

PAPER • OPEN ACCESS

Total Idle Time Density Function of $M/C_2/1$ Systems under $(0,k)$ Policy

To cite this article: Isnandar Slamet *et al* 2018 *J. Phys.: Conf. Ser.* **1028** 012226

View the [article online](#) for updates and enhancements.

You may also like

- [Analysing the cause of idle time in loading and unloading operation at Indonesian international port container terminal: Port of Tanjung Priok case study](#)
F Suprata, C Natalia and A Sugioko
- [VieSched++: A New VLB I Scheduling Software for Geodesy and Astrometry](#)
Matthias Scharfner and Johannes Böhm
- [Robust optimization of a radiotherapy pretreatment preparation workflow](#)
Nienke Hoffmans-Holtzer, Luuk Smolenaers, Manouk Olofsen-van Acht et al.



ECS
The
Electrochemical
Society
Advancing solid state &
electrochemical science & technology

DISCOVER
how sustainability
intersects with
electrochemistry & solid
state science research

Total Idle Time Density Function of M/C₂/1 Systems under (0,k) Policy

Isnandar Slamet¹, Ritu Gupta², Narasimaha R. Achuthan³, and Roger Collinson²

¹Department of Statistics, Universitas Sebelas Maret, Surakarta, 57126, Indonesia

²Department of Mathematics and Statistics, Curtin University, Perth, 6102, Australia

³Retired from Department of Mathematics and Statistics, Curtin University, Perth, 6102, Australia

isnandarslamet@staff.uns.ac.id

Abstract. The aim of this paper is to derive the probability density function (pdf) of the total idle time of busy period of M/C₂/1 queues operating under control policies through lattice path combinatorics (LPC) approach. The service distribution is approximated by Coxian two-phase distribution. We focus on deriving the pdf of total idle time of M/C₂/1 queues under (0,k) control policy, wherein the server goes on the vacation when the system becomes empty and re-opens for service immediately at the arrival of the kth costumer. We present an important result which is the theorem of the pdf of total idle time when system is in busy period that ends with a departure.

1. Introduction

Investigation on density function of busy period of a general M/G/1 has been done by many researchers. For examples, Agarwal *et al.* [1], Borkakarty *et al.* [2], Slamet *et al.* [3], and Slamet *et al.* [4]. The M/G/1 systems refer to the systems in which the mechanisms of arrival is assumed to follow Poisson distribution, service times is assumed to follow general distributions, and there is only one server available for service. Such investigations were based on Lattice Path Combinatorics (LPC) and general distribution was approximated by distribution called Coxian-phase type distribution.

In general the operation of a queueing system starts with certain initial starting conditions of the system. Often in many real life systems, after sufficient time has elapsed the system stabilizes and becomes free of initial conditions. Once the system stabilizes the queueing system is said to be in a steady-state. The queueing system is said to be in a transient state before reaching the steady-state. Most of the research work in queueing theory concerns with analyzing the output characteristics of a queueing system under steady state.[5-7] Many queueing systems do not operate long enough to reach steady state conditions and thus any investigation of the operating characteristics of such system depends heavily on investigating the queueing system under transient state.[8,9] Research on performances of Markov chain systems when the systems is in transient position has been conducted in wide origin ranging over several inter-disciplinary areas. Grassman [10], Krinik *et al.*, [11], Narahari and



Viswanadham [12], Doshi [13], and Sen and Jain [14] are among those researchers who published performance analysis of systems when there were in transient state.

The aim of this paper is to derive properties of transient state of the M/G/1 systems under (0,k) control policies. We approximate the general distribution by Coxian-2 phase distribution. This Coxian approximate distribution is very important distribution since one may find many distributions in which the distribution can be approached by Coxian distribution by adjusting its coefficient of variance. [1,2,3,4]

There are many queueing systems in our modern life when a server has to provide service according to the number of customers in the system. When the server finished serving primer customer, then it will take a rest. The server starts serving only when there are number of customers are in the systems. This server's policy can be found in computer network system, production and warehouse system, inventory system, communication and information technology system, and others. For such systems, we use to deal with vacation policy and a set of policy can be applied for server to return to its job. Doshi [13] presented performance analysis of many queueing systems under different vacations policy. It is important to note that such investigation on queueing system considering vacation mechanism resulted the model either in steady state or transient. We noticed that the solutions are in Laplace transforms which are intractable and closed form.

In section 2 we review Coxian two-phase distribution. The behavior of queueing systems is presented in section 3. The results of the research will be presented in section 4 followed by acknowledgment and references.

2. Coxian Two-Phase Distribution

The Coxian 2-phase distribution, characterized by parameters (μ_1, μ_2, β) describes duration until an event occurs in terms of a process consisting of 2 latent phases, leading to the Markovian structure.

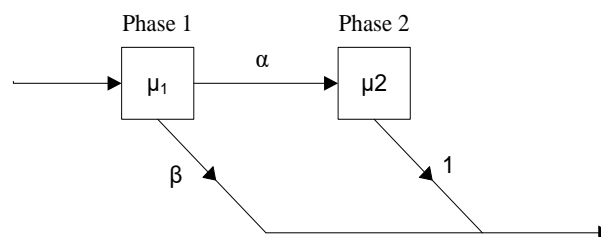


Figure 1. Coxian two-phase distribution

The Coxian distribution with two phases i.e. phase 1 and phase 2 is presented in Figure 1. In phase 1, the mean rate of serving in phase 1 is denoted by μ_1 and in phase 2, it is denoted by μ_2 . A customer may continues to receive service in phase 2 after having finished service in phase 1 with probability α . From the figure, we have α which is equal to $1-\beta$. Hence, β is probability of a customer leaves the system after finishing the service in phase 1.

3. Lattice Path Approach

When a customer joins the system then we consider an arrival happens in the system. After joining the queue, a customer receives the service. Finishing the service, a customer may leave the system or continue to the next server to get other service. When a customer has to leave the system after finishing the service, then we consider a departure of a customer from

the system. This can happen for a customer having finished the first service as well as the second service. When a customer has to switch to second service, then we consider a moving to second service with diagonal step happens. Based on this scenario, lattice path is the translation i.e. the step taken for an arrival which is a horizontal step, a departure which is a vertical step, and moving to next service which is a diagonal of $\sqrt{2}$ step.

This lattice path has been used in deriving the probability of density function and other properties of queueing systems in which different busy periods are considered. For this purpose, the time span t is divided into t/h intervals. This gives length h for each interval. The number of lattice paths then can be counted, and the density can be derived by considering its limit. [14]

4. Results and Discussion

In this section, we present an important result regarding total of idle time. For this idle time, we derive its corresponding probability density function when system is in busy period. We consider the system when a departure occurs in busy period. Other possibilities, through similar way, then one can achieve the corresponding pdf. We start deriving the pdf with defining the staying time. Next, we provide an example of lattice path representation on our queueing system under $(0,k)$ policy. The theorem is presented in the last of this section.

4.1 The staying time

The probability of staying time when there are i customers for each state is exponential. For given state, the probability is presented below:

$$P\{T_{n+1} - T_n > t \mid X_n = i\} = \begin{cases} e^{-(\lambda + \mu_u)t}, & \text{if a customer is undergoing phase } u \text{ of service, } u=1,2 \\ e^{-\lambda t}, & \text{if the system is idle} \end{cases}$$

4.2 $(0,k)$ control policy and its representation

The representation of a lattice path by considering $(0,k)$ control policy is presented in Figure 2.

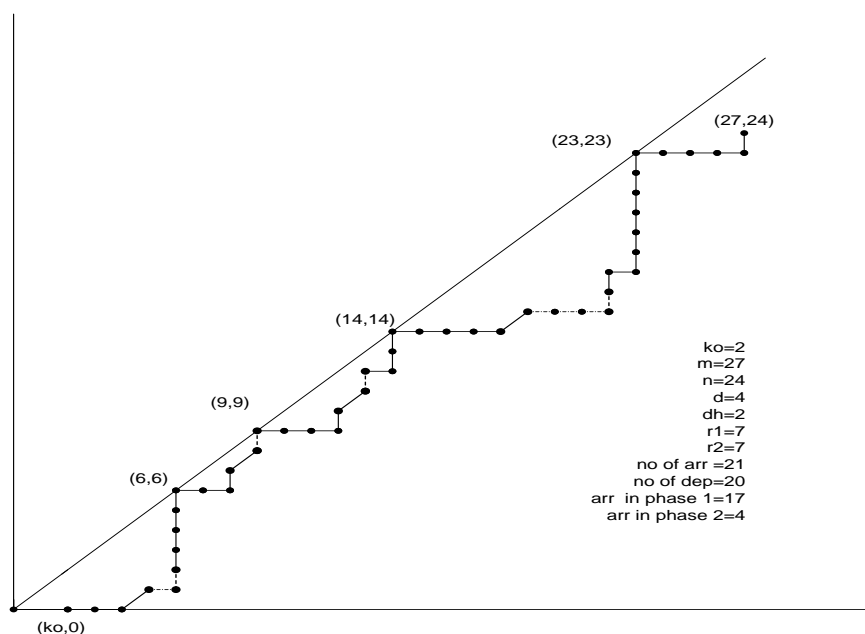


Figure 2. A $(0,k)$ policy for lattice path M/C2/1 Systems

This lattice path shows a sequence or steps in which the server starts when $k_0 = 2, k = 2$ and reaches the end point $(27, 24)$. Notice that the server is still active at end the point. At some points i.e. $(6, 6)$, $(9, 9)$, and $(14, 14)$ the number of arrivals and the number of departures is same. The lattice path stops in $(27, 24)$, therefore it won't reach the point $(27, 27)$. When the lattice path reaches point $(6, 6)$, the server does not provide any service until there are 2 customers awaiting for receiving service. When the second customer arrive, the server starts servicing the next customers. This service will stop when there is no more customer in the system. This happens in point $(9, 9)$. The similar procedure continuous until the lattice path reaches point $(27, 24)$. Below we derive the theorem that pdf.

Theorem. Let $f_{sk}(\tau)$ is the pdf of total time system in idle. Let system start with k_0 customers at time t . Consider the $(0, k)$ policy. For $\tau = \sum_{i=1}^s \tau_i$ = total idle time, then

$$f_{sk}(\tau) = \frac{e^{-\lambda\tau} (\lambda\tau)^{sk}}{\Gamma(sk + 1)}$$

Proof. Since τ the total idle time, then we have expression of the multiple integration as follows.

$$\int_{\tau_1} \dots \int_{\tau_s} \prod_{y=1}^s \frac{e^{-\lambda\tau_y} \lambda^k \tau_y^{k-1}}{\Gamma(k)} d\tau_y = \frac{e^{-\lambda\tau} \lambda^{sk}}{(\Gamma(k))^s} \int_{\tau_1} \dots \int_{\tau_s} \prod_{y=1}^s \tau_y^{k-1} d\tau_y$$

The above integration can be solved by first integrating the equation over τ_s , where $\tau_s = \tau - \tau_1 - \tau_2 - \dots - \tau_{s-1}$.

$$\int_0^{\tau - \tau_1 - \tau_2 - \dots - \tau_{s-2} - \tau_{s-1}} \tau_s^{k-1} d\tau_s = \frac{(\tau - \tau_1 - \tau_2 - \dots - \tau_{s-2} - \tau_{s-1})^k}{k}.$$

Further, dealing with integration over τ_{s-1} , the solution of second integration is

$$\int_0^{\tau - \tau_1 - \tau_2 - \dots - \tau_{s-2}} \frac{(\tau - \tau_1 - \tau_2 - \dots - \tau_{s-2} - \tau_{s-1})^k}{k} \tau_{s-1}^{k-1} d\tau_{s-1}$$

$$\text{Let } u = \frac{\tau_{s-1}}{\tau - \tau_1 - \tau_2 - \dots - \tau_{s-2}}, d\tau_{s-1} = \tau - \tau_1 - \tau_2 - \dots - \tau_{s-2} du$$

Solving the above integration and then calculate the integration over $\tau_{s-2}, \tau_{s-3}, \dots, \tau_2, \tau_1$ Finally we get

$$\int_{\tau_1} \dots \int_{\tau_s} \prod_{y=1}^s \frac{e^{-\lambda\tau_y} \lambda^k \tau_y^{k-1}}{\Gamma(k)} d\tau_y = \frac{e^{-\lambda\tau} \lambda^{sk}}{(\Gamma(k))^s} \frac{(\Gamma(k))^s}{\Gamma(sk + 1)} \tau^{sk}$$

which yields $f_{sk}(\tau) = \frac{e^{-\lambda\tau} (\lambda\tau)^{sk}}{\Gamma(sk + 1)}$. Hence we proved the theorem.

5. Conclusion

Based on the discussion above, the main result achieved is the pdf of the total idle time when system is in busy period. It can be said that lattice path can be applied for computing

the transient probabilities under control policy. This result is closed form which is more elegant comparing the traditional solution which is in the form of Laplace transform.

Acknowledgments

We would like to express our sincere gratitude to Universitas Sebelas Maret Surakarta for providing assistance to attend this conference.

References

- [1] M. Agarwal, K. Sen, and B. Borkakaty 2007 *Applied Mathematical Modelling*. **31**(10).
- [2] B. Borkakaty, M. Agarwal, and K. Sen 2009 *Applied Mathematical Modelling*.
- [3] I. Slamet, R. Gupta, N. R. Achutan, and R. Collinson 2016 *International Journal of Mathematical Models and Methods in Applied Sciences*. **Vol. 10**, ISSN: 1998-0140.
- [4] I. Slamet, R. Gupta, and N. R. Achutan 2014 Proc. of International Conference on Research, Implementation and Education of Maths and Sciences. ISSN 978-979-99314-8-1
- [5] D. R. Cox 1995 *Mathematical Proceedings of the Cambridge Philosophical Society*. 51
- [6] H. Takagi 1991 *Queueing Analysis : Volume 1: vacation and priority systems*.
- [7] M. F. Neuts 1989 *Structured Stochastic Matrices of M/G/1 Type and Their Applications*. Marcel Dekker, Inc., New York.
- [8] D. Gross and C.M. Harris 1998 *Fundamentals of Queueing Theory: 3rd Edition*. John Wiley & Sons, Canada.
- [9] J. A. Buzacott and J.G. Shanthikumar 1993 *Stochastic Models of Manufacturing Systems*. Prentice-Hall, Inc. Englewood Cliffs, New Jersey.
- [10] W. K. Grassman 1977 *Computers and Operations Research*. **4**(1).
- [11] A. Krinik 2002 *Journal of Statistical Planning and Inference*. **101**(1-2).
- [12] Y. Narahari, N. Viswanadham 1994 *IEEE Transaction on Robotics and Automation*. **10**(2).
- [13] B. T. Doshi 1986 *Queueing systems with vacations - a survey Queueing Systems*. 1.
- [14] K. Sen and J.L. Jain 1993 *Journal of Statistical Planning and Inference*. **34**(2).