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TOPICAL REVIEW

Photonic metamaterials: a new class of materials for manipulating light waves

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Abstract
A decade of research on metamaterials (MMs) has yielded great progress in artificial electromagnetic materials in a wide frequency range from microwave to optical frequencies. This review outlines the achievements in photonic MMs that can efficiently manipulate light waves from near-ultraviolet to near-infrared in subwavelength dimensions. One of the key concepts of MMs is effective refractive index, realizing values that have not been obtained in ordinary solid materials. In addition to the high and low refractive indices, negative refractive indices have been reported in some photonic MMs. In anisotropic photonic MMs of high-contrast refractive indices, the polarization and phase of plane light waves were efficiently transformed in a well-designed manner, enabling remarkable miniaturization of linear optical devices such as polarizers, wave plates and circular dichroic devices. Another feature of photonic MMs is the possibility of unusual light propagation, paving the way for a new subfield of transfer optics. MM lenses having super-resolution and cloaking effects were introduced by exploiting novel light-propagating modes. Here, we present a new approach to describing photonic MMs definitely by resolving the electromagnetic eigenmodes. Two representative photonic MMs are addressed: the so-called fishnet MM slabs, which are known to have effective negative refractive index, and a three-dimensional MM based on a multilayer of a metal and an insulator. In these photonic MMs, we elucidate the underlying eigenmodes that induce unusual light propagations. Based on the progress of photonic MMs, the future potential and direction are discussed.

Keywords: metamaterials, subwavelength optical devices

1. Introduction
Metamaterials (MMs) are an emerging group of artificial electromagnetic (EM) materials of subwavelength periodicity. MMs started to be developed around 2000 in the microwave frequency range. One of the features of MMs is that they manifest unusual EM responses that are rarely, if ever, found in existing solid materials. From an early stage, the primary objective was to attain a prominent magnetic response at higher frequencies than microwave.

A model was proposed in 1999 [1], which described a resonance in artificial roll structures of metal as a magnetic response, stimulating the fabrication of magnetic structures at microwaves. The artificial roll structure basically works as an electric inductance-capacitance circuit; the resonance is interpreted as a magnetic response and is described by the relative permeability $\mu$. The initial scheme of MMs was simple, as follows [2]. If one could fabricate isotropic artificial units exhibiting both electric and magnetic responses such as $\varepsilon = -1$ ($\varepsilon$: relative permittivity) and $\mu = -1$, then the refractive index $n$ would equal $-1$ because of the relation $n = -\sqrt{\varepsilon \mu}$. Note that, in the case of $\varepsilon < 0$ and $\mu < 0$, the EM waves are left-handed in contrast to the ordinary right-handed EM waves. This can be simply understood by transforming
Maxwell equations,
\[ \nabla \times \mathbf{E}(t, r) = -\frac{\partial \mathbf{B}(t, r)}{\partial t}, \]
\[ \nabla \times \mathbf{H}(t, r) = \frac{\partial \mathbf{D}(t, r)}{\partial t}, \]
where \( \mathbf{E} \) is electric field, \( \mathbf{H} \) is magnetic field, \( \mathbf{D} \) is electric displacement and \( \mathbf{B} \) is magnetic displacement. The constitutive equations in non-chiral media are expressed as
\[ \mathbf{D}(t, r) = \varepsilon_0 \varepsilon \mathbf{E}(t, r), \]
\[ \mathbf{B}(t, r) = \mu_0 \mu \mathbf{H}(t, r), \]
where \( \varepsilon_0 \) and \( \mu_0 \) are permittivity and permeability in vacuum, respectively. When the EM wave is a monochromatic plane wave, the electric field \( \mathbf{E}(t, r) \) has an exponential factor and is written as
\[ \mathbf{E}(t, r) = \mathbf{E} \exp(i \mathbf{k} \cdot \mathbf{r} - i \omega t), \]
where \( \mathbf{E} \) is polarization vector, \( \mathbf{k} \) is wave vector and \( \omega \) is angular frequency. By using equation (5), equations (1) and (2) are rewritten as
\[ \mathbf{k} \times \mathbf{E} = \omega \mu_0 \mathbf{H}, \]
\[ \mathbf{k} \times \mathbf{H} = -\omega \varepsilon_0 \mathbf{E}. \]
Figure 1(a) shows the spatial relation between the vectors \( \mathbf{k}, \mathbf{E} \) and \( \mathbf{H} \), where \( \mathbf{E} \) and \( \mathbf{H} \) are set to be parallel to the \( x \) and \( y \) axes, respectively. The Poynting vector \( \mathbf{S} = \mathbf{E} \times \mathbf{H}^\ast \) is given by
\[ \mathbf{E} \times \mathbf{H}^\ast = \frac{|\mathbf{E}|^2}{\omega \mu_0 \varepsilon^\ast} \mathbf{k}. \]
The observed Poynting vector \( \text{Re}(\mathbf{E} \times \mathbf{H}^\ast) \) is always \(+z\)-oriented, forming the right-handed set with the vectors \( \text{Re}(\mathbf{E}) \) and \( \text{Re}(\mathbf{H}) \). In figure 1(a), \( \mathbf{E} \) and \( \mathbf{H} \) are drawn as real-valued vectors, for simplicity. When \( \mu < 0 \) (real and negative), the wave vector \( \mathbf{k} \) is real and has the \(-z\)-direction as verified from equation (8). Then the vectors \( \mathbf{E}, \mathbf{H} \) and \( \mathbf{k} \) form a left-handed set (see figure 1(a)). Generally, the sign of refractive index \( n \) has two possibilities ( \( n = \pm \sqrt{\varepsilon \mu} \)), determined by the sign of \( \mathbf{k}/|\mathbf{k}| \) (in general, \( \text{Re}(\mathbf{k})/|\mathbf{k}| \)). Therefore, in the case of \( \varepsilon < 0 \) and \( \mu < 0 \), \( n \) is negative and satisfies \( n = -\sqrt{-\varepsilon \mu} \). Especially, \( n = -1 \) for \( \varepsilon = -1 \) and \( \mu = -1 \).

In 2001, a straightforward demonstration of a MM with negative refractive index was reported [3]. Figure 1(b) illustrates the concept of the basic unit (or \( \text{meta-atom} \)) of a MM with effective negative refractive index. The components were configured like an electric circuit: a metal (Cu) bar is responsible for the negative permittivity and a split ring resonator (SRR) is responsible for the negative permeability in the microwave medium. In the experiment, the two components formed a \( 5 \times 5 \) mm\(^2 \) unit cell and were separated by a glass board (semitransparent) of 250 \( \mu \)m thickness. Strictly, in figure 1(b), the real parts of \( \varepsilon \) and \( \mu \), written as \( \varepsilon_1 \) and \( \mu_1 \), respectively, are negative for the incident polarization parallel to the metal bar. A periodic array of the \( \text{meta-} \) atoms constituted with a MM of negative refractive index \( (n_1 < 0) \) at 10.5 GHz (3 cm wavelength) [3].

The demonstration of negative refraction triggered much research on MMs. Attempts were made to extend the effective \((\varepsilon, \mu)\) frame to higher frequency ranges, such as THz and optical frequencies (\( \sim \) PHz). At the initial stage, SRRs were extensively studied as a component exhibiting magnetic response and made it possible to obtain effective negative \( \mu \) at about 0.1 PHz \([4, 5]\), though the shift of working frequencies toward optical frequencies accompanied significant reduction in \( |\text{Re}(\mu)|\). It was theoretically and numerically examined how the magnetic response is preserved at optical frequencies in scaled-down SRRs \([6–8]\). The studies consistently reached a conclusion that the magnetic response associated with negative \( \mu \) is reduced at optical frequencies and disappears at about 0.3 PHz (1 \( \mu \)m wavelength).

Let us briefly describe why such disappearance of the magnetic response takes place. The permittivity of a Drude metal \( \varepsilon_D \) is given by equation (9) for a wide range of angular frequencies \( \omega \) \([9, 10]\):
\[ \varepsilon_D(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)}, \]
where \( \omega_p \) is plasma frequency and \( \gamma \) is damping factor. Figure 2 shows an example of the Drude permittivity using the parameters of Ag \([11]\). The Drude model is a good approximation for Ag in the frequency range shown in figure 2. Qualitatively, the real part (solid line) remains almost constant below \( 10^{13} \) Hz, implying that the scaling law is a good approximation and that MMs for these frequencies can be designed similarly to the microwave MMs. In contrast, the permittivity rapidly changes for optical frequencies, where the scaling law on EM eigenmodes does not strictly hold and significant changes are inevitable. A large reduction takes place for the effective \( \mu \) in MMs of U-shaped SRR array \([5–8]\). The dissimilarities of EM resonances in different frequency ranges were widely recognized, leading to the two major subfields of electromagnetics: microwave engineering and optics.
To extract the effective permittivity and permeability in MMs, the so-called retrieval procedure was proposed [12]. Figure 3 illustrates an optical configuration for a slab of thickness $d$. When an incident plane wave sheds on the slab at incident angle $\theta$, the complex reflectivity $r$ and transmissivity $t$ are related to the wave vector in the slab such that $\cos(k_{/d}) = R(r, t)$ where $R(r, t)$ is a rational expression of $r$ and $t$ [12]. The $z$-component of refractive index $n_z$ is related to the $k_z$ as $n_z = ck_z/\omega$ at the normal incidence ($c$ is the speed of light in vacuum). Therefore the refractive index $n_z$ is extracted from

$$n_z = \frac{c}{\omega d} \arccos[R(r, t)] \pm 2\pi m,$$

(10)

where $m$ is a non-negative integer. Since equation (10) has ambiguity in the real part, it is always necessary to show how to select a specific $m$ branch. If one could determine the value of $n_z$, the $y$ component $\mu_y$ is determined because the admittance $n_z/\mu_y$ is uniquely obtained, independent of equation (10). Finally the remaining unknown term $\varepsilon_x$ is determined from the dispersion equation $\varepsilon_x\mu_y = n_z^2$. The retrieval procedure assumes that the macroscopic material parameters $\varepsilon$ and $\mu$ are well defined in the medium, and provides a procedure to extract $\varepsilon$ and $\mu$ in a purely algorithmic manner. When the medium is isotropic, the parameters $\varepsilon$ and $\mu$ are fully determined in the procedure described above.

The macroscopic material parameters assume the long wave approximation to ensure the well-definedness. The physical meanings of the retrieval parameters in MMs have been much debated [13–16]. A recent reconsideration of the framework using the macroscopic material parameters is referred to in section 1.3.

In the optical experiment, effective refractive index was evaluated by probing the phase changes in reflected and transmitted light, which is usually done with an interferometer. There are two major interferometer designs for this purpose: one uses two walk-off prisms, making the two light paths detect the change of phase in traveling photonic MMs [17], and the other is the Michelson interferometer [18]. These techniques were often applied to extract the effective $\varepsilon$ and $\mu$ in photonic MMs. The most preferred photonic MMs and the nanofabrication techniques are described in section 1.1.

As mentioned above, the designs that work for microwaves can not be extended to photonic MMs just by scaling; photonic MMs need to be newly designed to realize novel features such as negative refractive index. There have been many proposals, but we here focus on the photonic MMs examined by experiment. One of the most well-known photonic MMs is fishnet MMs (see figures 4(a) and (b)), which were first introduced in 2005 [19, 20].

In this review, we summarize the present status of photonic MMs after a decade of progress, mainly based on experimental results and realistic simulations. ‘Realistic simulations’ means that the computations correspond to
experiments and are based on material parameters measured so far. We do not consider here purely theoretical implications relying on assumed material parameters and models such as effective medium theory because we intend to clarify what has been actually achieved without assumptions or idealization. We also address how photonic MMs are properly described without interpretations and models. After surveying the recent progress, we discuss the potential for photonic MMs in the near future.

During the rapid progress in MMs in the last decade, several review papers were published on various aspects: theoretical studies [21, 22], negative-refractive-index MMs [23–25], applications to super-resolution [26] and EM-wave absorbers [27]. These papers compiled the theoretical and experimental results reported at the time, so we do not attempt to collect all the results to date. Instead, we try to clearly present what has been established in photonic MMs by experiments and realistic simulations, and describe how to clarify the EM dynamics in photonic MMs without depending on models and assumptions.

We consider that the main wavelength range for photonic MMs extends from the visible to near-infrared range (roughly, up to 2 μm). We exclude most of the MMs designed for lower frequencies, such as SRR-based MMs. Some of the omitted topics can be found in reviews [21–27]. A major series of achievements in microwave MMs is not covered in this paper, in particular, transmission lines working at microwaves, which are electric circuits including novel left-handed MM elements. This subject was described in the monographs [28, 29].

This paper is organized as follows. Experimental and computational studies of photonic MMs are described in sections 1.1 and 1.2, respectively. In section 1.3, physical basics in the (ε, μ) frame are revisited with reference to a recent theory on macroscopic Maxwell equations. We extract lessons on the effective (ε, μ) frame in MM studies. Based on the fundamental issues in MMs described in section 1, we consider the achievement in photonic MMs in terms of effective refractive index in section 2. As an application of photonic MMs, subwavelength optical devices are reviewed in section 3. EM resonances in photonic MMs are described using the EM eigenmodes in section 4. Two typical MMs are addressed: a MM slab of fishnet structure in section 4.1 and a three-dimensional (3D) multilayered MM in section 4.2. A summary and future prospects are given in section 5.

1.1. Nanofabrication techniques

The fabrication of photonic MMs is supported by contemporary nanolithography techniques. The periodicity of MMs is required to be in the subwavelength range: for the incident wavelength λ, the periodicity of MMs should be smaller than λ/(2nε) where nε is the refractive index of the substrate or the medium surrounding the MM; otherwise, the diffraction-free condition is not satisfied.

Photolithography is useful for simple nanostructures. It has contributed to the initial fabrication of fishnet MMs of circular holes, that is, periodically perforated metal–insulator–metal (MIM) stacked layers [19, 20]. Figure 4(a) depicts a fishnet MM, omitting the substrate.

Patterns of most photonic MMs were drawn by electron beam (EB). EB lithography is a key process to produce periodic arrays of holes in MIM layers [30–36], cut-wire pairs [37, 38], SRRs [39, 40] and stacked SRRs [41]. Figures 4(b) and (c) illustrate a fishnet MM of rectangular holes and cut-wire pairs, respectively, while omitting the substrates. Figure 4 shows that the photonic MMs related to effective negative refractive index are MM composites, wherein Au and Ag are frequently used as approximate Drude metals, while Al2O3, MgF2 and SiO2 are typical insulators.

Focused-ion-beam (FIB) lithography was used to directly perforate stacked MIM structures containing about ten layers and produce relatively thick fishnet MMs [42, 43]. Negative refraction was observed in one of the fishnet MMs and the negative refractive index was evaluated from the refraction angle [42].

EB lithography and FIB techniques have a high (nm) precision but low throughput, and will hardly be employed in industrial processes. In addition, the top-down techniques are limited to producing photonic MMs of deep subwavelength thickness such that d/λ ∼ 1/10 (d is thickness of MMs). Small thickness hinders the analysis using equation (10).

Nanoimprint lithography has shown a rapid progress recently. Large-area photonic MMs (above 1 cm2) are fabricated in a high-throughput process [44], and its improvements will lead to industrial applications of photonic MMs.

1.2. Numerical methods

Numerical calculations are a basic tool in the studies of photonic MMs. Most MMs are periodic artificial structures, and therefore are described by Fourier-transformed Maxwell equations. Their solutions for diffractive gratings, while maintaining fast numerical convergence, proved elusive for many years. After various trials, a dexterous algorithm, namely the inverse-factorization rule, was finally conceived. Initial reports describing the algorithm for one-dimensional (1D) periodic structures appeared in 1996 [45–47]. Later, the inverse-factorization rule was extended to two-dimensional (2D) periodic structures in 1997 [48]. The method incorporating the inverse-factorization rule is called the Fourier modal method or rigorously coupled-wave approximation (RCWA) method.

In addition to an improved algorithm for solving Fourier-transformed equations, the solutions for 3D periodic structures need an algorithm for non-divergent matrix operations. 3D structures are generally regarded as stacked 2D structures. The transfer matrix method is an intuitive approach to dealing with 3D periodic structures (for example, [49]), but it cannot eliminate the exponentially divergent terms appearing in the matrix operations. To avoid the problematic behavior in numerical implementations, the scattering matrix method was conceived [50, 51]. The combination of the two improved numerical algorithms [48, 51] allowed to compute optical spectra such as reflection and transmission in photonic MMs.

Other methods were also employed. The finite element method was one of the most popular methods of evaluating
EM-field distributions in MMs. Spatial decomposition by grids is conducted automatically and is useful for maintaining the precision of computations. The algorithm for producing proper spatial grids can be used to solve Maxwell equations [52]. Several commercial software packages using the finite element method became available recently (for example, [53]).

The finite-difference time-domain (FDTD) method was sometimes used to evaluate EM-field distributions, but is used less often for photonic MMs. It is a major tool for modeling photonic crystals of semiconductors and dielectrics [54, 55].

All the numerical methods outlined above are based on classical electrodynamics, assuming that the composite materials respond locally, as expressed by equations (3) and (4). In metallic nanostructures, if quantum mechanical responses manifest themselves, the materials respond nonlocally and the classical methods become inaccurate. It was theoretically shown that nonlocal responses become distinctive when dimensions of metallic nanostructures, such as gap, are smaller than 3 nm [56–58]. In the photonic MMs addressed in this paper, such tiny structures are not included and hardly contribute to the optical responses, and thus the classical methods are valid.

1.3. Conventional EM theory and the reformulation

In solid state physics, the relative permeability \( \mu \) that represents magnetic response is usually taken as unity at optical frequencies [59]. This is because magnetic responses in solid materials originate from the spin states, whose resonances are far from the optical frequencies. It is extremely difficult to find a material exhibiting magnetic responses at optical frequencies. Materials of \( \mu < 0 \) and \( \epsilon < 0 \) were theoretically studied in the 1960s [60]; however, the corresponding solid materials have not been found. It was thus a challenge to produce artificial materials showing magnetic responses, which was one of the reasons why MMs attracted many researchers.

MMs revived the interest in the (\( \epsilon, \mu \)) frame. The well-known macroscopic constitutive equations are expressed by equations (3) and (4). The retrieval procedure [12] is an algorithmic way to evaluate the parameters \( \epsilon \) and \( \mu \). On the other hand, the electric and magnetic susceptibilities have a physical meaning as response functions in matter [59].

One theory recently reformulated the constitutive equations from the Hamiltonian containing EM fields and matter [61], the reconstruction was executed in a straightforward way. First, the starting point was the Hamiltonian of the EM field, matter and the interaction terms. The Hamiltonian equations resulted in the usual Maxwell and Newton equations. Second, the induced-current-density operator was generally given by the convolution of microscopic susceptibility and vector potential in the linear approximation (or the first-order perturbation). In this procedure, the variables \( (r, i) \) were Fourier-transformed into the set of \( (k, \omega) \). Third, to derive macroscopic EM equations, the long-wavelength approximation was additionally assigned to the equation for the current density \( \mathbf{I} \) and vector potential \( \mathbf{A} \). The macroscopic equation was finally expressed as

\[
\mathbf{I}(k, \omega) = \chi_{\text{EM}}(k, \omega) \cdot \mathbf{A}(k, \omega),
\]

where the susceptibility \( \chi_{\text{EM}} \) was \( k \)-expanded as

\[
\chi_{\text{EM}}(k, \omega) = \chi_{\text{EM}}^{(0)}(\omega) + |k|\chi_{\text{EM}}^{(1)}(k/|k|, \omega) + |k|^2\chi_{\text{EM}}^{(2)}(k/|k|, \omega) + \cdots .
\]

The zeroth-order term \( \chi_{\text{EM}}^{(0)} \) is a new form of the lowest-order response function, and can relate to \( \epsilon \) and \( \mu \) in media of high symmetry such as cubic symmetry. The first-order term \( \chi_{\text{EM}}^{(1)} \) is responsible for chirality in media [61]. Experimental tests for the new formulation are an intriguing issue from the viewpoint of fundamental science. Carefully designed MMs are good candidates for testing microscopic nonlocal responses described by equation (11); in this case, the long-wave approximation is not assumed.

The reformulation provides a few explicit lessons.

1. When one can safely use macroscopic parameters under the long-wavelength approximation, the parameter coming from the response function is uniquely determined, such as \( \chi_{\text{EM}} \). In solid materials, one can use the long-wavelength approximation for off-resonant states because the ratio of incident wavelength \( \lambda \) to the lattice constant \( a \) of a unit cell in solid crystals is typically \( \lambda/a \sim 10000 \).

2. The electric and magnetic resonances are far distant in energy in most of the solid materials studied so far. Therefore, the material response was uniquely attributable to a single parameter (\( \epsilon \) or \( \mu \)). The complicated situation that the electric and magnetic responses simultaneously take place at the same frequencies has not been experimentally examined in the (\( \epsilon, \mu \)) frame. A corresponding \( \epsilon-\mu \)-mixing situation was proposed for a medium including Er ions at optical frequencies [62] though there has been no corresponding experimental report to date.

3. The application of the effective (\( \epsilon, \mu \)) frame to MMs studies has two problems. One is that the long-wavelength approximation is not at all guaranteed: the ratio is typically \( \lambda/a \sim 3 \) in photonic MMs where \( a \) is the length of a unit cell in MMs. Actually, the EM fields in photonic MMs are very inhomogeneous as shown in section 4. Furthermore, experimental photonic MMs are often significantly thinner than the incident wavelength. This thinness can be a serious obstacle in selecting a proper \( m \) branch in equation (10). Another problem is that even if the long-wavelength approximation holds, the responses from MMs would be mostly nonlocal (or \( k \)-dependent), so that other higher-order parameters of \( \chi_{\text{EM}}^{(i)} \) (\( i \geq 1 \)) must be determined simultaneously. Consequently, another ambiguity appears.

To avoid these difficulties, a proper description resolving the EM eigenmodes in photonic MMs is presented in section 4.
2. Extended range of refractive index

In this section we survey photonic MMs that extend the range of refractive index. Most solid materials have a refractive index of 1–3.5 in the real part under off-resonant conditions. Photonic MMs can be designed to extend this range to larger than 5 or smaller than 1. As mentioned in section 1.1, an effective negative index in photonic MMs can be determined in practice by estimating the refraction angle. From here on, the effective refractive index is simply called ‘refractive index’.

2.1. Negative refractive index

A negative refractive index at optical frequencies was mostly obtained in fishnet MMs; it resulted from a resonant effect and therefore was associated with large optical loss. In the near-infrared region, the performance was relatively better: a typical figure of merit (FOM), defined by $-\text{Re}(n)/\text{Im}(n)$, was about 3 for the near-infrared frequencies without special treatment in the fabrication [30, 42], whereas it was smaller than 1 in the visible region [31, 33, 35]. By using FIB in fabrication procedures, an FOM of about 3 was obtained even in the visible region [43]. However, to substantially overcome the loss, an active fishnet MM was needed; thus an FOM of about 100 was demonstrated in a pump–probe experiment [36]. The refractive indices in MM slabs were extracted using the retrieval procedure described in section 1.

In thicker MMs where EM waves form the wave front, refractive index can be evaluated using the wave vector of refraction, as in ordinary solids; the negative refractive index in a thick fishnet MM was estimated from the angle of negative refraction in a MM prism [42]. The situation is similar to the unusual refraction in photonic crystals [63, 64]. In terms of light-wave propagation, there is no essential difference between MMs and photonic crystals, but it should be noted that MMs mainly manipulate plane waves whereas photonic crystals are mainly used to handle the guided waves inside slab structures [63, 64].

Another type of negative-refractive-index MM was reported from a realistic simulation [65] though it is less popular than the fishnet MMs. The MM had a multilayer structure of a metal and an insulator, and we call the MM stratified metal–insulator metamaterial (SMIM). In the SMIMs of Ag and HfO$_2$, it was numerically confirmed that negative refraction takes place for a Gaussian beam of violet light falling at oblique angle. The corresponding refractive index was estimated to be nearly $-1$ in the real part [65]. Another recent realistic design resulted in an isotropic refractive index close to $-1$ at a particular violet frequency in a Ag–TiO$_2$ SMIM [66].

Recently, a realistic simulation suggested that an array of coaxial metallic nanostructures with semiconductor embedded in annular slits can serve as a negative-refractive-index material that works in the visible range and is insensitive to incident angle [67]. The negative refractive index was determined from the refractive wave vector. Although the structure is more complicated than the fishnet MMs and is harder to produce, it will become viable owing to the progress in nanofabrication techniques.

2.2. High refractive index

High refractive index $n$ is often associated with slow light since the group velocity is equal to $c/n$ in non-dispersive media. It was theoretically shown in the 1960s [68, 69] that a unit structure of MIM has a slow-light mode, which is related to high refractive index. Figure 5(a) shows an illustration of the MIM structure. Figure 5(b) schematically presents dispersion diagrams of the lowest (or first) MIM waveguide mode with solid lines and the light cone (LC) with dashed lines. The first MIM waveguide mode is located far below the LC, implying that the EM wave in the I layer propagates in the same way as in materials of higher refractive index than the index of bulk material constituting the I layer.

Theoretically, the concept of MIM waveguides is simple [68, 69]. However, the corresponding experimental results were not reported until 2006 [70] when the MIM waveguide mode was confirmed; in the experiment, closed MIM waveguides were employed, which had a finitely long I layer along the $z$-axis in figure 5(a) and were stopped by the metal on one side.

Starting from MIM structures, the scheme to obtain high-refractive-index MMs is quite straightforward. A theoretical analysis for arrays of the MIM structures was carried out and suggested high-refractive-index materials [71]. Figure 5(c) schematically depicts an array of MIM structures of the periodicity $a$ and width $s$ of each I layer. The $x$-axis was set to be parallel to the periodic direction and the $y$-axis was parallel to the slits. The thickness of the MIM array was finite along the $z$-axis. It is important that the array of MIM layers has a highly transmissive waveguide mode, that is, transverse electric and magnetic (TEM) mode in terms of waveguide theory [72].

Figure 5(d) shows computed transmittance ($T$, blue) and reflectance ($R$, red) spectra in a typical MIM array. The incidence was set to be normal to the $xy$-plane and the polarization vector $\mathbf{E}$ was parallel to the $x$-axis. In the computation, the metal was assumed to be Ag and the insulator was SiO$_2$; the periodicity $a$ was 195 nm, the width $s$ was 5 nm and the thickness was 100 nm along the $z$-axis. Clearly, a prominent $T$ peak appears at 1.10 eV (1126.8 nm wavelength). The computation was implemented with the code based on the RCWA method for 1D periodic structures [46] and scattering matrix algorithm [51]. To conduct realistic computations, the permittivity of Ag was taken from [73], while for that of SiO$_2$, the typical value of 2.1316 was used.

Figure 5(e) shows a snapshot of electric field (color and vectors) on an $xz$ section; the excitation condition corresponds to the $T$ peak in figure 5(d). For the incident electric field of $E_x(\phi) = E_0 \cos(\phi)$, the phase $\phi$ was set to be $\phi = 0$ at the top edge in figure 5(e). The electric-field distribution was calculated by the finite element method [53]. It was found that the EM waves in the insulator slits propagate by a half wavelength and are highly enhanced. Thus, we were able to estimate the refractive index $n$ in the SMIM from...
the relation of $0.5 \times 1126.8/n = 100$ and obtained a high index $n \approx 5.6$, which is the real part of a complex value. From realistic numerical evaluations for other SMIMs, it was revealed that the refractive index is larger than the index of the I layer [74, 75]. This feature is useful for controlling the phase of light waves in subwavelength dimensions as shown in section 3, offering a way to access slow light.

Arrays of metallic bars are well known as wire-grid polarizers, which are standard polarizers for microwaves [76] and are commercially available from the microwave to optical frequency range. Historically, the initial concept of wire-grid polarizers was experimentally demonstrated by Hertz in 1888 [77]. Since transverse magnetic (TM) waves are transmissive and transverse electric (TE) waves are reflective, polarizations of EM waves are selected by the wire-grid polarizers.

2.3. Low refractive index

In contrast to high-refractive-index materials, a low refractive index of less than 1 is hard to achieve. In solid materials, low refractive index may be realized close to Lorentz-type resonances, which are typically excitons at optical frequencies [78]; however, the resonances are associated with large optical loss at room temperature due to spectral broadening. This means that low refractive index in ordinary solids is unsuitable for applications in optical devices.

A way of accessing low refractive index was found in a study of a SMIM [74]. SMIMs are naturally uniaxial media as shown in figure 6(a); the principal axis is parallel to the $x$-axis. For the light of $x$ polarization in figure 6(a), SMIM is a high-refractive-index material as described in section 2.2, yet it has a low refractive index for the $y$ polarization. Let us describe this feature through an example. Figure 6(b) presents reflectance spectra of a SMIM, which is composed of 30 nm Ag and 120 nm SiO$_2$ layers and is periodic along the $x$-axis. The reflectance spectrum at the $y$ polarization (red solid line) shows that the SMIM optically responds like a Drude metal in equation (9), suggesting the plasma frequency $\omega_p$ at 2.2 eV (arrow). The SMIM was expected to have $\text{Re}(n)$ close to 0 near the $\omega_p$. It was indeed substantiated by realistic calculations that the effective wavelength inside the SMIM, $\lambda/\text{Re}(n)$, above $\omega_p$ is longer than the incident wavelength $\lambda$ [75]. This result proved that the real part satisfies the relation of $0 < \text{Re}(n) < 1$. In addition, low-refractive-index SMIMs offer advantages of suppressing optical loss and typically have a small imaginary part such that $\text{Im}(n) \leq 0.1$.

As seen in the example of SMIMs, in order to attain a low refractive index, it is essential to move the plasma frequency $\omega_p$ into the optical frequencies by employing MMs. Qualitatively, the I layers serve to reduce the free-electron density in the unit cell of SMIMs, and so $\omega_p$, which is located at an ultraviolet frequency for pure Ag, shifts to optical frequencies in SMIMs as indicated by the arrow in figure 6(b).

Historically, the idea of modifying the electronic states in metals was tested in the 1980s, when superlattices of semiconductors attracted great interest in materials science. Superlattices of metals and insulators were investigated theoretically to control the dispersion of electronic states and the responses of dielectric function [79, 80]. The studies focused on the inner electronic states and were not extended.
3. State-of-the-art subwavelength optical devices employing photonic MMs

This section reviews optical devices employing photonic MMs, aiming to realize linear optical devices that can control light waves in subwavelength paths.

Generally, plane waves have two basic physical quantities: polarization vector \( \mathbf{E} \) and phase. When the right-hand side of equation (5) is expressed as \( \mathbf{E} \exp(i\phi) \), the factor \( \phi \) is the phase. Figure 7 schematically shows a fundamental correlation diagram of the two physical quantities and resultant optical devices. Selective transmission or reflection by the polarization vector is the basis of polarizers. Anisotropic phase manipulations make it possible to realize wave plates. If both polarization vector and phase are controlled, circular dichroic (CD) devices are produced. Photonic-MM-based wave plates, polarizers and CD devices are described in sections 3.1, 3.2 and 3.3, respectively.

3.1. Wave plates

Wave plates manipulate incident polarizations in desired ways. For instance, quarter wave plates transform an incident linear polarization to a circular polarization, and half wave plates rotate an incident linear polarization by 90°. The principle of wave plates is anisotropy of refractive indices in EM-wave media of high transparency. For EM plane waves propagating along the \( z \)-axis, the contrast of refractive indices \( n_x \) and \( n_y \) determines the efficiency of the wave plate. In solid materials known to date, strong anisotropy is represented by a difference of refractive indices and is at most 0.1. Various wave plates are commercially available, and their efficiency is limited by the anisotropy in media. Even the most efficient wave plates for visible light have a thickness of 100 \( \mu \)m order. Thus, very strong anisotropy in refractive indices is the key to producing efficient wave plates of subwavelength thickness.

Figure 6(b) shows that the SMIM has small reflectance and therefore becomes transparent for both incident \( x \) and \( y \) polarizations at higher frequencies (or photon energies) than the plasma frequency \( \omega_p \), which is indicated by an arrow. The \( \omega_p \) was determined from the \( y \)-polarized reflectance spectrum. Strong anisotropy in refractive indices \( n_x \) and \( n_y \) takes place in the SMIM at frequencies higher than \( \omega_p \). By tuning the thickness \( d \), the SMIM can be used as an efficient wave plate in the transmission configuration of figure 6(a) where the incident linear polarization is specified by the angle \( \psi \). In figure 6(b), at the thickness of 135 nm, the SMIM works as a quarter wave plate at the incident photon energy of 2.7 eV (460 nm wavelength) as shown in figure 6(c); when an incident plane wave of linear polarization \( \psi = 45^\circ \) normally sheds on the \( xy \) plane, the transmitted electric-field trajectory projected on the \( xy \)-plane exhibits a circular polarization [75]. Thus, a wave plate employing the SMIM made it possible to significantly reduce the thickness. In comparison with ordinary wave plates of 1 cm thickness, \( 7.4 \times 10^4 \)-fold miniaturization was obtained. Also, the thickness of 135 nm...
is subwavelength in comparison to the incident wavelength of 460 nm.

As the thickness approximately doubles, the SMIM of (Ag 30 nm/SiO₂ 120 nm) works as a half wave plate at 2.7 eV [75]. This half wave plate also has subwavelength thickness. In the near-infrared range, SMIMs of (Ag/Si) can serve as subwavelength wave plates [75]. Thus, wave plates of subwavelength thickness can be obtained at an arbitrary optical frequency by selecting a set of metal and insulator and by tuning their structural parameters.

As mentioned in section 2.2, SMIMs can serve as polarizers. This function is realized for frequencies lower than ωₚ, where the reflectance spectrum in figure 6(b) presents a definite contrast and implies that transmittance is also polarization selective. The wave plate introduced here is another function of the SMIM, which has not been achieved for more than a century since the introduction of wire polarizers by Hertz [77].

It is shown above that SMIMs can function as wave plates in the visible and near-infrared ranges. Other MMs could serve as subwavelength wave plates. A realistic simulation showed that a MM of skew stacked SRRs transforms incident linear polarizations to other elliptic or linear polarizations at infrared range [81], though the MM was neither a quarter nor a half wave plate. Note that the MM comprising a stack of SRRs replied on a resonant effect; therefore, the working frequency is more difficult to tune than that of the SMIMs above.

3.2. Polarizers

Polarizers are one of the most popular devices in EM engineering and optics. As mentioned in section 2.2, metallic wire-grid polarizers are widely known. SMIMs are a kind of wire-grid polarizer, but they need to be as thick as several wavelengths to obtain a good extinction ratio. A challenge in photonic MMs is to produce much thinner polarizers than those known to date, and to find a more efficient mechanism of polarization selection, which would contribute to the substantial miniaturization.

The new polarizer design as shown in figure 8 was experimentally found to be highly efficient [82]. Figure 8(a) is a top-view image by scanning electron microscope (SEM). Butterfly-shaped Ag nanostructures were arrayed in a square-lattice manner in the x-y-plane with the periodicity of 1.0 μm (horizontal black bar). Figure 8(b) depicts a stacked complementary (SC) structure, consisting of three layers, which are schematically decomposed in the right-hand side. The top layer is composed of Ag (gray) and air (white), the middle layer consists of an insulator with n = 1.54 (pale blue) and air, and the bottom layer is formed by the butterfly-shaped Ag and the insulator. The quartz substrate below the bottom layer is omitted in figure 8(b). The top and bottom layers are seen in the SEM image of figure 8(a). The second layer is a spacer between the top and bottom layers. Note that the top and bottom layers are complementary with respect to the Ag parts. The SC structure was produced by depositing Ag after nanofabrication of the insulator. The thickness of the top to bottom layers was 50, 150 and 50 nm, respectively, giving a total thickness of 250 nm.

Figure 8. A stacked complementary MM working as a subwavelength polarizer. (a) Top-view SEM image. The horizontal scale bar denotes 1.0 μm. (b) A schematic illustration of the unit cell of three-layer stacked structure. The top and bottom layers are complementary with respect to metal (gray). Pale blue denotes insulator with n = 1.54. (c) Measured transmittance spectra at the normal incidence under y (blue solid line) and x (red dashed line) polarizations.

Figure 8(c) shows transmittance spectra measured at the normal incidence to the x-y-plane. The transmittance spectra at the x and y polarizations are shown by the red dashed and blue solid lines, respectively. A high contrast of transmittance clearly appears at 0.7–0.8 eV; in particular, the extinction ratio of Tₓ/Tᵧ exceeds 1.5 × 10³ at 0.75 eV (1650 nm wavelength). The total thickness was only 1/6 of the incident wavelength and was in the subwavelength regime. It was also confirmed that the measured transmittance spectra in figure 8(c) agree with transmittance spectra calculated using the RCWA method [48] combined with the scattering matrix algorithm [51]. The permittivity of Ag was taken from [73].

The EM dynamics in subwavelength dimensions was examined by realistic simulations [83, 84] using the finite element method [53]. It was shown that enhanced evanescent EM waves (or plasmonic fields) in the top and bottom layers couple and form a local plasmon mode. The resonant mode is the source of the high contrast of transmittance.

In addition, it was found that by improving the design of the unit cell, a polarizer with an extinction ratio exceeding 10⁵ can be achieved [83]. Note that, in comparison with the wire-grid polarizers, the photonic MMs of SC structures are about ten times thinner for the same extinction ratio. It was also shown using the Babinet’s principle that SC
structures enhance extinction ratio and are intrinsically efficient polarizers [82].

3.3. CD devices

CD responses are often observed in nature. For example, jeweled beetles reflect light in a CD manner. It was recently reported that they have helically stacked microstructures in the wings [85]; the structures are composed of several layers and are several µm thick in total. Can photonic MMs work as more efficient CD structures than natural structures?

It was recently shown that arrays of metallic helical microstructures work as infrared CD devices [86, 87]. However, it is probably difficult to shift the working wavelengths to the visible range, for a similar reason to the scaling law concerning SRRs described in section 1.

In principle, circular dichroism is a consequence of the combination of phase shift in circular-polarized waves and polarization selection. It can be explicitly verified by using Jones matrices. Based on this principle, simple designs including photonic MMs were recently introduced [88]. Figure 9 shows the actual designs.

Figure 9(a) depicts a two-layer CD device. The first layer located at the positive z side is a quarter wave plate as described in section 3.1. The second layer includes arrays of square metallic rods; it is a polarizer, which transmits plane waves of the polarization vector E parallel to the (1, 1) direction in the xy-plane and terminates plane waves of E||[(1, −1). In the lower part in figure 9(a), the dimensions in the unit cell are shown in nm; gray denotes Ag and pale blue stands for SiO₂. The thickness of the first and second layers was 284 and 210 nm, respectively. The total thickness of the two-layer device was 494 nm.

Figure 9(b) shows the transmittance spectra of the two-layer device for right-handed circular (RHC) and left-handed circular (LHC) polarizations are displayed with blue solid and dashed lines, respectively. We here define the RHC polarization as the polarization forming a right-handed screw for the wave vector k and the LHC polarization as that forming a left-handed screw. Clearly, transmittance under the LHC polarization is suppressed at 1.4–1.5 eV while transmittance under the RHC polarization is about 80%. Thus, the two-layer device efficiently transmits only the RHC incidence. At 1.45 eV (855 nm wavelength) indicated by an arrow, the two-layer CD device works ideally. In figure 9(c), the polarization of transmitted light at 1.45 eV is plotted and exhibits a linear polarization parallel to the (1, 1) direction. It was shown that the two-layer device is an ultrathin (or subwavelength-thick) CD device that transforms the incident RHC polarization to a linear polarization [88]. The transmittance spectra were computed by the RCWA method [48] associated with the scattering matrix algorithm [51]. The permittivity of Ag was taken from [73].

Figure 9(d) shows a three-layer CD device, in which a third layer was added to the second layer in figure 9(a). The total thickness was 749 nm since the third layer was adjusted to 255 nm. The third layer serves as a quarter wave plate. In figure 9(e), transmittance spectra, evaluated similarly to figure 9(b), show the selection of RHC polarization, and optimal selectivity was achieved at 1.45 eV (arrow). Figure 9(f) shows a 3D plot of the polarization of transmitted light at 1.45 eV, exhibiting LHC polarization for the incident RHC polarization. It was thus substantiated that the three-layer CD device transforms the incident RHC polarization to the LHC polarization in the subwavelength path.

One of the advantages of CD devices employing photonic MMs is the simplicity of each component. It is therefore feasible to tune other working energies by adjusting the components. Moreover, other combinations can lead to subwavelength devices that are not described here; for example, it was numerically shown that skew-stacked wave plates form orthogonal polarization rotators, which rotate any linear polarization by 90° [89]. These advantages are important for producing new subwavelength optical devices with required functions.

3.4. Comparison of MMs with optical fibers and photonic crystals

Several photonic devices have subwavelength dimensions. Here we look at photonic crystal fibers and photonic crystals.

As is widely known, photonic crystal fibers induce super-continuum light [90]. Their clad core is usually a defect in the air-hole array of subwavelength periodicity and serves as a light guide. Although nonlinear processes generally have a small cross section, the propagation length in photonic crystal fibers is far longer than the wavelength of propagating light. Therefore, multiple nonlinear interactions result in transmitted light having a very broad spectral range, called super-continuum light. Photonic crystal fibers are opposite to photonic MMs in terms of the length of light paths, and suggests the ways to optimize nonlinear optical processes in photonic MMs.

Photonic crystals are notable for high-precision light manipulators [91–93]. By exploiting semiconductors that can be nanofabricated with high precision, the engineering of photonic bandgap and light waveguides has progressed significantly at telecommunication wavelengths (1.3–1.6 µm). Thus, photonic crystals have formed an established subfield in artificial EM crystals. On the other hand, photonic MMs include metal and are regarded as an emerging subgroup in artificial EM crystals. One of the features in photonic MMs is metallic-permittivity-based light manipulation, which is related to the highly efficient subwavelength control described in sections 3.1–3.3. Photonic MMs usually manipulate plane waves, in contrast to waveguide modes in semiconductor photonic crystals.

4. EM eigenmodes and unusual light propagations in photonic MMs

As described in section 3, refractive index is a useful quantity to characterize photonic MMs and is helpful in designing subwavelength optical devices quantitatively. Apart from their practical usefulness, a fundamental question still
remains: what is an appropriate way to clarify the properties of photonic MMs?

In this section, we address two main types of photonic MMs. One is a fishnet MM slab, which is far thinner than incident wavelengths. The other type is a 3D structure of stacked multilayer structure, which is analyzed on an example of a SMIM. In both cases, the EM eigenmodes are sufficiently clarified, and features of the MMs are explicitly shown without depending on models or assumptions.

Before proceeding to the eigenmode analysis, we briefly mention a novel type of light propagation, namely cloaking. The initial idea stemmed from the simple fact that wave vector $k$ can take arbitrary directions by choosing the set of material parameters $(\varepsilon, \mu)$. If one can prepare a specific $(\varepsilon, \mu)$ distribution around an object, it is possible to hide the object [94, 95]. This is how cloaking using MMs works. An experimental demonstration of cloaking was first reported for microwaves [96]. At optical frequencies, it was shown that a small object can be hidden from in-plane detection in a MM slab [97]. The concept of cloaking stimulated numerous trials, and many fascinating results were already reported. However, they relied on the assumption of extreme values of $(\varepsilon, \mu)$, and so the results are out of the scope of this paper.

4.1. Fishnet MM slabs

As described in section 1.1, most photonic MMs studied experimentally were slab structures with a typical thickness

Figure 9. Subwavelength CD devices. (a) Two-layer stacked structure, transforming a circular polarization to a linear polarization. Schematic (upper image) and the dimensions inside the unit cell in nm (lower image). (b) Transmittance spectra under RHC (blue solid line) and LHC (blue dashed line) incidences. (c) Electric-field trajectory of transmitted light, projected on the $xy$-plane. (d) Three-layer stacked structure, transforming a circular polarization to the counter-circular polarization. Schematic (upper) and the dimensions inside the unit cell (lower). (e) Transmittance spectra under RHC (blue solid line) and LHC (blue dashed line) incidences. (f) 3D plot of electric-field trajectory of transmitted light traveling along the wave vector $k$. 
Figure 10. (a) Schematic illustration of a free-standing fishnet MM slab, and its optical configuration. (b) Absorption spectra at TM polarization, shown with an offset. Incident angles vary from 0° to 20° (from bottom to top). (c) Dispersion diagrams, obtained from the absorption peaks. Left: TE modes. Right: TM modes. Oblique solid lines denote the light cone in air. The dotted line denotes the lowest mode in a homogeneous MIM (h-MIM) waveguide. The dashed line represents the reduced MIM (r-MIM) mode into the first Brillouin zone. (d) Time-averaged EM power flow in the I layer (left) and the xz sections (right). The excitation condition is indicated by arrows in (b) and (c). The definition of the power flow is given in the text. (e) Time-averaged EM power flow in a five-layers fishnet MM slab, containing three metal and two insulator layers. The excitation condition is similar to that in (d).

d of about λ/10 where λ is incident wavelength. The physical characteristics of thin periodic structures have been described for shallow gratings [98, 99] and photonic crystal slabs [100, 101]; the method was to resolve the in-plane EM eigenmodes in the slabs. Such analysis has rarely been conducted for photonic MM slabs. In this section, we show typical results for fishnet MM slabs based on the analysis on in-plane eigenmodes.

Several theoretical reports intended to clarify the origin of the negative refractive index in fishnet MMs [43, 102, 103]. They attributed the effective (ε, µ) to the electric and magnetic responses in the unit cell, as if ε and µ could be defined independently. As described in section I.3, the material parameters representing optical responses are in principle determined in a different manner. Another theory limited to normal incidence was reported [104]. Thus, a complete description of the fishnet MMs was lacking.

Figure 10(a) depicts the optical configuration for a fishnet MM slab comprising a perforated MIM (Au–Al2O3–Au) structure. The structural parameters such as the periodicity and diameter of holes were taken from [19]. An analysis for an experimental sample was briefly reported [105]. To show explicitly the EM eigenmodes while avoiding effects of the substrate, we here consider the MM slab to be free-standing in air, which was set such that the square lattice is parallel to the xy axes. The plane of incidence, defined by the wave vectors of the incident plane wave and the reflected wave, was set parallel to the xz-plane. In the optical configuration, the in-plane wave number \( k_{\parallel} \) induced by the incident wave vector \( k \) is equal to the \( x \) component \( k_x \). The in-plane eigenmodes are presented in the \( (k_{\parallel}, \omega) \)-plane.

Diffraction, if present, should be subtracted additionally from the right-hand side of equation (13). The optical spectra were computed by the numerical method similar to that used for figure 9(b). The permittivity of Au was taken from reference
homogenized medium. EM-power-flow properties favor thicker fishnet MMs owing to their simple inner EM dynamics. More thorough analysis for the fishnet MM slabs was very recently reported in [106].

The analysis described here was useful in resolving the resonant modes in another MM [84] and a plasmonic crystal [107]. Recently, a dispersion analysis for a left-handed antenna was also conducted at microwave frequencies [108]. It is thus likely that this analysis method will become a standard even for MMs, similarly to diffractive gratings and photonic crystal slabs.

Figure 11 shows schematic illustrations of wave dynamics at oblique incidence. Arrows denote wave vectors. Figure 11(a) depicts actual wave dynamics that a ray of oblique incidence shifts backward for the x-axis. From outside the SMIM, this effect seems to be negative refraction as drawn in figure 11(b). If it is assumed that the fishnet MM could be treated as homogeneous medium, the only one possible wave vector inside the homogenized fishnet MM is the vector indicated in figure 11(b). From comparison of the illustrations, it is obvious that we have provided a firm ground for the wave dynamics in fishnet MM slabs (see figure 10).

4.2. A 3D SMIM

3D MMs are usually produced by stacking layers. Here we consider a 3D MM having a stratified multilayer structure. In lossless multilayers, the eigenmodes were analyzed in the 1970s [109]. In photonic crystals, the eigenmode analysis was based on a lossless composite [54, 55]. In MMs, the composites are usually lossy because they include metals. It was therefore necessary to extend the analysis to a framework...
using complex permittivity in composites. The extended analysis was first conducted for SMIMs [110]. Figure 12(a) shows a schematic of a SMIM. EM fields in SMIMs can be treated in a rigorous manner by employing the transfer matrix method [49]. Although the eigenvalues in the transfer matrix have complex values, it was found that the Bloch state, defined by $K$ in the following equation

$$T \mathbf{v} = \exp(iKa) \mathbf{v},$$

is well defined and describes propagation along the $\pm z$ directions. Here, $T$ is the $2 \times 2$ transfer matrix and $\mathbf{v}$ is a set of EM-field components representing the $\pm z$ propagations; $\mathbf{v}$ consists of electric field $(E_x^+, E_y^+)$ and magnetic field $(H_x^+, H_y^+)$ for the TE and TM polarizations, respectively.

A thin metallic slab with thickness of several tens nm was recognized as an interesting element in MMs since it was pointed out that the slab can serve as an optical imaging plate with super-resolution [111]. This feature was confirmed in an experiment detecting near-field images at a near-ultraviolet frequency [112].

Theoretical studies [113, 114] suggested that SMIMs can function as artificial media for realizing super-resolution. Accordingly, experiment showed the positive results [115, 116]. In theoretical evaluations, SMIMs have often been explained based on effective medium theory [49, 115, 116], in which it was assumed that SMIMs are uniaxial media of effective permittivity tensors. Under this assumption, SMIMs have hyperbolic constant-frequency contours, which suppress diffraction and lead us to super-resolution lens, so-called ‘superlens’. However, the theory relies on the effective permittivity and/or permeability tensors. As pointed out in section 1, it is in principle difficult to justify this assumption. We here examine a SMIM in a rigorous manner by employing the transfer matrix.

Figure 12(b) shows reflectance spectra of a SMIM of (Ag 30 nm/SiO$_2$ 30 nm) at various incident angles for the TM (dashed lines) and TE (solid lines) polarizations. The SMIM was set to contain 2001 layers and can be regarded as having semi-infinite thickness. The top and bottom layers were set as SiO$_2$ layers of 15 nm thickness to ensure the symmetric unit cell along the $z$-axis. The reflectance spectra, computed by incorporating the scattering matrix algorithm [50], show qualitative differences for the TM and TE polarizations. In figure 12(a), incident waves at the TM and TE polarizations satisfy $H_{in} || y$ and $E_{in} || y$, respectively. The largely decreased

**Figure 12.** (a) Optical configuration for a semi-infinitely thick SMIM. Gray and pale blue denote metal (M) and insulator (I), respectively. (b) Reflectance spectra at various incident angles. (c) Transmission band at TM polarization (blue). The definition is described in the text, and $k_0$ is the wave number of light in vacuum. (d) Transmission band at TE polarization (red). (e) Constant-frequency contour of the TM mode at 3.5 eV, indicated by a purple dashed line in (c). Blue solid and dashed lines denote the real and imaginary parts of $k_z$, respectively. (f) Constant-frequency contour of the TE mode at 3.5 eV, indicated by a purple dashed line in (e). Red solid and dashed lines denote the real and imaginary parts of $k_z$, respectively.
reflectance is located at about 3.5 eV for the TM polarization and is shifted to higher energies for the TE polarization, implying that EM eigenmodes are definitely different for these polarizations.

Figure 12(c) shows the transmission TM band (blue). The horizontal axis is \( k_z / k_0 \), where \( k_0 \) is the wavenumber of light in vacuum, and the vertical axis is photon energy \( \hbar \omega \) in eV. The transmission band was obtained from the condition \( 0 < \text{Im}(K) < 0.8 \) for a wave propagating along the \( +z \)-direction. In SMIMs, transmissive bands are pseudo-photon bands defined by the relatively small imaginary part of \( K \). A wide transmission band appears above 3.0 eV, corresponding to the decrease of reflectance. As indicated in figure 12(b), the photon energy at which TM-polarized reflectance significantly decreases was almost independent of the incident angles; the \( R_{TM} \) behavior is consistent with the flat bottom edge of the transmission band. Also, the transmission band spreads for large \( k_z / k_0 \) such as \( |k_z / k_0| \geq 2 \), suggesting that the surface plasmon polaritons at the Ag/SiO\(_2\) interface contribute to the transverse light propagation.

Figure 12(d) shows the transmission TE band (red). Except for \( k_x = 0 \), as the TE-polarized reflectance spectra implied, the transmission TE band is distinct from the TM band in figure 12(c). The spectral shift of \( R_{TE} \) in figure 12(b) corresponds to the quadratic bottom of the TE band.

Figures 12(e) and (f) show constant-frequency contours at 3.5 eV for the TM and TE polarizations, respectively. Solid lines denote the real parts of \( k_z \) and dashed lines correspond to the imaginary parts. At the TM polarization, a concave contour appears in the real part. Concave contours are useful for achieving super-resolution in the transferred images because they suppress the diffraction. If one considers the constant-frequency contour of light in vacuum, expressed as \( (k_x / k_0)^2 + (k_z / k_0)^2 = 1 \), then it is obvious that the concave dispersion represents unusual light propagation in the SMIM. On the other hand, at the TE polarization, \( \text{Re}(k_z) \) decreases as \( \text{Re}(k_x) \) increases.

The present rigorous analysis offers a quantitative evaluation of optical loss in SMIMs. \( \text{Im}(k_x) \) is responsible for the exponential decay of Bloch waves. Figures 12(d) and 12(f) show that \( \text{Im}(k_x) \)’s increase as the value \( |k_z / k_0| \) increases. Although realistic evaluations of the optical loss in a superlens composed of SMIMs are often omitted in experiments [115, 116], the present analysis is helpful for a quantitative estimation of the loss and improvement of the structural details.

5. Summary and future expectations

We have outlined the progress in photonic MMs, based mainly on experimental results and realistic simulations. The extension of the range realized by refractive index was one of the achievements (section 2) and was exploited to design subwavelength optical devices in a practical manner (section 3). In particular, a subwavelength polarizer has been experimentally demonstrated. Subwavelength optical devices resulted in significant miniaturization. They will be a key application of photonic MMs.

We furthermore outlined how to properly describe photonic MMs without relying on any assumption and/or model (section 4). The method consists of resolving EM eigenmodes in photonic MMs and examining EM-field distributions. Two representative photonic MMs, a fishnet MM slab and a 3D SMIM, were illustrated based on the common method in artificial EM materials, and the features of the photonic MMs were clarified.

In the last decade, the effective (\( \varepsilon, \mu \)) frame was usually used in MM studies. However, its validity is quite limited and is hardly justified in most cases (section 1). By revisiting the fundamentals of EM fields and matter, a theory to reconsider the original (\( \varepsilon, \mu \)) frame in solid state physics was introduced and a few lessons for MMs were drawn (section 1.3).

Despite the rapid progress in photonic MMs, their uncovered potential seems to be quite limited. Most studies on photonic MMs were conducted in the linear response regime. In the near future, the diverse field of nonlinear MMs will be explored. As frequently mentioned, MM slabs are far thinner than incident wavelengths. To obtain large nonlinearity, MM slabs will need to be thicker, comparable in thickness to the coherent length in nonlinear processes. It would be a breakthrough to design and fabricate the thicker MM slabs.

MMs are a naturally dense array of artificial metallic nanostructures and therefore meet the requirement for EM-field rectification. It was recently reported that a memory device working at GHz frequencies incorporates SRRs for magnetic-field rectification [117]; the device suggests the potential of combining photonic MMs with existing electronic devices. Locally enhanced and direction-controlled EM fields can be managed with photonic MMs. Novel optoelectronic devices incorporating photonic MMs await invention.

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