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Quantum teleportation via a *W* state

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Abstract. We investigate two schemes of quantum teleportation with a W state, which belongs to a different class from the Greenberger–Horne–Zeilinger class. In the first scheme, the W state is shared by three parties, one of whom, called a sender, performs a Bell measurement. It is shown that the quantum information of an unknown state is split between two parties and recovered with a certain probability. In the second scheme, a sender takes two particles of the W state and performs positive operator valued measurements. For the two schemes, we calculate the success probability and the average fidelity. We show that the average fidelity of the second scheme cannot exceed that of the first one.

Contents

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1. Introduction

Entanglement is considered to be a fundamental resource of quantum information processing such as quantum cryptography [1], quantum teleportation [2], quantum computation [3] and so on. Since the seminal work of Bennett *et al* [2], there have been extensive works in the field of quantum teleportation in theory and experiment [4]–[6]. Entangled states, which are called quantum channels, make it possible to send an unknown state a long distance.

Quantum teleportation via bipartite pure entangled states can be classified into two types: standard teleportation (ST) and conclusive teleportation (CT) [7]–[9]. Maximally entangled states, i.e. Bell states, are used as quantum channels in standard quantum teleportation. On the other hand CT utilizes any pure entangled states. Due to the fact that non-maximally pure entangled states are used as quantum channels, the random outcome of the sender's measurement determines whether an unknown state is teleported surely or not.

Entanglement in three qubits is more complicated than that in two qubits. Recently, the entanglement of three qubits [10, 11] was classified into separable, biseparable, W, and Greenberger–Horne–Zeilinger (GHZ) states [12]–[14]. The GHZ class state cannot be transformed to the W class by the local operation and classical communication. Although many proposals have utilized the GHZ state in quantum teleportation [15]–[17] and some have suggested an implementation of the W state [18]–[21], teleportation using the W class state has only been studied in a few ways [22].

In this paper, we study two schemes of teleportation via a W state which are classified by two types of measurement. By calculating the total success probability and the average fidelity for each teleportation scheme, we can explore the entanglement properties of the W state. Because of the property of the W state [23], it is shown to be not always successful in recovering the unknown state.

In the first scheme, Alice teleports an unknown state to Charlie with the help of Bob. She performs the Bell measurement (BM) on the unknown state and her part of a W state. This scheme makes it possible to split and to recover the quantum information of the unknown state in the teleportation. After Alice's BM, the quantum information of the unknown state is split between Bob and Charlie. Depending on Bob's measurement on his part of the W state, Charlie recovers the unknown state with a certain probability of success.

In the second scheme, only Alice and Charlie participate in teleportation. She and he take two particles and one particle of the W state, respectively. Alice prepares a basis set of an unbalanced W state, the positive operator valued measurement (POVM) [24] on her three particles including the unknown state in teleportation. This type of the measurement is called an asymmetric POVM because the measurement basis is the W type with unequal weights. The asymmetric POVM is fully based on an original CT [7, 9] where the measurement set is built from an orthogonality between a measurement basis and a channel. From the calculation of success probability and average fidelity, we obtain the maximum values of the protocol, which cannot exceed those of the previous scheme.

The paper is organized as follows. In section 2, we briefly review quantum teleportation via the GHZ state. In section 3, we present two schemes of quantum teleportation with the W state. In section 4, the average fidelity of the two schemes is calculated. Finally, in section 5 we make some remarks on our results.

2. Quantum teleportation via GHZ states

We start with a brief review of quantum teleportation with a GHZ state in [15, 16]. It is similar to the ST with the Bell state, but three parties participate in quantum teleportation. In particular, Hillery, Bužek and Berthiaume emphasized quantum teleportation with the GHZ state as splitting and recovering of quantum information as a quantum secret sharing [15].

Suppose Alice wishes to teleport an unknown qubit to Charlie with the assistance of Bob. Alice begins with the unknown state

$$|\phi\rangle_{\mathrm{U}} = \alpha|z_-\rangle_{\mathrm{U}} + \beta|z_+\rangle_{\mathrm{U}}, \tag{1}$$

where $|\alpha|^2 + |\beta|^2 = 1$ and $z_\pm$ are the outcomes (+ or −) in measuring $\hat{\sigma}_z$. Alice, Bob and Charlie share the GHZ state given by

$$|\mathrm{GHZ}\rangle \equiv \frac{1}{\sqrt{2}}(|z_-z_-z_-\rangle + |z_+z_+z_+\rangle). \tag{2}$$

Alice combines the unknown state $|\phi\rangle_{\mathrm{U}}$ with her GHZ particle and performs the BM on her two particles. Before Alice's BM, the total state $|\Psi^{\mathrm{tot}}_{\mathrm{GHZ}}\rangle = |\phi\rangle_{\mathrm{U}} \otimes |\mathrm{GHZ}\rangle_{\mathrm{ABC}}$ can be written in terms of the Bell basis of her part

$$\begin{aligned}|\Psi^{\mathrm{tot}}_{\mathrm{GHZ}}\rangle = \tfrac{1}{2}[&|\Psi_+\rangle_{\mathrm{UA}}(\alpha|z_-z_-\rangle + \beta|z_+z_+\rangle) + |\Psi_-\rangle_{\mathrm{UA}}(\alpha|z_-z_-\rangle - \beta|z_+z_+\rangle)\\ &+ |\Phi_+\rangle_{\mathrm{UA}}(\beta|z_-z_-\rangle + \alpha|z_+z_+\rangle) + |\Phi_-\rangle_{\mathrm{UA}}(\beta|z_-z_-\rangle - \alpha|z_+z_+\rangle)],\end{aligned} \tag{3}$$

where the Bell basis states are given by

$$\begin{aligned}|\Psi_\pm\rangle_{\mathrm{UA}} &= \frac{1}{\sqrt{2}}(|z_-z_-\rangle_{\mathrm{UA}} \pm |z_+z_+\rangle_{\mathrm{UA}}),\\ |\Phi_\pm\rangle_{\mathrm{UA}} &= \frac{1}{\sqrt{2}}(|z_-z_+\rangle_{\mathrm{UA}} \pm |z_+z_-\rangle_{\mathrm{UA}}).\end{aligned} \tag{4}$$

After Alice's BM, the total state is projected into one of four states in equation (3). It is interesting that quantum information of an unknown state is split between Bob and Charlie at this moment and recovered at Charlie's location by Bob's measurement and Charlie's unitary operation.

Bob measures his GHZ particle in the x-direction and announces his outcome. With the information of two classical bits from Alice and one bit from Bob, Charlie can make a suitable unitary operation on his qubit to recover the original unknown state as follows:

$$\begin{aligned}&|\Psi_+\rangle_{\mathrm{UA}}|x_+\rangle_{\mathrm{B}} \to I, && |\Psi_+\rangle_{\mathrm{UA}}|x_-\rangle_{\mathrm{B}} \to \hat{\sigma}_z,\\ &|\Phi_+\rangle_{\mathrm{UA}}|x_+\rangle_{\mathrm{B}} \to \hat{\sigma}_x, && |\Phi_+\rangle_{\mathrm{UA}}|x_-\rangle_{\mathrm{B}} \to \hat{\sigma}_x\hat{\sigma}_z,\\ &|\Psi_-\rangle_{\mathrm{UA}}|x_+\rangle_{\mathrm{B}} \to \hat{\sigma}_z, && |\Psi_-\rangle_{\mathrm{UA}}|x_-\rangle_{\mathrm{B}} \to I,\\ &|\Phi_-\rangle_{\mathrm{UA}}|x_+\rangle_{\mathrm{B}} \to \hat{\sigma}_x\hat{\sigma}_z, && |\Phi_-\rangle_{\mathrm{UA}}|x_-\rangle_{\mathrm{B}} \to \hat{\sigma}_x.\end{aligned} \tag{5}$$

As a result, in the BM of Alice's part, the teleportation using the GHZ state is always successful.

On the other hand, suppose Alice and Charlie wish to perform teleportation via a GHZ state without Bob. Alice takes two particles of the GHZ state $|\mathrm{GHZ}\rangle_{\mathrm{AA'C}}$ and Charlie the other one. Teleportation can be implemented if Alice changes her set of measurements from BMs to the GHZ-type measurement as follows:

$$|\Psi_{\pm}^{\mathrm{GHZ}}\rangle_{\mathrm{UAA'}} = \frac{1}{\sqrt{2}}(|z_-z_-z_-\rangle \pm |z_+z_+z_+\rangle),$$
$$|\Phi_{\pm}^{\mathrm{GHZ}}\rangle_{\mathrm{UAA'}} = \frac{1}{\sqrt{2}}(|z_-z_+z_+\rangle \pm |z_+z_-z_-\rangle). \tag{6}$$

If we use a notation $|z_-\rangle_{\mathrm{A}}$ ($|z_+\rangle_{\mathrm{A}}$) in the place of $|z_-z_-\rangle_{\mathrm{AA'}}$ ($|z_+z_+\rangle_{\mathrm{AA'}}$), the protocol becomes exactly the same as ST. Therefore, we are able to perform a perfect teleportation via the GHZ state.

Furthermore, what about teleportation of a partially entangled GHZ state? In fact, Bandyopadhyay's work implies this problem in [8]. He introduced an assisted qubit, which has an equivalent coefficient to a partially entangled channel, and Alice measures the unknown qubit with the assisted qubit and her channel qubit. The scheme shows a method of teleportation via a partially entangled GHZ state.

3. Quantum teleportation via W states

3.1. Bell measurement

A similar idea of quantum teleportation applies to the following W state:

$$|W\rangle = \frac{1}{\sqrt{3}}(|z_+z_-z_-\rangle + |z_-z_+z_-\rangle + |z_-z_-z_+\rangle). \tag{7}$$

Alice also begins with an unknown state in equation (1). Alice, Bob and Charlie share the W state. Alice combines an unknown state with her W qubit and performs BM on these qubits. The basis vectors of Bell state operators in the z direction are represented by equation (4). Before Alice's measurement, the total state $|\Psi_W^{\mathrm{tot}}\rangle = |\phi\rangle_{\mathrm{U}} \otimes |W\rangle_{\mathrm{ABC}}$ can be expressed in terms of the Bell basis as follows:

$$\begin{aligned}|\Psi_W^{\mathrm{tot}}\rangle = \frac{1}{\sqrt{6}}[&|\Psi_+\rangle_{\mathrm{UA}}\{|z_-\rangle_{\mathrm{B}}(\beta|z_-\rangle_{\mathrm{C}} + \alpha|z_+\rangle_{\mathrm{C}}) + |z_+\rangle_{\mathrm{B}}(\alpha|z_-\rangle_{\mathrm{C}})\}\\
&- |\Psi_-\rangle_{\mathrm{UA}}\{|z_-\rangle_{\mathrm{B}}(\beta|z_-\rangle_{\mathrm{C}} - \alpha|z_+\rangle_{\mathrm{C}}) - |z_+\rangle_{\mathrm{B}}(\alpha|z_-\rangle_{\mathrm{C}})\}\\
&+ |\Phi_+\rangle_{\mathrm{UA}}\{|z_-\rangle_{\mathrm{B}}(\alpha|z_-\rangle_{\mathrm{C}} + \beta|z_+\rangle_{\mathrm{C}}) + |z_+\rangle_{\mathrm{B}}(\beta|z_-\rangle_{\mathrm{C}})\}\\
&+ |\Phi_-\rangle_{\mathrm{UA}}\{|z_-\rangle_{\mathrm{B}}(\alpha|z_-\rangle_{\mathrm{C}} - \beta|z_+\rangle_{\mathrm{C}}) - |z_+\rangle_{\mathrm{B}}(\beta|z_-\rangle_{\mathrm{C}})\}].\end{aligned} \tag{8}$$

After Alice's measurement, the total state is collapsed into one of the four states in equation (8). Unlike the ST with maximally entangled Bell state or with a GHZ state, the probabilities of the four outcomes depend on the unknown state. The probability p_{Ψ_+} that Alice's output state is $|\Psi_+\rangle$ is given by

$$p_{\Psi_+} = |_{\mathrm{UA}}\langle\Psi_+|\Psi_W^{\mathrm{tot}}\rangle_{\mathrm{UABC}}|^2 = \frac{1+|\alpha|^2}{6}. \tag{9}$$

Also, the probabilities of Alice's outcomes being $|\Psi_-\rangle$, $|\Phi_+\rangle$, and $|\Phi_-\rangle$ are $(1+|\alpha|^2)/6$, $(1+|\beta|^2)/6$, and $(1+|\beta|^2)/6$, respectively.

Bob's measurement outcome decides the success or failure of the teleportation. If Bob's outcome is z_+, the teleportation fails. Otherwise, Charlie can do the following operation on his W qubit to get the unknown state according to Alice's information:

$$\begin{aligned} &|\Psi_+\rangle_{\mathrm{UA}}|z_-\rangle_{\mathrm{B}} \to \hat{\sigma}_x, && |\Psi_-\rangle_{\mathrm{UA}}|z_-\rangle_{\mathrm{B}} \to \hat{\sigma}_x\hat{\sigma}_z, \\ &|\Phi_+\rangle_{\mathrm{UA}}|z_-\rangle_{\mathrm{B}} \to I, && |\Phi_-\rangle_{\mathrm{UA}}|z_-\rangle_{\mathrm{B}} \to \hat{\sigma}_z. \end{aligned} \tag{10}$$

Recently, this has been proposed as a scheme of teleportation via a W state [25]. In fact, they have suggested two ways of teleportation which have different probabilities, but we have pointed out that the two schemes give the same probability 2/3, which does not depend on coefficients α and β of the unknown state [26].

When Alice has an outcome state $|\Psi_+\rangle$, the probability that Bob measures an outcome z_- is $1/(1+|\alpha|^2)$. When we define a probability p_1 (p_2, p_3, p_4) as $|_{\mathrm{B}}\langle z_-|_{\mathrm{UA}}\langle\Psi_+|\Psi_W^{\mathrm{tot}}\rangle_{\mathrm{UABC}}|^2$ (with the substitutions of Ψ_+ by Ψ_-, Φ_+, Φ_-), the success probability of the scheme is given by

$$P_{\mathrm{bell}}^{\mathrm{suc}} = \sum_{i=1}^{4} p_i = 2\cdot\frac{1}{1+|\alpha|^2}\cdot\frac{1+|\alpha|^2}{6} + 2\cdot\frac{1}{1+|\beta|^2}\cdot\frac{1+|\beta|^2}{6} = \frac{2}{3}. \tag{11}$$

On the other hand, probabilities of failed teleportation are given by

$$\begin{aligned} p_5 &= |_{\mathrm{B}}\langle z_+|_{\mathrm{UA}}\langle\Psi_+|\Psi_W^{\mathrm{tot}}\rangle_{\mathrm{UABC}}|^2 = \frac{|\alpha|^2}{6}, \\ p_6 &= |_{\mathrm{B}}\langle z_+|_{\mathrm{UA}}\langle\Psi_-|\Psi_W^{\mathrm{tot}}\rangle_{\mathrm{UABC}}|^2 = \frac{|\alpha|^2}{6}, \\ p_7 &= |_{\mathrm{B}}\langle z_+|_{\mathrm{UA}}\langle\Phi_+|\Psi_W^{\mathrm{tot}}\rangle_{\mathrm{UABC}}|^2 = \frac{|\beta|^2}{6}, \\ p_8 &= |_{\mathrm{B}}\langle z_+|_{\mathrm{UA}}\langle\Phi_-|\Psi_W^{\mathrm{tot}}\rangle_{\mathrm{UABC}}|^2 = \frac{|\beta|^2}{6}, \end{aligned} \tag{12}$$

where the total probability of failure is $P_{\mathrm{bell}}^{\mathrm{fail}} = \sum_{i=5}^{8} p_i = 1/3$.

Even though the scheme does not perform POVMs [9], it can also be called a kind of CT by W states because a conclusive event occurs. The total success probability of teleportation is 2/3 in the scheme because the probability of a z_- outcome of Bob is 2/3 [26]. In fact, it is a natural conclusion that Bob's outcome decides whether the quantum channel between Alice and Charlie is entangled or not.

3.2. Asymmetric W-type measurement

Let us suppose only Alice and Charlie participate in quantum teleportation via the W state because we would like to compare a feature of teleportation by BM with that by POVM. Alice keeps two qubits of the W state and Charlie the other one. Alice performs three-qubit POVMs on one qubit of the unknown state and her two qubits of the W state. Notice that the protocol of CT of a d-dimensional state with POVMs is developed in a way of making a conclusive event [9]. Similarly, we are able to perform a method of POVMs without any loss of generality.

The total state of the unknown state and the W state can be expressed in terms of the bases as follows:

$$\begin{aligned} |\Psi_W^{\mathrm{tot}}\rangle_{\mathrm{UA'C}} = \tfrac{1}{2}\{&|\psi_1'\rangle(\alpha|z_-\rangle + \beta|z_+\rangle) + |\psi_2'\rangle(\alpha|z_-\rangle - \beta|z_+\rangle) \\ &+ |\psi_3'\rangle(\beta|z_-\rangle + \alpha|z_+\rangle) + |\psi_4'\rangle(\beta|z_-\rangle - \alpha|z_+\rangle)\}, \end{aligned} \tag{13}$$

where the basis vectors are represented by

$$
\begin{aligned}
|\psi_1'\rangle_{\mathrm{UAA}'} &= \frac{1}{\sqrt{3}}(|z_-z_-z_+\rangle + |z_-z_+z_-\rangle + |z_+z_-z_-\rangle),\\
|\psi_2'\rangle_{\mathrm{UAA}'} &= \frac{1}{\sqrt{3}}(|z_-z_-z_+\rangle + |z_-z_+z_-\rangle - |z_+z_-z_-\rangle),\\
|\psi_3'\rangle_{\mathrm{UAA}'} &= \frac{1}{\sqrt{3}}(|z_+z_-z_+\rangle + |z_+z_+z_-\rangle + |z_-z_-z_-\rangle),\\
|\psi_4'\rangle_{\mathrm{UAA}'} &= \frac{1}{\sqrt{3}}(|z_+z_-z_+\rangle + |z_+z_+z_-\rangle - |z_-z_-z_-\rangle).
\end{aligned}
\tag{14}
$$

For Alice's POVMs, we consider un-normalized vectors $|\tilde{\psi}_i'\rangle$ to satisfy a condition of $\langle\tilde{\psi}_i'|\psi_j'\rangle = \delta_{ij}$. Generally, there are three parameters in each $|\tilde{\psi}_i'\rangle$ but the condition reduces to one parameter such as a or a'. The basis vectors of the asymmetric POVM are represented by

$$
\begin{aligned}
|\tilde{\psi}_1'\rangle_{\mathrm{UAA}'} &= a|z_-z_-z_+\rangle + \left(\frac{\sqrt{3}}{2} - a\right)|z_-z_+z_-\rangle + \frac{\sqrt{3}}{2}|z_+z_-z_-\rangle,\\
|\tilde{\psi}_2'\rangle_{\mathrm{UAA}'} &= a|z_-z_-z_+\rangle + \left(\frac{\sqrt{3}}{2} - a\right)|z_-z_+z_-\rangle - \frac{\sqrt{3}}{2}|z_+z_-z_-\rangle,\\
|\tilde{\psi}_3'\rangle_{\mathrm{UAA}'} &= a'|z_+z_-z_+\rangle + \left(\frac{\sqrt{3}}{2} - a'\right)|z_+z_+z_-\rangle + \frac{\sqrt{3}}{2}|z_-z_-z_-\rangle,\\
|\tilde{\psi}_4'\rangle_{\mathrm{UAA}'} &= a'|z_+z_-z_+\rangle + \left(\frac{\sqrt{3}}{2} - a'\right)|z_+z_+z_-\rangle - \frac{\sqrt{3}}{2}|z_-z_-z_-\rangle.
\end{aligned}
\tag{15}
$$

The proper POVM set $\{\hat{M}_i\}$ is defined by

$$
\begin{aligned}
\hat{M}_i &= \lambda_{\mathrm{asym}}|\tilde{\psi}_i'\rangle\langle\tilde{\psi}_i'|,\\
\hat{M}_5 &= \mathbb{I}_{8\times 8} - \sum_{i=1}^{4}\hat{M}_i, \qquad (\langle\tilde{\psi}_i'|\psi_j'\rangle = \delta_{ij}),
\end{aligned}
\tag{16}
$$

where the real parameter $\lambda_{\mathrm{asym}} \geq 0$ and $i = 1, 2, 3, 4$. The set of this POVM also satisfies positivity and completeness for any quantum state $|\psi'\rangle$ as follows:

$$
\sum_{i=1}^{4}\lambda_{\mathrm{asym}}|\langle\tilde{\psi}_i'|\psi\rangle|^2 \leq 1 \qquad \text{for } \forall|\psi\rangle. \tag{17}
$$

For simplicity, let us assume that a is real and equal to a'. Since $\hat{M}_5$ should be a positive operator, a and λ_{asym} should satisfy one of the following conditions (figure 1):

$$
\begin{aligned}
&\text{(i)} \quad 0 < a \leq \frac{\sqrt{3}}{2} \qquad 0 < \lambda_{\mathrm{asym}} \leq \frac{2}{3}\\
&\text{(ii)} \quad \frac{\sqrt{3}}{2} < a \qquad 0 < \lambda_{\mathrm{asym}} \leq \frac{1}{4a^2 - 2\sqrt{3}a + \frac{3}{2}}.
\end{aligned}
\tag{18}
$$

We can easily calculate the probability p_i of each conclusive case and total success probability $P_{\mathrm{asym}}^{\mathrm{suc}}$,

$$
p_i = \langle\Psi_W^{\mathrm{tot}}|\hat{M}_i|\Psi_W^{\mathrm{tot}}\rangle = \frac{\lambda_{\mathrm{asym}}}{4} \tag{19}
$$

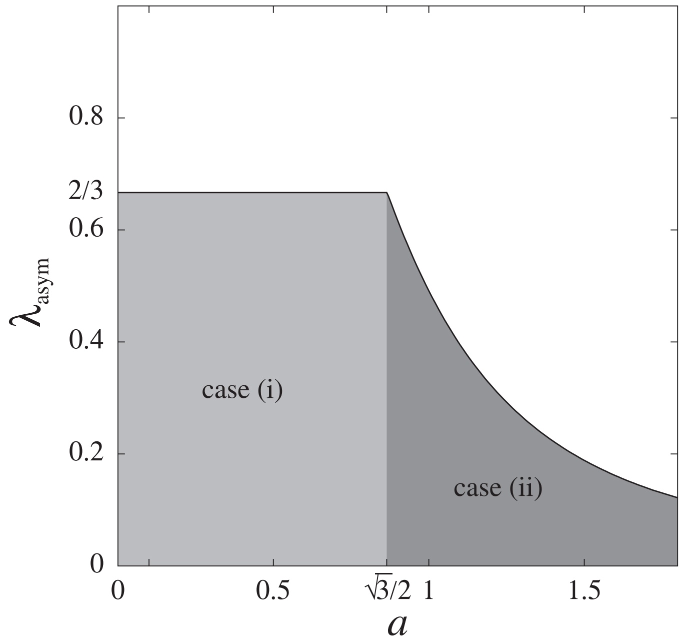

Figure 1. Parameter range of an asymmetric POVM (equation (18)).

$$P^{\mathrm{suc}}_{\mathrm{asym}} = \sum_{i=1}^{4} p_i = \lambda_{\mathrm{asym}} \leq \tfrac{2}{3}, \tag{20}$$

with total failure probability $P^{\mathrm{fail}}_{\mathrm{asym}} = 1 - P^{\mathrm{suc}}_{\mathrm{asym}}$. Since λ_{asym} is at most $2/3$ in figure 1, the optimal success of teleportation with an asymmetric POVM is $2/3$.

As a result of the two cases, although we optimize a parameter in Alice's part, the total success probability of each scheme in quantum teleportation has a limit value of $2/3$. Therefore, it can be characterized as a property of a W state when it is used in quantum teleportation.

4. Average fidelity of teleportation with W states

In any protocol of teleportation, it is necessary to average the fidelity over all possible input states. The average fidelity reflects how much information is transferred to the teleported state. The teleported pure state of the density operator $\hat{\rho}_i$ relies on the outcome i. The average fidelity is defined as

$$\bar{\mathcal{F}} \equiv \frac{1}{V} \int \mathrm{d}\vec{\Omega} \sum_i p_i(\vec{\Omega}) f_i(\vec{\Omega}), \tag{21}$$

where $f_i(\vec{\Omega}) = \langle \phi(\vec{\Omega}) | \hat{\rho}_i | \phi(\vec{\Omega}) \rangle$ and an unknown pure state $|\phi(\vec{\Omega})\rangle$ is parametrized by a real vector $\vec{\Omega}$ in the parameter space of volume V [27]. In the CT, we separate the average fidelity into two parts for easy calculation as follows:

$$\bar{\mathcal{F}} = \bar{\mathcal{F}}_{\mathrm{con}} + \bar{\mathcal{F}}_{\mathrm{inc}} = \frac{1}{V} \int \mathrm{d}\vec{\Omega} \left[\sum_{i=1}^{m} + \sum_{i=m+1}^{n+m} \right] p_i(\vec{\Omega}) f_i(\vec{\Omega}), \tag{22}$$

where m and n are the total numbers of conclusive events and inconclusive events.

To understand protocols clearly, we make a comparison of average fidelity in each case of quantum teleportation of the BM (III A) and the asymmetric POVM (III B). Although equation (22) contains integration of $\vec{\Omega}$, probability $p_i(\vec{\Omega})$ is independent of $\vec{\Omega}$ and $\frac{1}{V}\int d\vec{\Omega}\, f_i(\vec{\Omega})$ is 1 for a conclusive event and $1/2$ for an inconclusive event.

For the case of the teleportation by the BM, the success probability p_i ($i = 1, 2, 3, 4$) in equation (11) is $1/6$ and the sum of the failure probability p_i ($i = 5, 6, 7, 8$) in equation (12) $1/3$. Then, the average fidelity in the BM is

$$\bar{\mathcal{F}}_{\mathrm{Bell}} = \sum_{i=1}^{4} \frac{1}{6} \cdot 1 + \sum_{i=5}^{8} p_i \cdot \frac{1}{2} = \frac{5}{6}. \tag{23}$$

In the teleportation by an asymmetric POVM, the success probability of each conclusive event is $\lambda_{\mathrm{asym}}/4$ with fidelity 1. The average fidelity is

$$\bar{\mathcal{F}}_{\mathrm{asym}} = \sum_{i=1}^{4} \frac{\lambda_{\mathrm{asym}}}{4} \cdot 1 + (1 - \lambda_{\mathrm{asym}}) \cdot \frac{1}{2} = \frac{1}{2} + \frac{1}{2}\lambda_{\mathrm{asym}} \leq \frac{5}{6}. \tag{24}$$

Since λ_{asym} is smaller than $2/3$, the maximal average fidelity is also $5/6$ as a limit of the BM. Therefore, the method of the BM provides the maximum value of average fidelity as well as that of success probability between two types of teleportation via a W state.

5. Remarks

We have shown two schemes of quantum teleportation of an unknown qubit via a W state taking into account two kinds of measurement by Alice. In the first protocol, after Alice performs BMs on the unknown state and her channel state, the information of the unknown state is split between Bob and Charlie. But recovering the unknown state is not always successful, unlike quantum teleportation with a GHZ state. The second scheme of quantum teleportation is based on POVMs in Alice's part. The teleportation by POVM with two parties reveals no advantage compared with ST using an EPR state, but is helpful to explore the properties of quantum information processing via the W state because the BM scheme is based on split quantum information, but the POVM one is not.

We have also calculated the maximal success probability and the optimal average fidelity for each scheme. The maximum values of success probability and average fidelity are $2/3$ and $5/6$, respectively. Although W states are not particularly good for quantum teleportation, they can be used as a multi-channel in quantum teleportation because the average fidelity $5/6$ is greater than $2/3$.

In spite of the same values of the success probability and the fidelity in the teleportations, we would like to comment that the amount of classical communication is different in each case. The teleportation protocol via the W state by the BM needs three classical bits to achieve a successful teleportation, but the schemes by asymmetric POVM require three bits with consideration of a failure notice to a receiver.

Finally, we note a scheme of teleportation using a W state of different amplitudes as follows:

$$|\Psi_W\rangle = a|z_- z_- z_+\rangle + b|z_- z_+ z_-\rangle + c|z_+ z_- z_-\rangle, \tag{25}$$

where $|a|^2 + |b|^2 + |c|^2 = 1$. Following the schemes shown in section 3.1, we can easily guess a protocol of the BM from $|\Psi_W^{\mathrm{tot}}\rangle = |\phi\rangle_{\mathrm{U}} \otimes |\Psi_W\rangle_{\mathrm{ABC}}$. If Bob's measurement results in $|z_-\rangle$, Charlie also obtains a teleported state with a certain probability. On the other hand, if we consider quantum teleportation with the POVM, even in a simple case of $|a| = |c|$ and $|a| \neq |b|$, it seems to be difficult to find a POVM set.

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