

PAPER • OPEN ACCESS

Waveriders on the plane shock waves with maximum lift-todrag ratio

To cite this article: N Ostapenko and S Stradomskiy 2018 *J. Phys.: Conf. Ser.* **1129** 012027

View the [article online](#) for updates and enhancements.

You may also like

- [A dimensionality reduction algorithm for mapping tokamak operational regimes using a variational autoencoder \(VAE\) neural network](#)
Y. Wei, J.P. Levesque, C.J. Hansen et al.
- [The gust-mitigating potential of flapping wings](#)
Alex Fisher, Sridhar Ravi, Simon Watkins et al.
- [Statistical Modeling of Vehicles Collision When Using a DVR with the Function of Signaling about Dangerous Motion of the Vehicle to the Oncoming Lane](#)
A N Varnavsky



ECS
The
Electrochemical
Society
Advancing solid state &
electrochemical science & technology

DISCOVER
how sustainability
intersects with
electrochemistry & solid
state science research

Waveriders on the plane shock waves with maximum lift-to-drag ratio

N Ostapenko, S Stradomskiy

Institute of Mechanics of Lomonosov Moscow State University, 1 Mitchurinsky prosp.,
119192 Moscow, Russia

E-mail: sergei.stradomsky@yandex.ru

Abstract. The problem of the waverider's shape with maximum lift-to-drag ratio was formulated and solved. The waverider was constructed on a plane shock wave underplane symmetry and two isoperimetric conditions: the specific volume and the lift coefficient are given and fixed. The upper surface of the waverider is aligned with the oncoming stream and does not disturb it. The bottom surface consists of straight lines that make up the same angle with the undisturbed flow. The leading edge is a curve located in the plane of the shock wave and making an angle with the direction of the oncoming stream. If the leading edge does not come out to the base section, then its design provides lateral spacers of triangular shape with an angle between their edges. Besides the pressure, our model of interaction of the flow with the surfaces of the waverider accounts for the friction, which varies independently along each chord. In this problem we define the base section as two symmetrical flat surfaces. In such formulation, the optimization problem is considered for the first time. The distribution of the length of the chord of waverider in the plan is found by applying the numerical methods.

1. Introduction

The problem of a waverider that has a maximum lift-to-drag ratio for a certain set of isoperimetric conditions has been one of the central problems in aeromechanics for about half a century.

Modern studies of promising supersonic and hypersonic vehicles of high lift-to-drag ratio show that the idea of constructing lifting shapes on stream surfaces in certain well-known gas dynamic solutions put forward as early as forty years ago is still the leading concept in designing waveriders.

In [2], for the first time, the variational problem of the optimal waverider on plane shock waves was formulated and solved. The upper surface of the waverider is directed along the flow. The lift coefficient and specific volume of the waverider are given, as without conditions on the dimensions of the waverider, as well in the presence of it. In the formulation of the variational problem [2] there were considered the most important isoperimetric conditions for carrier aerodynamic forms that are the lift coefficient, the variable coefficient of friction along the length of the body, limitations on heat flux at the leading edge and the contribution of the blunted leading edge resistance to the full body resistance. It is shown that for independent functions describing the shape of the leading edge and the base section, the absolutely optimal body has a bottom surface in the form of a V-shaped wing. The purpose of this work is to determine the optimal shape of the waverider with a symmetrical relative to the central chord of the device, a straight bottom cut. This restriction for the bottom cut of waverider is useful in applications.



2. Geometry of a waverider constructed on a plane shock

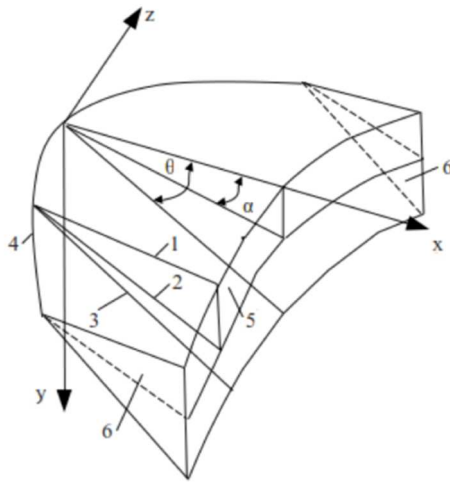


Figure 1. Waverider with side washers.

We will consider a waverider formed by three cylindrical surfaces subject to the plane symmetry $z = 0$ in x, y, z reference frame (figure 1). The upper surface 1 of the waverider is a stream surface of the undisturbed flow. The bottom surface 2 is a stream surface of the flow behind the oblique shockwave 3, which makes an angle θ with the undisturbed flow direction. The line of intersection of two cylindrical surfaces that are the leading edge 4 of the waverider is in the plane of the oblique shock wave 3. The angle α between the generators of the surfaces 1 and 2 originating from the same point on the waverider's leading edge is the angle of flow deflection in the oblique shock wave 3. The third cylindrical surface is the waverider's bottom cut 5 with generators perpendicular to those of the upper surface 1 in the planes $z = \text{const}$. In general case the waverider has side washers 6 with leading edges are located on the shock wave 3.

3. Formulation of the variational problem

The variational problem formulated below relates to the problem of the maximum lift-to-drag ratio of a vehicle (waverider) with two isoperimetric conditions.

The problem. Let the lift coefficient C_y and the specific volume of the waverider τ be preassigned:

$$C_y = \frac{F_y}{q_\infty S_{pl}}, \quad \tau = \frac{V}{(S_{pl})^{3/2}} \quad (1)$$

Here F_y is the waverider lift, V is the volume of the waverider, q_∞ is the velocity head, and S_{pl} is the area in plan.

In order to find the solution of the problem it is necessary to minimize the drag coefficient of the waverider:

$$C_x = \frac{F_x}{q_\infty S_{pl}} \quad (2)$$

where F_x is the waverider's drag.

The formulation of the variational problem for minimizing the functional $J(\eta)$ has the form,

$$J(\eta) = C_x(\eta) \rightarrow \min(\eta) = C_{x0}, \quad \tau(\eta) = \tau_0, \quad \eta \geq 0 \quad (3)$$

where $\eta(z)$ is the function defining the chord length by wingspan of the waverider in plan.

4. The method of finding the optimal solution

The numerical method of local variations, described in [3] for the problems with one isoperimetric condition, was chosen as a method for solving the variational problem. In this paper, we propose a modification of this method of local variations for a problem with two isoperimetric conditions one of which is dependent on the derivative of the required extremal η .

The modification of the method [3] is as follows: first, we discretize the interval $[0, b]$, and second at an arbitrary point with index j a weak variation of function η of the initial contour is given. In order to satisfy two isoperimetric conditions, it is necessary to carry out variations in two different points that do not coincide with the first one. In order to find the values of these variations, we

linearize the functions for the specific volume and lifting force by means of weak variations at two neighboring points different from j , which allows to approximate the derivative of the function η . Thus, we obtain two equations for two unknown variations. Such operations are carried out for each point of partition of the interval over which the function η is defined.

5. Results of the numerical solution of the problem

5.1. Optimal waveriders, whose shape consists of straight line segments, do not exist

Three parameters are introduced: η_0 is the chord length at point $z = 0$, η_b/η_0 is the ratio of the chord lengths at points with coordinates $z = b$ и $z = 0$ respectively, b/η_0 is the ratio of the half-span of the waverider to the chord length at the point $z = 0$ respectively, by which one can uniquely reconstruct a contour composed of straight lines for a given slope of the base section.

A system of three homogeneous linear algebraic equations is constructed with respect to the variations $\delta\eta_0$, $\delta(\eta_b/\eta_0)$, $\delta(b/\eta_0)$, which is consistent only if its determinant is zero. The calculations show that under no conditions the optimal contour does not belong to the class of straight lines.

5.2. The optimal form of the waverider depend only on the state of the boundary layer

The relations (1), (2) for the waverider drag coefficient and isoperimetric conditions can be written in equivalent forms B_1 and B_2 , from which it follows that the distribution of the chord over the scope of the optimal waverider does not depend on the Reynolds number. Thus, within the framework of the considered model of aerodynamic forces on a waverider, the extremal depends only on the state (laminar or turbulent) of the boundary layer given by the parameter m :

$$\widehat{C}_x = [C_x - (c_p + c_b)tg\alpha] \frac{1-m}{B_1} = \frac{I_3}{I_1} + \frac{B_2}{B_1} \frac{I_4}{I_1} + \frac{tg\theta(1+k B_2/B_1)}{(2-m)I_1} \eta_b^{2-m} \quad (4)$$

$$\widehat{C}_y = (c_p - C_y) \frac{1-m}{B_2 tg\alpha} = \frac{1}{I_1} (I_4 + \frac{ktg\theta}{2-m} \eta_b^{2-m}) \quad (5)$$

$$\widehat{\tau} = I_2 / I_1^{3/2} \quad (6)$$

where (see [2])

$$B_2/B_1 = (A_n / \sqrt[n]{Re_\infty}) / (A_n / \sqrt[n]{Re_1}) = [(u_1/u_\infty \cos \alpha) (\frac{\rho_1}{\rho_\infty})^{1+\omega} (\frac{p_1}{p_\infty})^{1+\omega}]^{1/n} \quad (7)$$

$$I_2 = \int_0^b \eta^2 dz, \quad I_1 = \int_0^b \eta dz, \quad I_3 = \int_0^b \eta^{1-m} \Phi_1, \Phi_1 = \sqrt{1+f'^2} \quad (8)$$

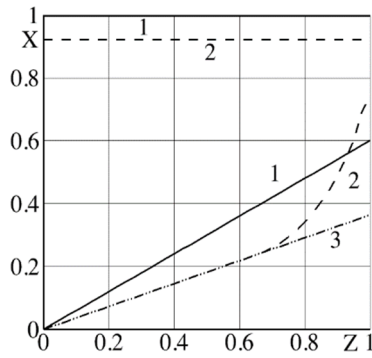
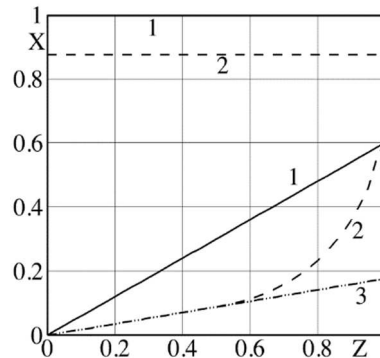
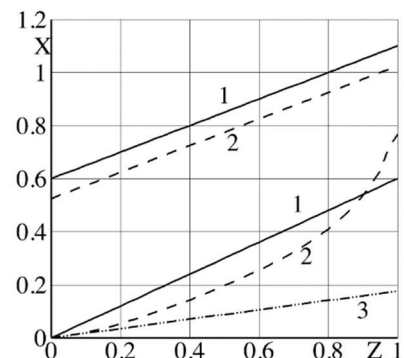
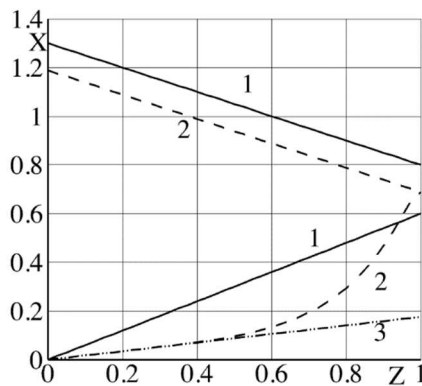
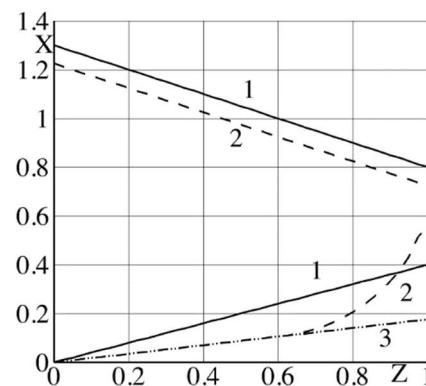
$$I_4 = \int_0^b \eta^{1-m} \Phi_2 dz, \Phi_2 = \sqrt{1+k^2 f'^2}, \quad k = \frac{\sin(\theta-\alpha)}{\sin \theta} < 1, \quad f(z) = tg\theta[\eta_0 + cz - \eta(z)] \quad (9)$$

Here, A_n is the coefficient, which depends on the parameters of the flow and the number n ($n = 2$ and $n = 5$ in the cases of laminar and turbulent boundary layer, respectively), $m = 1/n$, c_p and c_b are the pressure coefficient behind the plane shock wave and the bottom pressure coefficient, respectively, and ω is the exponent in the viscosity-temperature dependence.

5.3. Optimal contours with the heat flux limitation on the leading edge

According to [4], the main parameters determining the heat flux to the blunt leading edge of the wing are the Mach number of the undisturbed velocity component, normal to the leading edge and the radius of its blunting. In order to simulate the limiting conditions for the heat flux at the leading edge, it is sufficient to specify a lower limit for the sweep of the leading edge, angle γ . In other words, it is sufficient to introduce a top constraint for the Mach number normal to the leading edge of the

waverider, if blunting does not introduce fundamental changes in the lift and drag coefficients [2]. In the following figures, which correspond to the various restrictions on the heat flux with different slopes C of the base section, the results of the calculation for the following parameters are presented: $M = 3$, $m = 0.5$, $Pr = 0.71$, $\alpha = 5^\circ$, $\omega = 0.72$, and the waverider's wall temperature is equal to the equilibrium temperature.

Figure 2. Waverider with $\gamma = 20^\circ$.Figure 3. Waverider with $\gamma = 10^\circ$.Figure 4. Waverider with $\gamma = 10^\circ$.Figure 5. Waverider with $\gamma = 10^\circ$.Figure 6. Waverider with $\gamma = 10^\circ$.

In all figures, solid straight lines 1 show the initial shapes of the waverider in plan, the dashed curves 2 are the optimal shapes, and the dashed straight line 3 shows the sweep angle of the leading edge that determines the limitation on the heat flux. In figure 2 and 3 to 6, this angle is equal to 20° and $6-10^\circ$ respectively. Figures 2 and 3 show the results for $C = 0$, figure 4 for $C = 0.5$, and figure 5 and 6 for $C = -0.5$. As can be seen, under limitation for the heat flux, there is always a straight section at the leading edge of the optimal waverider. It is very important that in all cases, except of figure 5, the optimal waveriders has the side washers. All calculations were carried out for the half-size $b = 1$, since the dimensions of the waverider do not have a fundamental value, and, as shown in [5], its lift-to-drag ratio can be increased by increasing the characteristic size of the vehicle.

Table 1. Data for contours with heat flux.

Figure	$\widehat{C}_{y0}$	$\widehat{C}_y$	$\widehat{\tau}_0$	$\widehat{\tau}$	K_0	K
2	0.0715	0.0715	0.0275	0.0275	4.2282	4.3007
3	0.0715	0.0715	0.0275	0.0275	4.2282	4.3235
4	0.0714	0.0714	0.0242	0.0242	3.8945	3.9657
5	0.0715	0.0715	0.0316	0.0316	4.4977	4.5897
6	0.0715	0.0715	0.0312	0.0312	4.5574	4.6270

The numerical values of K are obtained for the Reynolds number $Re_\infty = 10^5$. The gain in the lift-to-drag ratio of the optimal waverider in comparison with the equivalent waverider with the initial shape in plan is from tenths to several percent in each case.

5.4. The influence of the Mach number on the optimal contour in plan

Figure 7 shows the dependence of the optimal contour on the Mach number. The curves 1, 2 and 3 show optimal contours for the number $M = 3, 5, 7$, respectively. In all three cases, the original contour was the same, $Re_\infty = 10^5$.

Table 2. Data for contours with different Mach number.

Contour	$\widehat{C}_y$	$\hat{\tau}$	K
1	0.0715	0.0278	4.383
2	0.0456	0.0273	3.411
3	0.0354	0.0288	2.914

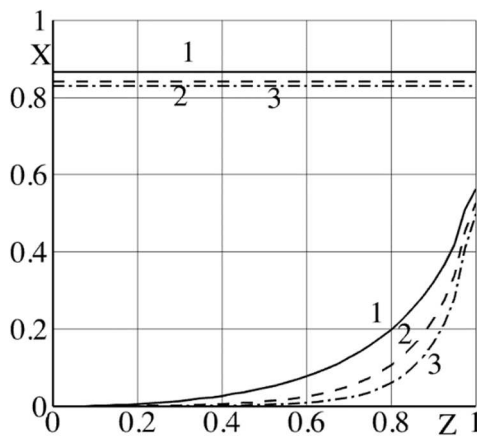


Figure 7. The influence of the Mach number.

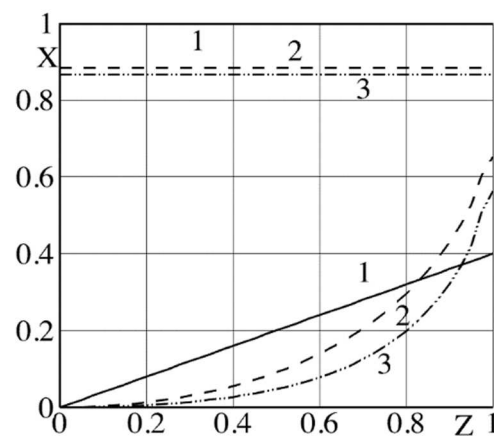


Figure 8. The influence of the state of boundary layer.

As can be seen, with an increase of the Mach number, the contour of the leading edge becomes more convex, and the lift-to-drag ratio decreases.

5.5. The influence of the state of boundary layer on the form of the optimal contour

In figure 8, the straight lines 1 indicate the original contour, which in both cases was the same. The results are shown for $M = 3$ and $C = 0$. The dashed lines 2 and 3 correspond to the optimal contours with the parameter $n = 5$ and $n = 2$.

5.6. The influence of the angle on the optimal contour.

In figure 9 the straight lines 1 indicate the initial contour, which in both cases was the same. The dashed curves 2 and 3 correspond to the optimal contours constructed for $\alpha = 20^\circ$ и 5° , respectively. Mach number $M = 3$. As can be seen, the effect of the angle α on the optimal contour is small. An increase in the lift-to-drag ratio of the optimal waverider is observed if the angle α is reduced (see also Table 3).

Table 3. Data for contours with different angle α .

Contour	$\widehat{C}_y$	$\hat{\tau}$	K
1 - 20°	0.440	0.1175	2.535
1 - 5°	0.0715	0.0282	4.296
2	0.442	0.1175	2.542
3	0.0715	0.0282	4.383

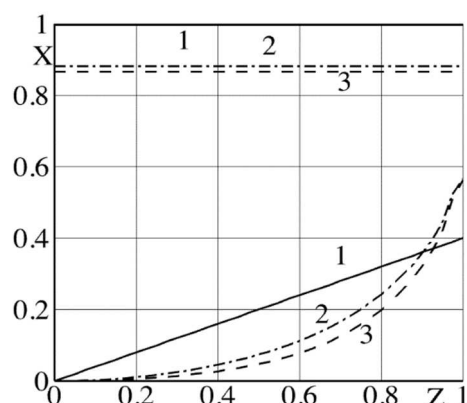


Figure 9. The influence of the angle α .

6. Summary

We have formulated and solved the problem of the waverider's shape with maximum lift-to-drag ratio. The waverider is constructed on a plane shock wave subject to plane symmetry and under two isoperimetric conditions: the specific volume of the waverider and the lift coefficient are given. In addition to the pressure our model of interaction of the flow with the surfaces of the waverider accounts for the friction, which varies on the plate independently along each chord from the leading edge to the base section. The upper surface of the waverider is aligned with the oncoming stream and does not disturb it. The bottom surface consists of straight lines that make up the same angle α with the undisturbed flow. The extremal that is the distribution of the length of the chord of the waverider in the plan by scope is found by using the numerical method of local variations adapted to the variational problem with two specified isoperimetric conditions. We have studied the shape of the optimal waverider in plan as function of the Mach number, the angle α , the boundary layer state, and restrictions on the heat flux at the leading edge.

Acknowledgements

The work was carried out with the partial financial support of the Russian Foundation for Basic Research (Project No. 18-01-00182).

References

- [1] Ostapenko N A and Stradomski S S 2017 Waveriders on the plane shock waves with maximum lift-to-drag ratio *Inter. Conf. «Modern problems of continuum mechanics» dedicated to the memory of academician Leonid Sedov in case of 110th anniversary of his birth, 13–15th November* (Moscow: Math. Inst. of RAS)
- [2] Ostapenko N A 2000 On the optimal shapes of waveriders constructed on plane shocks *Fluid Dyn.* **6** (35) 830–45
- [3] Chernous'ko F L 1965 A local variation method for the numerical solution of variational problems *USSR Computational Mathematics and Mathematical Physics* **4** (5) 234–4
- [4] Brykina I G, Gershbein E A and Peigin S V 1980 Laminar boundary layer on swept-back wings of infinite span at an angle of attack *Fluid Dyn.* **3** (15) 344–54
- [5] Ostapenko N A and Semenov A V 2008 Lift-to-drag ratio of the optimal waveriders on the plane shock waves *«Problems of modern mechanics»: to the 85th anniversary of academician G G Chernyi [collection]/ Lomonosov State University, Institute of Mechanics.* Under the editorship A A Barmin (Moscow: Moscow University; Omega-L) 408–31