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The Dynamic of Trapped Bose-Einstein Condensate Under Noise

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Abstract. It is known that Bose-Einstein Condansate is well described by the non-linear Schrodinger equation known as the Gross Pitaevskii Equation (GPE) with the macroscopic wave function which evaluates with time and space. Many studies have been performed on nonlinear properties in Bose-Einstein Condansate. In this study, we present some numerical results of the Gross-Pitaevskii Equation with the external potential under noise. The phase portraits and Poincaré sections of the system are simulated numerically both with and without noise.

1. Introduction

It is known that Bose– Einstein condensate (BEC) was predicted by Einstein and Bose in 1924 theoretically [1, 2]. But BEC was conceived for the first time experimentally by Cornell and Wieman in 1995 [3]. After the discovery of BEC experimentally, it has become an attracted topic for scientist. Physicists especially started to observe condensation using the relatively advanced cooling techniques. The condensate is well described by a mean field theory and a macroscopic wave function, solving the nonlinear Schrödinger equation so-called Gross– Pitaevskii equation (GPE) [4, 5] that includes a nonlinear term representing particle-particle interactions [6, 7]. The GPE is not easy to solve analytically due to the external trap potential and nonlinearity of interactions which makes the solutions rich so the numerical simulations are performed generally [7, 8, 9, 10, 11, 12]. In this paper, we investigate the chaotic solutions of the BEC system in the tilted Gaussian optical lattice potential under noise numerically.

2. Model

It is well known that BEC is well described by the Gross–Pitaevskii Equation (GPE) with the macroscopic wave function $\Psi = \Psi(x, t)$ which evaluates with time and space [4, 5]. The 1-D Gross-Pitaevskii equation is given as

$$i\hbar \frac{\partial}{\partial t}\Psi(x, t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x, t) + \left[V_{ext}(x) + g_{1D}\left|\Psi(x, t)\right|^2\right]\Psi(x, t), \tag{1}$$

where m is the mass of the atoms of the condensate, g_{1D} describes the interaction between atoms in the condensate and given by

$$g_{1D} = \frac{g_{3D}}{2\pi a_r^2} = 2a_s \hbar w_r, \tag{2}$$

$g_{3D} = \frac{4\pi \hbar^2 a_s}{m}$ where a_s is s-wave scattering length between atoms. It is positive for repulsive interaction and negative for attractive interaction (in our case $a < 0$). V_{ext} is the external trapping potential. We choose the external trapping potential as below,

$$V_{ext}(x) = V(x) + Fx. \tag{3}$$

Where $V(x)$ is the optical potential and F is the inertial force. This force, which generates a tilted optical potential, accelerates the atoms in the x direction and leads to the atoms tunnelling out of the traps [6, 13]. In this paper, we consider the optical potential as Gaussian optical lattice potential. We construct a potential which involve Gaussian peaks along the x direction in spatial phase by using Fourier transform procedure. Each Gaussian peaks described by Eq. 4. A is amplitude of each Gaussian peaks and μ and σ are the system parameters.

$$f(x) = Ae^{-\frac{(x-\epsilon)^2}{2\sigma^2}}, \tag{4}$$

in order to generate Gaussian pulse potential, we define a step length

$$xx = \frac{x_{max} - x_{min}}{n}, \tag{5}$$

x_{max} and x_{min} are maximum and minimum border for numerical calculation. n defines the step length. We create B matrix by evaluating $f(x)$ from x_{max} to x_{min} as below,

$$B = \begin{bmatrix} x_{min} & Ae^{-\frac{(x_{min}-\mu)^2}{2\sigma^2}} \\ \vdots & \vdots \\ x_{max} - xx & Ae^{-\frac{(x_{max}-xx-\mu)^2}{2\sigma^2}} \end{bmatrix} \tag{6}$$

The discreet Fourier Transform of B matrix is given in Eq. 7

$$C = \begin{bmatrix} Ae^{-\frac{(x_{min}-\mu)^2}{2\sigma^2}} e^{\frac{-2\pi i x_{min} k}{n}} \\ \vdots \\ Ae^{-\frac{(x_{max}-xx-\mu)^2}{2\sigma^2}} e^{\frac{-2\pi i (x_{max}-xx)k}{n}} \end{bmatrix} \tag{7}$$

From C matrix, we fnd EQ. 8 that generates one dimensional Gaussian Pulse potential.

$$V_{ext}(x) = \frac{C_1}{\sqrt{n}} + \frac{2}{\sqrt{n}} \sum_{j=2}^{j_{max}} Abs(C_j)Cos\left(w\frac{j-1}{xx}x - Arg(C_j)\right) \tag{8}$$

here $j_{max} = \frac{3xx}{2\pi\sigma} + 1$.

In order to obtain a simple description and a better understanding of the BEC dynamics, we consider Ψ as [12]

$$\Psi(x,t) = \Phi(x) e^{\frac{-i\mu t}{\hbar}}, \tag{9}$$

here μ is the chemical potential of the condensate and $\Phi(x)$ is a real function independent of time. $\Phi(x)$ is normalized to the total number of particles in the system, i.e.,

(a)   (b)
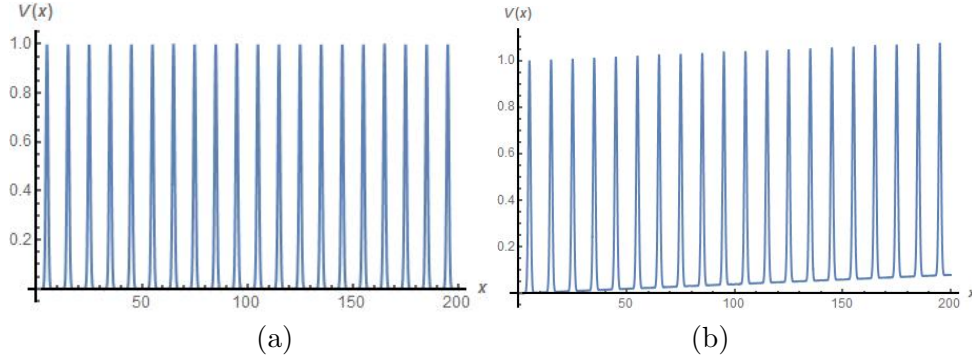
Figure 1. Gaussian Pulse Potential (a) without tilt term, (b) with tilt term.

$$\int |\Phi(x)|^2\, dx = N \tag{10}$$

where N is the particle number. Substitution of Eqs.(8) and (9) into Eq.(1)

$$\mu\Phi(x) = -\frac{\hbar}{2m}\frac{\partial^2}{\partial x^2}\Phi(x) + \left[V_{ext} + Fx + g_{1D}\Phi|x|^2\right]\Phi(x) \tag{11}$$

which can also be written in the following form where ($\upsilon_{ext} = \frac{2mV_{ext}}{\hbar^2}, \upsilon_2 = \frac{2mV_2}{\hbar^2}, \gamma = \frac{2m\mu}{\hbar^2}, \zeta = \frac{2mF}{\hbar^2}, \eta = \frac{2mg_0}{\hbar^2}$). Setting the solution of Eq. (11) of form

$$\Phi(x) = \phi(x)e^{i\theta(x)} \tag{12}$$

Inserting Eq. (12) into Eq. (11),

$$\frac{d^2\phi}{dx^2} + \phi\left(\frac{d\theta}{dx}\right)^2 = \left[\upsilon_{ext} + \zeta x - \gamma + \eta|\phi|^2\right]\phi, \tag{13a}$$

$$\frac{d}{dx}\left(2\phi^2\frac{d\theta}{dx}\right) = 0. \tag{13b}$$

Eq. (13b) denotes the existence of a flow density,

$$J = 2\phi^2\frac{d\theta}{dx} \tag{14}$$

If we put J into Eq. (13a), we have a nonlinear equation as below.

$$\frac{d^2\phi}{dx^2} + \frac{J^2}{4\phi^3} = \left[\upsilon_{ext} + \zeta x - \gamma + \eta|\phi|^2\right]\phi. \tag{15}$$

It is difficult to obtain the exact solution of Eq. (15) due to its complexity therefore numerical solutions were performed. For numerical solution we can reduce Eq. 15 in to first order coupled equations by adding an external white noise term

$$\dot{\phi_1} = \phi_2 \tag{16a}$$

$$\dot{\phi_2} = \frac{J^2}{4\phi_1^3} + \left[\upsilon_{ext} + \zeta x - \gamma + \eta|\phi_1|^2\right]\phi_1 + D\delta(x). \tag{16b}$$

Here D is the amplitude of white noise and δ produce the noise.

3. Numerical Results

We present some numerical results of the Gross-Pitaevskii Equation with the external potential under noise. In this section, we present the chaotic solutions of the BEC system with the Gaussian optical potential first without noise after under white noise. Fig.2 shows that the Poincaré sections with $\phi_1[0] = 0.1, \phi_2[0] = 0.8$ initial conditions. Fig.2 (a) display the signature of regular structure of system for the parameterssets are $\zeta = 10^{-6}$, $J = 0.8$, $w = 10\pi$, $\gamma = 0.5$, $\eta = -1$, $\sigma = 210.52$, $A = 0.2$ and (b) display chaotic structure of system for the parameterset $\zeta = 0.01$, $J = 0.8$, $w = 10\pi$, $\gamma = 0.5$, $\eta = -1$, $\sigma = 210.52$, $A = 0.2$

(a)

(b)

(c)

(d)
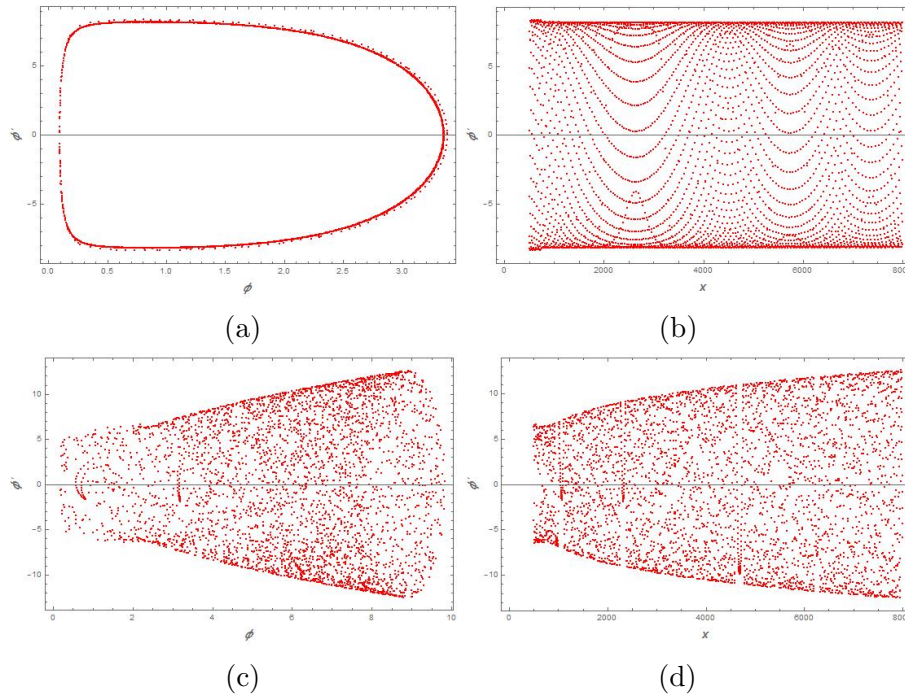
Figure 2. (a) Regular Poincaré sections (b) regular spatial evulotion of BEC for $\zeta = 10^{-6}$ (c) chaotic Poincaré sections (d) chaotic spatial evulotion of BEC for $\zeta = 0.01$. for the parametersets: $J = 0.8$, $w = 10\pi$, $\gamma = 0.5$, $\eta = -1$, $\sigma = 210.52$, $A = 0.2$, and the intial condition: $\phi_1[0] = 0.1, \phi_2[0] = 0.8$ without noise.

For chaotic cases we add white noise to the system. In Fig.3 we show Poincare display of BEC for (a) $D = 0.01$, (b) $D = 0.05$, (c)=$D = 0.1$, (d) $D = 0.2$ respectively for parameter sets: $\zeta = 0.01$, $J = 0.8$, $w = 10\pi$, $\gamma = 0.5$, $\eta = -1$, $\sigma = 210.52$, $A = 0.2$ and with same initial condition in Fig. 2.
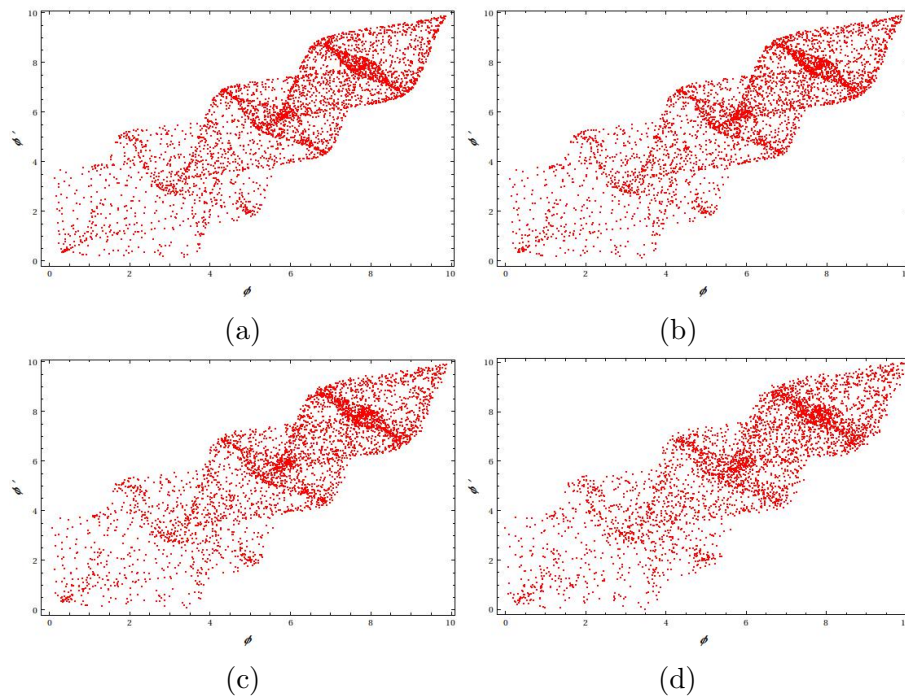
Figure 3. (a) Regular Poincaré sections (b) regular spatial evulotion of BEC for $\zeta = 10^{-6}$ (c) chaotic Poincaré sections (d) chaotic spatial evulotion of BEC for $\zeta = 0.01$. for the parametersets: $J = 0.8$, $w = 10\pi$, $\gamma = 0.5$, $\eta = -1$, $\sigma = 210.52$, $A = 0.2$, and the intial condition: $\phi_1[0] = 0.1, \phi_2[0] = 0.8$ without noise.

4. Conculusion

In summary, we have studied the BEC with the tilted Gaussian optical lattice potential under white noise. The chaotic numerical solutions of system are investigated by constructing the Poincaré sections depending on the system parameters. Numerical solutions indicate that the system continues to exhibit chaotic behaviors with different noise variances. The behaviors of system under noise show small changes in phase space for not large values of noise. These instabilities of the system refer to negative interatomic interactions.

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