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Abstract

Free-electron lasers (FELs) are a unique source of light, particularly in the x-ray domain. After the success of FELs based on conventional acceleration using radio-frequency cavities, an important challenge is the development of FELs based on electron bunching accelerated by a laser wakefield accelerator (LWFA). However, the present LWFA electron bunch properties do not permit use directly for a significant FEL amplification. It is known that longitudinal decompression of electron beams delivered by state-of-the-art LWFA eases the FEL process. We propose here a second order transverse beam manipulation turning the large inherent transverse chromatic emittances of LWFA beams into direct FEL gain advantage. Numerical simulations are presented showing that this beam manipulation can further enhance by orders of magnitude the peak power of the radiation.

1. Introduction

High intensity femtosecond x-ray light sources have enabled spectacular progress in the understanding of the phenomena of matter at the atomic scale [1, 2]. Since the discovery of the laser more than 50 years ago [3], several sources of coherent x-ray and short duration light pulses are now available. Presently, the highest peak intensities are reached on fourth generation free-electron lasers (FEL) using electron beams from conventional linear accelerators (CLA) in the magnetic field of undulators as gain medium. Present FELs operate for users in the VUV soft x-ray range [4–6] and in the Angström region with GW peak power level and 2–100 fs pulses [7, 8].

FELs rely on the resonant emission at wavelength $\lambda_r$ of an electron beam in the periodic magnetic field of an undulator. $\lambda_r$ is given by $\lambda_r = \lambda_u / (2\gamma^2) (1 + K_u^2 / 2)$, with $\gamma$ the normalized beam energy, $\lambda_u$ the undulator period and $K_u$ its deflection parameter. Up to now, state-of-the-art relativistic electron beams have been delivered by radio-frequency CLAs. With gradients of a few tens of MV/m, typical beams of 1 nC charge, $\pi$ mm.mrad normalized emittance, 0.01% energy spread, $1 \text{ GeV}$ energy are delivered within a few hundreds of meters. Facing the increasing demand of the scientific community, the development of compact FELs is a major challenge. The recent progresses in laser wakefield accelerators (LWFA) [9, 10] that efficiently produce high quality electron beams in very high electric fields, in a few hundreds of GV/m, have opened the possibility to design and to conceive compact FELs. Tremendous progress has been achieved during this last decade to improve LWFA performances that now deliver routinely quasi-monoenergetic electron beams [11–13] with a typical few kA current, a few fs bunch length [14], a few hundred MeV-1 GeV energy [15, 16], percent energy spread [17], mmrad divergence and $\pi \text{ mm.mrad}$ normalized emittance [18–20]. In a FEL, thanks to the electron–photon interaction, the resonant radiation is exponentially amplified along the beam path $S$ throughout the undulator. The output power follows [21]: $P(S) = P_0 \exp(S/L_g)$ with $P_0$ the input noise power coupled into the dominant amplified mode and $L_g$ the gain length given by:

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\[ L_g \propto \left(1 + \frac{\sigma_x^2}{\rho^2}\right)/\rho, \quad \rho \propto \left(\frac{\dot{I}}{\sigma_x \sigma_y}\right)^{1/3}. \]  

(1)

\( \rho \) is the FEL Pierce parameter, \( \dot{I} \) is the peak current, \( \sigma_\delta \) is the rms relative energy spread and \( \sigma_x, \sigma_y \) are respectively the rms horizontal and vertical averaged electron bunch sizes along the undulator. A detailed analytical formula of the FEL gain length can be found in [22]. The FEL peak power saturation is reached typically after \( 20 \, L_g \) [23] and is orders of magnitude higher than the simple undulator spontaneous emission power delivered by synchrotron radiation sources. For high gain FEL, the condition [21]:

\[ \sigma_\delta \ll \rho \]  

(2)

is required and thus a too large electron rms relative energy spread prevents strong amplification. The main difficulty resides in the intrinsic large divergence and energy spread delivered today by the LWFA beam that complicates the beam transfer [24–26] and strongly reduces the amplification gain process. In a LWFA-based FEL, a first step to cope with the initial large energy spread is to decompress the beam by means of a magnetic chicane [27] or to use a transverse gradient undulator [28].

An additional transverse chromatic beam manipulation is proposed here, coupled with the bunch decompression, so that a constant effective electron peak transverse density and sizes slip with the FEL optical wave inside the undulator and further distress the intrinsic large energy spread and divergence of these beams. While the decompression mitigates the large energy spread at the expense of the peak current, the chromatic manipulation turns the intrinsic large divergence into direct FEL gain advantage. According to simulations, this technique enables the further increase of the FEL peak, to compress the FEL pulse duration, all still within a compact layout.

2. Electron beam manipulations

The resulting four-stage layout is illustrated in figure 1. It includes a first stage composed of three high gradient permanent magnet quadrupoles to refocus the beam, a second stage composed of the chicane to decompress the beam and eventually to accommodate a mirror for a seed, a third stage composed of additional quadrupoles to operate the chromatic manipulation and the undulator. The chicane decompression is an energy demixing process, where the slice peak current \( \dot{I} \) and according relative energy spread \( \sigma_\delta \) decrease at the same rate, distressing the requirement of equation (2) [27]. Prior to being decompressed and injected into the undulator, the beam has to be refocused by means of a quadrupole triplet. The different electron trajectories through the refocusing stage, according to their energy, span the trace phase space distribution enlarging the emittances. From phase space geometry, the initial divergence drastically increases this effect. To limit it, the quadrupoles must be located as close as possible to the source at the cost of a very high gradient [29, 30]. The use of compact and very strong permanent quadrupole magnets enables a few hundred Tm \(^{-1}\) to be reached. Nevertheless, these effects are still important with respect to the initial emittance. The slice energy sorting of the chicane will then transfer the chromatic emittance into mismatch from slice to slice along the undulator and spoil the FEL process efficiency.

Up to second order, a general quadrupole transfer using the usual transport notation [31] limited to the horizontal plane is given by:

\[
\begin{bmatrix}
  x \\
  x_p
\end{bmatrix}
= \begin{bmatrix}
  \eta_{11} & \eta_{12} \\
  \eta_{21} & \eta_{22}
\end{bmatrix}
+ \delta \begin{bmatrix}
  \eta_{16} & \eta_{126} \\
  \eta_{216} & \eta_{226}
\end{bmatrix}
\begin{bmatrix}
  x_0 \\
  x_{p_0}
\end{bmatrix}
\]

(3)
where the first matrix \((r_{ij})\) of the right hand side stands for the linear part and the second matrix \((r_{ij6})\) stands for the chromatic second order perturbation. \((x_0, x_{0p}), (x, xp)\) are respectively the initial and final horizontal coordinates (position-angle) of one particle and \(\delta\) its relative energy deviation. An initial Gaussian distribution without any correlation of rms emittance \(\epsilon_{0}\) is assumed having a small initial rms size \(\sigma_{x0}\) and a large initial rms divergence \(\sigma_{xp0}\). Cancelling the term \((r_{12} = 0)\) of equation (3) enables the setting of the on-momentum particles \(\delta = 0\) to a waist \(\sigma = -\sigma_{x_{min}}\) as standard linear optic imaging of magnification \(r_{11}\). In addition, cancelling the chromatic term \((r_{236} = 0)\), the three rms associated momenta transfers according to their relative energy deviation are approximated by:

\[
\begin{align*}
\sigma_x^2(\delta) &\approx \sigma_{x_{min}}^2 + r_{26}^2 \sigma_{xp0}^2 \delta^2, \\
\sigma_{xp}^2(\delta) &\approx \frac{r_{26}}{r_{11}} \sigma_{xp0}^2 \delta, \\
\sigma_{xp}^2(\delta) &\approx \frac{1}{r_{11}} \sigma_{xp0}.
\end{align*}
\]

\(\sigma_x, \sigma_{xp}\) and \(\sigma_{xp}\) are respectively the rms size, divergence and cross term. They present an upright set of slice ellipses, as sketched in figure 2(a). According to equation (4), the energy slice emittances \(\epsilon(\delta) = \epsilon_0\) and the total geometrical emittance \(\epsilon_t\), integrated over the energy deviation, is given by:

\[
\epsilon^2_t \approx \epsilon_0^2 + \left( \frac{r_{26}}{r_{11}} \sigma_{xp0} \sigma_\delta \right)^2.
\]

The second term of the right hand side, defined here as the chromatic emittance \(\epsilon_{chrom}\), is drastically enhanced by the initial divergence. Each slice reaches its minimum size \(\sigma_{x_{min}}\) at the undulator position \(S(\delta)\) (figure 1(b)) according to:

\[
S(\delta) = -\frac{\sigma_{xp}(\delta)}{\sigma_{xp}(\delta)} = -\frac{r_{26}}{r_{11}} \delta.
\]

Thanks to the chicane energy sorting (of linear strength \(r_{56}\)) converting energy deviation (\(\delta\)) to the longitudinal position

\[
\Delta s = r_{56}\delta,
\]

the minimum \(\sigma_{x_{min}}\) slips along the bunch (focusing slippage) throughout the undulator. On the other hand, the FEL radiation wave is also known to slip along the bunch [32]. Both slippages can be synchronized so that the effective beam size, seen by the FEL, is always the minimum. In the exponential gain regime, the FEL slippage, \(\Delta s = \lambda_r/(3\lambda_u)S\) [32], combined with equations (6) and (7) gives the synchronization condition for the chicane strength:

\[
r_{56} = -\frac{\lambda_r}{3\lambda_u} r_{26}.
\]

As long as the initial beam divergence is large and limited to the second order term \(r_{226}\) cancellation, the synchronization condition (8) only depends on the focusing components of the transfer line and is free from any initial jitter such as position, pointing, divergence, length and energy spread. It is only weakly affected by the mean energy fluctuation (via the energy resonant emission \(\lambda_r\)). To operate this chromatic focusing slippage, in both horizontal and vertical planes, at least an additional triplet of quadrupoles is mandatory. A set of four quadrupoles gives more flexibility to also control both magnifications.
To summarize, including the vertical plane referenced by indexes three and four following the usual transport notation \[31\], the set of seven matrix transport conditions from the source to the undulator center are:

- set \( r_{12} = r_{34} = 0 \) to have a linear imaging;
- fix \( r_{13} = r_{35} = m \) to set a round beam and fix the synchronized \( r_{56} \);
- set \( r_{26} = r_{36} = 0 \) to apply the focusing slippage;
- fix \( r_{16} = r_{36} \) to equilibrate both horizontal and vertical chromatic emittance.

The two terms, \( r_{12} \) and \( r_{34} \), drive the chromatic emittances. They cannot be cancelled or reduced together by means of quadrupoles. At least, a transfer from one to the other is possible by varying the first refocusing quadrupoles. The synchronization conditions equation (8), in both planes, is still valid with appropriate magnifications leading to an elliptical beam.

The presence of possible strong vertical focusing (strong \( K_v \) and low energy) of a planar undulator may distort the vertical ‘focusing slippage’. There is always a positive chicane strength solution whatever the focusing, and in practice, the transverse effects of a four identical magnet chicane being negligible, a simple decompression scan should meet the synchronization condition. A significant effective transverse beam size reduction, and consequent gain length shortening according to equation (1) may be obtained.

### 3. Electron beam tracking

For this study, we consider a 400 MeV LWFA electron beam described as a 6D Gaussian bunch without any correlation having a total normalized rms emittance of \( 1 \mu \text{mm.mrad} \), a divergence of 1 mrad rms, a 1% rms relative energy spread with an rms bunch length of 1 \( \mu \text{m} \) and 4 kA peak current. These parameters are rather realistic, except the energy spread which is still a bit optimistic since experiments often show a few percent \[17\]. The FEL performances are compared for two transfer lines: the first one consisting of only one triplet followed by a chicane (referred to as the strong focusing case) and a second one with the additional quadruplet located downstream of the chicane, mandatory to control the second order chromatic terms in both planes (referred to as the chromatic matching case). The maximum quadrupole gradient is about 200 T/m in the first refocusing triplet and only 20 T/m for the additional ones. A state-of-the-art cryo-ready planar undulator of 5 m \[33\] is considered here, with a 15 mm period at 3 mm gap. The peak magnetic field reaches 1.5 T and the deflection parameter \( K_u = 2.12 \). The natural vertical focusing is then not negligible over 5 m. The electron beam transfer simulations are done in two steps: a first pass with the BETA code \[34\] fitting the first and a second order matching to fix the quadrupole strength followed by a sympletic 6D tracking pass based on perfect hard edge model magnets \[35\] from the source to the undulator exit. These trackings include second and every higher order optic aberrations that do not exhibit sensitive alteration to former beam manipulations limited to second order.

The longitudinal dynamics are very similar for both optics. Without chicane decompression, the peak current is slightly reduced from 4 (initial peak current) to 3.5 kA at the undulator by trailing particles induced by the large initial divergence. The beneficial chicane energy demixing effect is then slightly spoiled and is very sensitive to the initial divergence. The rms total normalized emittances for both planes are increased from 1 to about 5 \( \mu \text{mm.mrad} \) for the two optics and well estimated, within 5%, by the simplified equation (5). Both effects, trailing particles and chromatic emittances, take place mainly in the first triplet of quadrupoles where the beam divergences are still high. The two transfer lines only differ by their transverse optics as shown in figure 3. Layouts and rms beam envelopes are plotted in figure 3(a1)-(b1). Their associated transverse phase spaces, uprighted with respect to the energy deviation for the chromatic matching case, are shown in figure 3(a2)-(b2) at the undulator centre.

Figure 4 displays the resulting electron density pattern (define as \( I/(\sigma_x\sigma_y) \), for both optics, driving the FEL gain (equation (1)) for each slice along the bunch over the 5 m undulator. Three cases of chicane strengths (0, 0.5 and 1 mm) are compared. While the peak density is only localized at the undulator center for the strong focusing case (figure 4(a1)-(a3)), it is maintained constant along a large undulator fraction, thanks to the slipping waists in both planes, for the chromatic matching case (figure 4(b1)-(b3)). The chicane decompressing effect tilts, step by step, the density pattern from the tail towards the head of the bunch. If synchronized with the FEL wave slippage (equation (8)), an effective electron density increase is expected. According to equation (1), the FEL gain length should be reduced. In addition, the effective electron beam size is also constant ensuring a good overlap with the amplified FEL wave all along the undulator. With a chicane strength of 0.5 mm, the bunch is decompressed by a factor of 5 and reaches about 5 \( \mu \text{m} \) rms, accordingly the peak current drops down to 0.8 kA (figure 3(c2)). In both optic cases, the electron density reaches about 400 kA mm\(^{-2}\) at the undulator center.
In parallel, the energy demixing process also reduces the slice relative energy spread from 1 to about 0.2%, and the slice normalized emittances are reduced from 5 to about 1.8 π mm mrad. In terms of transverse Twiss parameters with a magnification of \( \beta_y = 0.9 \) in both planes, the chromatic matching gives here in both planes an effective low value of \( \beta_y \approx 0.9 \) for the synchronized FEL wave.

The synchronization relation of equation (8) is, up to the second order, independent from the initial relative energy spread and divergence as shown in figure 5. Compared to the nominal case with an energy spread of 1% and a divergence of 1 mrad (figure 5(a1)), the cases with larger relative energy spread and divergence exhibit the same transverse electron density patterns (or peak density slope) when varying the relative energy spread from 1 to 2% (figure 5(a2)) or the divergence from 1 to 1.4 mrad (figure 5(a3)). They are still synchronized but their density is lower by a factor of 2, respectively from direct peak current reduction or slice emittance increase.
In the simulations presented here, collective effects, such as space charge and coherent synchrotron radiation [36], are not included but have been estimated with ASTRA [37] and CSRtrack [38] codes. They are not too strong in this configuration and slightly increase the slice emittance within 10%. The magnet imperfections and misalignment are not included either. Particularly strong with permanent magnet technology, they may spoil the emittances and therefore the FEL process.

The concept of synchronizing the slice focusing all along the undulator together with the progress of the optical wave cannot be easily applied with an electron bunch from a conventional accelerator where correlated emittances are minimized. Indeed, it takes advantage of the natural energy position correlation of the electron beams generated from LWFA which could appear initially as an apparent drawback. It then facilitates the possible experimental demonstration of FEL amplification with an LWFA.

4. FEL simulations

FEL simulations are performed with a GENESIS code [39] at 40 nm in the XUV region. We also consider operation in the seeding configuration [40–42] using an injection of harmonics generated in gas in an FEL providing intense and coherent extreme ultraviolet light [43]. In GENESIS, the tracked beam is used as an external input file in time-dependent mode and a linear field tapering is applied to compensate the induced chicane linear energy chirp [44, 45]. The linear energy chirp is simply given here by \(1/f_{56} \). Expressed in relative energy deviation per millimeter, its value is rather large and ranges here from 1 to 5 mm\(^{-1}\). The four additional quadrupoles enables variation of the transverse magnification from 15 to 35 in both planes. For each magnification factor, the transfer line quadrupoles are fitted only once at the chicane strength giving the maximum FEL power and are left unchanged along the scan.

The output peak power versus the chicane strength is plotted in figure 6(a) seeded with 10 kW (considered as continuous). It exhibits an increase by about 2 orders of magnitude for the chromatic matching cases which reaches the saturation in the GW peak power range at the exit of the 5 m undulator. The chicane strength \(f_{56}\) sensitivity is not critical and gives a flat optimum over 0.2 mm. The maximum output power appears close to the chicane strength synchronization condition. Enlarging the magnification shifts this maximum towards larger chicane strengths (equation (8)). In this example, an optimum is reached for a chicane strength of \(f_{56} = 0.5 \) mm (blue curve, magnification of \(f_{11} = f_{53} = 20 \)). At \(f_{56} = 0.3 \) mm the focused slices slip slower than the FEL wave and at \(f_{56} = 0.7 \) mm the focused slices slip faster reducing the effective electron density and so the FEL output power. In both optics, without the chicane or using a very small decompression factor, the FEL gain remains extremely low. This results from the too large slice energy spread and, above all, the too short electron bunch length: the FEL wave simply slips rapidly out of the bunch, aborting the amplification process.

The FEL peak power evolution along the undulator is plotted in figure 6(b). The chromatic matching case has a higher and constant growth rate driven by the focusing slippage while the strong focusing case has FEL gain mainly at the undulator center where the electrons are focused. A deeper analysis from the GENESIS code output shows that, at the peak power location along the bunch, the electron transverse beam sizes exhibit large variations and differ from the FEL wave sizes for the strong focusing case (figure 7(a)). The overlap between the electrons and the FEL wave is then not optimum. In counter part, the effective rms electron and FEL wave transverse beam sizes are almost constant in both planes (here \(\sim 45 \) μm for electrons and \(\sim 50 \) μm for the FEL) for
the chromatic case over a large fraction of the undulator (figure 7(b)). The overlap between the electron and the FEL wave is then optimum. With $45 \mu m$ rms and a slice emittance of $1.8 \pi mm.mrad$, the Twiss parameters have a low value of $\beta_x = \beta_y \approx 0.9$. Consequently, the mean effective transverse density is increased by a factor of about 2. Still from the GENESIS output, the mean gain length of the chromatic case is about 0.4 m while it increases up to 0.8 m in the strong focusing case (figure 7(c)-(d)). A worthwhile reduction by a factor of 2 is reached here in this example. Empirical mean FEL gain length estimation [23], assuming a constant transverse beam size naturally provided by the chromatic matching, gives a very similar value of 0.4 m (figure 7(d)). The same gain length estimation for the strong focusing case gives a mean gain length of 0.6 m that is shorter than the 0.8 m provided by the GENESIS code simulation (figure 7(c)). The difference comes from the large effective electron transverse beam size variations in both planes, not included in the empirical estimation, that spoil the good continuous transverse overlap between the electrons and the FEL wave along the amplification process.

The output FEL temporal and spectral distributions also evolve in a smooth single spike Gaussian shape regime (figure 8). In addition, the chromatic matching case tends to reduce the FEL pulse length: it reaches its minimum around the synchronization condition where the peak power is maximum (i.e. FEL wave ‘canalization’ favoring the short single spike regime). The minimum pulse length is about twice as short in the chromatic matching mode with about $0.8 \mu m$ rms (6 fs fwhm) and is as short as the initial electron bunch (1 $\mu m$ rms) before decompression.

As previously mentioned, the chicane has a weak effect in the transverse optics (very similar to a simple drift without higher order aberrations) and the set of required conditions to operate the chromatic matching is weakly affected when varying the chicane strength. Figure 9 plots the comparison for the chromatic case and $r_{11} = r_{33} = 20$ of the output FEL peak power with (black dashed line) and without (blue line) refitting the quadrupoles. For the last case, the quadrupoles are only fitted with $r_{66} = 0.5 mm$ and the difference is almost negligible for experiment comfort.

The concept of synchronizing the slice focusing all along the undulator, together with the progress of the optical wave, relies on chromatic emittance built up in the earlier refocusing stage off the beam transfer line. The FEL peak power performance enhancement is still valid if the chromatic term ($\gamma c_{\text{chrom}}$) is at least equivalent or

![Figure 6](image_url)
larger than the initial emittance ($\gamma \epsilon_0$). From equation (5), limited to second order, this relation is:

$$\gamma \epsilon_{\text{chrom}} \approx \gamma \frac{\sigma_{\chi}^2}{\sigma_\theta_0^2} \sigma_\theta \geq \gamma \epsilon_0.$$  (9)

Here, with $\gamma \epsilon_0 = 1 \ \pi \text{.mm.mrad}$ and 1% rms relative energy spread, the large performance enhancement is still valid for divergence down to $\sim 0.4$ mrad rms. With an rms divergence reduced down to 0.1 mrad, the chromatic emittances are negligible and both optics become equivalent. High-energy (1 GeV) and low-emittance ($\gamma \epsilon_0 = 0.1 \ \pi \text{.mm.mrad}$) beams [9, 10], reduce the lower limit of divergence below 0.1 mrad rms. On the other
side, a large divergence, for instance up to 5 mrad rms, will drastically increase the emittance (here above 100 \( \pi \). mm.mrad) and the trailing particles effect. The principle of focusing slice synchronization advantage works although the FEL process is strongly spoiled, but the beam transfer itself having large transverse sizes, might be difficult due to the limited acceptance of the magnets and of the beam pipe.

An alternative to increase the electron transverse density by introducing additional focusing inside the undulator itself has been excluded because of technological challenges.

5. Conclusion

Based on the LWFA beams, magnets and undulators available today, we have demonstrated the possibility of further increasing the FEL peak power by two orders of magnitude turning the large inherent transverse chromatic emittance of those beams into direct FEL gain advantage. The possibility of reaching the GW peak power level at the femtosecond pulse duration over a 5 m undulator seems also feasible in the achievable seeded mode in the XUV wavelength range. Two quadrupole triplets flanking the chicane are needed, a more accurate focusing tuning is required and a variable permanent quadrupole gradient \[46\] or position \[29\] may then be suitable. The chicane strength synchronization tuning is large, only depends on the quadrupole working point and is free from any possible initial beam jitters making it rather robust. In addition, it seems to favor the single spike regime with shorter pulses. The transfer line layout is still rather compact as compared to a long conventional optics suppressing the chromatic emittance, and the feasibility is still to be validated for those beams. There are several other effects that were not included in this study as collective effects, magnet imperfections, misalignment and initial electron correlations. We believe that the chromatic focusing concept presented here is a viable scheme to drive FEL from LWFA beams with inherent large initial divergence and energy spread.

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