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Abstract. Avalanche flux penetration dynamics has been experimentally observed in a Josephson medium, a granular high-$T_c$ superconductor, while slowly increasing an external magnetic field. The observed voltage spikes are associated with the stepwise penetration of the field into the superconductor and obey the power-law size distribution. The results directly confirm the hypothesis of self-organized criticality in such a system.

Keywords: avalanches (experiment), electrical and magnetic phenomena (experiment), vortex matter (experiment)
1. Introduction

Granular high-$T_c$ superconductors are known to be strong-pinning multiply connected Josephson media that can be described by the critical state concept (see, e.g., [1, 2]). According to this concept proposed by Bean [3], flux penetration into a hard superconductor occurs via Abrikosov vortices that are pinned to the defects of various origins. At the same time, the magnetic pressure force is balanced by a pinning force similar to a dry friction force, and the supercurrent density adjusts to be always equal to the critical value.

The above theory assumes that the effective Josephson penetration depth is much larger than the characteristic size of superconducting granules, $\lambda_{\text{eff}} \gg a$. In this case, the system is a usual type-II hard superconductor with $H_{\text{c1}} \sim \Phi_0/\lambda_{\text{eff}}^2$ in extremely low fields, since $H_{\text{c1}}$ may be on the order of $10^{-3}$–$10^{-1}$ Oe (here, $\Phi_0$ is the flux quantum).

According to Ginzburg [4], the main parameter of a Josephson medium is the ratio of the characteristic granule area to the Josephson vortex area

$$V = \frac{a^2}{\lambda_{\text{eff}}^2} \ll 1, \quad \lambda_{\text{eff}}^2 = \frac{\Phi_0}{2\pi\mu_0\mu_{\text{eff}} j_c a},$$

(1)

where $\mu_{\text{eff}}$ is the effective permeability of the Josephson medium, $j_c$ is the critical current density. Taking $10^6$ A m$^{-2}$, $\mu_{\text{eff}} = 0.5$, and $a = 10$ $\mu$m, we obtain $V \approx 2$. Thus, criterion (1) is violated. Physically, this implies that each elementary loop formed by adjacent granules serves as a discrete pinning center for the flux quanta. Therefore, strong pinning is an intrinsic property of the system. In this case, continuum equations are inapplicable and the Josephson medium is described by equations with marked discreteness that are equivalent to those describing the system with self-organized criticality (SOC) [5].

According to SOC theory, giant dynamic systems under a small perturbation evolve to a critical state that is self-sustaining; i.e., its existence is insensitive to the variation of external parameters. Structurally, the critical state combines a large number of metastable critical states through which the system walks. An external perturbation drives the systems out of one such metastable critical state and generates a dynamic process (avalanche) after that the system passes to another metastable critical state. The avalanches may be either small or large, involving the whole system, but both are induced...
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by equally small perturbations. This behavior was called self-organized criticality, which manifests itself in a power-law size distribution of avalanches.

Ginzburg [4] showed that the self-organized critical state in the low-field electrodynamics of the Josephson medium appears \textit{ab initio} from Maxwell’s and Josephson equations, and, consequently, the avalanche dynamics is an intrinsic property of the system.

Numerical simulation of the self-organized critical state in various models of discrete superconductors was performed in a number of works (see, e.g., [6]–[9]), where it was shown that the size distribution of magnetic flux avalanches has the scaling form typical for SOC. Here, we emphasize the following important point: the condition $V \gg 1$ is very strong, because, according to the numerical simulation [9], SOC appears already at $V \approx 1$ (the values of $V = 0.6$ and $V = 1.2$ were used), i.e., when a single flux quantum corresponds to a single unit cell.

Thus, the appearance of a self-sustaining critical state (according to the Bean model [3]) and the characteristic scaling avalanche dynamics (according to the Bak model [5]) are the main attributes of the self-organized critical state in hard superconductors.

Avalanche vortex dynamics was recently observed in some superconductors in a number of works. This observation demonstrates the implementation of SOC. The power-law distribution of avalanche sizes was observed in NbTi alloy by means of a pickup coil [10], in Nb films with the use of the numerical method [11], in Nb by means of magneto-optical imaging and micro-Hall-probe techniques [12], and in the thin-film high-$T_c$ YBaCuO superconductor [13]. Altshuler and Johansen reviewed the methods used and the samples studied [14].

It is interesting to analyze parameter $V$ in (1) for the materials under investigation. Surdeanu \textit{et al} [15] used high-resolution magneto-optical imaging combined with the scaling analysis to study the flux penetration into the YBa$_2$Cu$_3$O$_{7-x}$ film with a defect size of 200 nm and a critical current of $j_c = 10^{11}$ A m$^{-2}$ in the field up to 17 mT. The value $V \approx 2$ implies the possible implementation of SOC in the system.

Aegerter \textit{et al} [13] observed SOC in the Bean state of the YBa$_2$Cu$_3$O$_{7-x}$ film of a thickness of $d = 80$ nm. Unfortunately Aegerter \textit{et al} did not present the film granularity and the critical current density. For $a = d$ and $j_c = 10^{11}$ A m$^{-2}$ for high-quality films, $V = 0.2$. In view of the numerical simulations [9], we may assume that the conclusions reached in [4] are applicable in this case.

Welling \textit{et al} [16] observed SOC in vortex avalanches in NbH$_x$ films. The size of the inhomogeneities, which are the clusters of a hydrogen-enriched phase and act as effective vortex pinning centers, was on the order of 0.1–1 $\mu$m. The $j_c$ value was not presented, but the assumption of a critical current density of $j_c \sim 10^8$–$10^9$ A m$^{-2}$ typical for the material results in $V \sim 1$ for large clusters.

Consequently, the concept of the Josephson medium with intrinsic pinning can be used as a theoretical model explaining the appearance of SOC in a number of works. At the same time, the flux dynamics in such natural Josephson media as granular superconductors has not yet been studied from the point of view of SOC implementation. Self-organized criticality in discrete superconductors has been observed only in artificial two-dimensional Josephson arrays [17].

Our previous theoretical and experimental investigation of granular superconductors in the range of low-field electrodynamics allowed us to determine the material parameters
required for the appearance of SOC and showed that SOC is accompanied by a number of characteristic phenomena.

It turned out that the YBa$_2$Cu$_3$O$_{7-x}$ ceramic prepared by a standard procedure with a grain size of $a \sim 3-10$ $\mu$m and a critical current density of $j_c \geq 30$ A m$^{-2}$ reveals such a behavior although $V$ is as small as 0.01–0.1. It is likely associated with the change (improvement) of intergranular contacts during the ceramic sintering and the corresponding enlargement of an elementary Josephson loop of the material up to tens of $\mu$m.

One of the above indications of SOC is the current–voltage characteristic (CVC) isotropy, i.e., its independence of the Lorentz force [4]. This was confirmed experimentally in [18]–[20]. Moreover, the behavior of the Josephson medium should exhibit strong fluctuations since the system dynamics is of the avalanche type. Such fluctuations were observed in the experimental study of voltage noise caused by the motion of flux quanta in granular superconductors in different geometries of the experiment [20, 21].

Nevertheless, it should be pointed out that the isotropy of CVC and voltage noise are indirect evidences of SOC; the direct evidence would be the observation of the scaling avalanche flux dynamics.

Thus, the experimental study of the flux dynamics particularly in a Josephson medium with strong internal pinning on discrete Josephson loops, which is exemplified by a granular high-$T_c$ superconductor, is of supreme importance for the confirmation of the self-organized critical state in such a system.

2. Experimental results and discussion

The study was carried out on the YBa$_2$Cu$_3$O$_{7-x}$ ceramic prepared by the standard procedure with a characteristic granule size of about 5 $\mu$m, a material density of 4.9 g cm$^{-3}$, and a transition temperature of $T_c = 90.5$ K. A 60-turn pickup coil made of 0.1 mm copper wire was wound around a cylindrical ceramic sample 15.0 mm long, of diameter 1.5 mm. Surrounding the sample with a double copper–permalloy shield allowed us to get rid of stray pickup and the Earth field, and to provide sample cooling in zero field. An external magnetic field was generated by the coil fed with a self-made high-accuracy integrating current source, which provided an accurate long-term linear field sweep. To reduce unwanted noise and stray pickup, the current source and preamplifier were powered from storage batteries.

The measured pickup coil voltage passed through a step-up transformer, preamplifier, low-pass filter with the cutoff $f_c = 6$ kHz, and main amplifier to the PC data acquisition board at a readout rate of 20 kHz. The overall gain of the circuit was $3.37 \times 10^6$. The accumulation time was as long as several hours, depending on the time constant of the current source, and the rate of the flux variation in the sample was down to one quantum $\Phi_0$ per second. The measurements were performed in helium atmosphere at liquid nitrogen temperature. Before each measurement, the sample was warmed up with a subsequent zero-field cooling.

To determine the range of low-field electrodynamics, the permeability and the second and third harmonics of the magnetization were measured using the technique proposed in [22, 23].

The inset in figure 1 presents the real and imaginary components of the magnetic permeability of the sample, indicating that the superconducting state at $T < 84$ K is
established throughout the whole sample. Figure 1 shows the dependence of the critical current density on the external field, which can be expressed as [24]

\[ j(H) = \frac{j_0 H_0}{H + H_0}, \]  

(2)

where \( j_0 = 40 \text{ A cm}^{-2} \) and the characteristic field is \( H_0 = 2.2 \text{ Oe} \). This dependence indicates that the range of low-field electrodynamics, where the critical current is field independent,

\[ j_c = j_0 = \text{const}, \]  

(3)

and the Bean model [3] is applicable, is \( H < 1 \text{ Oe} \). All the measurements were performed in this low-field region.

The jumps of the flux penetrating into the sample were detected as short unidirectional spikes of the voltage induced in the pickup coil. The apparatus sensitivity may be estimated in the following way: once the flux through the sample changes by \( \Delta \Phi = N \Phi_0 \), where \( N \) is the number of flux quanta in an avalanche, the voltage induced in the pickup coil with \( n \) number of turns during the response time \( \Delta t = 1/2\pi f_c \) of the measuring circuit is \( \Delta E \). Then, the number of flux quanta is calculated as

\[ N = \Delta E \Delta t / nK\Phi_0. \]  

(4)

Taking \( \Delta E = 1 \text{ V} \) (see figure 2), \( n = 60, \Phi_0 = 2.07 \times 10^{-15} \text{ Wb}, \Delta t = 40 \mu\text{s} \) and the gain \( K = 3.37 \times 10^6 \), we find \( N \approx 90 \). Thus, the apparatus allows us to observe avalanches of at least 100 flux quanta, which is comparable with the experimental resolution obtained by Field et al [10].

The following method was used for the reliable detection of the voltage spike: the whole record was divided into sections one second long containing 20,000 data points each, a standard deviation was determined within each section and the spike was identified as
Figure 2. Record in which the flux jump is observed. The dashed line indicates the cutoff level. The inset presents the shape of a voltage spike in detailed timescale. The field ramping rate is $1.13 \times 10^{-5}$ Oe s$^{-1}$, which corresponds to one flux quantum $\Phi_0$ per second.

Figure 3. Accumulated amplitude of voltage spikes at the field ramping rate corresponding to $1\Phi_0$ s$^{-1}$. The record accumulation time is 3000 s.

more than a fivefold exceeding of the standard deviation. Figure 2 presents one such section, where the flux jump was observed, and the inset shows the typical voltage spike with higher time resolution. The spikes were absent both above the transition temperature and in a constant magnetic field.

The fragments of the accumulated amplitude of spikes $S \sim \Sigma E_i$, where $E_i$ is the induced voltage exceeding the threshold value equal to five standard deviations, are shown in figures 3 and 4 for different rates of the field ramp-up. Thus, the quantity $S$ is proportional to the flux that has entered the sample. Obviously, the flux jumps of the random amplitude are observed.
Figure 4. Accumulated amplitude of voltage spikes at the field ramping rate corresponding to $13\Phi_0$ s$^{-1}$. The record accumulation time is 3600 s.

Figure 5. Amplitude distribution of voltage spikes. The slope of the linear fit is $k = -1.8$.

Figure 5 presents the amplitude distribution of the voltage spikes. The flux penetration is seen to occur in the form of jumps that obey a power-law amplitude distribution

$$P_E \sim E^{-k},$$

(5)

where $k = 1.8 \pm 0.2$. Such a scaling behavior is direct evidence of the existence of a self-organized critical state in the Josephson medium.

3. Summary

In summary, it is necessary to note that there is a row of unsolved problems in the study of SOC in the Josephson medium. One of them is a problem of the crossover between
the self-organized critical state and the usual critical state of type-II superconductors. At measurements near $T_c$, we observed that the number of small spikes increased, and, in contrast, the number of large avalanches decreased. The preliminary analysis of experimental data points to crossover from the power-law distribution to the exponential dependence of the avalanche sizes near $T_c$. However the lack of experimental statistics in this field has prevented research in this crossover region.

Another important and unsolved problem of SOC in the Josephson medium is the question of universality of the critical exponent $k$. There is no theoretical estimation of the $k$ value from first principles, so the experimental study of the flux avalanche distribution via the temperature, external magnetic field, and type of superconductor medium is a very pressing task.

Also, obviously, improvement of the experimental methods is necessary, because the region of spike amplitude is of about the order of its value. The circuit noise and the cutoff level restrict the small amplitudes of spikes, but saturation of the signal amplifier restricts the level of peak signal from the large flux avalanches. It will also allow us to study the problem of the dimensional scaling, since there are several typical lengths in the Josephson medium—the grain size $a$, the Josephson penetration length $\lambda_{\text{eff}}$ and the sample size.

Thus, self-organization of the critical state in a Josephson medium has been proved experimentally, but this is just the first step in the investigation of this wide ranging and unsolved problem.

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