result of this approximation with equation (7), we can express the longitudinal diffraction coefficient in terms of the spectral functions :

$$\begin{aligned} D_L^L(\theta) = \frac{e^{-i\pi/4}}{2\sqrt{2\pi}} \big(&\Sigma_1^1(-\cos\theta)\cos\theta + \Sigma_1^2(-\cos\theta)\sin\theta \\ &+ \Sigma_2^1(-\cos(\varphi-\theta))\cos(\varphi-\theta) + \Sigma_2^2(-\cos(\varphi-\theta))\sin(\varphi-\theta) \big) \end{aligned} \tag{14}$$

We have defined the spectral functions and have given an expression of the diffraction coefficient in terms of these functions. In the following section, we explain how these can be computed semi-analytically.

4. Semi-analytical resolution

In order to compute the spectral functions there are two steps. The first one is to determine a system of functional equations of which the spectral functions are solution. Then second is to use this system to prove that the spectral functions can be decomposed into two parts : a singular part and a regular part.

4.1. Functional equations

To determine a system of functional equations of which the spectral functions are solution, we begin by injecting decomposition (9) into the wedge boundary conditions (4). The boundary condition can be taken separately on each face and expressed using the corresponding coordinate system :

$$\begin{cases} B\big(v_1(x_1,0) + v_2(x_2\cos\varphi, x_2\sin\varphi)\big) = -Bv_L^{inc}|_{\mathcal{S}_1} \\ B\big(v_2(x_2,0) + v_1(x_1\cos\varphi, x_1\sin\varphi)\big) = -Bv_L^{inc}|_{\mathcal{S}_2} \end{cases} \tag{15}$$

(v_j^1, v_j^2) are the coordinates of v_j in the system (x_j, y_j).

Thanks to this decomposition, two new operators can be defined. B_1 is obtained by expressing the normal stress operator on $\mathcal{S}_j$ in terms of (x_j, y_j). This expression of the normal stress operator can then be projected onto the coordinate system $(O; e_{x_{3-j}}, e_{y_{3-j}})$ and expressed in terms of (x_{3-j}, y_{3-j}), yielding the operator B_2. The system of boundary conditions can then be written as :

$$\left\{ \begin{array}{l} B_1(v_1) + B_2(v_2) = -Bv_L^{inc}|_{\mathcal{S}_1} \\ B_1(v_2) + B_2(v_1) = -Bv_L^{inc}|_{\mathcal{S}_2} \end{array} \right. \tag{16}$$

Let us now take the Fourier transform of (16). To do so, the partial derivative of (10) with respect to x and y is computed for $y = 0, x \geq 0$. The expressions of these partial derivatives can then be inserted into B_1. The Fourier transform is then applied to B_1 :

$$\begin{aligned} \int_0^{+\infty} e^{-ix\xi} B_1(v_1)(x)\, dx &= \frac{1}{2}\mathrm{DM}(\Sigma_1)(\xi) \\ &= \frac{1}{2}\int_{\Gamma_0} \mathrm{DM}(\xi,\zeta)\Sigma_1(\zeta)\, \mathrm{d}\zeta \end{aligned} \tag{17}$$

with

$$\begin{aligned} \mathrm{DM}(\xi,\zeta) &= \frac{1}{2i\pi}\frac{1}{\xi-\zeta}\mathbf{dm}(\zeta) \\ &= \frac{1}{2i\pi}\frac{1}{\xi-\zeta}\begin{pmatrix} -1 & A(\zeta) \\ B(\zeta) & -1 \end{pmatrix} \end{aligned} \tag{18}$$

and

$$\begin{aligned} A(z) &= \frac{z}{\zeta_T(z)}(1 - 2\mu Q(z)) \\ B(z) &= -\frac{z}{\zeta_L(z)}(1 - 2\mu Q(z)) \\ Q(z) &= \zeta_L(z)\zeta_T(z) + z^2 \end{aligned} \tag{19}$$

The contour Γ_0 is represented in figure 2.

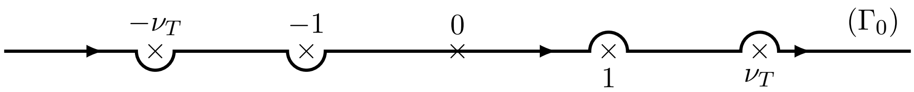

Figure 2: Integration contour Γ_0.

To compute the Fourier transform of operator the B_2, a new translation operator must be defined.

$$T_*(\xi = \nu_* \cos z) = \zeta\cos\varphi + \zeta_*(\zeta)\sin\tilde{\varphi} = \nu_*\cos(z + \tilde{\varphi}) \tag{20}$$

where the following notation is used:

$$\tilde{\varphi} = \left\{ \begin{array}{ll} \varphi & \text{if } \varphi < \pi \\ 2\pi - \varphi & \text{if } \varphi \geq \pi \end{array} \right. \tag{21}$$

This translation operator is well defined on the following domain, represented in figure 3.

$$\Omega_*^+ = \{\xi = \nu_*\cos z, 0 \leq \mathrm{Re} z < \pi - \tilde{\varphi}\} \tag{22}$$

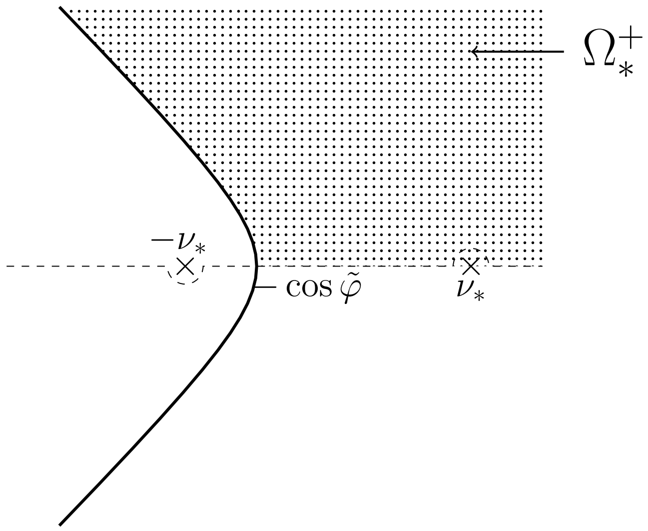

Figure 3: Domain Ω_*^+

The Fourier transform of B_2 is obtained in a similar manner as for B_1. The partial derivative of equation (10) with respect to x and y is computed this time for $x' = x\cos\varphi, y' = x\sin\varphi$. The expressions of these partial derivatives are then inserted into B_2. The Fourier transform is finally applied to B_2 :

$$\begin{aligned}\int_0^{+\infty} e^{-ix\xi} B_2(v_2)(x)\,dx &= \frac{1}{2}\mathrm{TM}(\Sigma_2)(\xi)\\ &= \frac{1}{2}\int_{\Gamma_0} \mathrm{TM}(\xi,\zeta)\Sigma_2(\zeta)\,\mathrm{d}\zeta\end{aligned} \tag{23}$$

with

$$\mathrm{TM}(\xi,\zeta) = \frac{1}{2\pi\mathrm{i}}\sum_{*=\mathrm{L,T}} \mathrm{D}_*(\xi,\zeta)\mathbf{tm}_*(\zeta, \mathrm{sgn}\ \sin\varphi) \tag{24}$$

$$D_*(\xi,\zeta) = \frac{1}{\xi - (\zeta\cos\varphi + \zeta_*(\zeta)\sin\tilde{\varphi})} = \frac{1}{\xi - T_*(\zeta)} \tag{25}$$

The matrices $\mathbf{tm}_L$ and $\mathbf{tm}_T$ are known explicitly. However, their expression being heavy, it is not reproduced here.

By taking the sum of these two operators DM and TM, called the diagonal matrix operator and the transfer matrix operator respectively, the Fourier transform of the boundary conditions on each face of the wedge is finally obtained. This is a system of functional equations solved by the spectral functions :

$$\begin{cases} \mathrm{DM}(\Sigma_1) + \mathrm{TM}(\Sigma_2) = \frac{\mathrm{W}_1}{\xi - \nu_\alpha\cos\theta_{\mathrm{inc}}} \\ \mathrm{TM}(\Sigma_1) + \mathrm{DM}(\Sigma_2) = \frac{\mathrm{W}_2}{\xi - \nu_\alpha\cos(\varphi - \theta_{\mathrm{inc}})} \end{cases} \tag{26}$$

The explicit expression of the residues in the right-hand side can be obtained by taking the Fourier transform of the right-hand side of (16).

This system of functional equations will be resolved to obtain an evaluation of the spectral functions.

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4.2. Method of resolution

The resolution of system (26) is done semi-analytically. This means that the solution Σ_j is the sum of two functions: a function y_j which is determined by an exact formula and a function X_j which is approached numerically.

The first step is to compute y_j, which is called the "singular part". First, the subsets $\{z_1^k\}_{k\geq 0}$ and $\{z_2^k\}_{k\geq 0}$ of $\mathbb{C}$ are defined recursively by Kamotski and Lebeau [2].

These sets are the poles of the spectral functions and correspond to the reflected (or multiply reflected) rays. Croisille and Lebeau [1] have shown that these sets of poles of the spectral functions are finite. Physically, this means that any incident ray on the wedge will eventually become an outgoing ray after a certain number of reflections.

Using these sets, Croisille and Lebeau [1] have also proven the following result :

Lemma 4.1. *There exist two functions y_1, y_2 being the finite sum of simple poles*

$$y_j(\xi) = \sum_k \frac{a_j^k}{\xi - z_j^k}, \quad a_j^k,\ z_j^k \in \mathbb{C} \tag{27}$$

such that u_1 and u_2 defined by

$$u_j(\xi) = \frac{W_j}{\xi - z_j} - \mathrm{DM(y_j)}(\xi) - \mathrm{TM(y_{3-j})}(\xi) \tag{28}$$

are analytical on $\mathbb{C}\backslash]-\infty,-1]$.

The functions y_1 and y_2 are the singular parts of the spectral functions Σ_1 and Σ_2. Their poles and residues are computed explicitly using a recursive procedure given by Croisille and Lebeau [1].

Let us now compute the "regular parts" X_1 and X_2 of the spectral functions, defined by :

$$X_j(\xi) = \Sigma_j(\xi) - y_j(\xi) \tag{29}$$

According to (28), these functions are solutions of the following system :

$$\begin{cases} \mathrm{DM(X_1)}(\xi) + \mathrm{TM(X_2)}(\xi) = \mathrm{u_1}(\xi) \\ \mathrm{TM(X_1)}(\xi) + \mathrm{DM(X_2)}(\xi) = \mathrm{u_2}(\xi) \end{cases} \tag{30}$$

The functions X_1 and X_2 are also analytical on $\mathbb{C}\backslash]-\infty,-1]$. They are approached numerically by a Galerkin collocation method. This new system is then evaluated at a finite number of points $\xi = b_1, ..., b_N$, yielding the following linear system of equations :

$$\begin{pmatrix} \mathbb{D} & \mathbb{T} \\ \mathbb{T} & \mathbb{D} \end{pmatrix} \begin{pmatrix} \mathbb{X}_1 \\ \mathbb{X}_2 \end{pmatrix} = \begin{pmatrix} \mathbb{U}_1 \\ \mathbb{U}_2 \end{pmatrix} \tag{31}$$

where the coefficients $\mathbb{D}_{lk}$ and $\mathbb{T}_{lk}$ can be computed exactly. However, the details of this computation are very long and technical and are not presented here. They will hopefully be the subject of a future publication dealing with any incident and scattered wave modes.

The semi-analytical computation of the spectral functions leads to the numerical evaluation of the diffraction coefficients, presented in the following section.

5. Numerical results and validation

In this section, the "acoustic limit" of the spectral functions code is taken by setting the longitudinal wave velocities to verify $c_L >> c_T$. This means that the parameters of the elastic model are set to simulate the diffraction of an acoustic wave.

We consider the example of a longitudinal wave incident with an angle $\theta_{inc} = 150^o$ on a wedge of angle $\varphi = 300^o$. The wedge is represented in figure 4.

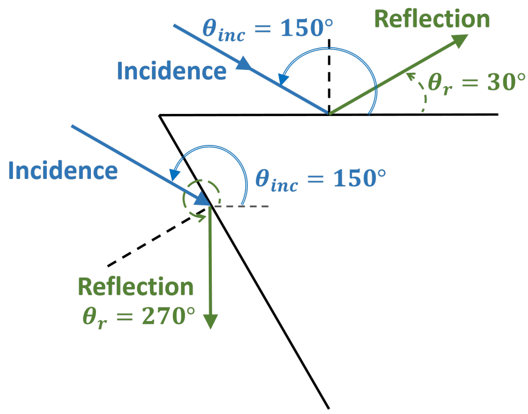

Figure 4: Incident and reflected rays on a wedge of angle $\varphi = 300^o$, $\theta_{inc} = 150^o$.

A far-field asymptotic evaluation of the longitudinal diffraction coefficient is computed for 200 observation angles using (14) and it is compared to the exact expression of the far-field diffraction coefficient of the scattering of a plane acoustic wave with a soft wedge, expressed by Sommerfeld [19]. The results are shown in figure 5. Note that uniform asymptotic models of acoustic wave scattering have been compared by Lü et al [20]. The results are shown in figure 5.

In figure 5, the continuous red line is the exact Sommerfeld solution and the blue dots are the values obtained using the spectral functions method. There is an excellent agreement between the results produced by each of these methods.

6. Conclusion

This communication deals with the modelling of the longitudinal wave scattered by a wedge impinged by a longitudinal plane elastic wave. After stating the problem precisely, we have shown that a far-field approximation of the elastic wave diffracted by a stress-free wedge can be obtained by computing a function called the diffraction coefficient. This coefficient is expressed in terms of two unknown functions Σ_1 and Σ_2 called the spectral functions.

The first numerical test has shown than this method is valid in the case of an acoustic wave. The results in the case of the acoustic limit of the elastic code promising and a full numerical and experimental validation is to come.

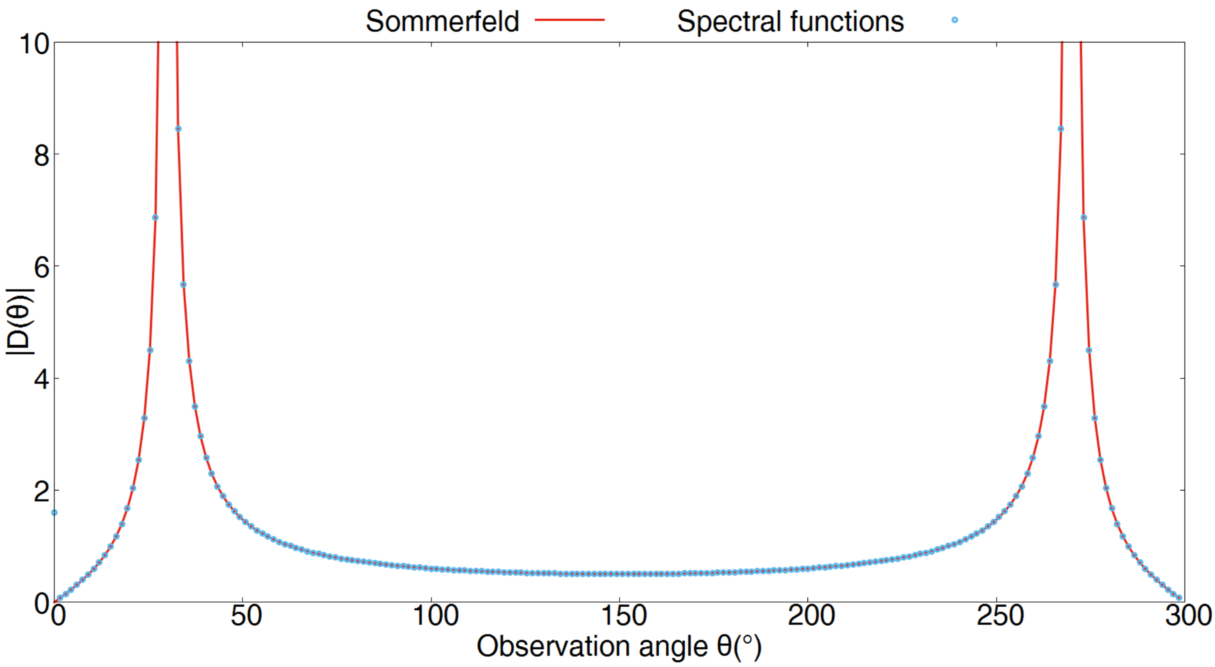

Figure 5: Diffraction coefficient computed with the spectral functions method and with the Sommerfeld method.

Future work on this method will include a validation of the two-dimensional diffraction of an elastic wave by a wedge, by comparing the results of the spectral functions code to other numerical methods, for angles both lower and higher than π. The method may then be extended to the case of the three-dimensional diffraction of a plane elastic wave by a wedge (oblique incidence) and to the case of a wedge with different boundary conditions.

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