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The computation method of rational multi-point forming of panel in the creep mode

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Abstract. The problems of modeling in the CAE-system of panels forming processes in the creep mode with the use of reconfigurable rod punch are considered. The geometrical and contact nonlinearity are taken into account in the model of the problem. For the formulation of rational deformation problems the damage parameter is taken as an optimization criterion. A discrete optimal control problem is formulated, which is solved by the method of dynamic programming with improvement by the method of local variations

1. Introduction

Technological problems of shaping large-sized products in the slow deformation mode are of great practical importance in modern domestic and foreign aircraft building [1-4]. Recently, the shaping of products from sheets and panels is considered using a reconfigurable rod punch (matrix). The shaping surface of the punch as well as the matrix formed by two coaxially arranged rods systems, each placed in an individual position by the numerical control, allows the adaptation of the tooling system to manufacturing parts of various configurations. In works [5-9] design features of rod systems are considered and the main approaches to definition of the loads acting on each rod element are proposed.

Accuracy of the part shape obtained by pressure processing technologies of materials under specified process parameters depends on accuracy of calculated and manufactured tooling shape (reconfigurable rod punch) that determines the anticipatory shape of the panel. The anticipatory shape of the panel should provide the given residual curvature of the panel after unloading. Thus, we arrive at the formulation of the inverse problem. The inverse shaping problem determines external forces and kinematic effects under which a strain process occurring under creep conditions over a given time interval leads to a prescribed residual configuration after elastic unloading [10-13].

The experimental results allow us to identify the dissipated work with the damage parameter [4]. In this case, optimal control problems can be formulated in shaping processes [14, 15].

The problem of finding the motion law of tooling rods, which provides minimal damage and a given residual shape of the panel after unloading, is considered in article.

2. Formulation and method for solving optimal forming problems

Let $V \subset R^3$ be a bounded domain with a sufficiently regular boundary S. The contact surface of rigid bodies (stamps) with deformable body is denoted through S_c ($S_c \subset S$). Denote by $u = (u_1, u_2, u_3)$ and $\tilde{u} = (\tilde{u}_1, \tilde{u}_2, \tilde{u}_3)$ the vectors of current and residual displacements of deformable body; $u, \tilde{u} \in [W_2^1(Q)]^3$,

$Q = V \times \{0 \le t \le T\}$. Denote by $\bar{u} = (\bar{u}_1, \bar{u}_2, \bar{u}_3)$ the vectors of displacements at the contact surface of rigid bodies; $\bar{u} \in [W_2^1(Q_c)]^3$, $Q_c = S_c \times \{0 \le t \le T\}$. The norm is given by

$$\| u \|_S = (u,u)^{1/2}{}_S = \left(\int_S \sum_{i=1}^3 u_i^2 dS \right)^{1/2}.$$

Consider the quasi-static shaping problem with allowance for small deformations and large displacements and rotations (general Lagrangian formulation), including creep strain and elastic unloading. The inverse problem of a kinematic shaping by contact rigid stamps can be represented in the form of a quasi-static variational principle with the functional [11]:

$$J(\dot{\tilde{u}}, \dot{u}, \dot{\tilde{u}}) = \frac{1}{2\varepsilon_1} \| \dot{\bar{u}} - \dot{\bar{u}}^* \|_{S_c}^2 + W_c + a(\dot{u}, \dot{u}) + a(\dot{\tilde{u}}, \dot{\tilde{u}}) + \frac{1}{2\varepsilon_2} \| \dot{\tilde{u}} - \dot{\tilde{u}}^* \|_S^2, \tag{1}$$

$$\varepsilon_1 > 0, \varepsilon_1 \to 0, \varepsilon_2 > 0, \varepsilon_2 \to 0,$$

where $\dot{\tilde{u}}^*$, $\dot{\bar{u}}^*$ is a given residual displacement rate and current contact displacement rate at the moment of time t; $t \in [0,T]$ is time of deformation under load. W_c is the quasistatic contact potentials obtained by imposing the contact conditions on the equations of body motion by method of Lagrange multipliers or by method of penal functions [16] and differentiation on t; potential form are given by

$a(\dot{u}, \dot{v}) = \int_V [\partial E(\dot{u}_{i,j}) / \partial \dot{u}_{i,j}] \dot{v}_{i,j} dV$, $a(\dot{\tilde{u}}, \dot{\tilde{v}}) = \int_V [\partial \tilde{E}(\dot{\tilde{u}}_{i,j}) / \partial \dot{\tilde{u}}_{i,j}] \dot{\tilde{v}}_{i,j} dV$,

$E(\dot{u}_{i,j}) = (1/2) c_{ijpl} \dot{\varepsilon}_{ij} \dot{\varepsilon}_{kl} - c_{ijpl} \dot{\varepsilon}_{ij} \dot{\varepsilon}_{pl}^c + (1/2) \sigma_{ij} \dot{u}_{p,i} \dot{u}_{p,j}$, $\tilde{E}(\dot{\tilde{u}}_{i,j}) = (1/2) c_{ijpl} \dot{\tilde{\varepsilon}}_{ij} \dot{\tilde{\varepsilon}}_{pl} - c_{ijpl} \dot{\tilde{\varepsilon}}_{ij} \dot{\varepsilon}_{pl}^c + (1/2) \rho_{ij} \dot{\tilde{u}}_{p,i} \dot{\tilde{u}}_{p,j}$

[17], c_{ijpl} are the components of symmetric tensor of elastic constants; $\dot{\varepsilon}_{pl}^c$ are the creep strain rates; $i, j, p, l = 1, 2, 3$; $\dot{\varepsilon}_{ij} = (1/2)(\dot{u}_{i,j} + \dot{u}_{j,i} + \dot{u}_{p,i} u_{p,j} + u_{p,i} \dot{u}_{p,j})$, $\dot{\tilde{\varepsilon}}_{ij} = (1/2)(\dot{\tilde{u}}_{i,j} + \dot{\tilde{u}}_{j,i} + \dot{\tilde{u}}_{p,i} \tilde{u}_{p,j} + \tilde{u}_{p,i} \dot{\tilde{u}}_{p,j})$.

The law of steady creep (Norton's law) [17]: $\dot{\varepsilon}_{ij}^c = \gamma s_{ij}$, $\gamma = (3/2) A \bar{\sigma}^{n-1}$, where s_{ij} are the components of the deviator of the stress tensor, $\bar{\sigma} = (3/2 s_{ij} s_{ij})^{1/2}$ is the effective stress (stress intensity), where A, n are the constants of the material.

The following iterative method for solving the inverse shaping problem in displacement of a contact surface is used:

$$\bar{u}_i^{k+1} = \bar{u}_i^k + \alpha^k (\tilde{u}_i^* - \tilde{u}_i^k) + \beta^k (\bar{u}_i^k - \tilde{u}_i^k), \tag{2}$$

where $0 < \alpha^k < 2$, $\beta^k \to 0$ at $k \to \infty$, $i = 1, 2, 3$. The proof of the convergence of this method and the approximate values of the coefficients are given in [12].

Thus, the method (2) is used to determine the final position of the tooling rods, which provides a given residual shape of the panel after unloading.

The problem of optimal deformation is formulated as follows [14]: it is required to find a way to deform an element of the medium for a given time T, so that at the time $t=T$, the specified creep strains $\dot{\varepsilon}_{ij}^{c*}$ are obtained and the damage parameter Ω is minimal. Minimizing the dissipation power for each time point will give the minimum value of the dissipated work $A = \int_0^T \sigma_{ij} \dot{\varepsilon}_{ij}^c dt$ and accordingly the damage parameter [4].

Thus, the mathematical formulation of optimal control problem includes equations of mechanics of a deformable solid, obtained from the stationary conditions of the functional (1), and the optimization criterion

$$J = \max_{x \in V} \int_0^T \sigma_{ij} \dot{\varepsilon}_{ij}{}^c dt \to \inf. \tag{3}$$

In the case of small deflections of plate, the optimal deformation is linear: $u_3 = (t/T)u_3^*$; in the case of large deflections, the optimal deformation is given by the nonlinear law $u_3 = (t/T)^{1/2} u_3^*$ [14,15].

Applying the finite-element method for solving variational problems [16, 17,18] constructed using the functional (1), we obtain a system of algebraic equations for the problem of deformation of the body during loading and a system of algebraic equations for the problem of deformation of the body during unloading:

$$^{t+\Delta t}\mathbf{K}^{(i-1)}\Delta\mathbf{U}^{(i)} = {}^{t+\Delta t}\mathbf{R}^{(i-1)}, \quad {}^{t+\Delta t}\hat{\mathbf{K}}^{(i-1)}\Delta\hat{\mathbf{U}}^{(i)} = {}^{t+\Delta t}\hat{\mathbf{R}}^{(i-1)}, \tag{4}$$

where $^{t+\Delta t}\mathbf{K}^{(i-1)}$, $^{t+\Delta t}\hat{\mathbf{K}}^{(i-1)}$ are the tangent stiffness matrices (in matrices $^{t+\Delta t}\mathbf{K}^{(i-1)}$ the additional elements which are formed from contact restrictions are already included), $^{t+\Delta t}\mathbf{R}^{(i-1)}$, $^{t+\Delta t}\hat{\mathbf{R}}^{(i-1)}$ are the vectors of internal and external forces. The superscript $t + \Delta t$ of a quantity indicate time for which it is calculated. The superscript $(i-1)$ indicate number of iteration at correction of the solution by Newton-Rafson's method. The solution at the next step is found through a formula $^{t+\Delta t}\mathbf{U} = {}^{t}\mathbf{U} + \Delta\mathbf{U}$. Once contact is detected, the degrees of freedom are transformed to a local system and a constraint is imposed such that $\Delta\mathbf{U}_{normal} = \mathbf{v} \cdot \mathbf{n}$, where $\mathbf{v}$ is the prescribed velocity of the rigid surface [19]. After that it is possible to find residual nodal movements $\tilde{\mathbf{U}} = \mathbf{U} + \hat{\mathbf{U}}$.

For an approximate solution of the optimization problem, the interval $[0,T]$ is divided into N parts: $0 < t_1 < t_2 < \ldots < t_N = T$. Taking into account the time-discrete equations of the step-by-step integration procedure (4) and the minimized functional (3) on time intervals, a discrete optimal control problem is formulated. In such a formulation, the problem can be solved by the method of dynamic programming [20,21].

Let the vector-function of the displacement of the contact bodies points on the boundary S_c be given in the form $\overline{\mathbf{U}}(t) = f(t)\overline{\mathbf{U}}^k$, where $\overline{\mathbf{U}}^k$ is the solution of the inverse problem by the method (2) with an arbitrary function $f(t)$.

On the set $G_k \equiv G(t_k)$ we take some discrete grid of points $u_{kp} \in G_k$; the set of all points of the chosen grid will be denoted by H_k ($k = 0,1,\ldots,N$) [20]. On two adjacent scales H_k and H_{k+1} take the points $a \in H_k$ and $b \in H_{k+1}$. Here, $u_{kp} \in G_k$; the possible positions of the contact bodies in time are taken as displacement of the plate.

On the interval time $[t_k,t_{k+1}]$ displacement, deformation and stress are determined from the first equation (4), when solving the plate deforming problem by moving contact bodies from position $a \in H_k$ to $b \in H_{k+1}$ according to defined function $f(t)$. It is necessary to find such a function $f(t)$ for which it will be executed (3) on the interval $[t_k,t_{k+1}]$. Thus, it is necessary to find a minimum of (3) over all possible sets of points $(u_{0p_0},u_{1p_1},\ldots,u_{Np_N})$, $u_{kp_k} \in H_k$, $k = 0,\ldots,N-1$.

This method reduces the amount of computation in comparisons with a simple search of all possible deformation ways, since in the calculation process the non-optimal trajectories are excluded. To obtain a more accurate solution, it is necessary to take a fine mesh ($N \to \infty$), in which case, of course, the calculation time increases significantly. Therefore, it is more efficient to use the method of local variations for improvement.

The method of local variations [20, 21] assumes that some path l_1 connecting scales H_0 and H_N is known. To determine the shorter path, the scales H_0, $H_1,\ldots,H_N$ are scanned successively. On the scale H_k, several points of the lines l_1 closest to the point are selected and the paths passing through these points are compared, and on the other sections the path remains unchanged. If there is a path with a shorter length, then this path is taken as the path l_1. Next, the points of the next scale are scanned H_{k+1}. The grid of the scale H_k before search by this algorithm should be reduced.

3. Numerical solutions

The solution of the optimal control problem by the method of dynamic programming is considered on the example of the forming of a square plate in a rod tool (figure 1).
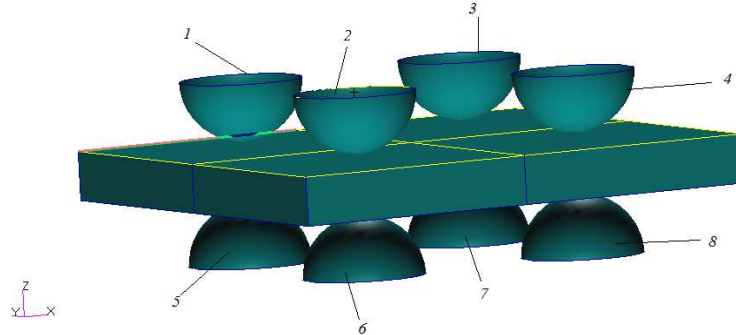
Figure 1. Model of the plate.

The material is isotropic and has the Young's modulus E = 7000 kg/mm^2 and Poisson's ratio v = 0.4. Steady state creep in the experiments is described by Norton's law with different coefficients B for each types of strain: compression $B_1 = 0.25 \cdot 10^{-14}(kg/mm^2)^{-n1}$ (h)$^{-1}$, $n_1 = 8$; tension $B_2 = 0.5 \cdot 10^{-14}$(kg/mm^2)$^{-n2}$(h)$^{-1}$, $n_2 = 8$. The deformation time $T = 260$ h.

The motion of the contact rods is given by the formula: $\bar{u}_{z,l}(t) = f(t)\bar{u}_{z,l}{}^*$, $l=1,\dots,8$. Here l is the number of contact body. For the given curvature of the plate, the displacements values of the contact bodies $\bar{u}_{z,l}{}^*$ are determined by the method (2).
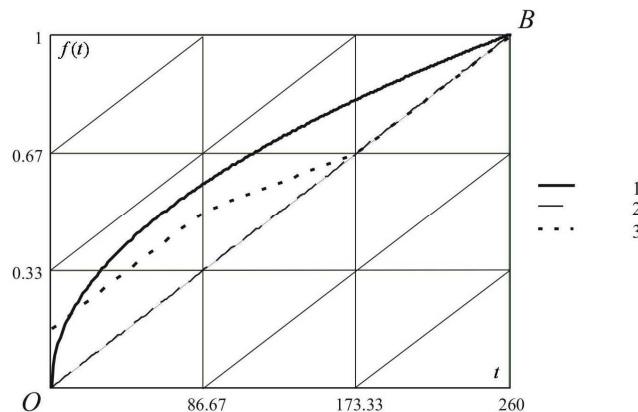
Figure 2. Functions of movement bodies contact.

We consider the case for $N = 3$ and the function $f(t)$ is the same for all rods. Possible options for the functions $f(t)$ connecting points O and B are shown in figure 2. Solving the problem using dynamic programming method will give a linear function (line 2, figure 2).

The solution is refined by the method of local variations [20,21]. To do this, points are added at the top and bottom of the found line at a distance of 1/6. Check for each t_k variant received by the broken line. As a result, the optimal broken line will be line 3. As can be seen from Figure 2, this line approaches the line 1 representing the function $f(t) = (t/T)^{1/2}$. Thus, as the grid is reduced, the optimal broken line calculated by this method will approximate the optimal curve obtained analytically.

Thus, the developed numerical method allows us to find rational shaping processes not only for an ideal plate, but also for such details as wing panels. The algorithm, in comparison with the one presented in [13], does not depend on the model geometry and material properties.

The developed algorithms can be used in industrial applications, allow to model and effectively evaluate the parameters of technological processes for manufacturing parts.

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