The CIPM list of recommended frequency standard values: guidelines and procedures

To cite this article: Fritz Riehle et al 2018 Metrologia 55 188

View the article online for updates and enhancements.
The CIPM list of recommended frequency standard values: guidelines and procedures

Fritz Riehle¹, Patrick Gill², Felicitas Arias³ and Lennart Robertsson³

¹ Physikalisch-Technische Bundesanstalt, Bundesallee 100, 38116 Braunschweig, Germany
² National Physical Laboratory, Hampton Road, Teddington, Middlesex, TW11 0LW, United Kingdom
³ Bureau International des Poids et Mesures, Pavillon de Breteuil, 92312 Sèvres Cedex, France

E-mail: fritz.riehle@ptb.de

Received 10 October 2017, revised 12 December 2017
Accepted for publication 20 December 2017
Published 14 February 2018

Abstract

A list of standard reference frequency values (LoF) of quantum transitions from the microwave to the optical regime has been recommended by the International Committee for Weights and Measures (Comité international des poids et mesures, CIPM) for use in basic research, technology, and for the metrology of time, frequency and length. The CIPM LoF contains entries that are recommended as secondary representations of the second in the International System of Units, and entries that can be used to serve as realizations of the definition of the metre. The historical perspective that led to the CIPM LoF is outlined. Procedures have been developed for updating existing, and validating new, entries into the CIPM LoF. The CIPM LoF might serve as an entry for a future redefinition of the second by an optical transition.

Keywords: secondary representation of the second, list of recommended frequencies, absolute frequency, future new definition of the second

1. Introduction and historical perspective

Since the redefinition of the unit of length in the International System of Units (SI) [1] by the 17th General Conference of Weights and Measures (Conférence générale des poids et mesures, CGPM) in 1983 [2] the metre has been defined via the adopted value of the speed of light in a vacuum \(c_0 = 299\,792\,458\,\text{m}\,\text{s}^{-1}\). The fixed numerical value for the speed of light \(c_0 = \lambda \cdot \nu\) links the vacuum wavelength \(\lambda\) and the frequency \(\nu\) of any plane electromagnetic wave. Consequently, each radiation whose frequency can be traced back to the primary standard of time and frequency, i.e. the caesium atomic clock, represents at the same time a unified standard of frequency, time and length.

In parallel with the redefinition of the metre, the 17th CGPM invited the International Committee for Weights and Measures (Comité international des poids et mesures, CIPM) to draw up instructions for the practical realization of the new definition of the metre, and to choose radiations which can be recommended as wavelength standards for the interferometric measurement of length and to itemise operating procedures for their use, and finally to pursue studies to improve these standards. These recommendations for the practical realization of the definition were generally referred to as the mise en pratique of the definition. In turn, the CIPM recommended that the metre be realized by one of the following methods:

(a) by means of the length \(l\) of the path travelled in a vacuum by a plane electromagnetic wave in a time \(t\): this length is obtained from the measured time \(t\), using the relation \(l = c_0 \cdot t\) and the value of the speed of light in vacuum \(c_0 = 299\,792\,458\,\text{m}\,\text{s}^{-1}\);

(b) by means of the wavelength in vacuum \(\lambda\) of a plane electromagnetic wave of frequency \(f\): this wavelength is obtained from the measured frequency \(f\), using the relation \(\lambda = \frac{c_0}{f}\) and the value of the speed of light in a vacuum \(c_0 = 299\,792\,458\,\text{m}\,\text{s}^{-1}\);

(c) by means of one of the radiations from the list of recommended radiations [2], whose stated wavelength in a vacuum, or whose stated frequency, can be used with the...
The invention of femtosecond frequency combs [14] had, further, coherent frequency measurement chains [13] and later the frequency stabilisation techniques, the development of phase to directly measure the frequency of any desired laser. Hence, that has access to such a device and a primary caesium clock capability of femtosecond frequency combs allows each laboratory more, two important consequences: firstly, the broad availability of such a device and a primary caesium clock.

The CIPM also recommended that, in all cases, any necessary corrections should be applied in order to take account of actual conditions such as diffraction, gravitation, or imperfection in the vacuum.

These three methods are essentially only two: a time of flight method and an interferometric method. The latter method uses a radiation of known vacuum wavelength that can be related to the SI frequency of the plane wave used in interferometry, either by a direct measurement or by reference to one of the recommended vacuum wavelengths of validated light sources. The mise en pratique for the definition of the metre was updated on several occasions by the Consultative Committee for Length (CCL) and its Mise en Pratique Working Group (MePWG) [3–6] thereby progressively improving the realization of the definition of the metre (figure 1). For practical length measurements, soon the uncertainty due to the realization of the length unit by optical wavelength/frequency standards became negligible: the practical measurement of the length of a gauge block in an interferometer is limited to about $10^{-8}$ [7–9], mostly determined by the properties of the artefact itself and the refractive index of air. Even for interferometric displacement measurements the diffraction correction will place a technical limit. As an example, consider the diffraction correction for a Gaussian beam of waist $w_0 = 0.1$ m and a wavelength of 500 nm which would amount to $6 \times 10^{-13}$ [10].

The use of laser cooling of absorbers [11, 12], improved frequency stabilisation techniques, the development of phase coherent frequency measurement chains [13] and later the invention of femtosecond frequency combs [14] had, furthermore, two important consequences: firstly, the broad availability of femtosecond frequency combs allows each laboratory that has access to such a device and a primary caesium clock to directly measure the frequency of any desired laser. Hence, the laser standards in the mise en pratique (method (c)) lose, to some extent, their importance by virtue of the direct realization by method (b). Secondly, the most advanced frequency standards in the mise en pratique had acquired low uncertainties that were orders of magnitude better than the uncertainties that could be made use of in length metrology. As a result, they became more interesting for other fields apart from length metrology, e.g. in basic research [15], ultra-high precision spectroscopy [16] or for optical atomic clocks. Consequently, in 2001 the mise en pratique was renamed ‘Practical realization of the definition of the metre, including recommended radiations of other optical frequency standards (2001)’ [5].

In general, it was expected that such optical frequency standards and other microwave frequency standards would demonstrate reproducibility and stability approaching that of primary caesium. It was considered that these systems could be used to realize the second; provided their accuracy was close to that of caesium, but accepting that their uncertainty could obviously be no better than the caesium uncertainty while the latter remained the primary frequency standard. Today, the most advanced optical frequency standards have evolved to optical clocks that outperform the best microwave clocks with respect to their uncertainty (figure 2) and instability.

Additionally, and most importantly, the femtosecond optical frequency comb offered solutions to the longstanding problem of a convenient and accurate clockwork that linked the optical and microwave regions and allowed for frequency comparisons between optical frequency standards with very different frequencies.

Consequently, in 2001 the Consultative Committee for Time and Frequency (CCTF) took note of the continuation of the caesium 133 definition of the second, but recognised that there were new atoms and ions being studied as potential optical frequency standards, facilitated by new optical-frequency measurement concepts that could allow the use of
optical transitions as practical frequency standards offering direct microwave outputs from such standards. One of these standards could provide the basis for a future definition of the second, and the CCTF focused on the desirability of reviewing accurate frequency measurements of such atom and ion transition frequencies made relative to the caesium frequency standard. As a result, the ‘Recommendation CCTF 1 (2001)’ [17] promoted the establishment of a list of ‘secondary representations of the second’ (SRS) where the documentation of uncertainty that applied to these SRS would be the same as those for primary caesium standards used to contribute to international atomic time (TAI).

Furthermore, the establishment of the set of SRS had significant implications for the list ‘Practical realization of the definition of the metre, including recommended radiations of other optical frequency standards (2001)’, formerly the *mise en pratique*. In order to avoid ambiguity in respect of radiations appearing on both lists with potentially differing levels of stated uncertainty, it was considered essential that the values for *mise en pratique* and SRS radiations be combined in a single list, where the CCTF would ratify new and existing radiations to be accepted as SRS, and the CCL would recommend new and existing radiations for realization of the definition of the metre. Subsequently, following the wishes of the CIPM, a Joint Working Group (JWG) of the CCL/CCTF was set up in September 2003 with experts from the CCL and CCTF, taking note of convergence of interests in work, to consider the criteria for adoption of a radiation as an SRS. The JWG—later renamed as the CCL–CCTF Frequency Standards Working Group (WGFS)—recommended in 2003 that the requirements should include a peer-reviewed uncertainty budget for the frequency of the radiation, and that the total uncertainty of the value should be no more than one order of magnitude larger than the best realizations of the primary frequency standards of that date [18]. In 2004 the CCTF (in ‘Recommendation CCTF 1 (2004)’ [19]) recommended using the rubidium-87 unperturbed ground-state hyperfine quantum transition frequency (6.8 GHz) as an SRS.

As a result of these deliberations, the CIPM concluded in its ‘Recommendation CI 1 (2006)’ [20] that a common list of ‘Recommended values of standard frequencies for applications including the practical realization of the metre and secondary representations of the second’ should be established. The CIPM took into account discussions of the CCL–CCTF JWG on the ‘Mise en Pratique’ of the definition of the metre and the secondary representations of the second’ in meetings at the International Bureau of Weights and Measures (Bureau international des poids et mesures, BIPM) in the years 2005 and 2006 on possible candidates to be included in this list as SRS. It furthermore recommended that the four optical transition frequencies at 1065 THz (\(^{199}\)Hg\(^+\)), 688 THz (\(^{171}\)Yb\(^+\)), 444 THz (\(^{88}\)Sr\(^+\)), and 429 THz (\(^{87}\)Sr) could be used as SRS and be included in the new list.

In 2007, the CCL recommended [21] to the CIPM an updated list of frequency values for the \(^{13}C_2H_2\) \((\nu_1 + \nu_3)\) band at 1.54 \(\mu\)m, the addition of frequency values for the \(^{15}C_2H_2D\) (2\(\nu_1\)) band at 1.54 \(\mu\)m, and the addition of frequency values for the hyperfine components of the P(142) 37-0, R(121) 35-0 and R(85) 33-0 iodine transitions at 532 nm, which were adopted by the CIPM as ‘Recommendation 1 (CI-2007)’ [22]. At the same meetings it was decided that an entry for unstimulated He–Ne lasers, operating on the 633 nm (3\(\delta_2\) – 2\(\delta_2\)) neon transition, be included in the list of standard frequencies (‘Recommendation 2 (CI-2007)’) [23] and that an accompanying paper with CCL authority be published [24].

In 2009, the CCTF and the CCL proposed updates to certain frequency values in the CIPM LoF. Three further radiations were included in the list for the first time. These were the \(^{88}\)Sr transition at 429 THz, the \(^{40}\)Ca\(^+\) quadrupole transition at 411 THz and the 518 THz clock transition in \(^{171}\)Yb. These updates were recommended by the CIPM the same year [25].

Similarly, updates were recommended by the CIPM in 2013 [26] and 2015 [27] (see also [28]) following the recommendations of the CCL [29] and the CCTF [30, 31], and by the CCTF in 2017 [32].

At the 2015 update a paradigm shift became necessary as a result of two developments. Firstly, a number of optical frequency standards demonstrated smaller fractional projected uncertainties than the best caesium atomic clocks, and secondly, with the optical frequency comb technique, ratios of two optical frequencies could be measured with uncertainties that supported the uncertainties of the best optical clocks. It has been shown that the relative frequency uncertainty of the optical and microwave outputs of a femtosecond laser frequency comb can be as low as \(8 \times 10^{-20}\) and \(1.7 \times 10^{-17}\), respectively [33, 34]. As a result of these developments, an increasing number of direct optical frequency ratios had been measured with uncertainties that were much lower than those of direct frequency measurements against the caesium atomic clock as the primary realization of the definition of the second. These measurements included \(^{27}\)Al\(^+/^{199}\)Hg\(^+\) [35], \(^{40}\)Ca\(^+/^{87}\)Sr [36], \(^{171}\)Yb\(^+\)(E3)\(^{171}\)Yb\(^+\)(E2) [37], \(^{199}\)Hg\(^+/^{87}\)Sr [38], and \(^{171}\)Yb\(^+/^{87}\)Sr [39]. The Al\(^+/\)Hg\(^+\) frequency ratio had already been used before to determine a new recommended value for the frequency of an optical frequency standard and a SRS [26]. With the combination of direct measurements against the caesium clocks and the optical frequency ratios, the whole body of frequency data represented at that time an overdetermined set of data. Margolis and Gill [40] proposed a method to determine the best values from such a set and their method was applied for the first time for the CIPM LoF in 2015. Robertson developed an alternative method based on a graph theory framework for closed loops [41]. The different approaches have been tested on the relevant levels to give the same results [42]. The application of the new procedure will be discussed in more detail in the next section.

In the meantime, more frequency ratios have been determined. In 2017 the CIPM decided to leave the responsibility for the recommendations to the CCTF and CCL depending on whether the particular entry is for SRS and other time and frequency applications, or for practical realizations of the metre, respectively [43]. To this end the WGFS sends proposals to the respective consultative committee (CC) which will then inform the other CC on its decision. In 2017 a new evaluation by the CCTF took place [32].
The CIPM LoF now itemised within this publication is fully up to date with the 2017 values as ratified by the CCTF and will be fully accessible from the BIPM [44] which will be the only relevant repository for all future recommended values. This repository also contains the source data file with all the entries that led to the recommendation and the information about the applied procedure.

2. List of recommended frequency standard values (CIPM LoF)

2.1. Properties of the CIPM LoF

The CIPM LoF at present already contains a large number of frequencies for different applications (figure 3). As discussed, some of those with the lowest uncertainties are used as SRS. A small group of four entries were recommended by the CCL as wavelength standards to realize the metre in interferometric length measurements (see table 1). Others find applications in current technology, e.g. in optical telecommunications [45]. Accurate frequency values are needed in basic science or the determination of fundamental constants [46]. The CIPM LoF is ordered according to frequency. The values of the frequency $f$ and of the vacuum wavelength $\lambda$ should be related exactly by the relation $\lambda \cdot f = c_0$ with $c_0 = 299\,792\,458\,\text{m}\,\text{s}^{-1}$, but the values of $\lambda$ are rounded.

Following a decision by the CIPM, the CIPM LoF is conceptually divided into two parts. The first part (‘active list’) includes radiations of high accuracy that are of use in the realization of optical frequencies and vacuum wavelengths. The second part of the list (‘frozen list’) includes radiations that are still deemed useful for various applications but may have larger uncertainties and which will in general have no future updates of their value. The webpage of the BIPM currently does not discriminate between standard frequency values belonging to the first or the second part of the CIPM LoF.

Each of the listed radiations can be replaced, without degrading the accuracy, by a radiation corresponding to another component of the same transition or by another radiation, when the frequency difference is known with sufficient accuracy. In some cases, e.g. iodine stabilized lasers or acetylene stabilized lasers, such frequency intervals between transitions and hyperfine components have been validated and recommended by the CIPM also. They are also given in the source data files [44].

One issue arising in respect of the future evolution of the CIPM LoF is the identification of criteria for inclusion of frequency values within the list. Given the powerful capability of femtosecond combs to compare optical frequencies at the $10^{-20}$ level, there is potentially a wide range of atomic reference transitions that could be included. However, it is not
considered desirable to proliferate the number of different entries within the list, and one general consideration is to examine the nature, usefulness and application of a prospective addition with respect to its metrological application. Thus, criteria might include the achieved level of uncertainty relative to the intended application. Relevant metrological applications include those in time and frequency, length and dimensional metrology, optical communication standards and applications in science and fundamental constants.

Furthermore, when some radiations already included in the list are considered unlikely to find any metrological application going forward, the precedent has already been established for the radiation to be moved from the ‘active’ list, to the ‘frozen’ list. It is anticipated that no further update in the frequency values within this ‘frozen’ list will be warranted, either on account of their relatively high uncertainty or their lack of application. However, it remains perfectly acceptable to make use of these values for specific applications where no user alternative is readily available, such as the use of spectral lamps for gauge block calibration within industry, or where the accuracy required is sufficiently low, such as those applications where the use of an unstabilised 633 nm He–Ne laser is appropriate.

Additionally, it remains open to the WGFS to recommend, after careful deliberation, deletion from the list in certain cases where no purpose continues to be served by that radiation.

2.2. Frequency standards commonly used for the realization of the definition of the metre by interferometry

The former list of recommended radiations originally contained five radiations of lasers stabilized to molecular absorption lines together with radiations of spectral lamps [2]. Subsequently, the number of radiations in this list increased and many of them were never used for practical length measurements (even if they could have been). When the CIPM LoF was established in 2005, the ‘Recommendation CCL2 (2005)’ [47] proposed ‘that the CCL may wish to select those frequencies which it considers important to highlight for use in high accuracy length metrology’. At the WGFS meeting on the 10–11 September 2007 the wavelengths at 633 nm, 543 nm and 532 nm were at this stage chosen as commonly used wavelengths (see table 1) but the meeting agreed to seek advice from the Working Group on Dimensional Metrology (WGDM) on this selection5.

In 2007, following a proposition from the CCL (‘CCL13 (2007)’ [49]) the CIPM recommended the unstabilised He–Ne laser at 633 nm for use in dimensional metrology. A more detailed guide relating to the use of 633 nm unstabilised lasers has been published subsequently [24].

In 2015 the CIPM—at the request of the CCL [50]—adopted the updates to the CIPM LoF [51], to include the 87Rb d/f crossover saturated absorption D2 line at 780 nm [52, 53] and the 531.5 nm saturated absorption a1 transition in molecular 127I2.

A recent detailed review about the transfer of the SI unit metre from the definition to practical length metrology can be found in [54].

2.3. Frequency standards recommended as SRS

As can be seen from figure 2 the estimated uncertainties obtained in realizing the unperturbed line centre of a transition are much lower for various atoms than the uncertainty that can be realized by the best atomic clocks based on the caesium hyperfine ground state. The lowest estimated uncertainties in the 10−18 range have been reported for the 87Rb optical lattice clock [55, 56], the 171Yb+ single-ion clock [57] or the 27Al+ quantum logic clock [58].

One has to discriminate carefully between these estimated uncertainties to realize the true line centre of the unperturbed transition and known frequencies in the SI. In the CIPM LoF following the recommendation of the CCTF in 2017, there are now one microwave transition (hyperfine transition in 87Rb) and eight optical frequency standards that are recommended as SRS (table 2) with estimated uncertainties as low as 4 × 10−18. This uncertainty is only a factor of about two larger than the uncertainties of the best primary caesium atomic clocks. In recent years, the 87Rb fountain clock at LNE-SYRTE has regularly contributed to TAI as can be seen from the time bulletin ‘Circular T’ [59] and from [60]. It has been shown that TAI could benefit well from optical clocks [61–63]. First attempts have also been made to include 87Sr optical lattice clocks.

### Table 2. SRS as of 2017.

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Fractional uncertainty</th>
<th>Transition</th>
</tr>
</thead>
<tbody>
<tr>
<td>6834682610.9043126</td>
<td>6 × 10−16</td>
<td>87Rb ground state hfs</td>
</tr>
<tr>
<td>429228004229873.0</td>
<td>4 × 10−16</td>
<td>87Sr neutral atom, 5s2 5S1/2−5p1/2 3P0</td>
</tr>
<tr>
<td>444779044095486.5</td>
<td>1.5 × 10−15</td>
<td>88Sr+ ion, 5s2 5S1/2−4d 3D2</td>
</tr>
<tr>
<td>518295836590863.6</td>
<td>5 × 10−16</td>
<td>171Yb neutral atom, 6s2 1S0−6s6p 3P0</td>
</tr>
<tr>
<td>6421214967726450.0</td>
<td>6 × 10−16</td>
<td>171Yb+ ion, 2S1/2−2P1/2</td>
</tr>
<tr>
<td>688358979309308.3</td>
<td>6 × 10−16</td>
<td>171Yb+ ion, 6s2 5S1/2−5d 3D2</td>
</tr>
<tr>
<td>1064721609899145.3</td>
<td>1.9 × 10−15</td>
<td>109Hg+ ion, 5d46s 5S1/2−5d6s2 3D3/2</td>
</tr>
<tr>
<td>1121015393207857.3</td>
<td>1.9 × 10−15</td>
<td>27Al+ ion, 3s2 3S1/2−3p5 3P2</td>
</tr>
<tr>
<td>1128575290808154.4</td>
<td>5 × 10−16</td>
<td>199Hg neutral atom, 6s2 1S0−6s6p 3P0</td>
</tr>
</tbody>
</table>

3. Guidelines for inclusion in the CIPM LoF and statement of associated uncertainty

Given the substantial rate of progress in frequency metrology and the rapid output of new measured frequency values made possible by femtosecond frequency combs, the WGFS has developed criteria and procedures for the inclusion of a new or updated frequency value in the CIPM LoF. These are, to a large extent, based on analysis from the previous CCL–CCTF JWG and CCL McPWG, but also incorporate criteria already adopted.

---

5 Document CCL07-03 in [21]. Minutes of the CCL-CCTF Frequency Standards Working Group Meeting (10–11 September 2007)
for the inclusion of primary frequency standards in TAI [64] in
the case of those radiations under consideration as SRS.

For each new evaluation—typically at intervals dictated by
the official meetings of the CCL and of the CCTF—the WGFS
summarizes the development and measurements all over the
world to be used either for considering updates of already rec-
commended frequencies, or possibly to be introduced as new
recommended frequencies. For any such value to be included,
the WGFS considers only the data that have been published in
peer-reviewed, international, scientific journals. It then makes
a thorough assessment of the value, and estimates an uncer-
tainty, in which the uncertainty published in that journal is
an important, but not the only, contribution. The WGFS applies a
Bayesian approach to make use of all available information to
estimate the uncertainty of each recommendation. Such addi-
tional information can result from a variety of sources. A few
examples can illustrate this. Sometimes, authors apply correc-
tions to their measurements, e.g. based on the measurement of
others or on theoretical data without uncertainties. The work-
ing group considers these data and sometimes feels the
need to increase these partial uncertainties which will affect the
total uncertainty. Sometimes, authors reference their values to
particular environmental conditions, e.g. at room temperature.
In this case corrections have to be applied to relate the mea-
sured frequency to an environment free of perturbations, which
can subsequently increase the former stated uncertainty. The
use of the same theoretical or experimental sensitivity coeffi-
cient for an applied correction to the measurements of dif-
ferent origin leads to a correlation effect which tends to reduce
the uncertainty if not correctly taken into account. Only a
few institutes have at their disposal primary caesium atomic
clocks that can realize the second with an uncertainty in the
few institutes have at their disposal primary caesium atomic

The criteria can be summarized as:

(i) The primary requirement for inclusion in the CIPM LoF
is the existence of a peer-reviewed publication (or at least
an official acceptance for publication by the journal) at
the time of consideration. This pre-supposes the suit-
ability of the radiation for frequency, length and other
precision metrology as determined by the WGFS.

(ii) When only one frequency value is available from a single
laboratory, the estimated standard uncertainty adopted
is typically a factor of three larger than the uncertainty
quoted in the published paper. Depending on the infor-
mation available to the WGFS at the time concerning
measurement data and conditions, the WGFS may con-
sider it appropriate to expand the uncertainty by a further
factor, or round the final result.

(iii) When two values are available (e.g. from a single labo-
atory at different times, or from two laboratories), the
frequency value adopted is the mean value weighted by
the respective published uncertainties. These uncertain-
ties are combined in quadrature, and then a factor of
two to three is applied to this combined value to give
the estimated standard uncertainty for the CIPM LoF. In
this way, more reliance is placed on the value with lower
uncertainty.

(iv) For frequencies with three or more data values submitted,
the value adopted is the weighted mean. For situations
where the values have individual uncertainties which are
of a similar magnitude (e.g. all within a factor of five), the
situation is such that statistical analysis can be applied,
but with some recognition that this may still not be a fully
robust procedure.

These rules have been applied over the last two decades for
individual frequency values derived from a direct comparison
with the caesium atomic clock. With the availability of high
accuracy direct frequency ratios between (mostly optical) fre-
quency standards, the above stated rules were amended:

(v) For frequencies linked to other frequencies in the CIPM
LoF by direct or indirect frequency comparisons with
sufficiently low uncertainty, the recommended frequency
value results from a least squares analysis of the relevant
data. The uncertainties include the estimated correlations
between the different measurements. Rules (ii)–(iv) are
applied accordingly to single measurements of frequency
ratios.

This procedure helps to cope with different aims, such as
the consistency of the frequency scale and the derivation
of realistic uncertainties that can be used in the commonly
accepted framework of the guide to the expression of uncer-
tainty in measurement (GUM) [65]. The global uncertainty
derived for the listed radiations needs to be estimated to
ensure consistency with future values and potentially tighter
uncertainties, and to avoid discrete steps in frequency value of a magnitude larger than the combined uncertainties of previous and future values. This is also important to ensure that discontinuities in the SI second are avoided if the new data is, for example, incorporated into a new definition of the second. Furthermore, the uncertainty of the recommended frequency will often be used as one input data point for an uncertainty budget with several other input data. Following the GUM [65] all independent contributions will be added in quadrature, tacitly assuming that the probability density of the particular contribution is Gaussian. This is only justified if the central limit theorem applies to a good approximation, which is definitively not the case if there are only one or two entries. In such a case the connection between the standard uncertainty with expansion factor $k = 1$, $k = 2$ and $k = 3$ and the confidence interval 68.27%, 95.45% and 99.73%, respectively, for an infinite number of measurements (degrees of freedom) completely breaks down. For one degree of freedom, the more appropriate Student’s $t$-distribution shows that the interval that encompasses a fraction of 68.27%, 95.45% and 99.73% of the distribution would have to be enlarged by a factor of 1.84, ~7 and ~7.8, respectively, as compared to the Gaussian distribution. By using the term ‘estimated standard uncertainty’, in general one thinks of 68% coverage. Here, the Student corrections for one or two measurements already make up a considerable fraction of the factors 2–3 applied by the WGFS.

Looking at the actual coverage in hindsight, if about every third value is actually outside these limits, this could give a hint about the validity of the adopted interpretation of the initial recommended value. Even though the data base of the CIPM LoF is still very small for such an investigation, in several cases our current best frequency estimate based on more measurements is very close to such a confidence limit of the first recommendation. Examples include $^{115}$In$^+$ and $^{88}$Sr. This observation lends support to the interpretation that the uncertainty of the recommended frequency values at these early stages can also be regarded as the typical estimated standard uncertainties.

It is interesting to compare this procedure with that of the Committee on Data of the International Council for Science (CODATA) [66]. This group calculates the weighted mean and uncertainty for measurements from several laboratories and normally takes a simple weighted mean and weighted uncertainty and then checks the chi-squared. If the data set is not consistent with the calculated distribution, the practice is to multiply the variances of all the measurements by the same multiplier (in some cases this has been as large as 15) and again take a simple weighted mean and weighted uncertainty. Again, the chi-squared is calculated to ensure that the calculated mean and uncertainty are consistent with the measurements. In contrast to the procedure of the WGFS, the CODATA group does not multiply the variances of only one or some of the measured values. The variances of all the measurements are all multiplied by the same value. If one measurement dominates over the others and is consistent with the others, then no multiplication is performed before taking the weighted mean” [67].

\[^{6}\text{See, e.g., appendix G of [65].}\]
laboratories to devise a recommendation, but did not consider the earlier values [71, 72] that were not consistent with later ones. After approval by the CCTF and CCL, the CIPM in 2006 recommended the frequency 429 228,004 229 877 Hz with an estimated fractional uncertainty of $1.5 \times 10^{-14}$, equivalent to 6.4 Hz. This frequency value and the assigned uncertainty are shown in figure 4 in the left section by the purple horizontal bar and pink area, respectively. At the meeting of the WGsS in 2009, four new frequency measurements were available [76–79]. Two of them came from the same laboratory (JILA [76, 78]) with the second one having a threefold reduced uncertainty. Hence, only the latter one was included, together with the two values from France and Japan [77, 79], to derive the weighted mean of 429 228 004 229 873.7 Hz with a fractional uncertainty of $1 \times 10^{-15}$ which was subsequently recommended by the CIPM [25]. This low uncertainty allowed the CIPM to recommend the Sr lattice clock transition as an SRS. Two new measurements [80, 81] were performed for the next evaluation in 2012 and a weighted mean of these five values sees the frequency value reduced by 0.3 Hz. The fractional uncertainty was kept at $1 \times 10^{-15}$ since the later measurements seemed to have a slightly lower value compared to the earlier ones. The new value was recommended by the CIPM in 2013 [26].

For the 2015 evaluation there were seven new measurements available [61, 82–87]. Together with the previous measurements, the new measurements (except for one) were used to derive a new recommendation based on a weighted mean. The measurement of Hachisu et al [86] was omitted because it was essentially based on the measurement of Falke et al [84]. In this evaluation the first optical frequency ratio measurements were also introduced, in the way described in more detail below. Frequency ratios connected the $^{87}$Sr value with the values of the $^{171}$Yb and the $^{199}$Hg transitions in lattice clocks and in the $^{40}$Ca$^+$ single-ion clock. Due to the large number of low uncertainty $^{87}$Sr data the inclusion of these frequency ratios did not have much influence on the $^{87}$Sr value itself, but were extremely helpful in tying down the uncertainties of other frequencies linked with the $^{87}$Sr values by the measured ratios.

The latest evaluation results from 2017 included five more direct frequency measurements with respect to the caesium clocks, and frequency ratio measurements with respect to other optical and microwave standards. The last two measurements (number 18 and 19) did not use a local primary frequency standard but were related to TAI. All the new measurements with low uncertainty were slightly below the recommendation of 2015. The outcome of the latest adjustment—to be discussed in more detail below—used all 19 values displayed in figure 4. As a result, the recommended frequency was reduced by 0.2 Hz and the fractional uncertainty was reduced to $4 \times 10^{-16}$. This uncertainty is not much higher than the relative uncertainty in realizing the SI Hz with the best primary caesium fountains. The estimated uncertainty was based on the comparison, via a fibre link, between primary standards [91] which included the uncertainties of the primary standards as well as the contribution of the fibre link.

4. Inclusion of optical frequency ratios

As pointed out above, the inclusion of optical frequency ratios and optical-to-microwave ratios has changed the evaluation procedure substantially. Besides a number of direct frequency measurements compared directly against the caesium atomic clock, there are a number of optical frequency ratios between optical atomic clocks that have been determined (figure 4) with much smaller uncertainties than would be possible if caesium clocks or other microwave clocks were involved. They include $^{27}$Al/$^{199}$Hg$^+$ [58], $^{40}$Ca$^+$/87Sr [36], $^{171}$Yb$^+$(E3)/$^{171}$Yb$^+$(E2) [37], $^{199}$Hg$^+$/87Sr [38], $^{171}$Yb$^+$/87Sr [92] or $^{199}$Hg$^+$/$^{87}$Rb [93]. Such frequency ratios have already been used to determine new recommended values for the frequencies of optical frequency standards and SRS [27]. Together with the direct absolute frequency measurements with respect to the caesium clocks, these frequency ratio measurements form an overdetermined set of data. It can be foreseen that optical frequency ratio measurements will involve an increasing number of the frequency standards (figure 5) and may also include new ones.

Margolis and Gill proposed and applied a least squares method to determine the ‘best’ estimates of the frequency values [40] from such a set of overdetermined measurements. All validated frequency measurements and frequency ratio measurements are prepared as frequency ratios with the direct frequency measurements against the caesium primary standard also expressed in frequency ratios. The fact that the input data set consists of frequency ratios makes this a non-linear least squares problem requiring linearization and iterations to find an acceptable solution. The adjusted frequency values can be used to determine other frequencies if the frequency ratio is to be measured later. Independent programmes are available and have been used to validate the codes. One of those devised by Robertsson [41] uses a slightly different conceptual approach which is based on the examination of closed loops in a graph theory framework [94]. Such closed loops can be easily recognized in figure 5, e.g. by the three-node single loop comprising $^{171}$Yb$^+$ (E2), $^{171}$Yb$^+$ (E3), $^{133}$Cs or the four-node loop comprising $^{17}$Yb$^+$, $^{87}$Sr$^+$, $^{199}$Hg$^+$, $^{133}$Cs. To circumvent the non-linearity of the ratios the logarithms of the frequencies are used, leading to a linear least squares problem. Similar to a three-cornered hat analysis, the logarithms of all frequency ratios should add up to zero. This provides a set of conditions which in a Lagrange multiplier scheme helps to identify the basis vectors for the residual space in the least squares calculation. A projection on this subspace gives the corrections in the experimental ratio values.

These methods for using all available experimental frequency data with their proper weights lead to a system of adjusted values that are more robust against outliers as compared to isolated frequency ratios. With the two methods discussed above, such outliers can even be identified, as has been demonstrated in [40]. If such outliers are not identified, the whole system of recommended frequencies can be affected by an erroneous frequency ratio measurement with underestimated uncertainty. In the same way, correlations between single measurements, if not properly identified, can have
microwave standards (133Cs and 87Rb) and optical standards (full
standard and the Rb microwave standard (dashed
measurements.
other standards used for the particular frequency measurements.
there is no single Cs clock and no single other standard but in
in the 2017 evaluation. The picture is simplified in the sense that
lines) and direct optical frequency ratios (dashed lines) as used
171Yb
similar effects. Consider the case of two frequency measure-
ments of, for example, 171Yb(E2), 171Yb(E3) in figure 5
performed at the same time against the same Cs clock. Any
increase in the Cs frequency will immediately lead to a cor-
relation coefficients whose square is used in the two methods
given above for the correlation matrix.
There might, however, be less-controllable sources of
correlations that are harder to quantify. It has been shown
recently that the SI-traceable measurement of an optical fre-
quency can be performed at the low 10^{-10} level without a local
primary standard, but referenced to TAI [90]. In this case any
frequency ratio measurement against a local caesium clock
that contributes to TAI during such a measurement will show
residual correlations with any other optical frequency standard
that is measured against TAI. For the time being, such a cor-
relation will be significantly reduced by the averaging pro-
cess used to generate TAI but will become more prominent if
two optical standards are directly measured against TAI at
the same time. It will become even more pronounced if optical
clock networks [95] for optical frequency ratios are employed
as a matter of course. This suggests that additional rules for
reporting both frequency measurements and frequency ratio
measurements are needed where, for example, the links and
the full data of the measurement period (including the start
and end times, the time of day (UTC) and the calendar day,
together with the relevant interruptions) are stated. The WGFS
is developing reporting guidelines that aim to take correlation
effects into account more fully.
Recent deliberations by the CCL–CCTF WGFS have also
considered the potential for inclusion of high accuracy fre-
cuency ratios within an additional appendix to the CIPM LoF.
Whilst any optical–Cs microwave frequency ratio necessarily
includes the uncertainty associated with the primary Cs fre-
cuency value, direct optical–optical frequency ratios of, for
example, SRS are capable of much lower uncertainty due to
their better reproducibility than the Cs standard and the capa-
bility of comb measurements at uncertainty levels even below
the optical reproducibilities. Such a procedure, however, has
not yet been decided.

5. Towards a new definition of the SI second

It was obvious for a long time that a much lower inherent
uncertainty, and a much higher relative stability, of the clock
frequency could be realized with clocks operating on an
optical transition rather than a microwave transition. The pro-
cess initiated in 2001 led to the establishment of SRS in order
to investigate their suitability for a future redefinition of the SI
second and to utilize them in the realization of TAI with the
prospect of improved time scales. Fifteen years later, nine SRS
are available (87Rb as a microwave standard and eight optical
SRS). The 87Rb standard and the Sr lattice clock at SYRTE are
beginning to contribute regularly to TAI and it has been shown
that a time scale can be established based on an optical clock
that is superior to one based on even the best caesium fountain
clocks [61–63]. By introducing more of the SRS and possibly
replacing less accurate clocks like hydrogen masers or cae-
sium beam clocks at the same time, the SRS could gradually
begin to improve the TAI and UTC time scales.

In the meantime, optical atomic clocks, optical long
haul links and the various methods of all-optical frequency
metrology are finding widespread applications and have even
led to the creation of novel fields like relativistic geodesy
[96, 97]. It is well known that according to General Relativity
two optical clocks in a different gravitational potential show
different frequencies [98] when compared, and this effect
has been taken into account for a long time when comparing
microwave atomic clocks in the international time scales TAI
and UTC. But with the achieved accuracy and stability of
optical atomic clocks it becomes possible to use this effect
to determine the difference in the gravitational potential of
two locations on Earth. In time and frequency metrology,
apart from the creation of better time scales, the distribution
of highly accurate and extremely stable optical frequencies
via fibres to many customers [99] may lead to new services
or allow synchronization of clocks over large distances. For
tests of fundamental theories or the question of the constancy
of fundamental constants [37, 100], optical atomic clocks are
the measuring devices of choice to grasp the first hints of new
physics. In space technology and astronomy, the ultra-precise
tracking of spacecraft and the improved reference systems for
very long baseline interferometry, respectively, will benefit
from the optical clocks. Thus, there is a growing community
that will benefit from a redefinition of the second in terms of

---

**Figure 5.** Measured frequency ratios between the 133Cs primary
standard and the Rb microwave standard (dashed–dotted line),
microwave standards (133Cs and 87Rb) and optical standards (full
lines) and direct optical frequency ratios (dashed lines) as used
in the 2017 evaluation. The picture is simplified in the sense that
there is no single Cs clock and no single other standard but in
reality there are many different realizations of the Cs second or
other standards used for the particular frequency measurements.
The numbers at the lines indicate the numbers of independent
measurements.
an optical clock transition, and so the questions of when the
time is right to redefine the unit of time, what the necessary
requirements are, and possible time scales for such a process
are becoming increasingly relevant and urgent [101–103].
The Working Group of Strategic Planning (WGSP) of the
CCTF has thus devised a roadmap to accompany this process,
which is outlined in the following section.

5.1. Milestones on a roadmap towards a redefinition
of the second

From figure 2, one expects that the uncertainties of optical
frequency standards will continue to reduce over the coming
years. The recent developments in novel excitation schemes
[104–106], 3D confinement of quantum absorbers [107], and
new strategies for the reduction of systematic shifts [108] lend
promise to this expectation that fractional uncertainties below
$10^{-18}$ could be reached. However, limited knowledge about
the exact gravitational potential suggests difficulties in the use
of practical time scales at this level on Earth. With fractional
inaccuracies in the $10^{-18}$ regime, the geopotential has to be
determined to the cm-level to account for gravitational red-
shift. From this point of view, the time would be right for a
new definition when the optical clocks have furnished proof
that their typical performances reach fractional uncertainties
of around $10^{-18}$, which would be roughly two orders of mag-
nitude lower than that expected of the best caesium fountains
of the time.

There are several ways to properly verify when such a
hundredfold improvement in the potential accuracy of optical
clocks over caesium primary clocks has been achieved. Optical
clocks of the same type, e.g. the Sr lattice clocks at SYRTE,
NPL and PTB can be compared with such an uncertainty in the
different laboratories linked via already-established fibre
links [95, 103, 109]. Frequency comparisons between remote
clocks can also be performed using transportable optical
atomic clocks [110, 111]. A third option for such a compar-
ison can be based on different measured frequency ratios and
the associated evaluations between remote optical clocks in
the way suggested below.

From these considerations, one could define the first two
milestones to be reached before a new definition can take
place. The time for a new definition is right when

1. at least three different optical clocks (either in different
laboratories, or of different species) have demonstrated
validated uncertainties of about two orders of magnitude
better than the best Cs atomic clocks of the time.
2. at least three independent measurements of at least one
optical clock from milestone 1 have been compared in
different institutes (with, e.g., $\Delta \nu/\nu < 5 \times 10^{-18}$) either
by transportable clocks, advanced links, or frequency
ratio closures.

To assure continuity between the present definition and
the new definition, the frequency of the selected optical clock
has to be measured with respect to the best caesium fountain
atomic clocks with uncertainties essentially determined by the
fountain clocks. Thus, the time for a new definition is right when

3. three independent measurements of the optical frequency
standards of milestone 1 with three independent Cs
primary clocks have been performed, where the measure-
ments are limited essentially by the uncertainty of these
Cs fountain clocks (with, e.g., $\Delta \nu/\nu < 3 \times 10^{-16}$).

It is highly desirable that optical clocks that have been
assigned the status of SRS contribute regularly to TAI in order
to improve the time scale and to further develop the tech-
nology and protocols of improved methods for comparisons.
This requires another milestone. The time for a new definition
is right when

4. optical clocks (SRS) contribute regularly to TAI.

To allow for closures and links between the dozen or more
different optical standards and their continuous use, a fifth
milestone would thus be desirable. The time for a new defin-
nition is right when

5. optical frequency ratios between a few (at least 5) other
optical frequency standards have been performed; each
ratio measured at least twice by independent laborato-
ries and agreement was found to better than, e.g., $\Delta \nu/\nu$
$5 \times 10^{-18}$.

It remains within the authority of the CIPM as to when it
will make a proposition to the CGPM for a redefinition. From
the current status it can be estimated that the new definition
could come into effect before 2030. After a redefinition, the
present standard of time and frequency would serve as an SRS
where the uncertainty to realize the second would be the same
as before. Improvements in the caesium atomic clocks would
then be evaluated regularly within the established framework
of a caesium clock as an SRS.

It should be noted that a number of the national metrology
institutes will have the ability to link a chosen species for a
new optical definition of the second to other optical clock spe-
cies accepted as SRS, by means of femtosecond comb and
optical fibre transfer techniques, without an increase in the
combined uncertainties above the level of a few times $10^{-18}$.
In this case, one would be able to realize the new definition
by means of these SRS with very little increase in uncertainty.

6. Conclusion

Optical frequency standards, first used as vacuum wavelength
standards for the realization of the metre in length metrology,
have evolved into optical clocks that now find their most prom-
inent application in frequency metrology. The clear demand in
these and other fields, with novel and unforeseen applications,
has led to a single list of recommended frequency standard
values for applications including the practical realization of
the metre and SRS. These frequency values and their uncer-
tainties have been determined with coherent procedures as
described in this publication. Even though the rapid progress
in optical frequencies has been less beneficial for dimensional
length measurements under ambient conditions, i.e. where the index of refraction matters, length measurements in space will also benefit from the new technologies and platforms that use optical frequencies. The newly-established analyses and procedures for deriving a coherent list of recommended frequencies from an overdetermined set of measurements are leading to a transparent and robust system of high reliability and low uncertainty. It is, furthermore, a solid basis that can lead to a future new definition of the SI unit of time, the second.

Acknowledgments

Extremely helpful discussions with the members of the CCTF WGFS, the CCTF WGSP and particular members of the CCL are gratefully acknowledged.

References

[2] Editor’s Note 1984 Documents concerning the new definition of the metre Metrologia 19 163–77
[36] Rosenband T et al 2008 Frequency ratio of $^{139}$Al and $^{181}$Hg single-ion optical clocks metrology at the 17th decimal place Science 319 1808–12
against a Sr lattice clock to verify the absolute frequency
measurement. Opt. Express 20 22034

[37] Godun R M et al 2014 Frequency ratio of two optical clock
transitions in $^{171}$Yb$^+$ and constraints on the time-variation
of fundamental constants Phys. Rev. Lett. 113 210801

[38] Yamanaka K, Ohmae N, Ushijima I, Takamoto M and
Katori H 2015 Frequency ratio of $^{199}$Hg and $^{88}$Sr optical
lattice clocks beyond the SI limit Phys. Rev. Lett. 114 230801

Onae A and Hong F L 2014 Frequency ratio measurement of
$^{171}$Yb$^+$ and $^{87}$Sr optical lattice clocks Opt. Express 22 7898–905

[40] Margolis H S and Gill P 2015 Least-squares analysis of clock
frequency comparison data to deduce optimized frequency
and frequency ratio values Metrologia 52 628–34

[41] Robertson L 2016 On the evaluation of ultra-high-precision
frequency ratio measurements: examining closed loops in a
graph theory framework Metrologia 53 1272–80

[42] Oates C 2017 private communication: an independent
programme was developed in Mathematica (NIST)

[43] International Committee for Weights and Measures 2017
Proceedings of the 106th meeting (16–17 and 20 October

[44] BIPM 2018 Recommended Values of Standard Frequencies
standardfrequencies.html)

WDM applications: DWDM frequency grid (www.itu.int/
rec/T-REC-G.694.1-201202-I/en)

[46] de Beauvoir B, Nez F, Julien L, Cagnac B, Biraben F,
Absolute frequency measurement of the 2S–8S/D transitions
in hydrogen and deuterium: new determination of the
Rydberg constant Phys. Rev. Lett. 78 440–3

[47] Recommendation CCL 2005 Consultative Committee
for Length (CCL), Report of the 12th meeting to the
International Committee for Weights and Measures (15–16
ccl/publications-cc.html)

Frequency Standards Working Group (Sèvres, 10–11
September 2007)

[49] Consultative Committee for Length (CCL) 2007 Report of
org/en/committees/cc/ccl/publications-cc.html)

[50] Consultative Committee for Length (CCL) 2015 Report of
the 16th Meeting (23–24 September 2015) pp 17–9 (www.
bipm.org/en/committees/cc/ccl/publications-cc.html)

[51] Recommendation CIPM/104-37 2015 Updates to the list of
standard frequencies Comité International des Poids et
Mesures, 104th Meeting (October 2015) (www.bipm.org/
en/committees/cipm/publications-cipm.html)

structure and absolute frequency of the $^{87}$Rb $^{5P_{3/2}}$ state
Opt. Lett. 21 1280

Rubidium-stabilized diode laser for high-precision
interferometer Opt. Eng. 43 909

[54] Schödel R 2016 Interferometry—how do I coax a length out of
light? PTB Mitt. 126 35

clock at $2 \times 10^{-18}$ total uncertainty Nat. Commun. 6 6896

[56] Ushijima I, Takamoto M, Das M, Ohkubo T and Katori H
2015 Cryogenic optical lattice clocks Nat. Photon. 9 185–9

[57] Huntemann N, Sanner C, Lipphardt B, Tamm C and Peik E
2016 Single-ion atomic clock with $3 \times 10^{-18}$ systematic
uncertainty Phys. Rev. Lett. 116 063001

[58] Chou C W, Hume D B, Koellemijt J C J, Wineland D J and
Rosenband T 2010 Frequency comparison of two high-
accuracy Al$^+$ optical clocks Phys. Rev. Lett. 104 070802

[59] BIPM 2018 FTP Server of the BIPM Time Domain
(www.bipm.org/jsp/en/TimeFtp.jsp)

[60] Abgrall M et al 2015 Atomic fountains and optical
clocks at SYRTE: status and perspectives C. R. Phys. 16 461–70

[61] Le Targat R et al 2013 Experimental realization of an
optical second with strontium lattice clocks Nat.
Commun. 4 2109

Realization of a timescale with an accurate optical lattice
clock Optica 3 563–9

[63] Ido T, Hachisu H, Nakagawa F and Hanado Y 2016 Rapid
evaluation of time scale using an optical clock J. Phys.: Conf.
Ser. 723 012041

[64] Parker T E 2012 Invited review article: the uncertainty in
the realization and dissemination of the SI second from a
systems point of view Rev. Sci. Instrum. 83 021102

[65] Joint Committee for Guides in Metrology 2008 Evaluation
of Measurement Data—Guide to the Expression of
Uncertainty in Measurement (www.bipm.org/en/
publications/guides/gum.html)

recommended values of the fundamental physical constants:
2014 Rev. Mod. Phys. 88 035009

[67] Bernard J 2017 private communication NRC

[68] Chartier J-M et al 1991 International comparison of iodine-
stabilized helium–neon lasers at $\lambda = 633\,\text{nm}$ involving
seven laboratories Metrologia 28 19–25

$^{127}$I$_2$-stabilized He–Ne lasers at $\lambda = 633\,\text{nm}$ Metrologia
29 331–9

[70] Ma L S, Picard S, Zucco M, Chartier J-M, Robertson L and
Windeler R S 2003 Direct measurement of the absolute
frequency of the international reference laser BIPM4
Metrologia 41 65–68

optical lattice clock Nature 435 321–4

Katori H 2005 Frequency measurement of a Sr lattice clock
using an SI-second-related optical frequency comb
linked by a global positioning system (GPS) Opt. Express
13 5253–62

[73] Ludlow A D, Boyd M M, Zelevinsky T, Foreman S M, Blatt S,
Notcutt M, Ido T and Ye J 2006 Systematic study of the
$^{87}$Sr clock transition in an optical lattice Phys. Rev. Lett.
96 030803

[74] Le Targat R, Bailiard X, Fouche M, Brusch A, Tcherbakoff O,
Rovera G D and Lemonde P 2006 Accurate optical lattice
clock with $^{87}$Sr atoms Phys. Rev. Lett. 97 130801

[75] Takamoto M, Hong F-L, Higashi R, Fujii Y, Imae M and
Katori H 2006 Improved frequency measurement of a
one-dimensional optical lattice clock with a spin-polarized
fermionic $^{87}$Sr Isotope J. Phys. Soc. Japan 75 104302

[76] Boyd M M, Ludlow A D, Blatt S, Foreman S M, Ido T,
Zelevinsky T and Ye J 2007 $^{87}$Sr lattice clock with
inaccuracy below $10^{-15}$ Phys. Rev. Lett. 98 083002

[77] Bailard X et al 2008 An optical lattice clock with spin-
polarized $^{87}$Sr atoms Eur. Phys. J. D 48 11–7

[78] Campbell G K et al 2008 The absolute frequency of the
$^{87}$Sr optical clock transition Metrologia 45 539–48

[79] Hong F-L et al 2009 Measuring the frequency of a Sr optical
lattice clock using a 120 km coherent optical transfer
Opt. Lett. 34 692–4

[80] Falke S et al 2011 The $^{87}$Sr optical frequency standard at PTB
Metrologia 48 399–407
[81] Yamaguchi A, Shiga N, Nagano S, Li Y, Ishijima H, Hachisu H, Kumagai M and Ido T 2012 Stability transfer between two clock lasers operating at different wavelengths for absolute frequency measurement of clock transition in $^{87}$Sr Appl. Phys. Express 5 022701
[86] Hachisu H and Ido T 2015 Intermittent optical frequency measurements to reduce the dead time uncertainty of frequency link Japan. J. Appl. Phys. 54 112401
[87] Tanabe T et al 2015 Improved frequency measurement of the $^{1}\text{S}_0-^{3}\text{P}_1$ clock transition in $^{87}$Sr using a Cs fountain clock as a transfer oscillator J. Phys. Soc. Japan 84 115002
[88] Lodewyck J et al 2016 Optical to microwave clock frequency ratios with a nearly continuous strontium optical lattice clock Metrologia 53 1123
[89] Hachisu H, Petit G and Ido T 2017 Absolute frequency measurement with uncertainty below 1 $\times 10^{-15}$ using international atomic time Appl. Phys. B 123 34
[91] Guéna J et al 2017 First international comparison of fountain primary frequency standards via a long distance optical fiber link Metrologia 54 348–54
[92] Nemitz N, Ohkubo T, Takamoto M, Ushijima I, Das M, Ohmae N and Katori H 2016 Frequency ratio of Yb and Sr clocks with $5 \times 10^{-17}$ uncertainty at 150’s averaging time Nat. Photon. 10 258–61
[95] Riehle F 2017 Optical clock networks Nat. Photon. 11 25–31
[99] Grosche G 2014 Eavesdropping time and frequency: phase noise cancellation along a time-varying path, such as an optical fiber Opt. Lett. 39 2545–8
[100] Huntemann N, Lipphardt B, Tamm C, Gerginov V, Weyers S and Peik E 2014 Improved limit on a temporal variation of $m_\gamma/m_\mu$ from comparisons of Yb$^+$ and Cs atomic clocks Phys. Rev. Lett. 113 210802
[101] Gill P 2011 When should we change the definition of the second? Phil. Trans. R. Soc. A 369 4109–30
[107] Campbell S L et al 2017 A Fermi-degenerate three-dimensional optical lattice clock Science 358 90