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Analytical analysis of fracture conductivity for sparse distribution of proppant packs

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Abstract
Conductivity optimization is important for hydraulic fracturing due to its key roles in determining fractured well productivity. Proppant embedment is an important mechanism that could cause a remarkable reduction in fracture width and, thus, damage the fracture conductivity. In this work a new analytical model, based on contact mechanics and the Carman–Kozeny model, is developed to calculate the embedment and conductivity for the sparse distribution of proppant packs. Features and controlling factors of embedment, residual width and conductivity are analyzed. The results indicate an optimum distance between proppant packs that has the potential to maintain the maximum conductivity after proppant embedment under a sparse distribution condition. A change in the optimum distance is primarily controlled by closure pressure, the rock elastic modulus and the proppant elastic modulus. The proppant concentrations and the poroelastic effect do not influence this optimum distance.

Keywords: proppant, embedment, conductivity, hydraulic fracturing

(Some figures may appear in colour only in the online journal)

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A_f</td>
<td>Area of fracture face, m^2</td>
</tr>
<tr>
<td>a</td>
<td>Radius of contact area, m</td>
</tr>
<tr>
<td>A</td>
<td>Flow area, m^2</td>
</tr>
<tr>
<td>A_i</td>
<td>Area of rock face, m^2</td>
</tr>
<tr>
<td>A_o</td>
<td>Contact area of single particle, m^2</td>
</tr>
<tr>
<td>C</td>
<td>Kozeny–Carman constant, dimensionless</td>
</tr>
<tr>
<td>D_2</td>
<td>Thickness of the upper and lower rocks, m</td>
</tr>
<tr>
<td>ΔD_1</td>
<td>Deformation of proppant packs, m</td>
</tr>
<tr>
<td>ΔD_2</td>
<td>Deformation of rock, m</td>
</tr>
<tr>
<td>E_i</td>
<td>Elastic modulus of proppant, Pa</td>
</tr>
<tr>
<td>E_2</td>
<td>Elastic modulus of rock, Pa</td>
</tr>
<tr>
<td>F</td>
<td>Applied force on proppant, N</td>
</tr>
<tr>
<td>F_1</td>
<td>Force located at L_{hf} &lt; x &lt; L_{hf}, N m^{-1}</td>
</tr>
<tr>
<td>F_c</td>
<td>Fracture conductivity, m^2.m</td>
</tr>
<tr>
<td>F_i</td>
<td>Force located at part i, N m^{-1}</td>
</tr>
<tr>
<td>f</td>
<td>Fitting parameters, dimensionless</td>
</tr>
<tr>
<td>H</td>
<td>Length in the direction of rows, m</td>
</tr>
<tr>
<td>H_p</td>
<td>Length of proppant pack in the direction of rows, m</td>
</tr>
<tr>
<td>h</td>
<td>Embedment depth, m</td>
</tr>
<tr>
<td>J</td>
<td>Layers of proppants, dimensionless</td>
</tr>
<tr>
<td>K</td>
<td>Stress intensity factor, Pa-m^{0.5}</td>
</tr>
<tr>
<td>K_i</td>
<td>Stress intensity factor at fracture tip induced by part i, Pa-m^{0.5}</td>
</tr>
<tr>
<td>k</td>
<td>Permeability, m^2</td>
</tr>
</tbody>
</table>
1. Introduction

Hydraulic fracturing is a reservoir stimulation method often applied to low permeability formations that would otherwise be uneconomical to produce. The process starts with the high-pressure injection of a fracturing fluid into a wellbore to create cracks. Fluids containing sand or other proppants suspended with the aid of thickening agents are then injected. When the hydraulic pressure is removed from a well, the sides of the fractures compress onto the proppants, creating a high-permeability pathway through which natural gas and petroleum flow more freely in deep-rock formations (figure 1).

The response of fractured wells is highly dependent on fracture conductivity. In conventional reservoirs, a high viscosity crosslinked fluid is used to create long and symmetric fractures. Proppants are believed to be placed uniformly in the fracture due to high viscosity and good proppant carrying capacity (El-M. Shokir and Al-Quraish 2009). Factors affecting the proppant placement have been widely studied (Clark and Guler 1983, Mobbs and Hammond 2001, Dontsov and Peirce 2014). In unconventional reservoirs, slick water is widely used to generate the fracture network. The fluid has low viscosity and poor proppant carrying capacity, resulting in some nonuniform distributions of proppants in the fracture network (Awoleke et al. 2016). Experimental and field tests both show that sparse distribution—such as a partial monolayer in secondary fractures (Fredd et al. 2001, Zou et al. 2015) or sparse distribution in channel fracturing (D’Huteau et al. 2011)—can improve the fracture conductivity. The variation of fracture conductivity under sparse distribution, therefore, is very important in optimizing the fracturing design in unconventional reservoirs.

Proppant embedment may significantly impair fracture conductivity due to a decrease in the fracture width (figure 2). Proppant embedment and deformation play significant roles in the stimulation response of hydraulic fracturing, especially in clay-rich, soft formations. Additionally, a stress shadow effect, caused by multiple fracture interactions, can increase the effective closure stress (Jin et al. 2010), resulting in enhanced proppant embedment. This is generally regarded as a geomechanical effect in an unconventional reservoir simulation and should be considered when assessing the production forecast in formations with low Young’s modulus (Yu and Sepehrnoori 2014).
Currently, it is still impossible to conduct in situ measurements of residual width and conductivity. Several experimental apparatus were developed to study the features of residual width and conductivity (Volk et al 1981, Hartley and Bosma 1985, Huitt and McGlothlin 1986, Lacy et al 1997, Lacy et al 1998, Wen et al 2007, Guo et al 2008, Zhang et al 2015). Based on the results of these experiments, empirical formulas to calculate residual width and conductivity were proposed.

In recent years, several analytical models have also been developed. Guo and Liu (2012) proposed an analytical model to describe the embedment features in soft rock. Their model, however, only considered closely packed cases. Khanna et al (2012) proposed a simplified approach for calculating the conductivity of narrow fractures (2012). Li (2012) adopted, including disregarding the surface roughness of fracture walls due to very high contact stresses. The proppant embedment and deformation are considered to be the primary mechanism of fracture width change and fracture conductivity loss. These assumptions limit the developed approach’s ability to accurately predict the conductivity. Even so, the results can provide insight into the features and factors that control the conductivity change induced by proppant embedment and deformation.

### 2. Model development

In hydraulic fracture, proppants are packed in multilayers, as shown in figure 3. J layers of proppants support a fracture width of \( w_j \). In the top view, the \( M \) columns and \( N \) rows of proppants result in an area of the fracture region \( A_f = L \times H \).

#### 2.1. Theoretical background

Heinrich Hertz has derived the solution of the contact between an elastic sphere and an elastic half-space (Johnson 1987), as illustrated in figure 4. An elastic sphere of radius \( R \) indents an elastic half-space to depth \( \delta \), and thus creates a contact area of radius

\[
a = \sqrt{R\delta}.
\]  

The applied force \( F \) is related to the displacement depth \( \delta \) by

\[
F = \frac{4}{3} E^* R \frac{1}{2} \delta^2
\]

where

\[
\frac{1}{E^*} = \frac{1 - v_1^2}{E_1} + \frac{1 - v_2^2}{E_2}
\]

where \( v_1 \) is Poisson’s ratio of proppants (dimensionless); \( v_2 \) is Poisson’s ratio of rock (dimensionless); \( E_1 \) is the elastic modulus of proppants (Pa); \( E_2 \) is the elastic modulus of rock (Pa).

The radius of the contact circle \( a \), depth \( \delta \) and the maximum contact pressure \( p_{\text{max}} \) are given by

\[
a = \left( \frac{3FR}{4E^*} \right)^\frac{1}{3}
\]

\[
\delta = \left( \frac{9F^2}{16RE^*v_2} \right)^\frac{1}{3}
\]

\[
p_{\text{max}} = \left( \frac{6FE^* v_2}{\pi^2 R^2} \right)^\frac{1}{3}
\]

The average contact pressure compressing proppants is related to the maximum contact pressure

\[
p_{\text{ave}} = \frac{2}{3} p_{\text{max}}.
\]
2.2. Even distribution of proppants

2.2.1. Residual width. Figure 5 shows a stress analysis of a proppant pack. The relationships between the load and stress satisfy the following equations:

\[ p_A = p_M N A \]

and

\[ p_{ps} = p_{ave} \]

where \( p_c \) is the closure pressure (Pa); \( p_o \) is the overburden pressure (Pa); \( p_f \) is the fluid pressure in a fracture (Pa); \( p_s \) is the pressure with which a single proppant acts on rock (Pa); \( A_f \) is the area of the propped fracture surface (m\(^2\)); \( A_s \) is the contact area of a single particle (m\(^2\)). \( A_f \) and \( A_s \) can be expressed as follows:

\[ A_f = 4MR^2 \]

Combining equations (8)–(11), \( p_s \) can be derived as

\[ p_s = \frac{4p_c R^2}{\pi a^2} \]

and

\[ p_s = p_{ave} \]

Combining equations (7), (12) and (13), the maximum contact pressure \( p_{max} \) can be expressed as

\[ p_{max} = \frac{3}{2} \frac{4p_c R^2}{\pi a^2} \]

Combining equations (4)–(6) and (14), the depth \( \delta \) and maximum contact pressure \( p_{max} \) can be derived as

\[ \delta = \frac{3p_c}{\pi E^r} R \]

\[ p_{max} = \frac{1}{\pi} \left( \frac{24p_c E^r}{E} \right)^{\frac{3}{2}} \]

The thickness of the upper and lower rocks is the same and assumed as \( D_2 \). The deformation of rock is reflected in the form of proppant embedment (Li et al. 2014). Therefore, the embedment depth \( h \) can be expressed as

\[ h = \delta + \Delta D_2 = \left( \frac{3p_c}{E^r} \right)^{\frac{3}{2}} R + \frac{p_c}{E^2} D_2 \]

where \( h \) is the embedment depth, (m); \( \Delta D_2 \) is the deformation of rock (m).

The deformation of proppant packs satisfies the following equation:

\[ \Delta D_1 = \frac{p_{ave} w_0}{E_1} \]

where \( \Delta D_1 \) is the deformation of proppant packs (m); \( w_0 \) is the initial width of a fracture (m). Combining equations (7), (17) and (18), the change in the fracture width \( \Delta w_f \) can be
obtained by

\[ \Delta w_f = \Delta D_1 + 2h = \frac{2p_{\text{max}}}{3E_1}w_{f0} + 2\left(\frac{3p_c}{E^*}\right)^{\frac{3}{2}}R \]

\[ + \frac{p_c}{E_2}D_2 \]  

and

\[ w_f = w_{f0} - \Delta w_f \]  

where \( \Delta w_f \) is the change in fracture width (m); \( w_f \) is the final fracture width (m).

### 2.2.2. Fracture conductivity

Results from discrete element modeling indicate that the Carman–Kozeny model can be used to calculate the permeability of proppant packs (Mollanouri Shamsi et al 2015, Sanematsu et al 2015). The permeability is described as follows:

\[ k_c = \frac{\phi^3}{36C(1 - \phi)^p}(2R)^2 \]  

where \( \phi \) is the porosity of a sample; \( C \) is the Carman–Kozeny constant. For beds packed with spherical particles, the constant \( C \) is equal to 5. The fracture conductivity can be obtained by

\[ F_c = k_f \times w_f. \]

### 2.3. Uneven distribution of proppants

#### 2.3.1. Residual width

The proppant packs may have a sparse distribution in a hydraulic fracture, especially in channel fracturing (D’Huteau et al 2011). A simple sparse distribution is illustrated in figure 6.

A two-proppant distance ratio (\( R_{s1}, R_{s2} \)) can be defined as

\[ R_{s1} = \frac{H_p}{L_p} = \frac{2NR}{H} \]  

\[ R_{s2} = \frac{L_p}{H_p} = \frac{2MR}{L} \]  

where \( L_p \) and \( H_p \) are the length and height of a proppant pack (m). If the two packs are close to each other, \( R_{s1} \) and \( R_{s2} \) are equal to unity; if not, they are smaller than unity.

#### 2.3.2. Fracture conductivity

A representative unit in figure 6 is shown in figure 7. Conductivity of this unit is developed in this section. The other unit can be derived using the same process. In this figure, \( q \) is the flow rate (m\(^2\) s\(^{-1}\)), \( u \) is the velocity (m s\(^{-1}\)), \( k \) is the permeability (m\(^2\)), and \( \Delta p \) is the pressure difference (Pa). The subscripts (1, 2, 3) represent the different sections in this figure.

According to Darcy’s Law

\[ q = uA = \frac{kA \, dp}{\mu \, dx} \]  

where \( A \) is the flow area (m\(^2\)); \( \mu \) is the viscosity of a fluid (Pa s). According to the relationship between the pressure
difference in sections 1 and 2, the equivalent permeability of sections 1 and 2 is

$$\frac{1}{k_{eq}^{1.2}} = \frac{1}{k_1} (1 - R_{o2}) + \frac{1}{k_2} R_{o2}. \quad (30)$$

According to the relationship between the flow rate in sections 1 and 3, the equivalent permeability of the three sections is

$$k_{eq}^{1.2.3} = k_{eq}^{1.2} R_{s1} + k_3 (1 - R_{s3}). \quad (31)$$

The permeability of the different sections is

$$k_2 = \frac{\phi^3}{36C(1 - \phi)^2} d^2$$

$$k_1 = k_3 = \frac{w_2^2}{12}. \quad (32)$$

Subsequently, fracture conductivity can be obtained by

$$F_c = k_{eq}^{1.2.3} \times w_1. \quad (33)$$

### 3. Discussions

#### 3.1. Model verification

To verify the new model, it is necessary to compare the new model with the existing model and experimental data. Lacy et al. (1998) and Lu et al. (2008) measured the proppant embedment and width change at different closure pressures. The experimental data they reported were used to test the new model in this paper and the model developed by Li et al. (2014). Li et al. (2014) summarized the known experimental conditions as listed in table 2.

The comparison of the theoretical values of proppant embedment with experimental data from Lacy et al. (1998) is shown in figure 8. The values calculated with the new model and the model by Li et al. (2014) are very close. The two theoretical models’ values are smaller than the experimental data. The reason for the smaller values has been discussed by Li et al. (2014).

The fitting parameters were introduced in equation (17):

$$h = f\left[\left(\frac{3P_c}{E_2}\right)^2 + \frac{P_c D_2}{E_2}\right]. \quad (34)$$

#### Table 2. Basic parameters.

<table>
<thead>
<tr>
<th>Data source</th>
<th>$v_1$</th>
<th>$v_2$</th>
<th>$E_1$ (MPa)</th>
<th>$E_2$ (MPa)</th>
<th>$R$ (mm)</th>
<th>$D_2$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lacy et al (1998)</td>
<td>0.13</td>
<td>0.13</td>
<td>21306</td>
<td>1172</td>
<td>0.3175</td>
<td>20</td>
</tr>
<tr>
<td>Lu et al (2008)</td>
<td>0.144</td>
<td>0.144</td>
<td>35000</td>
<td>28770</td>
<td>0.3175</td>
<td>15</td>
</tr>
</tbody>
</table>

#### Figure 8. Comparison of theoretical values of proppant embedment with experimental data from Lacy et al. (1998).

The same fitting parameters were introduced into the data provided by Lu et al. (2014). The comparisons of the modified models are shown in figure 9. The revised models fit well with the experimental data and the calculated values of these two models almost overlap with each other. The correction factors and regression coefficients of the revised models in figure 9 are listed in table 3.

#### Table 3. Fitting results.

<table>
<thead>
<tr>
<th>Model</th>
<th>$f$</th>
<th>$R^2$</th>
<th>$f$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>New model</td>
<td>1.314</td>
<td>0.92</td>
<td>16.49</td>
<td>0.97</td>
</tr>
<tr>
<td>Li et al (2014)</td>
<td>1.322</td>
<td>0.92</td>
<td>21.14</td>
<td>0.95</td>
</tr>
</tbody>
</table>
models are shown in figure 10. Similarly, the two models fit well with the experimental data; the fitting parameters are also listed in table 3. The results indicate that the new model can match the experimental data in all the cases studied, with the same or slightly better fitting results as compared with the model by Li et al. (2014).

A fracture width change is required in the calculation of fracture conductivity. Consequently, the width change is also compared with the modified model and experimental data.

The comparisons of the modified models and experimental data from Lacy et al. (1998) are shown in figure 11. The two modified models’ values are both smaller than the experimental data; the deviation of the model values from the experimental data, however, is not large and follows the trend of the experimental data.

The comparisons of the modified models and experimental data from Lu et al. (2008) are shown in figure 12. The new modified model in this paper can match most of the data until the closure pressure increases to 40 MPa. At the same time, the modified model results of Li et al. (2014) are smaller than the experimental data. The results demonstrate that the new modified model can be used to calculate the width change.

3.2. Application to the sparse distribution of proppant packs

3.2.1. Changes in embedment, width and conductivity. In this section, the new models modified by two sets of experimental data are used to predict the embedment, fracture width change and conductivity in the sparse distribution of proppant packs. The results of the new model modified by the data from Lacy et al. (1998) is referred to as case A; the other, with the data from Lu et al. (2008), is referred to as case B.

Embedment can be obtained with the modified new model at closure pressure $P_c = 30$ MPa, as shown in figure 13. With an increase in $R_{s1}$, embedment decreases, as expected. The change in embedment is greater when $R_{s1}$ is small because, with an increase in $R_{s1}$, the pressure applied on

Figure 10. Comparisons of modified models and experimental data from Lu et al. (2008).

Figure 11. Comparisons of the modified models and experimental data from Lacy et al. (1998).

Figure 12. Comparisons of the modified models and experimental data from Lu et al. (2008).

Figure 13. Embedment under different proppant distance ratios.
the rock and proppant packs reduces. From figures 13 and 14, the trends indicate that $R_{s1}$ has a significant influence on embedment and fracture width.

Furthermore, the stress intensity factor at the crack tip would yield useful information on fracture closure especially when the proppants are unevenly and sparsely distributed within the fracture. The stress intensity factor ($K_f$) is used in fracture mechanics to predict the stress state near the fracture tip. When the fracture is open, a positive value of $K_f$ exists at the fracture tip. Otherwise, a negative or zero value of $K_f$ indicates the fracture closure.

The stress intensity factor $K_f$ at the fracture tip by a point force $F_1$ can be obtained from the following expression (Wells 1965):

$$K_f = \frac{F_1}{2\sqrt{\pi L_{HF}}} \int_{-L_{HF}}^{L_{HF}} \frac{L_{HF} + x}{L_{HF} - x} \, dx.$$  \quad (35)

An approximate formula can be expressed as

$$K_{fi} = \frac{F_i}{2\sqrt{\pi L_{HF}}} \frac{L_{HF} + x}{L_{HF} - x}$$  \quad (36)

$$K = \sum_{i=1}^{N_{HF}} K_{fi}$$  \quad (37)

where $K$ is the stress intensity factor (Pa·m$^{0.5}$); $K_f$ is the stress intensity factor at the fracture tip induced by part $i$ (Pa·m$^{0.5}$); $F_1$ is the force located at $L_{HF} < x < L_{HF}$ (N·m$^{-1}$); $F_i$ is the force located at part $i$ (N·m$^{-1}$); $L_{HF}$ is the fracture half length (m); $x$ is the position along the fracture length (m); $N_{HF}$ is the number of parts divided in hydraulic fracture (dimensionless).

Under a sparse distribution, the stress intensity factor for a narrow fracture filled with proppants has two limits. On one side, in the proppant free part of the fracture, the stress intensity factor is zero as the fracture faces have no cohesion (Neto and Kotousov 2013). This case will happen when there is no proppant or highly compressible proppant. On the other side, when the whole fracture is continuously propped by incompressible particles, the stress intensity factors at the tip of the fracture have a certain value, $K_0$. The hydraulic fracture is divided in to $N_{HF}$ parts. Then $K_0$ of each part can be obtained by equation (36) and then the integrated value can be achieved by equation (37).

Figure 15 shows the relationship between the stress intensity factor and $R_{s1}$ at different fracture lengths. The $K_0$ is a normalized stress intensity factor, which is normalized against $K_0$. The relationships presented in figure 15 clearly show the expected tendency of $K$ approaching $K_0$, when the fracture is fully filled with proppants ($R_{s1} = 1$). This case also corresponds to the maximum residual width of the fracture.

Fracture conductivity after proppant embedment and deformation can also be obtained, as shown in figure 16. The fracture conductivity is normalized against the conductivity of a channel with an opening equal to the initial fracture width, which is $w_{00}^2/12$. The two cases have the same trend. With an increase in $R_{s1}$, the fracture conductivity first increases to a certain value and then decreases. In other words, there is an optimal distance which has maximum conductivity after proppant embedment and deformation. This is because increasing the proppant distance ratio has two different effects on conductivity: (1) a reduction in proppant deformation and embedment due to more proppants supporting the closure pressure, which decreases the loss of conductivity, and (2) an increase in the loss of conductivity due to a
reduced void volume which is occupied by more proppants. The two mechanisms control the loss of conductivity, resulting in the complex behavior shown in figure 16. As seen from this simulation, when the proppant distance ratio is less than 0.2 and 0.4 for the two cases, respectively, the reduction in proppant deformation and embedment mainly controls the change in conductivity. When the proppant distance ratio increases and exceeds a certain value, the reduction of a void volume primarily controls the change in conductivity. The curve trend is the same as the calculations done by Khanna et al (2012), where the conductivity is computed by an Ansys CFX with a monolayer of proppants. A reasonable agreement exists between the model developed in this paper and their model.

The two cases have the same trend and, therefore, in the following sections, the basic parameters from Lacy et al (1998), shown in table 2, are used to study the effect of different influencing factors. If not specified otherwise, the closure pressure is 30 MPa and $R_{s2} = 1$.

3.2.2. Effect of closure pressure. Figure 17 shows a relationship between conductivity and closure pressure ($P_c$) at different $R_{s1}$. As shown in figure 17(a), the conductivity decreases with the closure pressure for all the cases. As shown in figure 17(b), the curve trends under four closure pressures are slightly different, especially at the inflection point. With an increase in the closure pressure, the inflection point moves to the right, meaning that $R_{s1}$ with maximum conductivity increases. In other words, with an increase in the closure pressure, the proppant packs should be designed to be closer to each other in order to achieve the maximum conductivity. Under a high closure pressure, the conductivity of a smaller $R_{s1}$ reduces to almost zero. This type of proppant placement will be closed due to proppant embedment and deformation.

3.2.3. Effect of rock elastic modulus. Figure 18 shows a relationship between conductivity and rock elastic modulus ($E_2$) at different $R_{s1}$. As shown in figure 18(a), with an increase in $E_2$, the conductivity increases, and the incremental amplitude decreases. When $E_2$ is large, a further increase in $E_2$ has little effect on conductivity. As shown in figure 18(b), with an increase in $E_2$, the inflection point moves to the left, which means that $R_{s1}$ with maximum conductivity decreases. Essentially, with an increase in $E_2$, the proppant packs can be
designed more diffusively to each other; with an increase in $E_2$, the ability to resist embedding increases, while the deformation of rock under compression also decreases, so the fracture width can be better maintained. Thus, the maximum fracture conductivity can be achieved with a larger distance between proppant packs.

### 3.2.4. Effect of proppant elastic modulus

Figure 19 shows a relationship between conductivity and proppant elastic modulus ($E_1$) at different $R_{s1}$. As shown in figure 19(a), with an increase in $E_1$, the conductivity increases and the incremental amplitude decreases. The trend is the same as for $E_2$ in figure 18(a). When $E_1$ is large, a further increase in $E_1$ has little effect on conductivity. As shown in figure 19(b), with an increase in $E_1$, the inflection point moves to the left, which means that $R_{s1}$ with maximum conductivity decreases. In other words, with an increase in $E_1$, the proppant packs can be designed more diffusively to each other, because with an increase in $E_1$, the deformation of proppants under compression decreases, so the fracture width can be better maintained.

### 3.2.5. Effect of proppant concentration

Parameter $J$ reflects the proppant concentration in a fracture. Figure 20 shows a relationship between conductivity and proppant concentration ($J$) at different $R_{s1}$. As shown in figure 20(a), with an increase in $J$, the conductivity, as well as the incremental amplitude, increases. The results demonstrate that the relationship is a power law. As shown in figure 20(b), with an increase in $J$, the inflection point does not move obviously, which means that the $R_{s1}$ with maximum conductivity remains the same for the four cases. The proppant concentration does not show a discernable influence on the optimal distance.

### 3.2.6. Effect of poroelastic effect

During hydraulic fracturing, the fluid pressure near the wellbore is high, while it is small near the tip. The two faces of the hydraulic fracture are not parallel to each other. The fracture width reduces from wellbore to the fracture tip along the longitudinal direction. The initial fracture width was

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**Figure 19.** Effect of proppant elastic modulus on conductivity. (a) the conductivity increases with the increased proppant elastic modulus; (b) the optimal distance ratio shows a slight decrease with an increase in proppant elastic modulus.

**Figure 20.** Effect of proppant concentration on conductivity. (a) the conductivity increases with the increased proppant concentration; (b) the optimal distance ratio does not show an obvious change with the variation of proppant concentration.
calculated by an analytical model (PKN) \cite{Perkins1961, Nordgren1972}. It is assumed that the dynamic fracture width calculated using the PKN model is completely filled with proppants. Values for embedment and conductivity were obtained by the model proposed in this paper.

Figure 21 shows a relationship between conductivity and position along the fracture length direction at different fracture shapes. The conductivity in parallel fractures is a constant, while it decreases along the fracture length direction in the elliptical shape. Generally, fracture conductivity is a constant for a single fracture in a reservoir simulation. Figure 21, however, shows that the possible actual case is totally different from a constant. This difference may have an impact on simulated well productivity.

In the production process, after the fluid diffusion from the porous matrix to the fracture and then to the wellbore, the poroelastic effect occurs. On the one hand, the pore pressure decrease and the effective closure pressure increase will increase the embedment depth. On the other hand, fluid flow within the fracture causes a pressure drop ($\Delta p_{\text{f}}$) along the fracture length direction. In order to better illustrate the effect of poroelastic, an elliptical shape was used.

Because there is pressure drop inside the fracture. The fluid pressure at the tip is higher than that at the near wellbore region. Therefore, the effective stress increases from the fracture tip to near the wellbore region. With an increased pressure drop, the effective stress near the wellbore increases. Therefore, the conductivity near the wellbore is more sensitive to the pressure drop, as shown in figure 22(a). However, the varied amplitude is limited, which does not show an effect on the optimal distance (figure 22(b)).

4. Conclusions

Based on the study, the following conclusions can be drawn.

(1) New mathematical models have been derived to calculate changes in fracture width and conductivity with consideration of the sparse distribution of proppant packs.
(2) With an increase in the proppant distance ratio, the fracture conductivity first increases to a certain value and then decreases. An optimum proppant distance ratio maintains the maximum conductivity after proppant embedment and deformation.
(3) A change in the optimum proppant distance ratio increases with closure pressure and decreases with the rock elastic modulus and proppant elastic modulus. There is no obvious change with the variation of proppant concentration.
(4) The stress intensity factor at the fracture tip increases with the increased proppant distance ratio.

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