PAPER • OPEN ACCESS

Two Dimensional Finite Element Based Magnetotelluric Inversion using Singular Value Decomposition Method on Transverse Electric Mode

View the article online for updates and enhancements.

Two Dimensional Finite Element Based Magnetotelluric Inversion using Singular Value Decomposition Method on Transverse Electric Mode

Tiffany Tjong[1], Lisa Yihaa' Roodhiyah[1], Nurhasan[1], Doddy Sutarno[1]

[1] Electromagnetic Induction Lab., Physics of Complex System Div., Faculty off Mathematics and NaturalSciences, Institut Teknologi Bandung, Indonesia

E-mail : tiffany11002@students.itb.ac.id

Abstract. In this work, an inversion scheme was performed using a vector finite element (VFE) based 2-D magnetotelluric (MT) forward modelling. We use an inversion scheme with Singular value decomposition (SVD) method toimprove the accuracy of MT inversion.The inversion scheme was applied to transverse electric (TE) mode of MT. SVD method was used in this inversion to decompose the Jacobian matrices. Singular values which obtained from the decomposition process were analyzed. This enabled us to determine the importance of data and therefore to define a threshold for truncation process. The truncation of singular value in inversion processcould improve the resulted model.

1. Introduction

Magnetotelluric (MT) is an electromagnetic method whichis used to map electrical resistivity distributionin the subsurface. MT method involves measuring electric field and magnetic field on the surface of the earth. The data obtained usually presented as apparent resistivity and phase data [1].To derive electrical resistivity value from MT data, inversion method can be used. Inversion inMT usually are ill-posed where small changes in data could drastically affecting the inversion results [2]. Finite element discretization method is a good method to be used in electromagnetic field problems such as MT.The solution obtained from finite element method satisfying the physical concept of electromagnetic field [3].The usage of singular value decomposition (SVD) has been used to the ill-posed problems to stabilize the iteration process within the inversion [2].SVD is able to produce more precise and robust inversion result [4].There are some works which have applied SVD to the MT inverse problems such as [5],[6],[7].The previous inversion works mostly usefinite difference discretization. Within this work, we applied SVD inversion method to a vector based finite element (VFE) MT modelling for transverse electric (TE) mode.

2. Inversion with Singular Value Decomposition

The inversion scheme described in this work isheavily based on the work of [1].In general, the relationship of model parameter and data can be described as :

$$\mathbf{d} = \mathbf{g}(\mathbf{m}) \tag{1}$$

where **d** is a vector containing several data which are apparent resistivity and phase as a function of frequency and the location of measurement.The vector**m** is the model parameters vector. The model

parameter in this case is true electrical resistivity for each cell in VFE discretization. **g(m)**is the forward modelling which produce a set of calculated data for a certain set of **m**.Basically, the inversion problem is to obtain the correct set of **m** which generates a set of **g(m)** that fit with the measurement data **d**.In simple inversion scheme, **m** is determined by using iterative method which involves estimating initial value and a perturbation of the value for each iteration. The perturbation of **m**is defined as follow :

$$\Delta \mathbf{m} = \left(\mathbf{J}^{\mathrm{T}} \mathbf{J} \right)^{-1} \mathbf{J}^{\mathrm{T}} \boldsymbol{\varepsilon} \tag{2}$$

where **ε** is the error between measured data and calculated data and **J** is Jacobian matrix. Jacobian matrix **J** is also termed as the sensitivity matrix. Equation (2) is used when **J** is not singular. For inversion with SVD, Δ**m** is obtained by directly inverting **J** :

$$\Delta \mathbf{m} = \mathbf{J}^{+} \boldsymbol{\varepsilon} \tag{3}$$

J^{+} is the generalized inverse of Jacobian matrix and is defined by :

$$\mathbf{J}^{+} = \mathbf{V} \, \mathbf{S}^{+} \, \mathbf{U}^{\mathrm{T}} \tag{4}$$

V, **S**, and **U** are the matrices which is a result of matrix **J** decomposition using SVD method [8]. Matrix **S** is a diagonal matrix with the components are the singular value of **J**. **S**$^{+}$ is the invert matrix of **S**.

3. Results and Discussion

The inversion was performed on a homogenous model with several different singular value threshold. We use a synthetic model to test the inversion program. The resistivity value of the synthetic model is 100 Ohm-m.Initial resistivity value was set to 5 Ohm-m. Figure 1 shows the inversion result for singular value threshold ranging from 0.001 to 1. The lower singular value threshold (0.001 and 0.01) producing model with mean percentage errorvalue of 3.7657e+10 % and 685.8591 %. The higher threshold value also producing less accurate model with error value 59.0965%. The optimal model is attained with the threshold ranging from 0.02 to 0.1. The threshold for truncating the singular value influences the result of the inversion.
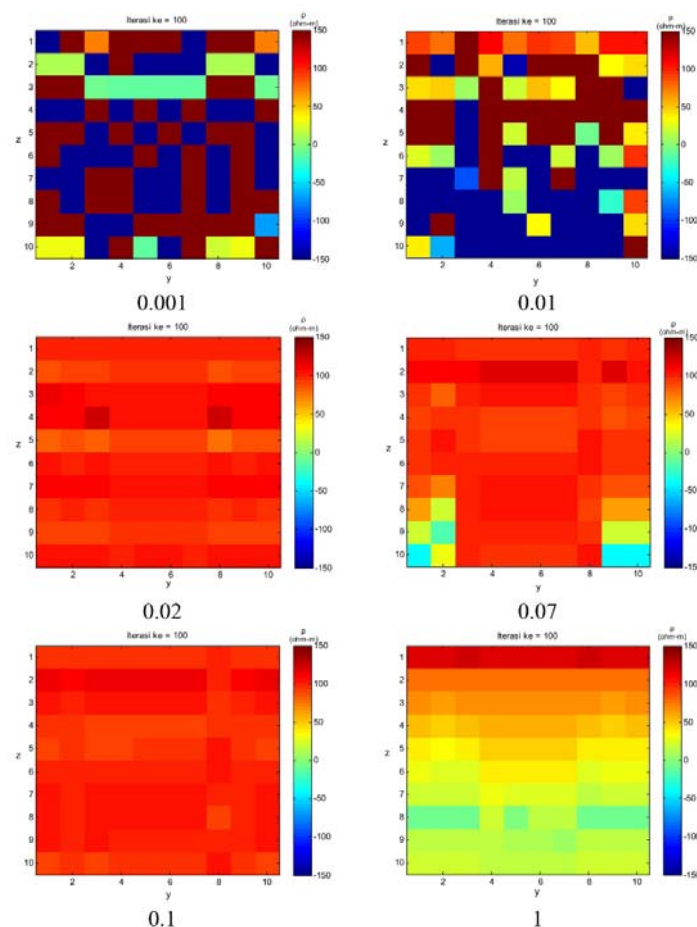
Figure 1. Inversion result with different singular value threshold for homogenous model

4. Conclusion

We have developed an inversion program using singular value decomposition method to solve the ill-posedness of magnetotelluricproblem.The test using homogenous model shown a similarity with the synthetic model. Truncation of singular values also affecting the inversion result. For homogenous model shown in this work, the optimum value to obtain the appropriate result is between 0.02 to 0.1.

Acknowledgments

References

[1] Cagniard, L., 1953. Basic theory of the magneto-telluric method of geophysical prospecting. *Geophysics*, *18*(3), pp.605-635.
[2] Jupp, D.L.B. and Vozoff, K., 1975. Stable iterative methods for the inversion of geophysical data. *Geophysical Journal International*, *42*(3), pp.957-976.
[3] Kordy, M., Wannamaker, P., Maris, V., Cherkaev, E. and Hill, G., 2016. 3-D magnetotelluric inversion including topography using deformed hexahedral edge finite elements and direct

solvers parallelized on SMP computers–Part I: forward problem and parameter Jacobians. *Geophysical Journal International*, *204*(1), pp.74-93.

[4]   Lines, L.R. and Treitel, S., 1984. Tutorial: A review of least-squares inversion and its application to geophysical problems. *Geophysical prospecting*, *32*(2), pp.159-186.

[5]   Jupp, D.L. and Vozoff, K., 1977. Two-dimensional magnetotelluric inversion. *Geophysical Journal International*, *50*(2), pp.333-352.

[6]   Kalscheuer, T. and Pedersen, L.B., 2007. A non-linear truncated SVD variance and resolution analysis of two-dimensional magnetotelluric models. *Geophysical Journal International*, *169*(2), pp.435-447.

[7]   Candansayar, M.E., 2008. Two-dimensional inversion of magnetotelluric data with consecutive use of conjugate gradient and least-squares solution with singular value decomposition algorithms. *Geophysical Prospecting*, *56*(1), pp.141-157.

[8]   Jackson, D.D., 1972. Interpretation of inaccurate, insufficient and inconsistent data. *Geophysical Journal International*, *28*(2), pp.97-109