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Gravity-related spontaneous wave function collapse in bulk matter

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Abstract
In the DP-model, gravity-related spontaneous wave function collapses suppress Schrödinger cat states which are conceptually problematic especially for gravity and space-time. We derive the equations of the model for the hydrodynamic-elastic (acoustic) modes in a bulk. Two particular features are discussed: the universal dominance of spontaneous collapses at large wavelengths, and the reduction of spontaneous heating by a slight refinement of the DP-model.

Keywords: spontaneous wave function collapse, Newton gravity, gravity related decoherence

1. Introduction

After Schrödinger’s famous thought experiment, superpositions of macroscopically different quantum states are called Schrödinger cat states (or cats, simply). Their existence in nature would be problematic, particularly for our concept of gravitation and space-time. A gentle modification of the superposition principle might suppress cats. Consider a massive system in a quantum state \(|f_1\rangle\) of well-defined spatial mass distribution \(f_1\), and consider another state \(|f_2\rangle\) as well. If \(f_1\) and \(f_2\) are ‘macroscopically’ different, the superposition...
represents a cat. We quantify the measure of ‘catness’ as
\[
\ell_G^2 = -U_{11} - U_{22} + 2U_{12}, (2)
\]
where \( U_{ij} \) are the formal Newton interaction potentials between the mass distributions \( f_i, f_j \), for \( i, j = 1,2 \) in turn:
\[
U_{ij} = -G \int f_i(\mathbf{r}) f_j(\mathbf{r}') \frac{d\mathbf{r}d\mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|}. (3)
\]

A spontaneous collapse of the cat (1) is then postulated:
\[
\frac{|f_1\rangle + |f_2\rangle}{\sqrt{2}} \rightarrow \text{either} \quad |f_1\rangle \quad \text{or} \quad |f_2\rangle, (4)
\]
with the decay time
\[
\tau_G \sim \frac{\hbar}{\ell_G^2}. (5)
\]

This is the central postulate in the gravity-related (G-related) spontaneous collapse model, also called DP-model after its proponents [1–7]. The greater the catness (2), i.e., the difference between \( f_1 \) and \( f_2 \), the shorter the cat’s decay time is. Penrose and the author derived the structure (2) of \( \ell_G \) independently, using different heuristic arguments.

From the above postulated collapse, it follows that the pure cat state becomes the mixture of its two components:
\[
\frac{|f_1\rangle + |f_2\rangle}{\sqrt{2}} \rightarrow \frac{|f_1\rangle |f_1\rangle + |f_2\rangle |f_2\rangle}{2}. (6)
\]

Accordingly, we talk about G-related spontaneous decoherence—an intrinsically related mechanism to the spontaneous collapses. The dynamics of spontaneous decoherence is simpler, being in terms of master equations for the density matrix \( \hat{\rho} \) of the system in question; we take this approach in the forthcoming sections. (We come back to the important distinction between collapse and decoherence in the summary.)

Since catness (2) would diverge for point-like constituents, a certain spatial cut-off \( \sigma \) is needed. The smaller the cut-off \( \sigma \), the stronger will be the proposed spontaneous decay. For a strong decay, one chooses \( \sigma = 10^{-12} \) cm, to allow for a mass density resolution as fine as the size of the nuclei [3]. Unfortunately, the detailed dynamics of the spontaneous collapses leads to a constant rate of kinetic excitation for all microscopic constituents. This has been a basic problem, first pointed out in [8].

Traditionally, the DP-model used to be applied to single macroscopic degrees of freedom (d.o.f.) like the center of mass (c.o.m.) of a bulk. The present work derives the DP-model for the hydrodynamic-elastic (acoustic) d.o.f., opening new perspectives.
2. G-related decoherence

Let us start from a many-body system of Hamiltonian

$$\hat{H} = \sum_{a} \frac{\hat{p}_a^2}{2m_a} + \sum_{a,b} V(\hat{x}_a - \hat{x}_b),$$  \hspace{1cm} (7)

where $m_a, \hat{p}_a, \hat{x}_a$ are the mass, and the canonical variables, respectively, of constituents. In the DP-model, the von Neumann evolution equation of the quantum state $\hat{\rho}$ is modified by the G-related decoherence term $D\hat{\rho}$:

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar} [\hat{H}, \hat{\rho}] + D\hat{\rho}.$$  \hspace{1cm} (8)

This is the master equation of the DP-model where $D = D^\dagger$ is proportional to the Newton gravitational constant $G$:

$$D\hat{\rho} = -\frac{G}{2\hbar} \int \left[\hat{f}_a (\mathbf{r}), \left[\hat{f}_b (\mathbf{r}'), \hat{\rho}\right]\right] \frac{d\mathbf{r}d\mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|}.$$  \hspace{1cm} (9)

The key quantity is the smoothened mass distribution operator:

$$\hat{f}_a (\mathbf{r}) = \sum_{a} m_a g_{\sigma}(\mathbf{r} - \hat{x}_a),$$  \hspace{1cm} (10)

where $g_{\sigma}(\mathbf{r})$ is the central Gaussian distribution of width $\sigma$. The finite width plays the role of cut-off since for point-like constituents $D$ would diverge otherwise. We assume $\sigma \sim 10^{-12}$ cm which is about the nuclear size. In Fourier representation, using (10), the G-related spontaneous decoherence (9) takes this form:

$$D\hat{\rho} = -\frac{G}{2\hbar} \int \frac{4\pi e^{-k^2\sigma^2}}{k^2} \sum_{a,b} m_a m_b \left[e^{i\mathbf{k}\mathbf{x}_a} - e^{-i\mathbf{k}\mathbf{x}_b}\right] \frac{dk}{(2\pi)^3}.$$  \hspace{1cm} (11)

Due to the decoherence term in (8), the total energy (7) is not conserved. We can determine its Heisenberg time-derivative, yielding a number:

$$\frac{d\hat{H}}{dt} = D\hat{H} = D \sum_a \frac{\hat{p}_a^2}{2m_a} = \frac{1}{2\sqrt{4\pi}} G\hbar \sigma^{-3} M;$$  \hspace{1cm} (12)

where $M = \sum_a m_a$ is the total mass. For bulk matter, say for $M = 1 g$, the rate of spontaneous energy gain is cca. 100 erg s^{-1} which would cause a gross eternal warming up, much too higher than in other collapse models.

Heating is an annoying feature of all spontaneous collapse models. From (12) we realize that it is the kinetic energy of each constituent which is increasing at a constant rate $\sim G\hbar m_a \sigma^{-3}$. To further characterize it, we define the ‘nuclear’ density

$$f_{\text{nucl}} = \frac{m_{\text{av}}}{(4\pi \sigma^2)^{3/2}},$$  \hspace{1cm} (13)

where $m_{\text{av}}$ is the average constituent mass. Let us consider the classical (non-quantum) frequency
\[ \omega_G^{\text{nucl}} = \sqrt{4\pi Gf^{\text{nucl}}/3} \sim 1\text{kHz} \quad (14) \]

of the ‘Newton oscillator’ [9] in ‘nuclear’ density. Now we can write the rate of spontaneous energy increase per microscopic d.o.f. as

\[ \frac{1}{2}\hbar (\omega_G^{\text{nucl}})^2 \sim 10^{-21} \text{erg s}^{-1}. \quad (15) \]

This is an extreme small value, but for an Avogadro number of constituents it yields too much heating [8], like 100 erg s\(^{-1}\), as we said above.

### 3. Acoustic mode decoherence

To make Schrödinger cats decay, which the DP-master equation (8) is good for, the G-related spontaneous collapses of the macroscopic d.o.f. matter. For macroscopic d.o.f. it is plausible to take the hydrodynamic-elastic (acoustic) ones. In close-to-equilibrium states they decouple from the microscopic d.o.f, therefore the dynamics of acoustic d.o.f. becomes autonomous, the corresponding effective quantum state \( \hat{\rho} \) satisfies a closed evolution equation with an effective Hamiltonian \( \hat{H} \). As we shall see, also the DP-decoherence term (11) induces a closed form for the acoustic d.o.f.

First, let us define the Hamiltonian part of the dynamics for a homogeneous bulk of mass \( M \), volume \( V \), and mass density \( f^0 = M/V \).

\[ f^0 = \frac{M}{V}. \quad (16) \]

We start form the notion of displacement field known, e.g., from the theory of elasticity [10]. We introduce the quantized displacement field \( \hat{u}(r) \) together with the canonically conjugated momentum field \( \hat{\pi}(r) \), satisfying the canonical commutators:

\[
\begin{align*}
[\hat{u}_i(r), \hat{u}_j(r')] &= 0, \\
[\hat{\pi}_i(r), \hat{\pi}_j(r')] &= 0, \quad (i, j = 1, 2, 3) \\
[\hat{u}_i(r), \hat{\pi}_j(r')] &= i\hbar \delta_{ij}\delta(r - r').
\end{align*}
\]

\[ (17) \]

We assume that the macroscopic excitations of our bulk are quantized acoustic (sound) waves. For long wavelengths, they satisfy linear dynamics with the following Hamiltonian:

\[ \hat{H} = \int \left( \frac{1}{2f^0} \hat{\pi}^2 + \frac{f^0}{2} c_\ell^2 (\nabla \hat{u})^2 \right) \mathbf{dr}, \quad (18) \]

where \( c_\ell \) is the longitudinal sound velocity. For simplicity, we have restricted the calculations to the longitudinal modes satisfying \( \nabla \times \hat{u} = 0 \).

Second, let us determine the G-related spontaneous decoherence of the acoustic modes. To this end, we re-express the decoherence \( D (9) \) in function of the displacement field \( \hat{u}(r) \). We disregard the electronic constituents because of their small mass. We write the coordinate operators of the nuclei into this form:

\[ \hat{x}_a = \mathbf{x}_a + \hat{u}(\mathbf{x}_a), \quad (19) \]

where \( \mathbf{x}_a \) are the fiducial positions. If, furthermore, we assume that the displacements \( \hat{u}(r) \) are much smaller than \( \sigma \) then in \( D \) the cross-terms between different nuclei can be ignored and, in
Fourier representation (11), the Taylor expansion \( \exp[i\mathbf{k}\hat{\mathbf{u}}(\mathbf{x}_a)] \approx 1 + i\mathbf{k}\hat{\mathbf{u}}(\mathbf{x}_a) \) applies:

\[
D\hat{\rho} = -\frac{G}{2\hbar} \int \frac{4\pi e^{-k^2\sigma^2}}{k^2} \sum_a m_a^2 \left[ \hat{\mathbf{u}}(\mathbf{x}_a), [\hat{\mathbf{u}}(\mathbf{x}_a), \hat{\rho}] \right] \frac{dk}{(2\pi)^3}
\]

\[
= -\frac{G}{2\hbar} \frac{1}{3\sqrt{4\pi\sigma^2}} \sum_a m^2_a \left[ \hat{\mathbf{u}}(\mathbf{x}_a), [\hat{\mathbf{u}}(\mathbf{x}_a), \hat{\rho}] \right]
\]

\[
\approx -\frac{G}{2\hbar} \frac{1}{3\sqrt{4\pi}} m^2_{av} f^0 \int \left[ \hat{\mathbf{u}}(\mathbf{r}), [\hat{\mathbf{u}}(\mathbf{r}), \hat{\rho}] \right] d\mathbf{r}. \tag{20}
\]

The symbol \( m^2_{av} \) stands for the average squared mass of the nuclei. Let us define the ‘nuclear’ density as

\[
f^{nucl} = \frac{m^2_{av}/m_{av}}{(4\pi\sigma^2)^{3/2}}, \tag{21}
\]

slightly different from (13), the same order of magnitude though. Similar will be the corresponding Newton oscillator frequency \( \omega^0_{G}^{nucl} \tag{14} \). Using it, we obtain the final form of the G-related decoherence term of the acoustic modes:

\[
D\hat{\rho} = -\frac{1}{2\hbar} f^0 \left( \omega^0_{G}^{nucl} \right)^2 \int \left[ \hat{\mathbf{u}}(\mathbf{r}), [\hat{\mathbf{u}}(\mathbf{r}), \hat{\rho}] \right] d\mathbf{r}. \tag{22}
\]

According to equations (8,18, 22), the decoherence master equation of the acoustic modes reads

\[
\frac{d\hat{\rho}}{dt} = \frac{1}{2\hbar} \int \left( -\frac{i}{f^0} \left[ \hat{\pi}^2, \hat{\rho} \right] - if^0 c_r^2 \left[ (\nabla \hat{\mathbf{u}})^2, \hat{\rho} \right] - f^0 \left( \omega^0_{G}^{nucl} \right)^2 \left[ \hat{\mathbf{u}}, [\hat{\mathbf{u}}, \hat{\rho}] \right] \right) d\mathbf{r}. \tag{23}
\]

Recall that \( \hat{\mathbf{u}}(\mathbf{r}), \hat{\pi}(\mathbf{r}) \) are effective canonical variables of the long wavelength acoustic modes. This feature will be elucidated in Fourier representation.

### 3.1. Fourier representation

Let us expand the canonical variables in terms of discrete Fourier components \( \hat{\mathbf{u}}_k = \hat{\mathbf{u}}_{-k}^\dagger \) and \( \hat{\pi}_k = \hat{\pi}_{-k}^\dagger \):

\[
\hat{\mathbf{u}}(\mathbf{r}) = \frac{1}{\sqrt{V}} \sum_k \hat{\mathbf{u}}_k e^{i\mathbf{k}\cdot\mathbf{r}},
\]

\[
\hat{\pi}(\mathbf{r}) = \frac{1}{\sqrt{V}} \sum_k \hat{\pi}_k e^{i\mathbf{k}\cdot\mathbf{r}}, \tag{24}
\]

satisfying the discrete canonical commutation relationships

\[
[\hat{u}_{ki}, \hat{u}_{lj}^\dagger] = 0, \quad [\hat{\pi}_{ki}, \hat{\pi}_{lj}^\dagger] = 0, \quad (i, j = 1, 2, 3)
\]

\[
[\hat{u}_{ki}, \hat{\pi}_{lj}^\dagger] = i\hbar \delta_{ij}\delta_{kl}. \tag{25}
\]
The Hamiltonian (18) and spontaneous decoherence (22) read, respectively:

\[ \dot{H} = \frac{1}{2} \sum_{k} \left( \frac{1}{f^0} \hat{\pi}_k^{\dagger} \hat{\pi}_k + f^0 c^2 \hat{k}^2 \hat{u}_k^{\dagger} \hat{u}_k \right), \tag{26} \]

\[ D\dot{\rho} = -\frac{1}{2\hbar} \sum_{k} f^0 \left( \omega_{G}^{\text{nucl}} \right)^2 \left[ \hat{u}_k^{\dagger} \hat{u}_k \right]. \tag{27} \]

The master equation (23) takes the following form for the acoustic Fourier modes:

\[ \frac{d\dot{\rho}}{dt} = \frac{1}{2\hbar} \sum_{k} \left( -\frac{i}{f^0} \left[ \hat{\pi}_k^{\dagger}, \hat{\pi}_k, \hat{\rho} \right] - i f^0 c^2 \hat{k}^2 \left[ \hat{u}_k^{\dagger}, \hat{u}_k, \hat{\rho} \right] - f^0 \left( \omega_{G}^{\text{nucl}} \right)^2 \left[ \hat{u}_k^{\dagger}, \hat{u}_k, \hat{\rho} \right] \right). \tag{28} \]

Now we can calculate the heating rate:

\[ \frac{d\dot{H}}{dt} = D\dot{H} = \sum_{k} D \frac{\hat{\pi}_k^{\dagger} \hat{\pi}_k}{2f^0} = \sum_{k} \frac{3}{2} \hbar \left( \omega_{G}^{\text{nucl}} \right)^2. \tag{29} \]

Observe that each acoustic mode undergoes the same tiny heating similar to the heating (15) found for the individual d.o.f. of each constituent.

### 3.2. C.o.m. decoherence

The c.o.m. motion of the bulk is decoupled from the internal acoustic modes. Let us read out the dynamics of the c.o.m. position \( \hat{x} \) and momentum \( \hat{p} \) from (24):

\[ \hat{x} = \frac{1}{\sqrt{V}} \hat{u}_0, \]

\[ \hat{p} = \sqrt{V} \hat{\pi}_0, \tag{30} \]

where we set the fiducial c.o.m. position to the origin. We identify the c.o.m. parts of the master equation (28): the free body kinetic Hamiltonian \( \hat{p}^2/2M \) and the standard G-related position decoherence

\[ D_{\text{c.o.m.}} \dot{\rho} = -\frac{1}{2} M \left( \omega_{G}^{\text{nucl}} \right)^2 \left[ \hat{x}, \left[ \hat{x}, \hat{\rho} \right] \right]. \tag{31} \]

The c.o.m. dynamics is thus governed by the autonomous master equation

\[ \frac{d\hat{\rho}_{\text{c.o.m.}}}{dt} = -\frac{1}{\hbar} \left[ \hat{p}^2, \hat{\rho}_{\text{c.o.m.}} \right] - \frac{1}{2\hbar} M \left( \omega_{G}^{\text{nucl}} \right)^2 \left[ \hat{x}, \left[ \hat{x}, \hat{\rho}_{\text{c.o.m.}} \right] \right]. \tag{32} \]

in full accordance with the old derivations in the DP-model [1–3]. If we calculate the heating rate we get

\[ D_{\text{c.o.m.}} \frac{\hat{p}^2}{2M} = -\frac{1}{4\hbar} \left( \omega_{G}^{\text{nucl}} \right)^2 \left[ \hat{x}, \left[ \hat{x}, \hat{p}^2 \right] \right] = \frac{3}{2} \hbar \left( \omega_{G}^{\text{nucl}} \right)^2 \sim 10^{-21} \text{ erg s}^{-1}. \tag{33} \]

This is the same extreme small value (15) that we obtained universally for each individual constituent or, alternatively, for each acoustic mode (29).
3.3. Universal dominance of G-related decoherence

In bulk matter, the DP-model yields a certain simple universal behaviour of spontaneous decoherence. In the master equation (28), consider the magnitudes of the harmonic potential and the G-related decoherence terms, respectively. Both of them are quadratic in the displacements \( \hat{u}_k \). Although their structure is different, we see that the harmonic potential becomes suppressed by the G-related decoherence term for small wave numbers \( k \) such that

\[
\omega_G \ll \omega_{\text{nucl}}.
\]  

(34)

In solids, e.g., the typical range of sound velocity is \( c_\ell \sim 10^5 \text{ cm s}^{-1} \), the above condition means wavelengths larger than \( \sim 1 \text{ m} \). The master equation (28) for these modes takes the following form:

\[
\frac{\text{d}\hat{\rho}}{\text{d}t} = \frac{1}{2\hbar} \sum_{\text{1} \leq k \leq \text{1 m}} \left( \frac{-i}{f^0} \left[ \hat{n}_k^\dagger \hat{n}_k, \hat{\rho} \right] - f^0 \left( \omega_{\text{G}}^{\text{nucl}} \right)^2 \left[ \hat{u}_k^\dagger, [\hat{u}_k, \hat{\rho}] \right] \right). 
\]  

(35)

In oscillatory modes of wavelength \( \gg 1 \text{ m} \), the G-related decoherence dominates over the directional force. Suppose we have a bulk of rock as big as 100 m. Consider a sub-volume inside, with size about a few meters at least. Then the c.o.m. of this inside body behaves as if the body were a free-body subject to c.o.m. spontaneous decoherence, like in section 3.2. The directional force from the behalf of the environmental rock is not absent, of course. On a time scale much longer than spontaneous decoherence’s, it will keep the inside body close to its fiducial position.

3.4. Strong spontaneous decoherence at low heating

Consider the master equation (28) of the DP-model for the acoustic modes. As we said in section 3.1, each mode undergoes the heating rate (15). Is it possible, by some refinement of the DP-model, to reduce the spontaneous heating but to retain the strength of decoherence in the macroscopic d.o.f.?

Let us choose a larger cut-off \( \sigma \), say hundred times the 'nuclear' size. The ominous parameter \( f_{\text{nucl}} \) (13, 21) would drop by six orders of magnitude, resulting in six orders of magnitude reduction of heating rate at the price of the same reduction of the strength of spontaneous collapses. The critical size, \( \sim 1 \text{ m} \) in section 3.3, where c.o.m. DP-collapses become faster than the directional forces, will increase by six orders of magnitude. So we cannot play much with the cut-off \( \sigma \).

Instead, we can play with the number of acoustic modes. Suppose that short wave acoustic modes are not subjected to G-related spontaneous collapses. For instance, let us set this limit to \( \lambda = 10^{-5} \text{ cm} \), i.e., we replace the standard master equation (28) by the following version:

\[
\frac{\text{d}\hat{\rho}}{\text{d}t} = \frac{-i}{2\hbar} \sum_k \left( \frac{1}{f^0} \left[ \hat{n}_k^\dagger \hat{n}_k, \hat{\rho} \right] + f^0 c_\ell^2 k^2 \left[ \hat{u}_k^\dagger \hat{u}_k, \hat{\rho} \right] \right) - \frac{1}{2\hbar} \sum_{\text{1} \leq k \leq \lambda} \left( f^0 \left( \omega_{\text{G}}^{\text{nucl}} \right)^2 \left[ \hat{u}_k^\dagger, [\hat{u}_k, \hat{\rho}] \right] \right). 
\]  

(36)

Since \( \lambda \) is three orders of magnitude larger than the internuclear distance in common bulk matter, the number of spontaneously heated acoustic modes drops by nine orders of magnitude compared to the number of the nuclei. The spontaneous heating rate of 1 g will reduce to \( 10^{-7} \text{ erg s}^{-1} \) instead of 100 erg s\(^{-1} \) found in section 2. The strength and dynamics of G-related
spontaneous collapses of the long wavelengths acoustic modes, including the c.o.m. as well, remain the same as before.

4. Summary

We have derived the DP-model of G-related spontaneous collapses for the hydrodynamic-elastic (acoustic) d.o.f. of bulk matter. To ensure strong significance of collapses, we chose the minimum plausible cut-off $\sigma$ which is about the nuclear size. This leads to the dominance of the spontaneous collapses over the elastic forces inside common condensed matter for wavelengths larger than about 1 m. The warming up, an annoying side-effect of spontaneous collapses, will considerably drop if we ascribe spontaneous collapses to the really macroscopic acoustic modes only. This modification does not influence the usual predictions of the model concerning the collapse in macroscopic d.o.f.

For simplicity’s sake, we worked out the master equations of spontaneous decoherence of the acoustic modes. We spared the now straightforward derivation of the stochastic (jump [1] or diffusive [3]) Schrödinger equations of G-related spontaneous collapses for the acoustic modes. As we mentioned in the Introduction, spontaneous decoherence and collapse are to be distinguished conceptually. Spontaneous decoherence is the testable local effect of spontaneous collapse. It can be mimicked by and it is usually masked by environmental decoherence. Continued laboratory efforts are trying to suppress these environmental effects [11–18]. On the contrary, collapse is global effect, cannot be mimicked or masked by the environment however noisy it is [9]. In any current models of spontaneous collapse [19], collapse itself is never detectable, only the resulting spontaneous decoherence is, as emphasized in [20]. To let spontaneous collapses be testable, recent extension of the DP-model has shown interesting theoretical and experimental perspectives [9, 20–22].

On the spontaneous decoherence of the acoustic modes, derived in the present work, we remark that it might be influenced or even masked by the modes’ higher order coupling to the microscopic d.o.f. inside the bulk or from the behalf of the environment. Further related investigations are certainly needed.

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