Journal of Physics: Conference Series

PAPER • OPEN ACCESS

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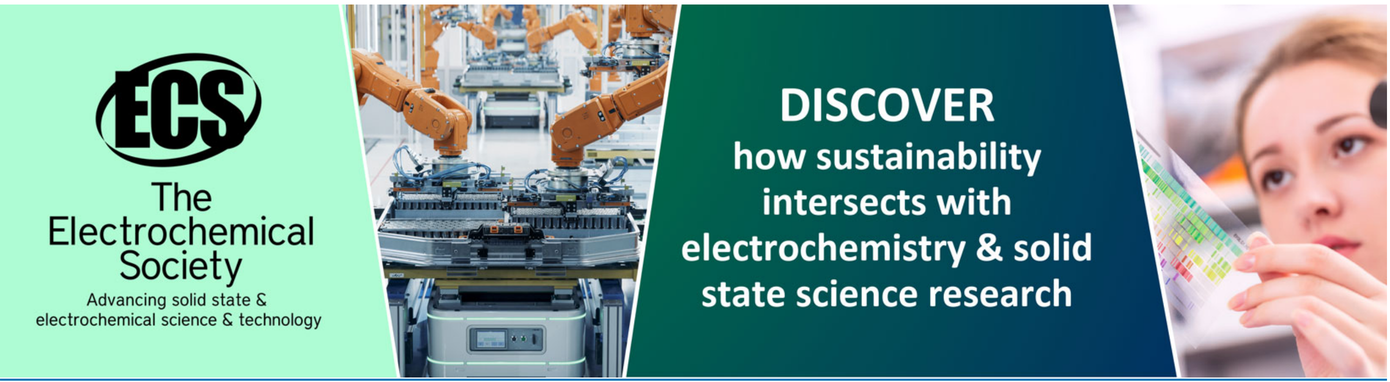
To cite this article: V O Bondarev 2018 *J. Phys.: Conf. Ser.* **1129** 012006



Single-mode flutter of an elastic plate in the presence of the boundary layer

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Abstract. The flutter of an elastic plate in a viscous supersonic gas flow is investigated. The influence of viscous perturbations of the boundary layer on the single-mode flutter is studied considering two different types of a boundary layer profile: the generalized convex boundary layer profile and the profile with a generalized inflection point. It is shown that in the case of the convex layer for thick boundary layers the plate is fully stabilized. In the case of a profile with the generalized inflection point, the thickening of the layer first yields the increase of the growth rates of the perturbations, and the growth is greater than in the inviscid approximation. Numerical simulation of two-dimensional supersonic gas flows with boundary layers (that can have a destabilizing effect on the flutter of an elastic surface) over different curved surfaces is performed. The flow patterns over them are studied: regions of these surfaces, over which stable boundary-layer profiles with a generalized inflection point are formed, are found, and their boundaries are obtained.

1. Introduction

The panel flutter is a dangerous aeroelastic instability of the aircraft skin panels. It was shown earlier, that along with well-known flutter of the coupled-mode type [1], which arises at high supersonic speeds, a single-mode flutter [2], excited at trans- and low supersonic speeds can also develop. Single-mode flutter cannot be detected and investigated by the piston theory, usually used in supersonic aeroelasticity. These results were confirmed numerically and experimentally [3].

At present, the investigation of the boundary layer effect on the loss of aeroelastic stability and the influence on a single-mode flutter is an important and insufficiently studied problem. In papers [4, 5] the influence of the boundary layer of an arbitrary form on panel flutter is investigated in the approximation of the inviscid shear layer (the Reynolds number $R = \infty$). It is shown that the effect of the boundary layer on the single-mode flutter depends on the type of the boundary profile, namely, the influence is different for the generalized convex profile and a profile with the generalized inflection point. The inclusion of the effect of viscosity into boundary layer model was performed in the study [6], where viscous boundary layer perturbations at large but finite Reynolds numbers were considered.

In this paper the results of a problem investigation in the inviscid approach are presented. The influence of viscous perturbations of the boundary layer on a single-mode panel flutter is described. Simulation and analysis of two-dimensional supersonic flows with boundary layers (which have a destabilizing effect on the flutter of panels, constituting the curved surface of the aircraft) are performed.

2. Influence of the boundary layer perturbations on the stability of an elastic finite plate

In this section the influence of the boundary layer perturbations on the stability of an elastic finite length plate in supersonic gas flow at large but finite Reynolds numbers is described.

2.1. Formulation of the problem

An elastic plate of finite length, built into an absolutely rigid plane, is considered. One side of the plate is exposed to gas flow, on the surface of the plate a boundary layer is formed (figure 1). It is assumed that the plate represents a skin panel of a flight vehicle, and the local velocity and temperature distributions in the boundary layer are known from the analysis of the steady flow around the vehicle. The problem is investigated in 2-D formulation, all variables do not depend on y. The boundary layer is assumed to be laminar.

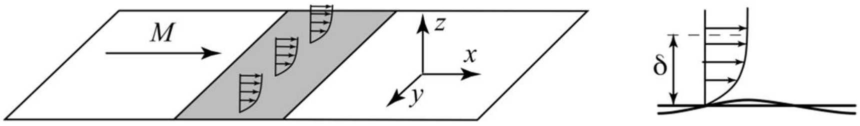

Figure 1. Gas flow over elastic plate.

The motion of the plate is described by the Kirchhoff-Love small deflection plate theory:

$$D\frac{\partial^4 w}{\partial x^4} - M_w^2\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial t^2} + p(x, t, z = 0) = 0, \tag{1}$$

where $w(x, t)$ is the plate deflection, D is its stiffness, M_w^2 is the dimensionless in-plane tension force, and $p(x, t, z)$ is the flow pressure disturbance induced by the plate motion.

Two boundary conditions must be specified at each plate edge. For example, these could be clamping, pinning, or free-edge boundary conditions.

The perturbations of perfect viscous gas with the boundary layer-type mean flow are described by the linearized Navier-Stokes equations. The no-slip and adiabatic conditions along the moving plate surface are considered.

In this paper, the asymptotic criterion of the global instability [7] is applied for studying the stability of an elastic finite plate in a gas flow. According to this criterion the analysis of a finite length system (plate) can be reduced to the study of infinite-system waves.

2.2. Dispersion relation in case of an infinite length plate

Considering the case of an infinite length plate, small perturbations in the form of travelling waves are imposed on the system $\Psi(z)e^{i(kx-\omega t)}$ in the Cartesian coordinate system x, y, z, where k and ω are the wave number and frequency, respectively.

Substituting the deflection of the plate in the form of a traveling wave $w(x, t) = e^{i(kx-\omega t)}$, and pressure disturbance $p(x, t, z) = \pi(0)e^{i(kx-\omega t)}$ into (1), the explicit form of the dispersion relation for an infinite plate in a boundary-layer flow is obtained

$$F(k, \omega) = Dk^4 + M_2^2k^2 - \omega^2 + \pi(0) = 0, \tag{2}$$

in which the pressure perturbation, acting on the plate surface, is found from the linearized dimensionless system of equations for gas perturbations [8]. For $R \to \infty$ the general solution of this system consists of a combination of six linearly independent solutions: two regular solutions, transforming into the solutions of the Rayleigh equation as $R = \infty$, and four solutions of WKB (Wentzel-Kramers-Brillouin) type [8].

2.3. The global instability criterion

Kulikovskii [7] showed that the stability criterion for systems of large finite length is generally different from the criterion for an infinite system. To briefly describe the Kulikovskii global instability criterion the spatial roots $k_j(\omega)$ of the dispersion equation for an infinite system are numbered in the order of decrease of $\mathrm{Im}k_j(\omega)$ for $\mathrm{Im}\omega \gg 1$: $\mathrm{Im}k_1 > \mathrm{Im}k_2 > 0 > \mathrm{Im}k_3 > \mathrm{Im}k_4$.

Next, the roots are splitted into two groups: the first is such that $\mathrm{Im}k_j(\omega) > 0$, j = 1, 2; the second is such that $\mathrm{Im}k_j(\omega) < 0$, j = 3, 4 as $\mathrm{Im}\omega \to +\infty$.

Let us define Ω curve in the ω-plane by the following equation

$$\mathrm{Im}k_2(\omega) = \mathrm{Im}k_3(\omega). \tag{3}$$

According to global instability criterion, as the length of the system L tends to infinity, its eigenfrequencies tend to the Ω curve, which corresponds to the single-mode flutter of the plate [4].

Thus, the instability criterion of long finite systems is as follows: the system is unstable if a piece of the Ω curve is located in the $\mathrm{Im}\omega > 0$ half-plane.

2.4. Results

The results obtained through Kulikovskii criterion [5] show that in case of the generalized convex boundary layer profiles (with $\left(u'/T\right)' < 0$ for $z \in [0; \delta)$, where u, T are the velocity and temperature profiles of the undisturbed flow, δ is the thickness of the boundary layer) the increase of the layer thickness yields the increase of the frequencies of growing eigenmodes and the decrease of their growth rates. For sufficiently thick boundary layers, the plate becomes fully stabilized; however, this occurs at thicker boundary layers than in inviscid shear-layer approximation.

In case of the boundary layer profile with a generalized inflection point z_i, where $\left(u'/T\right)' = 0$, the thickening of the layer first yields the increase of the growth rates accompanied by the widening of the frequency range of growing eigenmodes, and the growth rates are greater than in the inviscid approximation. For higher thicknesses, growth rates decrease and tends towards 0 as $\delta \to \infty$.

3. Numerical simulation of two-dimensional supersonic gas flows with boundary layers over different surfaces

In this section the numerical simulation of two-dimensional supersonic gas flows with boundary layers over different curved surfaces is performed in order to find and analyze stable boundary layer profiles with a generalized inflection point. Calculations are performed through the Ansys CFX by the finite volume method.

3.1. Formulation of the problem

Let us consider the formulation of the problem in terms of the configuration №1.1. The maximum dimensions of the computational domain are: height is 1 m, length is 1 m; its curved lower part is the combination of two flat intersecting surfaces (straight lines) that form an angle (figure 2a). The flow is laminar; the Navier-Stokes equations are solved within the computational domain.

In addition, configurations №2.1, №3.1 and №4.1 (figure 2b-d) are considered. In the configuration №2.1 the lower part of the domain is the straight line which intersects the parabola, in the configuration №3.1 it consists of parts of three surfaces: the straight line, the parabola and the straight line. In the configuration №4.1, the flow over a flat bottom surface with an injection of the boundary layer is simulated.

Also, configurations №1.2, №2.2, №3.2 and №4.2 are analyzed, they are coincide with configurations № 1.1 – 4.1, increased a hundred times in size.

3.2. Boundary conditions

In the calculations the boundary conditions are as follows: normal velocity $u_\infty^* = 495$ m/s, pressure $P_\infty^* = 101325$ Pa and temperature of the gas T_∞^* = 273.15 K (script "*" denotes dimensional parameters) are set at the inlet; the no-slip and adiabatic conditions are given at the bottom surface; $P^* = P_\infty^*$ and $T^* = T_\infty^*$ are set at the top boundary of the domain.

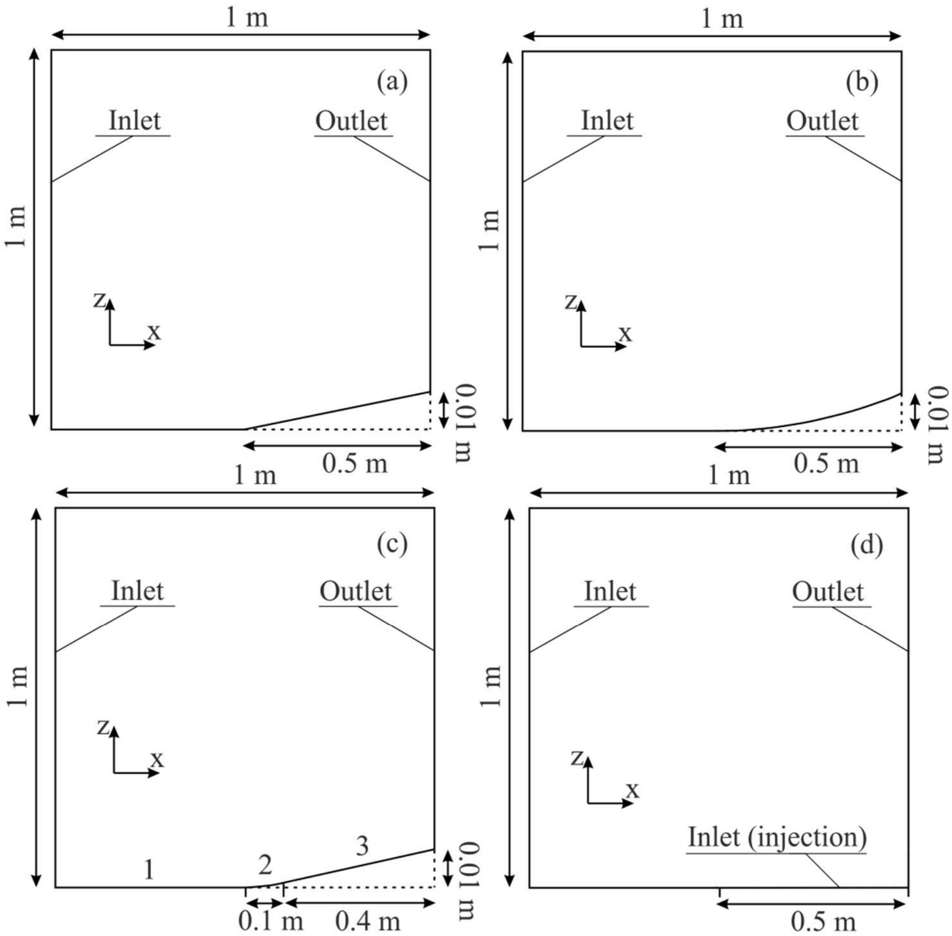

Figure 2. Configurations №1.1 (a), №2.1 (b), №3.1, the figures 1, 2, 3 denote surfaces of the plane, parabolic cylinder and plane, respectively, (c), №4.1 (d).

The boundary conditions for the configurations №4.1 and №4.2 are similar to conditions described above, except that the bottom surface is divided into two identical parts: at the first one the no-slip and adiabatic conditions are given, at the second one the velocity of the injection $u_2^* = 1$ m/s, pressure $P^* = P_\infty^*$ and temperature of the gas $T^* = T_\infty^*$ are set.

3.3. Results of the calculations

Similar flow patterns are observed in the configurations №1.1, №2.1, №3.1, №1.2, №2.2, №3.2: the profile with a generalized inflection point in the supersonic part ($u(z_i) < M - 1$) is formed immediately from the input edge of the curved bottom surface ($x^* = 0$). As the distance from this edge increases to x_1^* (which depends on the configuration), the boundary layer develops, however, the velocity at the inflection point $u(z_i)$ almost does not change. For $x_1^* < x^* < x_2^*$, the velocity $u(z_i)$ begins to grow and, reaching x_2^* (which depends on the configuration), the generalized inflection point moves to the subsonic part of the layer ($u(z_i) > M - 1$). The results of the calculations are shown in table 1, where δ_1^* is the thickness of the boundary layer at point x_1^*.

Unlike the previous configurations, in cases №4.1 and №4.2 the flow is unsteady, starting from a certain point: periodic vortices are moving along the plate, caused by the Kelvin-Helmholtz instability in the mixing layer of the injection and main flow.

Table 1. The boundaries of regions (of curved surfaces) over which a profile with a generalized inflection point in the supersonic part is formed.

Configuration	x_1^*	x_2^*	δ_1^*	$[x_1^*; x_2^*]$
№1.1	450 mm	475 mm	0.6 mm	25 mm
№2.1	480 mm	515 mm	0.6 mm	35 mm
№3.1	480 mm	505 mm	0.6 mm	25 mm
№4.1	380 mm	405 mm	0.6 mm	25 mm
№1.2	47.2 m	48.2 m	6.3 mm	1.0 m
№2.2	49.9 m	51.4 m	6.5 mm	1.5 m
№3.2	49.8 m	50.8 m	6.5 mm	1.0 m
№4.2	45.3-46.6 m	46.6-47.6 m	6.1 mm	1.0-1.3 m

From the results, using different computational grids, it follows that the positions of the points x_1^* and x_2^* in the configuration №4.1 are almost independent of time. In case №4.2, although these points move, the length of the segment $[x_1^*; x_2^*]$ varies slightly from 1.0 m to 1.3 m.

Thus, the numerical calculations confirm the existence of the stable laminar boundary layer profiles with a generalized inflection point over a series of curved surfaces (configurations №1.1–№4.2).

4. Conclusion

It is shown, that the influence of the boundary layer on the single-mode flutter is different for two types of the layer. In case of the generalized convex boundary layer profiles the increase of the layer thickness yields the decrease of growth rates of disturbances. For sufficiently thick boundary layers, the plate becomes fully stabilized; however, this occurs at thicker boundary layer than in inviscid shear-layer approximation.

In case of the boundary layer profile with the generalized inflection point the effect of layer on the flutter is fundamentally different: the thickening of the layer first yields the increase of the growth rates, and the growth rates are greater than in the inviscid approximation. Only in thick boundary layers growth rates decrease and layer has a stabilizing effect.

Numerical simulation of two-dimensional supersonic gas flows with boundary layers over different curved rigid surfaces (described in section 3.1) is performed by the finite volume method. The flow patterns over them are studied: searched regions of these surfaces, over which stable boundary-layer profiles with a generalized inflection point are formed, are found, and their boundaries are obtained.

According to Section 2, the results show that if the elastic plate is built in one of the curved surfaces in the searched region, then the boundary layer can have a destabilizing effect on this plate.

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