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# Effects of spacetime topology and curvature on the resonance interatomic energy

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**Abstract.** We study, using the formalism proposed by Dalibard, Dupont-Roc, and Cohen-Tannoudji, the resonance interatomic energy (RIE) of two identical two-level static atoms in a symmetric/antisymmetric entangled state, which are coupled to massless scalar fields, in a number of different spacetimes. We first show that the presence of a boundary in a flat Minkowski spacetime can dramatically modify the RIE of the two static atoms, resulting in an enhanced or weakened and even nullified RIE, as compared with that in the unbounded case; we then show that the RIE of the two atoms in the spacetime of a Schwarzschild black hole can be sharply affected by the spacetime curvature on one hand, but on the other hand it is surprisingly undisturbed by the Hawking radiation of the black hole; we finally show that the RIE of the two static atoms in the spacetime with an infinite and straight cosmic string (the so called cosmic string spacetime) is sensitive to the nontrivial topological structure of the spacetime, making the RIE of the two static atoms behave in a manner very similar to that near a perfectly reflecting boundary in a flat Minkowski spacetime.

## 1. Introduction

Dispersion energy between atoms occurs for two polarizable neutral atoms in a vacuum, as a result of the interaction between the atoms and fluctuating quantum fields. When two atoms are prepared in different eigenstates, the exchange of real photons can be involved and the dispersion energy is called the resonance interatomic energy.

Ever since the pioneering work of Casimir and Polder on the retarded interatomic energy between two neutral ground-state atoms [1], there has been considerable progress on the resonance interatomic interaction. On one hand, the resonance interaction energy between two uncorrelated atoms which are respectively coupled to vacuum electromagnetic fields with one atom in the ground state and the other in the excited state has been extensively studied over the past decades [2–13]. For this type of interaction, both oscillatory and monotonic behaviors of the resonance interatomic energy with respect to the interatomic separation in the retarded region are predicted, depending on whether there is an irreversible excitation transfer between the two atoms during the interaction process [11]. On the other hand, the resonance interatomic energy between two atoms in an unfactorizable state has also attracted much attention in recent years. For two identical atoms prepared in a symmetric/antisymmetric entangled state



and in interaction with the vacuum electromagnetic field, the resonance interatomic energy decays as  $R^{-1}$  ( $R$  denoting the interatomic separation) at large interatomic separations. This behavior is of a much longer range as compared with the resonance interatomic energy of two atoms in a factorizable state and the nonresonance interatomic energy between two ground-state atoms. Recently, it has been shown that noninertial motion of atoms induces some peculiar modifications for the resonance interatomic energy of two identical atoms prepared in a symmetric/antisymmetric entangled state with a constant separation [14, 15], and the possibility of modulating the resonance interatomic energy of such two static atoms in nanostructured materials such as a photonic crystal [16, 17] or by using a perfectly reflecting plane boundary [18–20] has also been discussed. In addition, the resonance interaction between atoms has also been widely investigated in relation to the resonance energy transfer between molecules [21–23]. Nowadays, not only it plays an important role in many physical processes [24–27], but also even goes to the core of some biological phenomena [9, 21, 28].

The aforementioned research works are about the resonance interatomic energy of atoms in the Minkowski spacetime. In recent years, there is growing interest in the radiative properties of atoms in interaction with quantum fields in curved spacetimes. Since the propagation of quantum fields is now modified as compared to the Minkowski spacetime, interesting features show up in quantum effects associated with it. The energy shifts and excitation rates of static atoms coupled to quantum fields in the Schwarzschild spacetime have been found to be modified by the spacetime curvature and the Hawking radiation of a black hole [29–34]; the response rate of particle detectors and the variation rate of energy of static or noninertial atoms in the cosmic string spacetime have been shown to be crucially dependent on the relative position of the detectors' or atomic trajectories with respect to the cosmic string, revealing that the nontrivial topology of the cosmic string spacetime has important effects on the atomic radiative properties [35–40]; and the occurrence of spontaneous excitation of static or freely falling atoms in the de Sitter spacetime is demonstrated to be possible due to the peculiar thermal nature of the de Sitter spacetime [41], which also has a profound influence on the Lamb shift of a single atom [42].

In this paper, we are interested in the resonance interatomic energy of two identical atoms prepared in an unfactorizable state (the symmetric/antisymmetric entangled state) and coupled to the massless scalar fields in vacuum in a number of spacetimes. We use the formalism proposed by Dalibard, Dupont-Roc and Cohen-Tannoudji [43, 44](DDC) to calculate separately the contributions of field fluctuations and atomic radiation reaction to the resonance interatomic energy of the two atoms in the Minkowski, Schwarzschild and the cosmic string spacetimes. By comparing the results in the three different backgrounds, we show how the Schwarzschild spacetime curvature and the nontrivial topology of the cosmic string spacetime affect the resonance interatomic energy, and reveal the intrinsic relation between characters of the curved spacetimes and the resonance interatomic energy.

The paper is organized as follows. In section 2, we consider the resonance interatomic energy of two identical atoms in a symmetric/antisymmetric entangled state and coupled to massless scalar fields near a perfectly reflecting plane boundary, and discuss the effect of the presence of the boundary in the flat spacetime on the resonance interatomic energy. In section 3, we study the resonance interatomic energy of two static atoms aligned radially outside a spherical black hole, and show that the resonance interatomic energy of the two atoms can be greatly weakened because of spacetime curvature, but is oblivious to the Hawking radiation of the black hole. In section 4, we show that the resonance interatomic energy of two atoms near an infinite and straight cosmic string is crucially dependent on the relative positions of the two atoms with respect to the string, and it behaves very much like its counterpart in the case of two atoms located near a perfectly reflecting boundary in a Minkowski spacetime. We finally give the conclusions in section 5. Throughout the paper, we adopt the natural units, i.e.,  $\hbar = c = 1$ .

## 2. Resonance interatomic energy in the Minkowski spacetime

We first consider the resonance interatomic energy of two identical two-level atoms which are coupled to the massless scalar field in the Minkowski spacetime. We designate “ $g/e$ ” representing the atomic ground/excited state, and assume that the two atoms are prepared in a symmetric/antisymmetric entangled state:  $|\psi_{\pm}\rangle = \frac{1}{\sqrt{2}}(|g_A e_B\rangle \pm |e_A g_B\rangle)$ . Then the Hamiltonian of the system composed by the two atoms and the field can be described by

$$H = \omega_0 \sigma_3^A(\tau) + \omega_0 \sigma_3^B(\tau) + \sum_{\vec{k}} \omega_{\vec{k}} a_{\vec{k}}^{\dagger} a_{\vec{k}} \frac{dt}{d\tau} + \lambda(\sigma_2^A(\tau)\phi(x_A(\tau)) + \sigma_2^B(\tau)\phi(x_B(\tau))), \quad (1)$$

in which  $\tau$  denotes the atomic proper time which coincides with the coordinate time  $t$  for the present case,  $a_{\vec{k}}^{\dagger}$  and  $a_{\vec{k}}$  are the creation and annihilation operators of the scalar field,  $\lambda$  is the coupling constant, and  $\sigma_2^{\xi}(\tau)$  ( $\xi = A, B$ ) and  $\sigma_3^{\xi}(\tau)$  are two pseudospin operators in the Hilbert space of the internal degrees of freedom of atom  $\xi$ .

Using the above Hamiltonian, we can derive the Heisenberg equations of motion for the dynamical variables of the atoms and the field, whose solutions are then divided into the free parts and the source parts. The field operator is then decomposed into a free field and a source field accordingly, in terms of which we can calculate the variation rate of the Hamiltonian of both atoms. Following the idea proposed by Dalibard, Dupont and Cohen-Tannoudji [43, 44], we first derive for both atoms the effective Hamiltonian of the contribution of the free field (the contribution of field fluctuations) and the contribution of the source field (the contribution of atomic radiation reaction), then we calculate the average values of them over the state of the two atoms,  $|\psi_{\pm}\rangle$ , and the vacuum state of the field. Such a treatment gives rise to the energy shift for both atoms, and the part of these energy shifts which is interatomic-separation-dependent is what we call the resonance interatomic energy. In the very recent years, this formalism has been extensively exploited to study the resonance interaction between two atoms in a symmetric/antisymmetric entangled state and the quantum fields in various situations [14, 15, 18–20, 33, 45–47].

The resonance interatomic energy of such two identical atoms is found to be thoroughly contributed by the atomic radiation reaction, but irrelevant of the contribution of vacuum fluctuations:

$$\delta E = -i\lambda^2 \int_{\tau_0}^{\tau} d\tau' C^{AB}(\tau, \tau') \chi^F(x_A(\tau), x_B(\tau')) + A \rightleftharpoons B \text{ term} \quad (2)$$

in which  $\chi^F(x_A(\tau), x_B(\tau'))$  and  $C^{AB}(\tau, \tau')$  are respectively the linear susceptibility function of the scalar field and the symmetric correlation function of the two atoms defined as

$$\chi^F(x_A(\tau), x_B(\tau')) = \frac{1}{2} \langle 0 | [\phi^f(x_A(\tau)), \phi^f(x_B(\tau'))] | 0 \rangle, \quad (3)$$

$$C^{AB}(\tau, \tau') = \frac{1}{2} \langle \psi_{\pm} | \{ \sigma_2^{Af}(\tau), \sigma_2^{Bf}(\tau') \} | \psi_{\pm} \rangle. \quad (4)$$

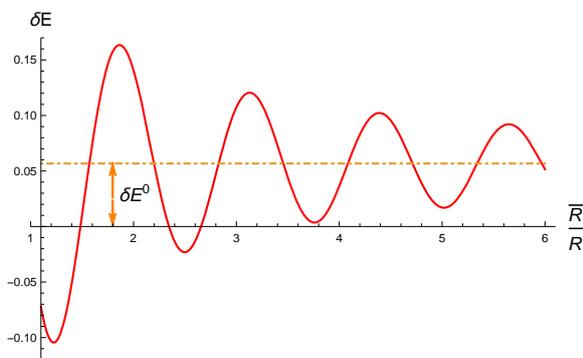
We now consider the resonance interatomic energy of two identical atoms prepared in the symmetric/antisymmetric entangled state near a perfectly reflecting plane boundary with interatomic separation  $R$ . By use of the above formulas, the resonance interatomic energy can be finally simplified to be [19]

$$\delta E = \mp \frac{\lambda^2}{16\pi} \left( \frac{\cos(\omega_0 R)}{R} - \frac{\cos(\omega_0 \bar{R})}{\bar{R}} \right) \quad (5)$$

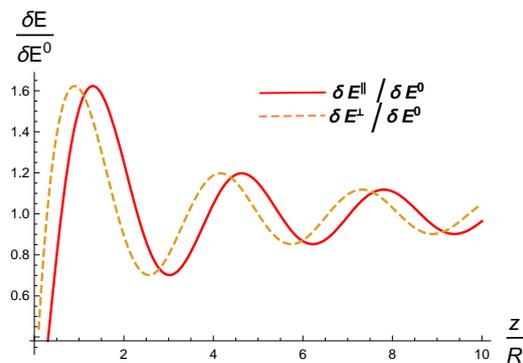
with  $\bar{R}$  denoting the distance between one atom and the image of the other atom, and “ $\mp$ ” on the right corresponding to  $|\psi_{\pm}\rangle$ . The above reveals that the resonance interatomic energy and thus

the interatomic force of two identical atoms in a symmetric entangled state is equal in magnitude but opposite in sign as compared with those of two identical atoms in an antisymmetric entangled state. Equation (5) shows that the resonance interatomic energy near the boundary is composed of two parts. The first part is completely the same as the counterpart in an unbounded space and the second part depends crucially on the atomic positions relative to the boundary and thus represents the effect of the presence of the boundary.

The following figures help to exhibit the effect of the presence of the boundary on the resonance interatomic energy. As shown in figure 1, the resonance interatomic energy of two atoms near a perfectly reflecting boundary,  $\delta E$ , can be greater or smaller, and even be nullified, as compared with its counterpart in an unbounded space,  $\delta E^0$ , depending on the relative positions of the two atoms with respect to the boundary (the ratio between  $\bar{R}$  and  $R$ ). With the increase of the atoms-boundary separation, the resonance interatomic energy oscillates around the value of its counterpart in a free space, and the oscillation amplitude decreases with the increase of atoms-boundary separation (the resonance interatomic energy shown in figure 2 for example).



**Figure 1.** The comparison of the resonance interatomic energy of two atoms with transition frequency  $\omega_0 = 1.549 \times 10^{16} \text{ s}^{-1}$  and with a fixed interatomic separation  $R = 9.674 \times 10^{-8} \text{ m}$  near a perfectly reflecting boundary. The ordinate is of unit  $\mp \frac{\lambda^2 \omega_0}{16\pi}$ .



**Figure 2.** The atoms-boundary-separation dependence of the relative resonance interatomic energy,  $\frac{\delta E^{\parallel/\perp}}{\delta E^0}$ , of two atoms aligned with their separation parallel/perpendicular to the boundary.  $z$  is the boundary-atom (the one closer to the boundary) separation, and  $\omega_0 R = 1$ .

### 3. Resonance interatomic energy in the Schwarzschild spacetime

In this section, we study the resonance interatomic energy of two identical atoms prepared in a symmetric/antisymmetric entangled state and coupled to the massless scalar field in the Schwarzschild spacetime. In the spherical coordinates, the metric in this spacetime can be described by

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \frac{dr^2}{1 - \frac{2M}{r}} - r^2(d\theta^2 + \sin^2\theta d\varphi^2). \quad (6)$$

For this spacetime, there are usually three vacuum states of physical interest: the Boulware, Unruh and Hartle-Hawking vacua. The Boulware vacuum corresponds to our familiar concept of an empty state for large radii; the Unruh vacuum corresponds to an outgoing flux of blackbody radiation at the black-hole temperature; and the Hartle-Hawking vacuum corresponds to the state of black hole radiation in equilibrium with an infinite sea of blackbody radiation. We are concerned with the resonance interatomic energy of two atoms in these different vacua. For simplicity, we consider the case of two atoms located with their separation along the radial

direction in the exterior region of the spherical black hole. Then the trajectories of the atoms with a constant separation  $L$  can be written as

$$t_A = t, \quad r_A = r + L/2, \quad \theta_A = \theta, \quad \varphi_A = \varphi, \quad (7)$$

$$t_B = t, \quad r_B = r - L/2, \quad \theta_B = \theta, \quad \varphi_B = \varphi \quad (8)$$

with  $2(r - 2M) > L$ .

The resonance interatomic energy of the two atoms can be studied by using the DDC formalism as in the preceding section. However, as the differential equation of the radial function of the scalar field in the Schwarzschild spacetime is often difficult to solve, here we are mainly concerned with the resonance interatomic energy in two asymptotic regions, i.e., at spatial infinity and near the horizon. When  $2(r - 2M) \gg L$  near the horizon or  $L \ll 2r^2/M$  at infinity, we find that the proper time of the two atoms can be approximated by  $\tau_A \approx \tau_B \approx t(1 - 2M/r)^{1/2}$  to the leading order of  $L$ , and we can use formula (2) to calculate the resonance interatomic energy. However, it is worth pointing out here that for this case the linear susceptibility function of the field operator,  $\chi^F(x_A(\tau), x_B(\tau'))$  in equation (2), should be replaced by its counterpart in the Schwarzschild spacetime. After lengthy simplifications, we find the resonance interatomic energy in these two regions of all the three vacua [45]

$$\begin{aligned} \delta E = & \mp \frac{\mu^2}{128\pi^2} \int_0^\infty \frac{d\omega}{\omega} \left( \frac{1}{\frac{\omega}{\sqrt{g_{00}}} + \omega_0} + \frac{1}{\frac{\omega}{\sqrt{g_{00}}} - \omega_0} \right) \\ & \times \left( \sum_l (2l+1) \vec{R}_{\omega l}(r_A) \vec{R}_{\omega l}^*(r_B) + \sum_l (2l+1) \overleftarrow{R}_{\omega l}(r_A) \overleftarrow{R}_{\omega l}^*(r_B) \right. \\ & \left. + \sum_l (2l+1) \vec{R}_{\omega l}(r_B) \vec{R}_{\omega l}^*(r_A) + \sum_l (2l+1) \overleftarrow{R}_{\omega l}(r_B) \overleftarrow{R}_{\omega l}^*(r_A) \right) \quad (9) \end{aligned}$$

in which  $g_{00} = (1 - \frac{2M}{r})$ , and  $\vec{R}_{\omega l}(r)$  and  $\overleftarrow{R}_{\omega l}(r)$  are the radial functions of the outgoing and ingoing modes originating from the horizon and infinity respectively.

By use of the approximation of the summation concerned with the radial functions of the outgoing and ingoing modes at two different spatial regions [45], the above resonance interatomic energy can be further simplified. We find that at infinity, it can be approximated by the sum of two parts. One part is resulted from the backscattering effect of the outgoing modes of the field off the spacetime curvature, and it is much smaller than the other part, which is almost the same as the counterpart of the resonance interatomic energy in an unbounded Minkowski spacetime. Thus at infinity, the resonance interatomic energy of the two atoms approaches its counterpart in a free Minkowski spacetime. When the two atoms are fixed near the horizon with  $2(r - 2M) \gg L$ , similarly, the resonance interatomic energy can also be simplified to be the sum of two parts, and one of them is resulted from the backscattering effect of the ingoing modes off the spacetime curvature, and its value is much smaller than the other part which is much smaller than the counterpart of the resonance interatomic energy in an unbounded Minkowski spacetime. Thus near the horizon, the resonance interatomic energy is much smaller than its counterpart in a free Minkowski spacetime.

Comparing the resonance interatomic energy of the two atoms with other atomic radiative properties in the same Schwarzschild spacetime, we find some sharp contrasts: the resonance interatomic energy of the two atoms in a symmetric/antisymmetric entangled state near the horizon is much smaller than its counterpart in a free Minkowski spacetime, while for other atomic radiative properties, such as the Lamb shift and the spontaneous excitation rate of a single static atom near the horizon, they are almost the same as their counterparts in a free Minkowski spacetime [29–32]; Besides, the resonance interatomic energy of two atoms in a

symmetric/antisymmetric entangled state in the Schwarzschild spacetime is the same in the Boulware, Unruh and Hartle-Hawking vacua, and it is irrelevant of the Hawking radiation of a black hole, while the Lamb shift and the spontaneous excitation rate of a single static atom in the three vacua are distinct, as they are significantly influenced by the radiation in the spacetime (such as the Hawking radiation of a black hole) [29–32]. The origin of this distinction can be traced back to the fact that the resonance interatomic energy of two identical atoms correlated by a symmetric/antisymmetric entangled state is fully contributed by the radiation reaction of the two atoms, which is insusceptible to the thermal state of the field; while for other atomic radiative properties, such as the Lamb shift and the excitation rate of a single atom, both field fluctuations and atomic radiation reaction contribute. Though the contribution of atomic radiation reaction is insusceptible to the thermal state of the field, the contribution of field fluctuations is crucially dependent on the state (thermal or nonthermal) of the field. This is also the very reason why the resonance interatomic energy of two uniformly accelerated atoms with constant interatomic separation and in a symmetric/antisymmetric entangled state is usually completely the same in the instantaneous inertial frame and in the coaccelerated frame [15].

#### 4. Resonance interatomic energy in the Cosmic string spacetime

We now consider the resonance interatomic energy of two identical two-level atoms prepared in a symmetric/antisymmetric entangled state and fixed near an infinite straight cosmic string [19]. Using the cylindrical coordinates  $(t, r, \theta, z)$  and assuming that the string is extended along the  $z$  axis, the spacetime metric follows

$$ds^2 = dt^2 - dr^2 - r^2 d\theta^2 - dz^2, \quad (10)$$

in which  $0 \leq \theta < 2\pi/\nu$ ,  $\nu = (1 - 4G\mu)^{-1}$  with  $G$  and  $\mu$  being the Newton constant and the mass per unit length of the string. The positions of the two static atoms can be represented by  $(t_\xi, r_\xi, \theta_\xi, z_\xi)$  with  $\xi = A, B$ .

The resonance interatomic energy of the two atoms can be calculated by formula (2) but with the linear susceptibility function of the field replaced by the counterpart in the cosmic string spacetime. The resonance interatomic energy can be finally expressed to [19]

$$\begin{aligned} \delta E = & \mp \frac{\lambda^2 \nu}{32\pi^2} \sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} d\kappa \int_0^{\infty} dk_{\perp} \frac{k_{\perp}}{\omega} e^{i\kappa z} e^{i\nu m \Delta\theta} \\ & \times J_{\nu|m|}(k_{\perp} r_A) J_{\nu|m|}(k_{\perp} r_B) \left( \frac{1}{\omega + \omega_0} + \frac{1}{\omega - \omega_0} \right), \end{aligned} \quad (11)$$

in which  $\Delta\theta = \theta_A - \theta_B$ ,  $z = z_A - z_B$ , and  $J_{\nu|m|}(k_{\perp} r)$  denotes the first kind Bessel function. This expression of the resonance interatomic energy of two atoms near an infinite straight cosmic string is very general, and it is usually difficult to simplify. However, for some special cases, analytical results are obtainable.

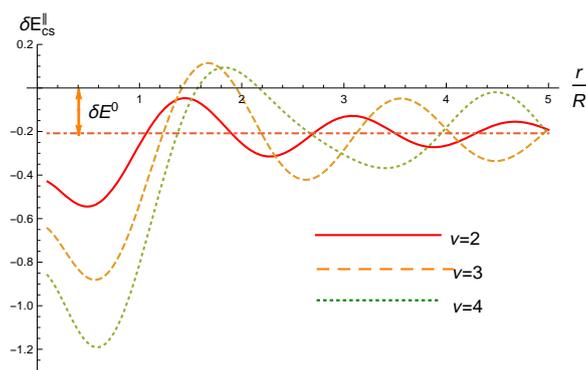
When  $\nu = 1$ , the above resonance interatomic energy reduces to the counterpart in an unbounded Minkowski spacetime. This consistency comes out naturally as the cosmic string spacetime reduces to the Minkowski spacetime when  $\nu = 1$ . For a general value of the parameter  $\nu$ , if at least one of the atoms is located on the string, the resonance interatomic energy of the two atoms is accurately  $\nu$  times of its counterpart in an unbounded Minkowski spacetime. As the value of  $\nu$  predicted in typical GUT-models is slightly larger than unity, the resonance interatomic energy of the two atoms is slightly stronger than its counterpart in an unbounded Minkowski spacetime. For the case of  $\nu$  being an integer larger than unity, the case of which analytical results are often obtainable and thus very useful for understanding the

intrinsic relation between the nature of the cosmic string spacetime and the atomic resonance interatomic energy, the resonance interatomic energy, equation (11), can be further simplified:

$$\delta E = \mp \frac{\lambda^2}{16\pi} \sum_{n=0}^{\nu-1} \frac{\cos(\omega_0 R_{n,\nu})}{R_{n,\nu}} \quad (12)$$

with  $R_{n,\nu} = \sqrt{z^2 + r_A^2 + r_B^2 - 2r_A r_B \cos(\Delta\theta + 2\pi n/\nu)}$ . The term on the right of the above resonance interatomic energy corresponding to  $n = 0$  is exactly the same as the counterpart of the resonance interatomic energy in an unbounded Minkowski spacetime, and other terms with  $n \neq 0$  are crucially dependent on the relative positions of the two atoms with respect to the string. This property of the resonance interatomic energy is reminiscent of the resonance interatomic energy of two identical atoms fixed near a perfectly reflecting boundary in the Minkowski spacetime (see equation (5)). Actually the resonance interatomic energy in these two cases is very similar. As shown in the following figure, the resonance interatomic energy of the two atoms near the string oscillates with the separation between the atoms and the string, and the oscillation amplitude decreases with increasing atoms-string separation. As compared with its counterpart in an unbounded Minkowski spacetime,  $\delta E^0$ , it can be greater or smaller and even be nullified. These similarities of the resonance interatomic energy in the two cases can be traced back to the nontrivial topology of the cosmic string spacetime. The cosmic string spacetime is locally flat but globally nontrivial, thus field modes propagating inside are “restricted” by the special structure of the spacetime, and the radiative properties of atoms in interaction with quantum fields in this spacetime exhibit some boundary-like behaviors.

Despite the similarities, the resonance interatomic energy in the cosmic string spacetime is also characterized by its peculiar properties. As shown in figure 3, when the two atoms are located very close to the string, the resonance interatomic energy is almost  $\nu$  times of its counterpart in an unbounded Minkowski spacetime; and the oscillation amplitude of the resonance interatomic energy with the atoms-string separation is closely dependent on the value of the parameter  $\nu$ : the larger the value of  $\nu$ , and the severe the oscillation. Thus in principle, it is possible to distinguish different cosmic string spacetimes via the resonance interatomic energy of two identical atoms in a symmetric/antisymmetric entangled state.



**Figure 3.** The resonance interatomic energy of two atoms fixed with constant separation  $R$  parallel to an infinite and straight cosmic string. The atom-string separation for both atoms is denoted by  $r$ , and  $\omega_0 R = 2$ . The ordinate is of unit  $\mp \frac{\lambda^2 \omega_0}{16\pi}$ .

## 5. Conclusions

We studied the resonance interatomic energy of two identical two-level atoms prepared in a symmetric/antisymmetric entangled state and in interaction with quantum massless scalar

fields in three circumstances: near a perfectly reflecting boundary in a Minkowski spacetime, in a Schwarzschild spacetime and near an infinite and straight cosmic string respectively. We found that the resonance interatomic energy in the Schwarzschild spacetime is modified by the spacetime curvature, while it is never disturbed by the Hawking radiation of a black hole, which results in the same resonance interatomic energy in the Boulware, Unruh and Hartle-Hawking vacua. This property of the resonance interatomic energy of two identical atoms in the Schwarzschild spacetime is in sharp contrast to other radiative properties of atoms (such as the Lamb shift and the spontaneous excitation rate of a single atom), and the reason can be traced back to the fact that the resonance interatomic energy of two atoms in a symmetric/antisymmetric entangled state is wholly contributed by the atomic radiation reaction which is irrelevant of the thermal state of the field and independent of the field fluctuations which are susceptible to thermal radiation. For the cases of two atoms located near a perfectly reflecting boundary and near an infinite and straight cosmic string, we found that the properties of the resonance interatomic energy are very similar, as in both cases the resonance interatomic energy oscillates with the atoms-boundary/atoms-string separation with the oscillation amplitude decreasing with increasing atoms-boundary/atoms-string separation. Such a boundary-like behavior of the resonance interatomic energy in the cosmic string spacetime results from the nontrivial topology of the cosmic string spacetime. Besides the similarities, the resonance interatomic energy of the atoms in the cosmic string spacetime is also characterized by its peculiar properties, making it in principle possible to discern different cosmic string spacetimes via the resonance interatomic energy.

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