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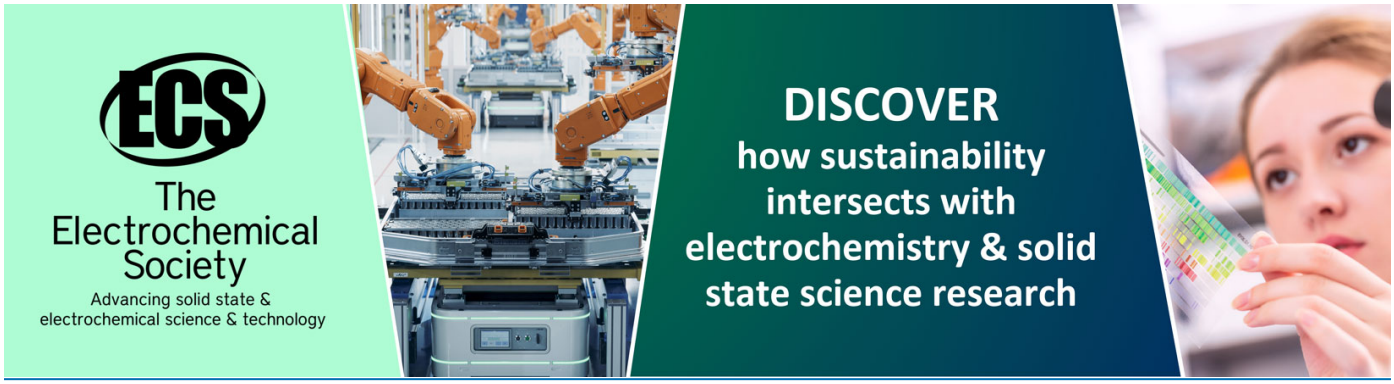

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Correction of the capability index computation based on the uncertainty of the check standard in measurement process

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Abstract. The determination of measurement quality is generally solved by the means of establishing the measurement uncertainty. In the recent years the quality of measurement, especially in industrial applications has been determined either by uncertainty but also by the capability indexes. The presented article deals with the effects of check standards uncertainty on the calculation of capability indexes.

1. Introduction

Process capability indices have been proposed in the manufacturing industry. Due to their effectiveness and simplicity of use, the process capability indices C_g, C_{gk} have been popularly accepted in the measurement processes as management tools for evaluating and improving measurement process quality. The assessment of the measurement process capability is carried out by means of the check standard (CS). Most research works related to C_g and C_{gk} assume no CS errors. However, such an assumption inadequately reflects real situations. Especially in production, a measurement error is sometimes considered, and it is assumed that it has PDF $N(0,\sigma_G^2)$ [1, 5]. The overview of the works is given in [6]. For measurement processes, the assumption that the CS error has $N(0,\sigma_{CS}^2)$ often may not be correct. The error of the check standard may also have a component that is constant for repeated measurements. Then we suppose PDF CS errors $N(\mu_{CS},\sigma_{CS}^2)$. Unfortunately, to this issue is not paid sufficient attention.
Conclusions drawn regarding measurement process capability are therefore unreliable and misleading.
In this paper, we conduct a sensitivity investigation for the measurement process capability index C_g and C_{gk} in the presence of CS errors (the systemic component in particular). We will show that the empirical value of capability indexes due to error control standard measure can overestimate the capability of the measuring process, which can result in an improper assessment of process capability. We will also propose correction factors for adjusting the values of the capability indexes.

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2. Theory

Let us assume that the measurement process has a distribution $X \sim N\left(\mu,\sigma^2\right)$, the check standard has a distribution $X_{\mathrm{CS}} \sim N\left(\mu_{\mathrm{CS}},\sigma_{\mathrm{CS}}^2\right)$ and the measurement result Y originating from the CS will have the following distribution $Y \sim N\left(\mu_Y+\mu_{\mathrm{CS}},\sigma_Y^2=\sigma^2+\sigma_{\mathrm{CS}}^2\right)$. Assumption of normality is not strict, practically speaking; all that is required is that the distribution of measurements is bell-shaped and symmetric. The parameter λ_{A} is defined by the following equation:

$$\lambda_{\mathrm{A}}=\frac{2\sigma_{\mathrm{CS}}}{U} \tag{1}$$

where

- U is the required uncertainty of the measurement process,
- λ_{A} represents the relative fluctuations of CS values during check measurements on this standard relative to the uncertainty of the entire process.

3. Capability index C_{g}

Capability index C_{g} is defined:

$$C_{\mathrm{g}}=\frac{U}{2\sigma} \tag{2}$$

The empirical capability index C_{g}^Y will be obtained by swapping the μ for μ_Y and σ for σ_Y. By applying this change, we will obtain the following equation:

$$C_{\mathrm{g}}^Y=\frac{U}{\sqrt{\sigma^2+\sigma_{\mathrm{CS}}^2}}=\frac{U}{2\sigma}\frac{1}{\sqrt{1+\sigma_{\mathrm{CS}}^2/\sigma^2}}=C_{\mathrm{g}}\frac{1}{\sqrt{1+\lambda_{\mathrm{A}}^2C_{\mathrm{g}}^2}} \tag{3}$$

The following relation can then be assumed from the equation (3) $1+\lambda_{\mathrm{A}}^2C_{\mathrm{g}}^2=1\Big/\left(1-\lambda_{\mathrm{A}}^2\left(C_{\mathrm{g}}^Y\right)^2\right)$.

The calculated index C_{g}^Y has always a lower value that the index C_{g}. This means that if the calculated index C_{g}^Y is acceptable, so is the real index as well C_{g}. Also, the knowledge of CS uncertainty caused by the fluctuation of values by its use makes it possible to evaluate the capability index. When assessing the capability of a measurement process, the relationship in (3) allows the correction of the capability index C_{g} by a correction factor $1\Big/\left(1-\lambda_{\mathrm{A}}^2\left(C_{\mathrm{g}}^Y\right)^2\right)$, under the condition that we know λ_{A}. The information regarding the parameter λ_{A} isn't always easy to determine and therefore we can settle for the fact that the calculated capability index C_{g}^Y is always smaller that the real capability index C_{g}. For the measured values Y_i the capability index C_{g}^Y is going to be calculated from empirical data according to the following equation:

$$\hat{C}_{\mathrm{g}} = \frac{U}{2s} \tag{4}$$

where $s^2 = \frac{1}{n}\sum_{i=1}^{n}\left(Y_i - \bar{Y}\right)^2$ and $\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$ are the most credible estimates of σ_Y^2 and μ_Y.

4. Capability index C_{gk}

We have defined the capability index C_{gk} according to the following equation:

$$C_{\mathrm{gk}} = \frac{U - |\delta|}{2\sigma} = \frac{U}{2\sigma} - \frac{|\delta|}{2\sigma} = C_{\mathrm{g}} - \frac{|\delta|}{U} C_{\mathrm{g}} = C_{\mathrm{g}}(1 - v) \tag{5}$$

The parameter v is the proportional deflection of the measurement process $v = \frac{|\delta|}{U}$, where $\delta = \mu - m$ and m represents the middle of the interval consisting of uncertainty borders U_1, U_2.

If the required uncertainty should be symmetrical, then $|U_1| = |U_2|$ and $m = \mu_{\mathrm{CS}}$. In this case μ_{CS} represents the "true" value of the CS, which is the desired final value.

In this scenario we will obtain the empirical capability index C_{gk}^{Y} by swapping the μ for μ_Y and σ for σ_Y. If we apply all the previously mentioned point the relationship between the "real" capability index C_{gk} and empirical capability index C_{gk}^{Y} will have the following form:

$$C_{\mathrm{gk}}^{Y} = \frac{U - |\delta| - \mu_{\mathrm{CS}}}{2\sqrt{\sigma^2 + \sigma_{\mathrm{CS}}^2}} = \frac{U - |\delta| - \mu_{\mathrm{CS}}}{2\sigma\sqrt{1 + \lambda_{\mathrm{A}}^2 C_{\mathrm{g}}^2}} = C_{\mathrm{gk}} \frac{1 - \frac{\gamma}{1 - v}}{\sqrt{1 + \lambda_{\mathrm{A}}^2 C_{\mathrm{g}}^2}} \tag{6}$$

where $\gamma = \frac{\delta_{\mathrm{CS}}}{U}$ and $\delta_{\mathrm{CS}} = \mu_{\mathrm{CS}} - x_{\mathrm{CS}}$, while x_{CS} is the nominal value of CS (indicated values by CS). This means that the empirical capability index can have a smaller or higher value than the "true" capability index. With the increase of λ_{A}, (which is a consequence of increasing CS data fluctuations based on the overall uncertainty of CS), C_{gk} increases as well. On the other hand, with the increase of MS systematic error the C_{gk} has an increasing tendency as well. This is valid for the case where the systematic error of CS (δ_{CS}) deflects in the same direction as the measurement process δ.

In the case that the systematic error CS δ_{CS} has an opposite deflection direction as the measuring process δ, the γ will be negative. In this case we must consider the worst case and multiply the empirical capability index with a factor of $\sqrt{1 + \lambda_{\mathrm{A}}^2 C_{\mathrm{g}}^2} \Big/ \left(1 + \frac{|\gamma|}{1 - v}\right)$. To apply this correction knowledge of the CS uncertainty sources are required. This means to know the contribution to the uncertainty originating from the fluctuations of the values generated by the CS and the systematic error of the CS. To gather and define this information can be in some cases complicated and time consuming. In most cases we assumed that considerable part of the CS uncertainty will manifest itself by the fluctuation of its generated values. If this is true, then we can assume that γ is negligible and that the λ_{A} is represented by the whole relative uncertainty of CS. If the considerable part of the CS uncertainty is manifested by in the systematic error, then we can assume that the λ_{A} is negligible and that the γ

(relative deflection of CS) is represented by the whole relative uncertainty of CS. The possible differences between the calculated and "real" probability index that are dependent of the CS uncertainty values can be seen in Figure 1.

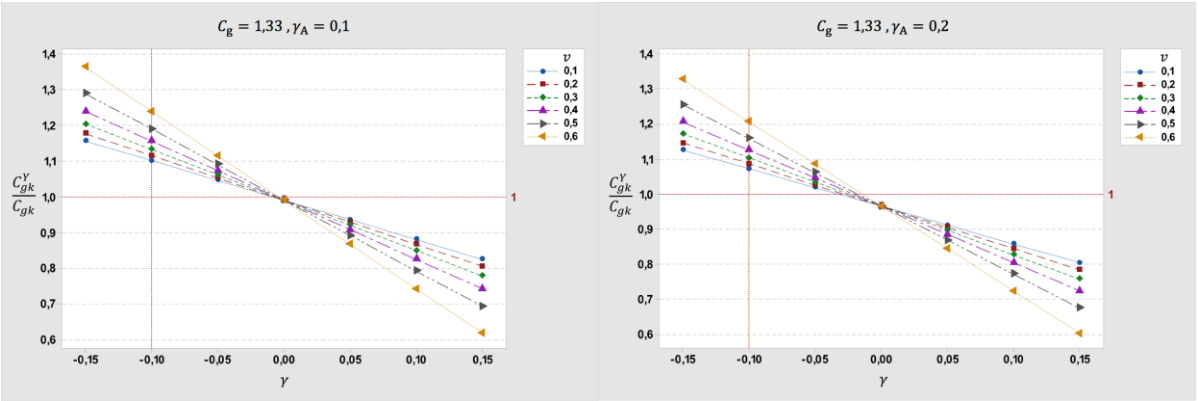

Figure 1. Share of experimental indexes to "real" for the situations when $\lambda_A = 0{,}1$ and $\lambda_A = 0{,}2$.

For the measured values Y_i the probability index $\hat{C}_{gk}^{Y}$ will be calculated from an empirical data according to the following equation:

$$\hat{C}_{gk}^{Y} = \frac{U - \left|\overline{Y} - x_{CS}\right|}{2s} \tag{7}$$

where s identical as in the section 3.

Conclusion

In this paper, we analysed the uncertainty effect of check standard on the measurement capability indexes value C_g and C_{gk}. Using large-scale simulation studies, we have demonstrated the undesirable effect of the check standard errors on capability indices. Then we provided correction factors that allow us to correct the empirical value. The results showed that the correction procedures adequately reduce the undesirable effects of the check standard errors. Correction of empirical capability indexes is very important at index values close to one.

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