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PAPER

Weak-value amplification for Weyl-point separation in momentum space

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E-mail: hailuluo@hnu.edu.cn**Keywords:** photonic spin Hall effect, weak measurements, spin–orbit coupling

Abstract

The existence of Weyl nodes in the momentum space is a hallmark of a Weyl semimetal (WSM). A WSM can be confirmed by observing its Fermi arcs with separated Weyl nodes. In this paper, we study the spin–orbit interaction of light on the surface of a WSM thin film. Our results show that the spin-dependent splitting induced by the spin–orbit interaction is related to the separation of Weyl nodes. By proposing an amplification technique called weak measurements, the distance of the nodes can be precisely determined. This system may have an application in characterizing other parameters of WSM.

1. Introduction

Weyl fermions have been proposed and long studied in quantum field theory, but this kind of particles has not yet been observed as a fundamental particle in nature. Recent research found that Weyl fermions can appear as quasiparticles in a Weyl semimetal (WSM) [1–5]. WSM is a new state of material that hosts separated band touching points—Weyl nodes—with opposite chirality [6–9]. The Weyl nodes come in pairs in bulk Brillouin zone when time-reversal or inversion symmetry is broken. Recently, ideal Weyl nodes was achieved in artificial photonic crystal structures [10]. WSM has attracted much attention due to its many exotic properties induced by the Weyl nodes, such as anomalous Hall effect [1, 11], surface states with Fermi arcs [12, 13], peculiar electromagnetic response [14, 15], and negative magneto-resistivity [16–19]. A WSM can be proved by observing its Fermi arcs with separated Weyl nodes. The experiment to observe Weyl nodes in TaAs or MoTe₂ by angle-resolved photoemission spectroscopy was reported [20, 21]. Due to the experimental resolution and spectral linewidth, the nodes in other WSM materials, such as NbP and WTe₂, may become difficult to be directly observed [22–24].

In this paper, we provide an alternative method to demonstrate the existence of Weyl-point separation in momentum space. Assume the WSM contains only a pair of Weyl nodes with broken time reversal symmetry [1, 25, 26], as illustrated in figure 1. The projection of the two Weyl nodes connects the ending points of Fermi arc on the Brillouin zone surface. The separation of the nodes is along the k_z direction. When the electromagnetic wave incidents on the surface without Fermi arc states, the spin–orbit interaction of light occurs and manifests itself as tiny splitting of left- and right-circular components. This phenomenon within visible wavelengths is known as photonic spin Hall effect [27–29]. We find that the interaction in WSM is still very weak, and a method called quantum weak measurements is introduced for detection [30–33].

The concept of weak measurements was proposed in the context of quantum mechanics [34–36]. There are three key elements in a weak measurement schema, namely, preselected state, postselected state, and weak coupling between the observation system and measuring pointer. In our weak measurement scheme, the spin–orbit interaction of light on WSM surface provides the weak coupling. Figure 2(a) shows the interaction weakly separates the left and right-circularly polarized components of the light with both in-plane and transverse shifts. The polarization state of light is taken as the system state. Labeling the preselected state and postselected state are $|i\rangle$ and $|f\rangle$, respectively, the result of the whole system called weak value is given by

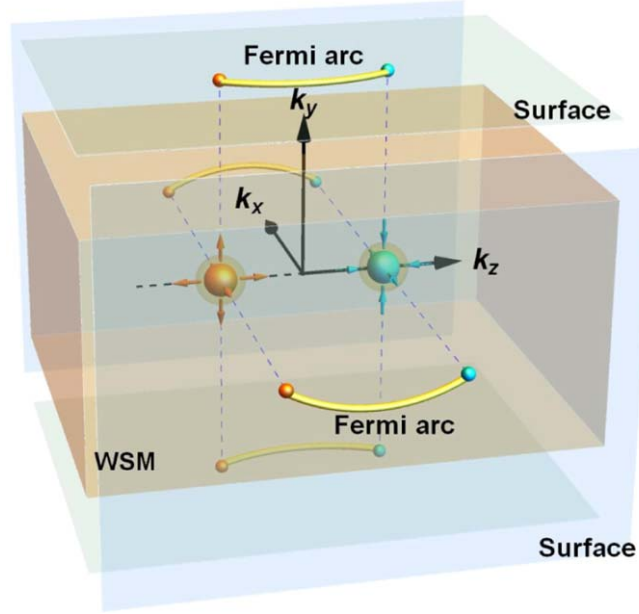


Figure 1. Illustration of a Weyl semimetal with Fermi arcs on its surface of Brillouin zone connecting projections of a pair of Weyl nodes with opposite chirality. The Weyl nodes are shown as the orange and cyan pellets with outward and inward arrows, behaving as magnetic monopoles with topological charges. Note that the surfaces perpendicular to k_z axis does not support arcs.

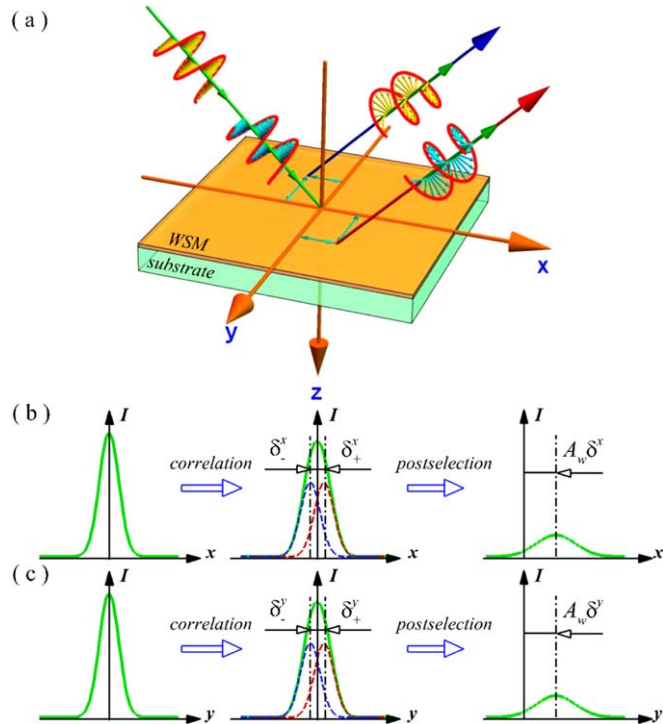


Figure 2. (a) Schematic of photonic spin Hall effect induced by spin-orbit interaction of light in a WSM-substrate system. Weak measurements with preselected and postselected ensembles for the enhancement of (b) in-plane shift and (c) transverse shift are introduced. The weak coupling separates the wave function into two spin components, which is denoted by red dotted line and blue dotted line. $\delta_x^{s,y}$ and $\delta_y^{s,y}$ indicate the small shifts of left- and right-circularly polarized components, respectively. The final pointer position is proportional to A_w after postselection. The green solid lines indicate the shapes of total wave function.

$$A_w = \frac{\langle f | \hat{A} | i \rangle}{\langle f | i \rangle}, \quad (1)$$

in which $\hat{A}$ is the system operator of an observable. By making $\langle f | i \rangle \rightarrow 0$, the weak value becomes very large and outside the eigenvalue range of the observable. We use this feature to detect the tiny shifts induced by the

photonic spin Hall effect. This process is described in figures 2(b) and 2(c). Weak value A_w can amplify tiny in-plane (x) shift or transverse (y) shift to a large final pointer position. The amplified shift is proportional to the weak value. In the following, we discuss the spin-orbit interaction of light reflected on WSM-substrate interface.

2. Theory and discussion for spin-orbit interaction of light

We establish a model to describe the spin-orbit interaction in the WSM-substrate system. The refractive index of substrate we set is $n = 1.5$. A WSM film with thickness d is placed on a substrate. Two Weyl nodes are separated by a wave vector $\pm \mathbf{b} = \pm(0, 0, b)$ in the Brillouin zone. b is in units of $2\pi/a$ throughout this paper with a representing the lattice spacing. Parameters for WSM are chosen as $d = 100$ nm, $v_F = 10^6$ m s⁻¹, and $a = 3.44$ Å. v_F is the Fermi velocity. A monochromatic Gaussian beam with beam waist $w_0 = 27$ μm and wavelength $\lambda = 633$ nm impinges from air to this system.

For the bounded beam, the polarization states of different angular spectrum components can be written as $|H(k_i)\rangle$ and $|V(k_i)\rangle$. To denote central wave vector of wavepacket, the coordinate frames (x_i, y_i, z_i) and (x_r, y_r, z_r) are used, where the subscripts i and r , respectively, represent incident and reflected beam. After reflecting at the air-WSM-medium interface, the rotations of polarizations for each angular spectrum components are different. Introducing the boundary condition $k_{ry} = k_{iy}$, the total action of the reflection can be described by $[|H(k_r)\rangle |V(k_r)\rangle]^T = M_R [|H(k_i)\rangle |V(k_i)\rangle]^T$, where M_R is

$$\begin{bmatrix} r_{pp} - \frac{k_{ry} \cot \theta (r_{ps} - r_{sp})}{k_0} & r_{ps} + \frac{k_{ry} \cot \theta (r_{pp} + r_{ss})}{k_0} \\ r_{sp} - \frac{k_{ry} \cot \theta (r_{pp} + r_{ss})}{k_0} & r_{ss} - \frac{k_{ry} \cot \theta (r_{ps} - r_{sp})}{k_0} \end{bmatrix}. \quad (2)$$

r_{ab} is the Fresnel reflection coefficients of the WSM-substrate system with a and b standing for either p or s polarization. Consider a horizontal polarization state of incidence. The state with Taylor series expansion of the Fresnel coefficients r_{pp} and r_{sp} after reflection becomes

$$\left(r_{pp} - \frac{k_{rx} \partial r_{pp}}{k_0 \partial \theta} \right) |H(k_r)\rangle + \frac{k_{ry} \cot \theta (r_{sp} - r_{ps})}{k_0} |H(k_r)\rangle - \frac{k_{ry} \cot \theta (r_{pp} + r_{ss})}{k_0} |V(k_r)\rangle + \left(r_{sp} - \frac{k_{rx} \partial r_{sp}}{k_0 \partial \theta} \right) |V(k_r)\rangle. \quad (3)$$

Here, $k_0 = \omega/c$ is the wave vector in vacuum and θ is the angle of incidence. We analyze equation (3) in spin basis to reveal the splitting of spin components. The total momentum wave function in the spin basis with $r_{ps} = r_{sp}$ is obtained as

$$|\psi_{\pm}\rangle = \frac{1}{\sqrt{2}} \left[r_{pp} - \frac{k_{rx} \partial r_{pp}}{k_0 \partial \theta} \mp i \left(r_{sp} - \frac{k_{rx} \partial r_{sp}}{k_0 \partial \theta} \right) \pm i \frac{k_{ry} (r_{pp} + r_{ss}) \cot \theta}{k_0} \right] |\pm\rangle |\Phi\rangle. \quad (4)$$

$|\Phi\rangle$ is the beam with a Gaussian distribution.

Based on equation (4), a calculation by taking into account the paraxial approximation yields the in-plane spatial and angular shifts as

$$\langle \Delta_{\pm}^x \rangle = \mp \text{Re} \left[\frac{\partial r_{sp} / \partial \theta \pm i \partial r_{pp} / \partial \theta}{(r_{pp} \mp i r_{sp}) k_0} \right], \quad (5)$$

$$\langle \Theta_{\pm}^x \rangle = \mp \frac{1}{z_R} \text{Im} \left[\frac{\partial r_{sp} / \partial \theta \pm i \partial r_{pp} / \partial \theta}{(r_{pp} \mp i r_{sp}) k_0} \right], \quad (6)$$

where the z_R is the Rayleigh length. There also exist shifts along the transverse direction

$$\langle \Delta_{\pm}^y \rangle = \mp \text{Re} \left[\frac{(r_{pp} + r_{ss}) \cot \theta}{(r_{pp} \mp i r_{sp}) k_0} \right], \quad (7)$$

$$\langle \Theta_{\pm}^y \rangle = \mp \frac{1}{z_R} \text{Im} \left[\frac{(r_{pp} + r_{ss}) \cot \theta}{(r_{pp} \mp i r_{sp}) k_0} \right]. \quad (8)$$

The results in equations (5)–(8) is the simplest form to describe the behavior of spin-orbit interaction of light. The real and imaginary parts of shift in angular spectrum representation correspond to the spatial and angular

shifts [37]. From above equations, the spin–orbit interaction of light is influenced by the reflection coefficients. Therefore, a discussion of the reflection coefficients can give a insight into the spin–orbit interaction. The reflection coefficients of WSM-substrate system are related to the distance of Weyl nodes.

To obtain the Fresnel reflection coefficients, the boundary conditions for electromagnetic field and the Ohm's law should be taken into account [38–41]. Assuming the electric (magnetic) fields in air and substrate are, respectively, represented by $\mathbf{E}_1$ and $\mathbf{E}_2$ ($\mathbf{H}_1$ and $\mathbf{H}_2$), the boundary conditions are $\hat{n} \times (\mathbf{E}_2 - \mathbf{E}_1) = 0$, $\hat{n} \times (\mathbf{H}_2 - \mathbf{H}_1) = \mathbf{J}_s$. $\hat{n} = -\hat{z}$ is the unit vector normal to the WSM-substrate interface, and $\mathbf{J}_s = \sigma_{ij} \mathbf{E}$ is the surface current density. σ_{ij} denotes the surface conductivity tensor in WSM with $i, j = x, y$. Solving the boundary condition expressions, the coefficients are given by

$$r_{pp} = \frac{\alpha_+^T \alpha_-^L + \beta}{\alpha_+^T \alpha_+^L + \beta}, \quad (9)$$

$$r_{ss} = -\frac{\alpha_-^T \alpha_+^L + \beta}{\alpha_+^T \alpha_+^L + \beta}, \quad (10)$$

$$r_{ps} = r_{sp} = -2 \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{k_{iz} k_{tz} \sigma_{xy}}{\alpha_+^T \alpha_+^L + \beta}, \quad (11)$$

where $\alpha_{\pm}^L = (k_{iz} \varepsilon \pm k_{tz} \varepsilon_0 + k_{iz} k_{tz} \sigma_{xx} / \omega) / \varepsilon_0$, $\alpha_{\pm}^T = k_{iz} \pm k_{tz} + \omega \mu_0 \sigma_{yy}$, and $\beta = \mu_0 k_{iz} k_{tz} \sigma_{xy}^2 / \varepsilon_0$. $k_{iz} = k_0 \cos \theta$ and $k_{tz} = nk_0 \cos \theta_t$. θ_t is the refraction angle; n is the refractive index of the substrate; ε_0, μ_0 are permittivity and permeability in vacuum; ε is the permittivity of substrate; $\sigma_{xx,yy}$ and $\sigma_{xy,yx}$ are the longitudinal and Hall conductivities, respectively.

The above boundary condition is valid for WSM ultra-thin film ($a \ll d \ll \lambda$). The film thickness in the WSM film can be chosen as $d \sim 100$ nm. In this limit, the existence of Weyl nodes can lead to giant Kerr and Faraday rotations, which are larger than those in topological insulators [25]. For topological insulator, the energy gap can remain almost unchanged until the thickness of a film is reduced to several nanometres [42]. The large polarization rotation in WSM is governed by the optical conductivity in WSM. For thin film WSM, the conductivity is given by $\sigma_{ij} = d\sigma_{ij}^B$ with σ_{ij}^B being the conductivity of the bulk WSM. The bulk conductivity σ_{ij}^B is obtained from the Kubo formalism. For a general case of multi-Weyl semimetal in order J , its nodes is stabilized by protection of point group symmetry [26, 43]. And $J = 1$ corresponds to WSMs. Near two Weyl nodes located at $\pm \mathbf{b} = b\hat{z}$, the Hamiltonian of WSMs can be written as

$$H = E_0 \left(\frac{k_- \sigma_+}{k_m} + \frac{k_+ \sigma_-}{k_m} \right) + \hbar v_F q_z \sigma_z, \quad (12)$$

where $k_{\pm} = k_x \pm ik_y$, $\sigma_{\pm} = \frac{1}{2}(\sigma_x \pm i\sigma_y)$ with σ representing the Pauli matrices, and $q_z = k_z \mp b$ is the effective wave vector along the k_z direction. E_0 and k_m are material-dependent parameters related to energy and momentum, respectively. For brevity, we set them as $E_0 = k_m = 1$ throughout this paper.

For Weyl points, the most significant feature is the monopole-like structure of the Berry curvature. Based on the calculation from equation (12), we plot the Berry curvature shown in figure 3. The sources and sinks of Berry curvature depend on the positions of the two Weyl points. The influence on the spin–orbit interaction of light by Berry curvature varies with the separation of Weyl notes. This is closely related to the optical conductivity in WSM.

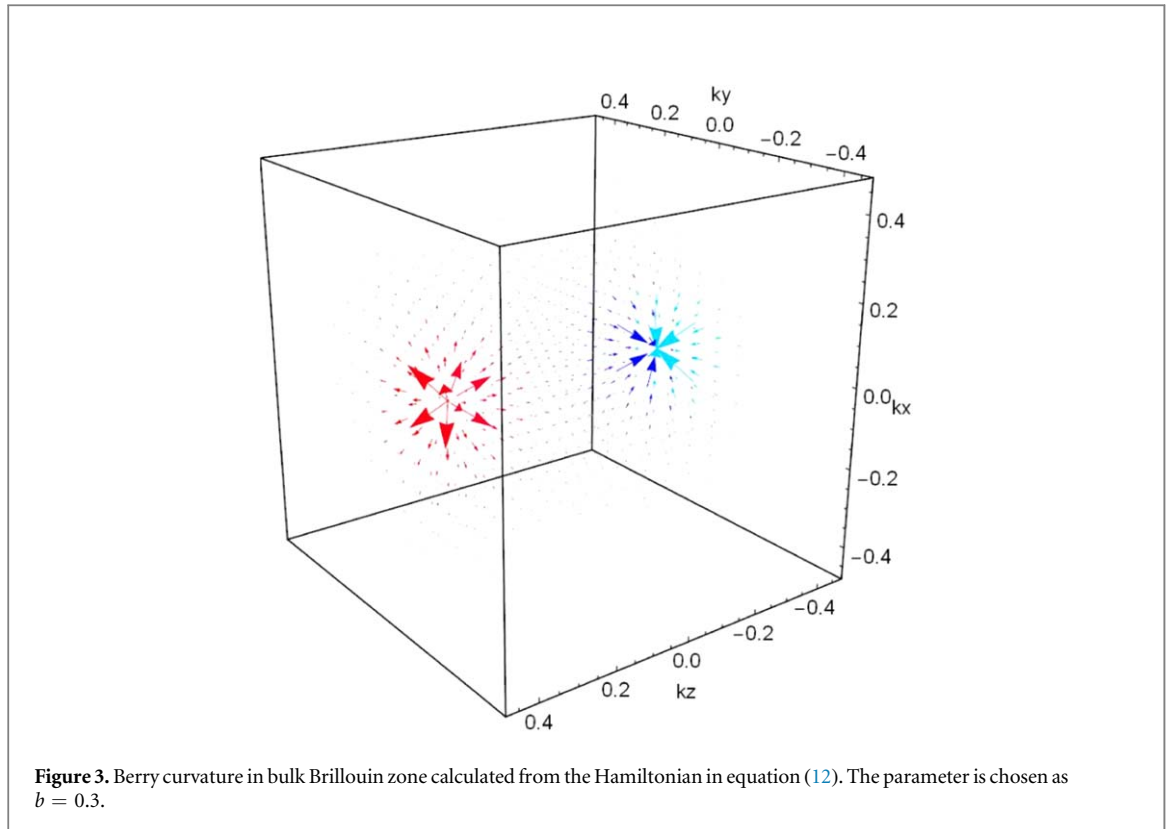
We next consider optical conductivities obtained from the low-energy Hamiltonian. We calculate the conductivity of WSMs from the Kubo formula in noninteracting limit. The real parts of the conductivities in the clean limit at zero temperature are given by

$$\text{Re}[\sigma_{xx}^B] = \frac{e^2 |\omega|}{24\pi \hbar v_F}, \quad (13)$$

$$\text{Re}[\sigma_{xy}^B] = be^2 \left[\frac{1}{2\pi^2 \hbar} + \frac{\omega^2}{24\pi^2 \hbar v_F^2 (k_c^2 - b^2)} \right]. \quad (14)$$

$\omega_c = v_F k_c$ with v_F and k_c representing Fermi velocity and the momentum cutoff along k_z axis, respectively. For the longitudinal conductivity, the intraband contribution of the optical conductivity in the undoped case (the chemical potential $\mu = 0$) can be neglected. The imaginary parts of the conductivities are obtained by introducing the Kramers–Kronig relation [44], we have

$$\text{Im}[\sigma_{xx}^B] = -\frac{e^2 \omega}{24\pi^2 \hbar v_F} \ln \left| \frac{\omega^2 - \omega_c^2}{\omega^2} \right|, \quad (15)$$



$$\text{Im}[\sigma_{xy}^B] = - \left[\frac{e^2 b}{2\pi^3 \hbar} + \frac{e^2 \omega^2 b}{24\pi^3 \hbar v_F^2 (k_c^2 - b^2)} \right] \ln \left| \frac{\omega_c - \omega}{\omega_c + \omega} \right| - \frac{e^2 \omega \omega_c b}{12\pi^3 \hbar v_F^2 (k_c^2 - b^2)}. \quad (16)$$

The optical response of the WSM changes with photon energy due to the dynamic conductivity. The conductivity of WSM shows a characteristic frequency dependence. Equations (13)–(16) hold at the low frequency limit $\omega < \omega_c$ [45, 46]. The corresponding theoretical predictions for the optical conductivity have been experimentally verified [47]. For other materials such as topological insulators, the dynamic conductivity arises as a function of temperature and photon energy in the surface states [48]. We see that the Hall conductivity brings about the influence of the Weyl nodes. If $b = 0$, indicating the annihilation of Weyl nodes, the conductivity σ_{xy} vanishes. And the Fresnel reflection coefficients reduce to a general case [40].

A direct result of the spin–orbit interaction of light on WSM’s surface is the beam shift in equations (5)–(8). We analyse the beam shift based on the optical conductivity and the reflection coefficient. Due to the complex optical conductivity, the reflection coefficient associated with the location of Weyl nodes are also complex numbers. The imaginary part of the complex reflection coefficient is related to the angular shift. To further discuss it, we write the coefficients as $r_{ab} = R_{ab} e^{i\varphi_{ab}}$ with R_{ab} and φ_{ab} labeling modulus and phase, respectively. Since the condition of the conductivity is at low frequency limit, the wavelength λ we chose is 633 nm, with the corresponding energy of the photons 1.96 eV.

In figure 4, the modulus for three different distances of the Weyl nodes are plotted as a function of incident angle. For a small separation of the Weyl nodes ($b = 0.05$), the behaviors of the reflection coefficients are nearly the same as the case without WSM film. With a larger b , the angle of $|r_{pp}| = 0$ vanishes. Such an angle in the case with zero crossing reflection coefficients is known as the Brewster angle. Near this incident angle, the action of spin–orbit interaction may become peculiar, such as resulting in a very large spin-dependent splitting [32, 49]. As the separation increases, r_{sp} and r_{ss} become large gradually, but the influence of the Weyl nodes to r_{ss} is not obvious between $b = 0.25$ and $b = 0.45$. To show the contribution of the imaginary parts to the reflection coefficients, figure 4(d) is provided for $b = 0.25$. The imaginary parts of the reflection coefficients, which can lead to angular shift, is not small enough to be neglected. We point out that for other b there also exists non-negligible imaginary part in the optical conductivity.

The spatial and angular shifts are related to the real and imaginary parts in equations (5)–(8). As the reflection coefficients are complex numbers, the shift contains both spatial and angular components. To show how the WSM impact on the spin–orbit interaction of light in the system, we discuss the shifts based on the film thickness d and the separation of the Weyl nodes represented by the parameter b . At a fixed angle of incidence $\theta = 50^\circ$, the influence of film thickness on beam shifts is shown in figures 5 and 6. Both spatial and angular shifts show a small change with the thickness d . The optical conductivity of WSM film is proportional to the thickness d . If $d = 0$, indicating a system without WSM, we have $r_{sp} = 0$. This case can lead to the transverse spatial shift as

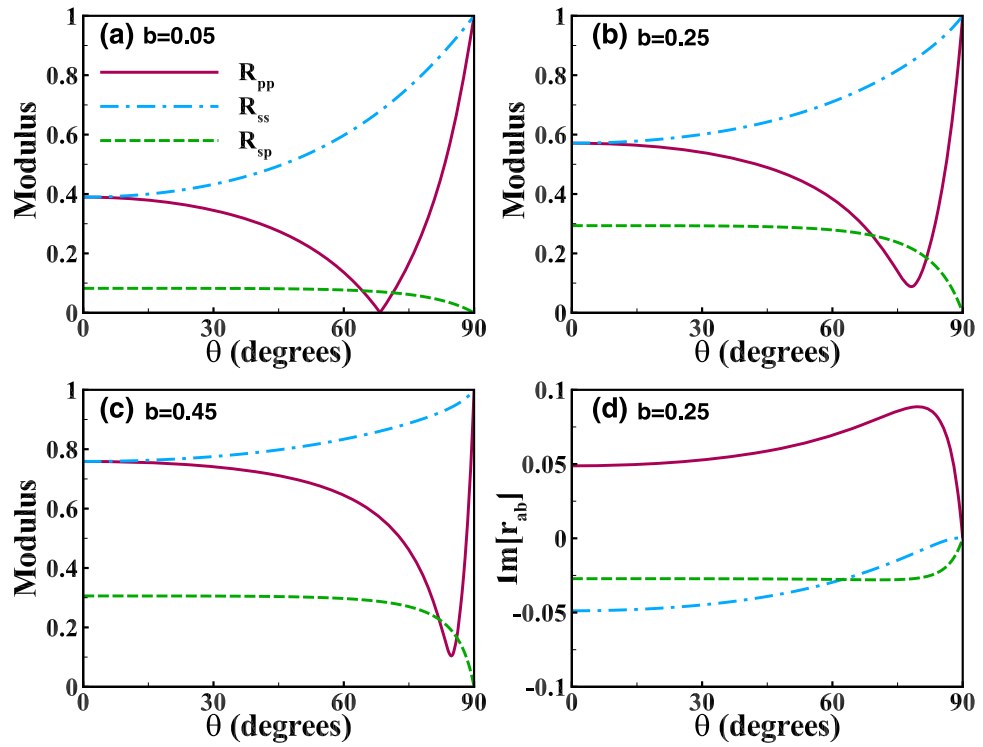


Figure 4. Modulus of reflection coefficients of the WSM-substrate system as a function of incident angle θ for (a) $b = 0.05$, (b) $b = 0.25$, and (c) $b = 0.45$. (d) Imaginary part of the coefficients with $b = 0.25$.

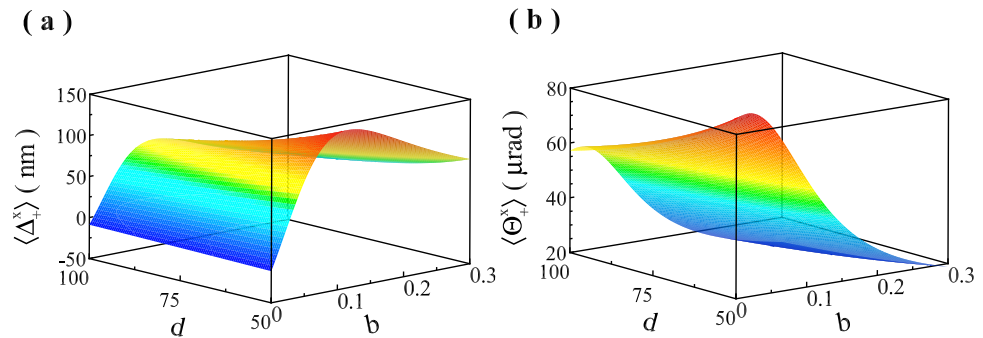


Figure 5. In-plane (a) spatial and (b) angular shifts as a function of film thickness d and parameter b (in units of $2\pi/a$) associated with the location of the Weyl node. We assume an incident beam with $|H\rangle$ polarization. The angle of incidence of the light beam is set as 50° . The refractive index of substrate is $n = 1.5$.

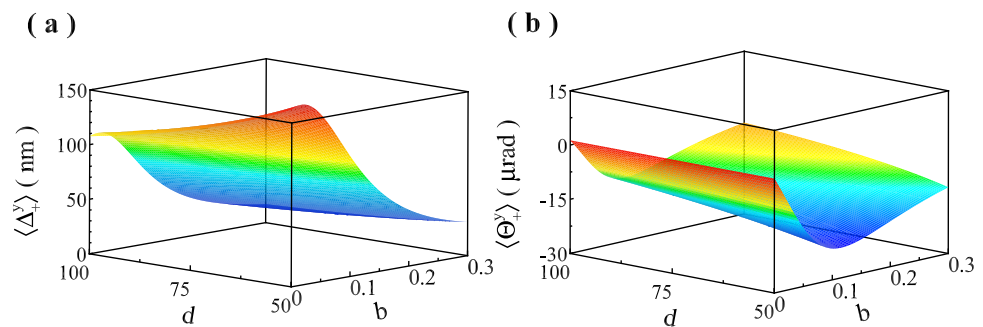


Figure 6. Transverse (a) spatial and (b) angular shifts as a function of film thickness d and parameter b .

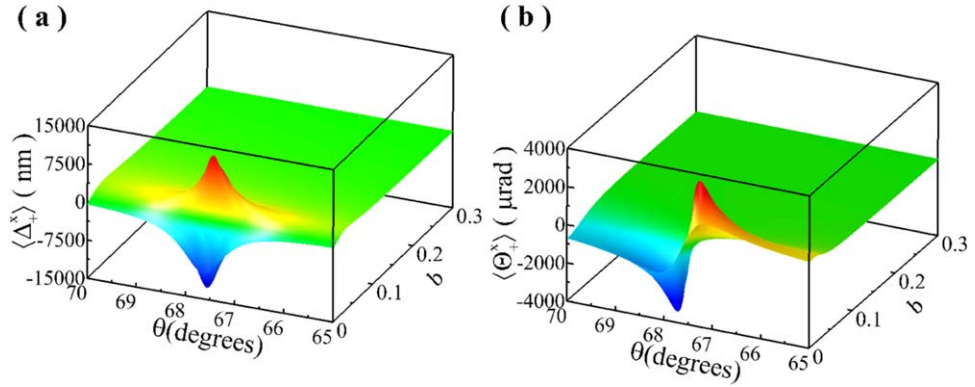


Figure 7. In-plane (a) spatial and (b) angular shifts as a function of incident angle θ and parameter b .

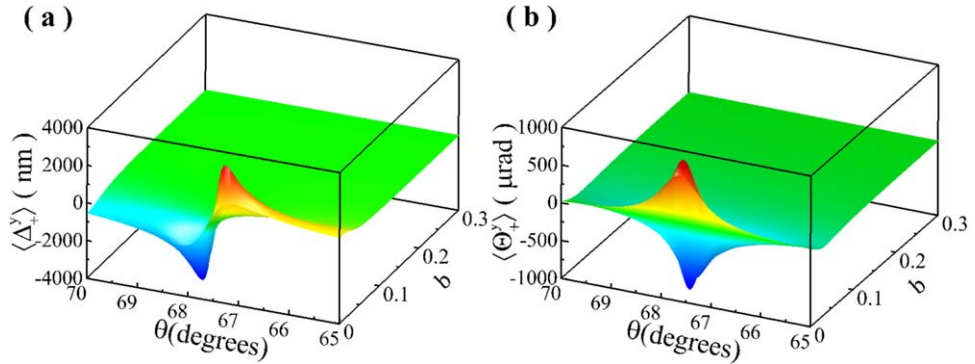


Figure 8. Transverse (a) spatial and (b) angular shifts as a function of incident angle θ and parameter b . The parameters are the same as in figure 7.

well [30–32], but it does not have in-plane splitting. Note that for some specific angles, such as Brewster angle, the shifts may become more sensitive to the variation of thickness. Under this situation, the optical effect combined with weak measurements can be applied to characterize the thickness of materials, such as identifying graphene layers [50, 51] and measuring nanometal film thickness [49]. The WSM coating affects the magnitude of the splitting with varied separation of Weyl nodes. From equation (8), the WSM coating is also responsible for the existence of transverse angular shift due to the nonzero cross-polarization coefficient and the complex r_{pp} and r_{ss} . The angular shift in figure 6(b) is nearly zero since we have $r_{sp} = 0$ with the separation of Weyl nodes $b = 0$.

Incident angle is an important factor to influence the coupling strength of the spin–orbit interaction of light. We next discuss the shifts related to incident angle θ and parameter b . The result in equations (5)–(8) is the approximate form of the behavior of spin–orbit interaction. If the separation of the Weyl nodes becomes zero, the terms containing r_{ps} vanish, and the expressions are invalid near the Brewster angle. Under such a situation, the small variations of the Fresnel coefficients should be taken into account. The precise shifts at initial position ($z_r = 0$) can be obtained by directly calculating the intensity centroid of reflected light beam. The precise result only modifies the shift in the limit that the Weyl points disappear ($b = 0$). For $b > 0$, it is almost the same as the one given by the approximate formula.

With the Weyl nodes, the transverse spatial shift can be very large. From equation (5), the in-plane spatial shift originates from the nonzero coefficient r_{sp} and the complex r_{pp} . r_{sp} is a quantity related to the separation of the Weyl point. A giant in-plane shift about $12 \mu\text{m}$ at $b \approx 0.02$ can be realized in the vicinity of the Brewster angle, as shown in figure 7(a). This is much larger than the case of transverse shift in figure 8(a), where the maximal value can reach about 3000 nm . Due to the divergence of whole light beam, it is still difficult to detect the giant shifts and measuring the separation of Weyl nodes. The sensitivity of the shifts to the parameters of WSM near the Brewster angle can improve precision during the weak measurement progress [49]. The sharp peak for the spatial shift occurs at $\theta \approx 67.5^\circ$ due to the term $(r_{pp}^2 + r_{sp}^2) \approx 0$ in equations (5) and (7). For both

in-plane and transverse angular shifts, $b = 0$ can also lead to a peak value near the Brewster angle. There is another peak for the transverse angular shift at $b \approx 0.02$, like the case of the in-plane spatial shift.

3. Quantum weak value amplification

Generally, the spin-orbit interaction of light on WSM surface is weak. In this section, we introduce quantum weak measurements to amplify the beam shifts for observation. This optical effect was first detected by weak measurements in air-glass system [30]. The spin-orbit interaction provides the weak coupling in weak measurements. For various reflecting surfaces, the effect is sensitive to their parameters. Therefore, using this system, some important parameters can be precisely determined, such as graphene layers and thickness of metal film [49, 50]. Here, we propose to use a similar experimental setup in [49–51] to investigate the Weyl nodes in WSM. A Gaussian beam generated by a laser passes through the first Glan laser polarizer to produce an incident state $|H\rangle$. The beam experiences a spin splitting after interacting with the WSM-substrate interface. This splitting containing both spatial and angular shifts provides the weak coupling in the weak measurement scheme, and the preselected state is $|i\rangle = F|H\rangle$. F is the reflection matrix given by

$$\begin{bmatrix} r_{pp} & r_{ps} \\ r_{sp} & r_{ss} \end{bmatrix}. \quad (17)$$

The preselected state $|i\rangle$ in the spin basis is

$$|i\rangle = \cos v|+\rangle + e^{i\gamma} \sin v|-\rangle. \quad (18)$$

$v = \arccos(|r_{pp} - ir_{sp}| / \sqrt{|r_{pp} - ir_{sp}|^2 + |r_{pp} + ir_{sp}|^2})$ and $\gamma = \arg(r_{pp} - ir_{sp}) - \arg(r_{pp} + ir_{sp})$. The weak value is naturally determined by preselected and postselected states. To reach a large weak value, we use the combination of a quarter-wave plate and the second Glan laser polarizer with suitable angles to postselect the final state [51]. After the beam goes through this combination, the postselected state is

$$|f\rangle = \sin v|+\rangle - e^{i(\gamma-2\phi)} \cos v|-\rangle, \quad (19)$$

in which the small deviation angle ϕ is also called as postselected angles. Since we deal with the spin basis, the observable $\hat{A}$ in equation (1) is Pauli operator

$$\sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \quad (20)$$

From above preselected and postselected states, the weak value σ_w is calculated as [51, 52]

$$\sigma_w = \frac{\langle f|\sigma_3|i\rangle}{\langle f|i\rangle} = i \cot \phi, \quad (21)$$

Theoretically, both the real and imaginary parts of weak value can enhance the tiny observable. The purely imaginary weak value σ_w can effectively amplifies the spatial shift due to the free evolution of the wave function in the momentum space, while the enhancement effect for the angular shift is very weak [53]. As a result, weak value σ_w can be measured by amplified pointer shift $\langle x, y \rangle$, which is obtained as

$$\left(\frac{z_r |\langle \Delta_+^{x,y} \rangle - \langle \Delta_-^{x,y} \rangle|}{z_R} - \frac{z_R |\langle \Theta_+^{x,y} \rangle - \langle \Theta_-^{x,y} \rangle|}{2} \right) \cot \phi. \quad (22)$$

The propagation-dependent shift in equation (22) can be effectively amplified in far field. We detect this beam shift in the experiment by a charge-coupled device. To achieve a large σ_w , one can make $\phi \rightarrow 0$ as much as possible. But in fact the shift has a maximum value when the postselected angle is close to zero. Under this situation, a modified weak measurements is required [54].

We plot the amplified shifts as functions of incident angle θ in figure 9 by setting the postselected angle ϕ as 0.2° . A large amplified factor $z_r \cot \phi / z_R \approx 79200$ can be obtained for spatial shifts. Assuming the weak value is real, the amplified factor $\cot \phi$ is only about 286 due to no propagation enlargement. Therefore, the imaginary weak value here is a good choice to detect the spatial displacement. The angular shift can be detected by a purely real weak value. We discuss four different separations of Weyl nodes, $b = 0.1$, $b = 0.2$, $b = 0.3$, and $b = 0.4$. In addition, the WSMs with thickness $d = 50$ nm and $d = 100$ nm are compared. The result shows that the in-plane and transverse shifts amplified by this factor can reach dozens of micrometers, which is detectable in the experiment. The enlargement of weak value for different film thickness is effective. The curves for different b appear peaks at different angles of incidence. This makes it feasible to detect the separations of Weyl nodes. On the other side, the beam shift is also sensitive to the thickness d . Therefore, our result may have applications in precision metrology of the thickness of WSMs. For the in-plane shift, a peak about $150 \mu\text{m}$ is achieved. The change of the shift with b become larger in the case of in-plane shift. Near some angle of incidence, the difference between two curves can reach about $250 \mu\text{m}$. This situation may be helpful to determine b , or even other parameters of WSM. In the experiment, we determine the

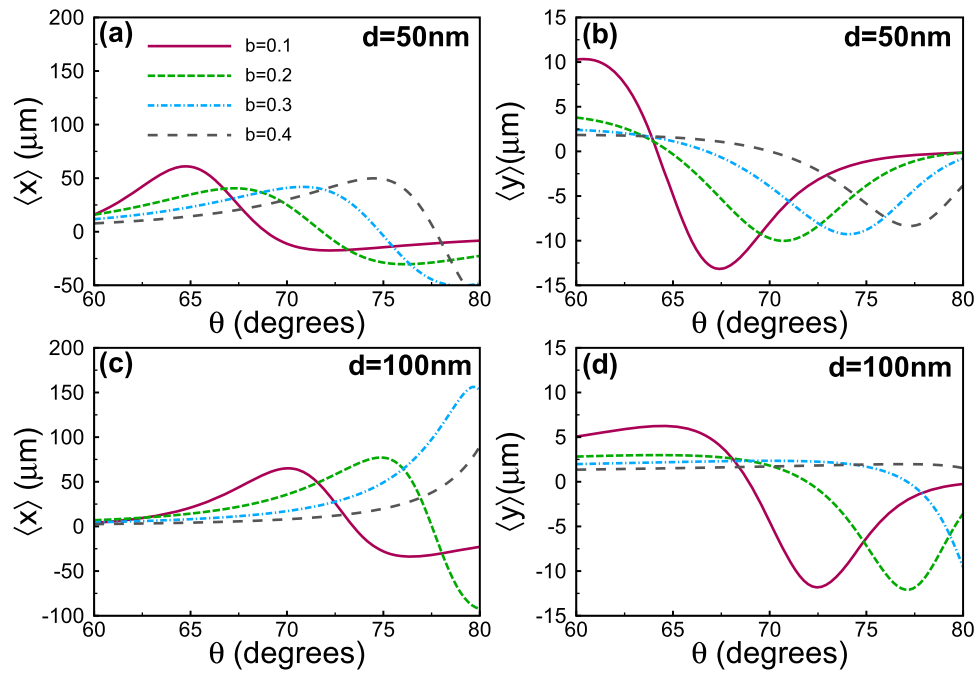


Figure 9. In-plane and transverse shifts amplified by purely imaginary weak value are shown as functions of incident angle θ . The postselected angle ϕ is chosen as 0.2° , and propagation distance z_r is 1000 nm.

Weyl-point separation by the curve of the shift as a function of b . For a WSM, its parameters, such as thickness, optical conductivity, and the order J of multi-WSM, may also influence to the shifts. Using the measured separation b , we can further examine the shift with variation of angle of incidence to avoid the influence and confirm the result [49]. Recently optical experiments on WSMs are still very limited [47, 55]. The wavelength we consider here is at the visible range that is accessible in an experiment.

4. Conclusion

In conclusion, we have theoretically discuss the spin-orbit interaction of light reflected on the WSM-substrate interface. We predict both in-plane and transverse shifts with the presence of WSM. The WSM we consider contains a pair of Weyl nodes, and the light incidents on the surface of WSM that does not support Fermi-arc electronic states. The analysis shows that the spin-dependent in-plane spatial and transverse angular shifts originate from the existence of Weyl points. Introducing a purely imaginary weak value, the spatial shifts can be effectively enlarged by a factor of $z_r|\sigma_w|/z_R \approx 8 \times 10^4$, which is 276 times larger than the one with real weak value. Due to the sensitivity to the wave vector $\pm \mathbf{b}$, measuring beam shift could become an alternative way to determine the distance of Weyl nodes in momentum space. Our results may open up a new experimental possibility for the investigations of optical responses into WSM.

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