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Fractional step two-phase flow lattice Boltzmann model implementation

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Abstract. A hybrid method that combines an advanced lattice Boltzmann model with traditional finite difference techniques is developed. An operator splitting technique is applied to separate the numerical scheme into two subproblems. Each subproblem is solved with different techniques, namely lattice Boltzmann and finite differences. This method is applied for the first time to a two-phase flow simulation case. Major simplifications are obtained in the lattice Boltzmann implementation. Simulations are presented to show these improvements and to validate the technique with kinematic viscosity down to $10^{-4}$ in lattice Boltzmann units. This technique does not add instabilities, and in contrast improves the stability behavior compared to reducing the relaxation time.

Keywords: bubbles and drops, rheology and transport properties, soft interfaces
1. Introduction

Lattice Boltzmann (LB) methods have been proposed for the simulation of two-phase flow [5]. However, for high resolution methods there is yet no LB model based upon purely kinetic theory to binary mixtures for gas–liquid systems. Some two-phase flow models incorporate an interparticle potential [15, 16]. Only recently the LB fundament has been expanded to cover gas mixtures [2, 1]. We focus mainly on the free energy approach since it has shown great potential for working at high density ratios and for eliminating numerical spurious currents [6, 9, 10]. Besides, the free energy approach proposes a thermodynamic framework suitable for studying interfacial physics. A review of the thermodynamic models can be found in Wagner [19].

The lattice Boltzmann schemes have limitations with regard to the Reynolds numbers that can be simulated, i.e. problem-dependent instabilities appear for low single-relaxation-time values. To counteract this limitation the Multi-Relaxation-Time (MRT) collision operator was proposed instead of the Bhatnagar–Gross–Krook (BGK) model [14]. However, no major improvement was reported and problem-dependent instabilities still arise for low viscosities.

It should be pointed out that these LB models can simulate density ratios of 1000 [12]. However, this is only achieved when a Crank–Nicholson scheme is applied to the time integration [11]. This Crank–Nicholson scheme is not only introduced to achieve second order in the time integration, but also to achieve unconditional stability when simulating systems with large density gradients. However, the implementation of such a scheme is difficult and needs twice as many computations as normal schemes. Besides, the density gradient derivatives in the first half-step are computed in a different way than those of the second half-step [11].

We will show here that it is not necessary to use the complete Crank–Nicholson scheme if we solve the LB model with a fixed relaxation time where the viscosity is no longer a free parameter. The contribution of the present work consists of a hybrid method
that combines an advanced lattice Boltzmann model with traditional finite difference techniques. This extra step is introduced for the following three reasons. First, as an alternative option for correcting the stress tensor. Secondly, to increase the maximum allowable Reynolds number by the use of a viscous stress correction [17]. Thirdly, to simplify the costly Crank–Nicholson double-step.

The present work is organized as follows. First we present the proposed lattice Boltzmann model. Next, we review the concept of operator splitting and show how this is applied in the present case. Finally we examine some numerical results and sum up the new findings.

2. The lattice Boltzmann model

The lattice Boltzmann model used is outlined in this section.

By using the present lattice Boltzmann model, the following macroscopic single-field formulation is indirectly solved:

\[
\frac{\partial \phi}{\partial t} + \nabla \cdot (\phi \vec{u}) = M \nabla^2 \mu_\phi \tag{1}
\]

\[
\frac{1}{\rho} \frac{\partial p_{pf}}{\partial t} + \nabla \cdot \vec{u} = 0 \tag{2}
\]

\[
\rho \frac{\partial \vec{u}}{\partial t} + \rho \vec{u} \cdot \nabla \vec{u} = -\nabla p_{pf} + \nabla \cdot \mathbf{T} - \rho \nabla \mu_\phi \tag{3}
\]

where \(\vec{u}\) is the flow velocity vector, \(p_{pf}\) is the pressure, \(\mathbf{T}\) is the stress tensor, \(M\) is the Cahn–Hilliard mobility and \(\mu_\phi\) is the chemical potential defined to model the interfacial behavior [18, 21, 22]. \(\rho\) is the density calculated through an order parameter defined as \(\phi \equiv \rho - \bar{\rho}\), where \(\bar{\rho} = \frac{1}{2}(\rho^*_h + \rho^*_l)\) is an intermediate density defined as the average of the high and low densities. \(\rho^*_h\) and \(\rho^*_l\) are the densities which minimize the bulk energy \(\Psi_0\).

The chemical potential can be defined in terms of the bulk energy

\[
\Psi(\rho, \nabla \rho) = \Psi(\phi, \nabla \phi) = A(\phi + \phi^*)^2(\phi - \phi^*)^2 + \frac{\kappa}{2} |\nabla \phi|^2 \tag{4}
\]

\[
\mu_\phi = \frac{\partial \Psi_0}{\partial \phi} - \kappa \nabla^2 \phi = A(4\phi^3 - 4\phi^* 2\phi - \kappa \nabla^2 \phi \tag{5}
\]

where \(A\) is the amplitude of the excess free energy of the interface double-well model and \(\kappa\) is a coefficient that weights the free energy according to the density gradients (Cahn–Hilliard) and the bulk phase density. Each phase will stabilize to either a positive or negative \(\phi^* = \frac{1}{2}(\rho^*_h - \rho^*_l)\). The coefficients in the chemical potential equation (5), \(A\) and \(\kappa\), depend on the system and can be found in the literature [12].

The equation system (1)–(3) is modeled with two lattice Boltzmann distribution functions. In the present work we are only interested in studying the momentum exchange, thus we provide only information on the pressure and momentum distribution modeled with

\[
f^{t+1}_{i,x+\epsilon} = f^t_{i,x} + \Omega_i f + \mathcal{F}_i \quad \text{D2Q9}. \tag{6}
\]

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Here \( f_{i,x}^t \) is the \( i \)th density function (or pseudoparticle) in node \( x \) at time \( t \), and \( \vec{e}_i \) the discrete velocity vector of the pseudoparticle \( i \). The collision term, \( \Omega_i \), and the forcing terms, \( \mathcal{F}_i \), are widely used in lattice Boltzmann schemes for modifying or adding extra terms to the equation system (1)–(3). Different expressions can be used for these source terms \([7,12]\). In this work the single-relaxation-time \( \tau_f \) (BGK) model is used for the collision terms defining the collision operator \( \Omega_i \) for the \( f \) distributions as

\[
\Omega_i f_i = \frac{\text{eq} f_i - f_i}{\tau_f}
\]

where

\[
\text{eq} f_i = 3 p_{pf} w_i + \rho \mathcal{V}_i(\vec{u})
\]

is the equilibrium distribution function. The subscript \( i \) denotes each discrete velocity for the D2Q9 model. The weights, \( w_i \), are \( w_1 = \frac{4}{9} \), \( w_{2,3,4,5} = \frac{1}{9} \) and \( w_{6,7,8,9} = \frac{1}{36} \). The auxiliary function \( \mathcal{V}_i \) has a particular notation to emphasize the fact that it is of order \( \mathcal{O}(u) \):

\[
\mathcal{V}_i(\vec{u}) = w_i (3 e_{i,\alpha} u_\alpha - \frac{3}{2} u^2 + \frac{9}{2} u_\alpha u_\beta e_{i,\alpha} e_{i,\beta})
\]

where Greek letters as subscripts correspond to the Cartesian coordinate directions where the Einstein summation convention is used. The kinematic viscosity is given by \( \nu = \tau_f c_s^2 \). From the moments of \( f \) it is possible to calculate the fluid variables.

The term \( \mathcal{F}_i \) for the pressure and momentum distribution consists of two contributions accounting for the density–pressure decoupling (DP) correction and the interfacial tension forces (ITF). The DP term decouples the thermodynamic relation between density and pressure. This relation is normally computed automatically by the LB model as the ideal gas law. Now, the \( p_{pf} \) has no relation to a thermodynamical concept of pressure. The decoupling term is

\[
\text{DP} \mathcal{F}_i = (\vec{e}_i - \vec{u}) \cdot \nabla \rho \mathcal{V}(\vec{u}).
\]

A potential version of the ITF is used. This approach was studied for phase-field applications by \([8]\) to eliminate parasitic currents. It was introduced into the LB methodology by \([9]\).

The chemical potential is included into the pressure tensor \([21]\) as \( \rho \nabla \mu_\phi = \nabla (\rho \mu_\phi) - \mu_\phi \nabla \rho \). Here the two terms on the RHS are easier to implement than the LHS. The first is introduced inside the pressure tensor as a modifier to the dynamic pressure and the second term is then added as a force term. This is simple since the only gradients needed are the density gradients which are already known when calculating the DP term. This scheme denoted as \( \text{(p)} \) for including the potential inside the pressure tensor is then obtained as

\[
\Phi = \phi \mu_\phi + p_{pf}
\]

\[
\mathcal{A}_i = \frac{27}{4} p_{pf} - \frac{15}{4} (\Phi) \quad i = 1
\]

\[
\mathcal{A}_i = 3 \Phi \quad i \in [2,9]
\]

\[
\text{eq} f_i = \mathcal{A}_i w_i + \rho \mathcal{V}_i(\vec{u})
\]

\[
\text{ITF} \mathcal{F}_i = \frac{(\vec{e}_i - \vec{u})}{c_s^2} \cdot \mu_\phi \nabla \phi (w_i + \mathcal{V}(\vec{u})).
\]
The derivatives of the chemical potential are not needed, while the derivative of the product $\phi\mu$ is calculated by the lattice Boltzmann scheme indirectly. It should be noted that the zero-order moment of the distribution does not change and it only represents the pressure. For the reasons mentioned above this treatment of the chemical potential inside the pressure term was the one adopted in the present work.

The distribution $\hat{f}$ should be integrated using a Crank–Nicholson scheme [12]:

$$f_{i,x+e}^{t+1} - f_{i,x}^{t+1} = \frac{1}{2} \frac{f_{i,x+e}^{t+1} - \text{eq} f_{i,x+e}^{t+1}}{\tau_f} - \frac{1}{2} \frac{f_{i,x}^{t} - \text{eq} f_{i,x}^{t}}{\tau_f} + \frac{1}{2} (F_{i,x+e}^{t} + F_{i,x}^{t}).$$

(12)

It was proposed to achieve this by introducing a temporal distribution $\hat{f}$:

$$f_{i,x+e}^{t+1} - f_{i,x}^{t+1} = \frac{1}{2} (\Omega_{i,x+e}^{t+1} + \Omega_{i,x}^{t+1}) + \frac{1}{2} (F_{i,x+e}^{t} + F_{i,x}^{t})$$

$$\hat{f}_{i,x}^{t} = f_{i,x}^{t} - \frac{1}{2\tau_f} (f_{i,x}^{t} - \text{eq} f_{i,x}^{t}) + \frac{1}{2} (F_{i,x}^{t})$$

$$\hat{f}_{i,x+e}^{t+1} = \hat{f}_{i,x}^{t}$$

$$f_{i,x+e}^{t+1} = \hat{f}_{i,x+e}^{t+1} - \frac{1}{2\tau_f + 1} (\hat{f}_{i,x+e}^{t+1} - \text{eq} f_{i,x+e}^{t+1}) + \frac{\tau_f}{2\tau_f + 1} (F_{i,x+e}^{t+1}).$$

(13)

The fluid variables are then calculated from the moments of $\hat{f}$:

$$p_{pf} = \sum_i \hat{f}_i + \frac{1}{2} \bar{u} \cdot \nabla \phi \quad \rho \bar{u} = \sum_i \hat{f}_i \bar{e}_i + \frac{1}{2} \bar{F}.$$

(14)

3. Operator splitting

We apply an operator splitting technique valid for time-dependent problems [13] following the Shu et al [17] approach, i.e. solving both

$$\int_t^{t^{+}} \frac{\partial \bar{u}}{\partial t} \, dt = \int_t^{t^{+}} \left( -\bar{u} \cdot \nabla \bar{u} - \frac{1}{\rho} \nabla p_{pf} + \frac{1}{\rho} \nabla \cdot \mathbf{T}_{LB} - \nabla \mu \phi \right) \, dt$$

and

$$\int_t^{t^{+}} \frac{\partial \bar{u}}{\partial t} \, dt = \int_t^{t^{+}} \left( \frac{1}{\rho} \nabla \cdot (\mathbf{T} - \mathbf{T}_{LB}) \right) \, dt.$$

(15)

(16)

Here, by summing these two equations together, the momentum balance equation (3) is recovered with the desired viscous stress tensor $\mathbf{T}$. The velocity $\bar{u}$ is defined at time $t$, $u^+$ is used for the value at the final step $t^+ = t + \Delta t$ and $u^*$ is defined at time $t^*$ for the inner problem. The second step will be referred to as the viscosity correction step. For the stress tensor we use $\mathbf{T} = \mu (\nabla \bar{u} + (\nabla \bar{u})^T) - (\frac{2}{3} \mu - k)(\nabla \cdot \bar{u})$.

The LB scheme can, in (weakly) incompressible fields, have the shear modeled as either the gradient of the momentum $\mathbf{T}_{LB} = \nu_{LB}(\nabla \rho \bar{u} + (\nabla \rho \bar{u})^T)$ or the velocity $\mathbf{v}_{LB} = \nu_{LB} \mu_{LB}((\nabla \bar{u} + (\nabla \bar{u})^T)$.

We propose to solve the Navier–Stokes momentum equation (2) in two steps: first order explicit in time finite differences (section 4) for the outer problem equation (16),

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and by lattice Boltzmann for the inner problem equation (15). For the LB model, a fixed relaxation time $\tau_f = \frac{1}{2}$ is used in equation system (13) reducing it to

$$
\hat{f}_{i,x+e}^{t+1} = eq f_{i,x}^t + \frac{1}{2}(\mathcal{F}_{i,x}^t).
$$

Regarding the implementation, the present hydrodynamic scheme is based on a D2Q9 distribution and requires only nine variables per node to be used in one step for storing the $f_i$ distribution. In the other step the density, pressure, velocities, chemical potential and density derivatives are stored instead. Two memory blocks are in principle needed, see figure 1.

In the first step, the kinetic step, the auxiliary distribution $\hat{f}_i$ is calculated using the hydrodynamic variables, i.e. equation (17). This is a first-neighbor computation since collision and shifting steps are merged. A second biased derivative stencil (first neighbors also) is used here for equation (10). The second step, the hydrodynamic step, calculates the moments and their derivatives from the distribution $\hat{f}_i$, i.e. equation (14). The operator splitting approach is introduced on the hydrodynamic scale, thereby modifying $\vec{u}^* \rightarrow \vec{u}^\ddagger$. For cache performance optimization, the two memory blocks are merged into one following a grid compression idea [20].

4. Finite difference scheme

We use a first-order explicit in time finite difference technique for solving equation (16). The solution with two different stencils are studied, namely central differences (C) and a so-called stable stencil based on a least squares quadric fitting [3] for antidiffusion problems (SS). This last family of stencils computes correctly the actual curvature of the surfaces and at the same time counteracts the formation of ripples of a wavelength of two lattice units. This is achieved by weighing information from second neighbors.

We reproduce here the (SS) Laplace operator for one dimension:

$$
\nabla_{SS} f_i \equiv \frac{2f_{i-2} - f_{i-1} - 2f_i - f_{i+1} + 2f_{i+2}}{7h^2}
$$

Figure 1. Representation of both kinetic and hydrodynamic memory blocks and steps.
and for two dimensions:
\[
\nabla_{SS} f_{i,j} = -28 f_{i,j} - 15 \left( f_{i-1,j} + f_{i+1,j} + f_{i,j-1} + f_{i,j+1} \right) \frac{77h^2}{77h^2} - 2 \left( f_{i+1,j+1} + f_{i-1,j+1} + f_{i-1,j-1} + f_{i+1,j-1} \right) \frac{77h^2}{77h^2} + 12 \left( f_{i-2,j} + f_{i+2,j} + f_{i,j-2} + f_{i,j+2} \right) \frac{77h^2}{77h^2}
\]

where \( i \) and \( j \) correspond to the indices for the \( x \) and \( y \) dimensions, respectively.

The SS operators are second order with an error larger than the central difference stencil, but more stable when solving ill-posed problems, like the inner problem, equation (15). However, the system in equations (15) and (16) is well posed. For the finite difference problem to solve, equation (16), is recast into

\[
T - T_{LB} = \begin{pmatrix}
\mu \frac{\partial u}{\partial x} - \nu_{LB} \rho \frac{\partial u}{\partial x} & \mu \left( \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \right) - \nu_{LB} \rho \left( \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \right) \\
\mu \left( \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \right) - \nu_{LB} \rho \left( \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \right) & \mu \frac{\partial u}{\partial y} - \nu_{LB} \rho \frac{\partial u}{\partial y} - \nu \frac{\partial u}{\partial y} - \nu \frac{\partial u}{\partial y}
\end{pmatrix}
\]

(19)

For each velocity component we then obtain

\[
u_x^+ - u_x^* = \Delta t \left[ 2 \left( \frac{\partial u}{\partial x} - \nu_{LB} \frac{\partial \rho}{\partial x} \right) \frac{\partial u_x}{\partial x} + 2 \left( \mu - \nu_{LB} \rho \right) \frac{\partial^2 u_x}{\partial x^2} \right] + \left( \frac{\partial \mu}{\partial y} - \nu_{LB} \frac{\partial \rho}{\partial y} \right) \left( \frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y} \right) + \left( \mu - \nu_{LB} \rho \right) \left( \frac{\partial^2 u_y}{\partial xy} + \frac{\partial^2 u_x}{\partial y^2} \right)
\]

\[
u_y^+ - u_y^* = \Delta t \left[ 2 \left( \frac{\partial \mu}{\partial y} - \nu_{LB} \frac{\partial \rho}{\partial y} \right) \frac{\partial u_y}{\partial y} + 2 \left( \mu - \nu_{LB} \rho \right) \frac{\partial^2 u_y}{\partial y^2} \right] + \left( \frac{\partial \mu}{\partial x} - \nu_{LB} \frac{\partial \rho}{\partial x} \right) \left( \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right) + \left( \mu - \nu_{LB} \rho \right) \left( \frac{\partial^2 u_x}{\partialyx} + \frac{\partial^2 u_y}{\partial x^2} \right)
\]

(20)

where it is possible to have any interfacial viscosity \( \mu = f(\rho) \) [4].

5.1. Stratified Poiseuille flow: linearity of shear

A stratified Poiseuille flow is investigated for a liquid to gas density ratio of 1000. This is a flow situation with velocity only in the \( y \) direction driven by a constant and uniform force \( g \). This problem balances the external force with the shear forces at the flow boundaries. A sketch of the domain and the velocity profile is given in figure 2. It is expected that in each phase the fluid velocity will present different curvatures since the dynamic viscosities are different in the bulk of each phase. We examine the case of a microfluid, i.e. a fluid where its interface thickness is comparable to the characteristic length of the problem.
The interface thickness is defined as $w = 3/32 h$ and the density profile is initialized with $\rho(x) = \phi^* \tanh(2(x-x_0)/w) + \bar{\rho}$, with $x_0$ being the interface center location coinciding with the algebraic mean of the density. The density profile is shown in both scales: linear (left) and logarithm (right) in figure 3. We propose that for non-negligible shear problems the geometric mean of the density describes better the change in the velocity profile. The geometric mean is located at

$$x_G = x_0 + \frac{w}{2} \tanh^{-1} \left( \frac{\sqrt{\rho_h \rho_l} - \bar{\rho}}{\phi^*} \right).$$

Since $\rho_h > \rho_g$, the $x_G$ is located always in the less dense phase. This is mathematically known as the *inequality of arithmetic and geometric means* statement.

Consider the case where $T = T_{\text{LB}}$, i.e. without viscosity correction and with constant kinematic viscosity. Figure 4 shows the derivative of the velocity times the fluid density $T_{xy}/\nu = \rho(\partial u_y/\partial x)$ where it can be noted that the shear is linear as expected. Variations are present near the boundaries and around the gas–liquid interface (as shown by a dashed line). The multiscale expansion requires that all characteristic lengths should be greater...
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Figure 4. The shear along the $x$ direction ($\rho \partial u_y/\partial x$). The first 3/4 of the domain is liquid and the last 1/4 is gas. The interface is marked as a vertical dashed line, its thickness is 3/32 of the entire $x$ domain: (a) 32, (b) 64 and (c) 96. The three cases shown in each figure corresponds to: without viscosity correction ($\mu_{LB}$), viscosity correction using central differences (C) and stable stencils (SS).

than unity in order to obtain a better simulation. Consequently the variations at the interface are expected to decrease when increasing the interface thickness together with the domain as shown by the continuous lines in the series from (a) to (c) in figure 4.

The kinematic viscosity is decreased by a factor of 10 through the viscosity correction step, i.e. $\nu = 0.1\nu_{LB}$. The non-continuous lines in figure 4 correspond to the cases with viscosity correction. For both the finite difference stencils used, the same linearity as without correction was observed. However, expanding the domain does not seem to have the same positive effect around the interface as with the uncorrected case.

5.2. Stratified Poiseuille flow: constant viscosity

For validation of the case shown in figure 2 we have studied the particular case of constant $\mu$. This implies different corrections for each phase and the interface. A quantification of error is possible since the analytical solution can be computed. The analytical solution for this problem is the solution of the Newtonian flow, i.e. constant dynamic viscosity. The problem then reduces to

$$\frac{d^2 u_y}{dx^2} = \frac{1}{\mu} g.$$  \hspace{1cm} (23)

For a distance, $h$, between the solid surfaces and constant external force $g$:

$$u_y(x) = \frac{1}{2\mu} g (x^2 - hx)$$ \hspace{1cm} (24)

with maximum velocity

$$U_y = -\frac{gh^2}{4\mu}.$$ \hspace{1cm} (25)

The model was run until it reached steady state. Figure 5 shows a comparison between simulation and analytical solutions. For figure 5(a) the density ratio is 100 and the
dynamic viscosity is 1. This represents six times the viscosity of the gas phase and 6% of the liquid phase viscosity. It is seen that very good agreement is achieved. The largest corrections are on the liquid phase where we can define the critical viscosity correction ratio as $(\rho l \nu_{LB}/\mu_l)$.

On the other hand, if the liquid phase viscosity correction ratio is further increased to, for example, a density ratio of 1000 and viscosity of 0.1, a significant discrepancy is observed as shown in figure 4(b). This represents a case with 60% of the viscosity in the gas phase and 0.06% of the liquid phase viscosity which is a difference of five orders of magnitude.

We propose to quantify this variation with the viscosity correction ratio $(\rho l \nu_{LB}/\mu_l)$. The error is defined as the average value of the error in each point normalized to $U_y$. The error in each point is the absolute value of the difference between the velocity of the analytical result and that computed by the method. In figure 6, the changes in error with the viscosity correction ratio for both finite difference stencils studied are shown.

Figure 7 shows the convergence error of the scheme as a function of problem size. For increasing sizes of the problem the second-order dependence is recovered since it is the order of both the LB and finite difference schemes. However, for smaller problem sizes a lower-order dependence is found. It should be pointed out that for all the cases tested the code was stable.

6. Droplet oscillation

This section intends to show the stability of transient problems in general and for different viscosity ratio regimes. Simulations driven by interfacial tension forces are used as examples.

In the example a deformed droplet is released in a square domain surrounded by periodic boundary conditions. The droplet oscillates due to capillary forces until all the

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energy is dissipated into viscous forces, see the sketch in figure 8. It is important to remark that, even though the viscous damping is very slow, no spurious velocities appear in later stages of the simulation. Thus, no parasitic currents are present, regardless of the value of the viscosity. An over-damping regime is observed for high viscosity values as shown in figure 9(a).

All the solutions were stable and oscillations appeared when the damping effect was reduced. In this regime (figure 9(b)) the kinematic energy is partially converted into interfacial potential energy when the droplet is deformed. When the droplet is spherical, the kinetic energy is at a maximum. All the regimes are stable. However, the error introduced increases for lower viscosities. This constitutes a limitation of the technique, although no instabilities were observed.
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Figure 8. Stages of droplet in oscillation. The maximum capillary forces $F_c$ are for the maximum curvature in the maximum deformation stage. In addition, the maximum velocities are present when the droplet is at minimum deformation.

Figure 9. Droplet oscillation with different viscosities with $(\mu_l/\mu_g) = 500$. (a) Damping avoid oscillations, (b) droplet oscillations appears when reducing the viscosity.

7. Conclusion

A hybrid scheme has been proposed for two-phase flow simulations using a lattice Boltzmann method and finite differences. It has been shown that this method is able to simulate a higher Reynolds number situation through reducing the viscosity for density ratios up to three orders of magnitude. The same was shown for the case of an oscillating droplet with a kinematic viscosity lower than $10^{-4}$ LB units.

Simulations are provided to show the linearity of the stress with and without a viscosity correction. A convergence study was performed for the viscosity correction case, and a transient and fully two-dimensional problem was run to show stability.

Regarding the implementation, we have shown that it is compact and reutilizes variables, i.e. the present hydrodynamics scheme is based on a D2Q9 distribution and only requires nine variables per node for storing both hydrodynamic quantities and the lattice Boltzmann distribution. Furthermore the lattice Boltzmann method implements the second biased finite differences [12] using first-neighbors information only.

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