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Stability of model flocks in turbulent-like flow

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Abstract. We report numerical simulations of a simple model of flocking particles in the presence of an uncertain background environment. We consider two types of environmental perturbations: random noise applied separately to each particle, and spatiotemporally correlated 'noise' provided by a turbulent-like flow field. The effects of these two types of noise are very different; surprisingly, the applied flow field tends to destroy the global order of the flocking model even for vanishingly small flow amplitudes. Local order, however, is preserved in smaller sub-flocks, although their composition changes dynamically. Our results suggest that realistic perturbations must be considered in assessing the stability of models of collective animal behavior, and that random noise is not a sufficient proxy.

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1. Introduction

Coordinated, collective motion of animal groups is incredibly prevalent in nature. In part because it is so generic, this emergent phenomenon has engaged and fascinated scientists from many disciplines. Biologists seek deeper insight into the pressures that drive collective behavior and the advantages it provides for individuals and populations; physicists and applied mathematicians hope to tease out general principles of self-organization that may be valid for other complex systems as well as the fundamental laws that govern biology; and engineers aim to exploit swarming behavior to design biomimetic multi-agent systems (in both hardware and software) that accomplish complex tasks with minimal explicit control.

Many strategies have been proposed to model the dynamics of animal aggregations, ranging from simple cellular automata to nonlinear, nonlocal partial differential equations [1–4]. The most common choice, however, is to treat the individuals in the aggregation as self-propelled particles that interact via simple local rules—typically, some combination of long-range attraction, short-range repulsion and intermediate-range alignment [5, 6]. For flocks (i.e., aggregations in which the individuals all primarily move in the same direction, as opposed to, for example, randomly moving swarms [7]), the alignment interaction is typically assumed to dominate. And indeed, these simple models appear to reproduce features observed in laboratory experiments fairly well. Buhl *et al* [8], for example, found that the behavior of marching locusts in an annular arena was well described by a simple agent-based model.

The natural world, however, is a complex, fluctuating place. Real animal aggregations nevertheless manage to maintain cohesion even in uncertain environments. Thus, it is reasonable to hypothesize that flocking conveys stability against environmental perturbations. This kind of stability is also highly desirable in biomimetic applications: a collective control strategy, for example, is useless if it cannot remain viable in a real, fluctuating environment. But current studies of flocking have rarely looked at the response of models to realistic perturbations; rather, the behavior of the model is studied in isolation. The quality of the model is determined purely based on the emergence of large-scale structure that is qualitatively reminiscent of behavior observed in the real world [1]. But before these models can be used for design purposes or to draw conclusions about the behavior of actual biological systems, their stability against realistic perturbations must be fully assessed.

The stability of flocking models in the presence of noise has certainly been studied previously [5]; typically, however, this noise is expressed as uncorrelated, independent random processes attached to each individual in the flock. But real animals live in fluid environments. For macroscopic animals, such as birds, fish or insects, the flows they experience will typically be at least somewhat turbulent—and thus, the 'noise' they feel from the turbulent fluctuations in their environment will be spatially and temporally correlated, even though the flow may fluctuate rapidly in time. This correlation may in turn lead to unexpected effects in the model.

We studied the effects of strongly fluctuating but spatiotemporally correlated flows on the stability of flocks using a simple model system. We used the classic flocking model first introduced by Vicsek *et al* [5], and embedded the flocks in a turbulent-like flow field produced by a kinematic simulation (KS) [9–11]. Although this flow is not a true solution of the Navier–Stokes equations, it fluctuates in space and time yet has finite spatiotemporal correlations and has an energy spectrum that matches the expected form for turbulent flows. We find that the flocks are strongly affected by the flow, even when the flow velocities are very small compared to the intrinsic speed of the particles, and that the effects of flow are qualitatively

distinct from random noise. Depending on the parameters of the simulation, flow can either suppress (when the tendency to flock is strong) or enhance (when the tendency to flock is weak) the global ordering of the flocks. Global order alone, however, is not sufficient to characterize the behavior we observe. In general, the flow tends to break large flocks dynamically into distinct clusters, retaining local order even when the global order breaks down. We identify two conditions that appear to be necessary for this process to occur: splay in the velocity field, and a nonuniform distribution of particles. Our results show that the response of flocking models to spatially correlated noise is qualitatively different from stochasticity applied to each individual particle, and suggest that the external environment cannot be neglected in assessing and validating models of collective animal behavior.

We begin below by describing in detail the models we use for both the particles and the flow in section 2. In section 3, we discuss our results, including changes to the global ordering of the flocking particles (section 3.1), the breakup of the flock into dynamical clusters (section 3.2) and the conditions under which a flock will be broken (section 3.3). Finally, we summarize and conclude in section 4.

2. Model

2.1. Flocking particles

The particles in our simulation obey the classic Vicsek flocking model [5], extended to three spatial dimensions [12]. The particles are spherical, point-like and have no inertia, but each carries an intrinsic velocity vector $\mathbf{v}_p$ with a constant magnitude v_p. The particles also feel the background flow field, and interact with each other; in this way, the direction of the intrinsic velocity can change, although its magnitude is fixed.

At each timestep, the position of the ith particle is updated according to

$$\mathbf{x}^{(i)}(t + \Delta t) = \mathbf{x}^{(i)}(t) + \mathbf{v}_p^{(i)}(t)\Delta t + \mathbf{v}_f(\mathbf{x}^{(i)}(t), t)\Delta t, \tag{1}$$

where $\mathbf{v}_f(\mathbf{x}, t)$ is the fluid velocity field. The direction of the intrinsic velocity vector $\mathbf{v}_p^{(i)}$ is updated by averaging the directions of motion of all the neighboring particles in a sphere of radius r_{int} around $\mathbf{x}^{(i)}(t)$; that is,

$$\mathbf{v}_p^{(i)}(t + \Delta t) = v_p \frac{\sum_{j \in r_{int}} \mathbf{v}^{(j)}(t)}{\left| \sum_{j \in r_{int}} \mathbf{v}^{(j)}(t) \right|}, \tag{2}$$

where $\mathbf{v} = \mathbf{v}_p + \mathbf{v}_f$ is the total velocity (intrinsic plus flow) of each particle. We note that the particles do not rotate with the flow vorticity; rather, they simply immediately move in the direction specified by the combination of the flow and their neighbors. Also note that even though the magnitude of each particle's intrinsic velocity is fixed, particles may move faster than v_p (and at different speeds from their neighbors) due to the effect of the background flow.

At the beginning of each simulation, we distribute N particles randomly in the computational domain, which we take to be a triply periodic box of side length 2π, with random orientations. The initial number density of the particles is thus $n_0 = N/(2\pi)^3$. The behavior of the flocks is determined by two nondimensional parameters [12]: the ratio of the intrinsic particle speed v_p to the background flow speed v_f, and the ratio of the average volume per particle (given by n_0^{-1}) to each particle's interaction volume (given by $(4/3)\pi r_{int}^3$). For convenience, we define this volume ratio to be $\zeta \equiv n_0^{-1}/[(4/3)\pi r_{int}^3]$.

2.2. Flow field

Real, macroscopic animals such as birds or fish typically live in turbulent environments. Thus, we wish to study the effects of a complex, multi-scale flow field with finite correlations in space and time on our model flocks. Simulating true turbulence (including, e.g., correct power-law scaling relationships or intermittency effects) is, however, not necessary for our purposes. Instead, we generate 'turbulent-like' flow fields using KS.

KS produces a flow field that is unsteady, multi-scale and correlated in time and space. Though it is not a solution of the Navier–Stokes equations, it is constrained to have a Kolmogorov-like energy spectrum that scales as $E(k) \sim k^{-5/3}$, where E is the spectrum and k is a wavenumber. We follow previous work and build the velocity field from a sum of Fourier modes as [9–11]

$$\mathbf{u}(\mathbf{x}, t) = \sum_{n=1}^{N_k} \left[\mathbf{A}_n \times \hat{\mathbf{k}}_n \cos(\mathbf{k}_n \cdot \mathbf{x} + \omega_n t) + \mathbf{B}_n \times \hat{\mathbf{k}}_n \sin(\mathbf{k}_n \cdot \mathbf{x} + \omega_n t) \right]. \quad (3)$$

N_k is the number of modes in the simulation, and allows us to specify the spatiotemporal complexity of the flow. $\hat{\mathbf{k}}_n$ is a unit vector in the direction of the wave mode $\mathbf{k}_n$, and is chosen randomly. The directions of $\mathbf{A}_n$ and $\mathbf{B}_n$ are also chosen randomly, under the constraint that they be orthogonal to $\hat{\mathbf{k}}_n$, in order to preserve incompressibility. Their amplitudes are given by

$$A_n^2 = B_n^2 = \frac{2}{3} E(k_n) \Delta k_n, \quad (4)$$

where Δk_n is the spacing between wave modes. This spacing is given by $\Delta k_n = (k_{n+1} - k_{n-1})/2$ for $2 \leqslant n \leqslant N_k - 1$, with $\Delta k_1 = (k_2 - k_1)/2$ and $\Delta k_{N_k} = (k_{N_k} - k_{N_k-1})/2$. To complete the specification of the flow, we need to choose a form for the energy spectrum $E(k)$; as stated above, we choose a Kolmogorov inertial range spectrum. Finally, the time dependence of the flow is given by $\omega_n = \lambda \sqrt{k_n^3 E(k_n)}$, where the nondimensional constant λ is of order 1 and determines the flow time scale.

In the results shown below, we generated KS flows with $N_k = 100$. In all cases, the wavelength of the largest mode was the size of the computational box (2π), while the wavelength of the smallest mode was $2\pi/10\,000$.

3. Results

3.1. Global order

The simplest way to characterize the global ordering of the particles is by considering the overall polarization of the aggregation; that is, by measuring how well aligned the velocity vectors of all the particles are. To do so, we follow previous work [5] and define an order parameter

$$\phi = \frac{\left| \sum_{i=1}^{N} \mathbf{v}^{(i)} \right|}{\sum_{i=1}^{N} \left| \mathbf{v}^{(i)} \right|}. \quad (5)$$

Note that $\mathbf{v}$ here is the *total* velocity vector of each particle, not just its intrinsic velocity. Since this is a vector sum, ϕ vanishes when the velocities point in random directions. When the particles are perfectly aligned, however, ϕ saturates to unity.

We measured the average order parameter $\langle \phi \rangle$, where the average is taken over time, after an initial transient, for three different cases: (i) particles moving with no background flow (that
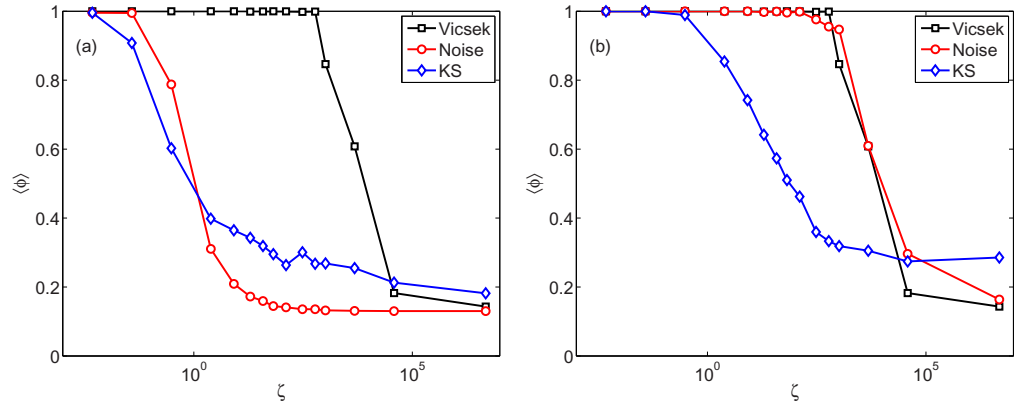
Figure 1. Global order parameter $\langle \phi \rangle$ (averaged over time after an initial transient) as a function of ζ, the ratio of the volume per particle to the interaction volume, for two cases: (a) $v_f/v_p \approx 0.1$ and (b) $v_f/v_p \approx 10^{-4}$. Even when the flow speed is very small, the global order is quickly destroyed. Flow is qualitatively different from uncorrelated noise.

is, a pure, noiseless Vicsek model); (ii) particles subjected to uncorrelated random noise drawn from a uniform distribution with amplitude v_f; and (iii) particles moving in a KS flow field. We treat the noise exactly like a 'flow', and couple it to the particles just as the flow is in equation (1), so that the particles have both a deterministic and a stochastic component to their motion in this case. The random noise applied to each particle is sampled independently for each particle at each timestep. In figure 1(a), we show the variation of $\langle \phi \rangle$ as a function of ζ, the ratio of the volume per particle to the interaction volume. As this parameter becomes large, the particles rarely interact with one another. Data in figure 1(a) is shown for relatively high-speed background flow, where the typical flow speed v_f is similar to the intrinsic particle speed v_p, with $v_f/v_p \approx 0.1$. The results in figure 1(a) show that, unsurprisingly, adding either significant noise or a strong flow tends to destabilize the flocks, leading to a significant decrease in the global polarization. At very small r_{int}, however, the flow field can lead to a slight increase in polarization, since the KS flow field is smooth in space and time.

More surprising, however, are the results shown in figure 1(b). Here, we show data for the case of $v_f \approx 10^{-4} v_p$, where the effects of the flow might be expected to be negligible. And simple noise is indeed negligible: the variation of the order parameter in the presence of such a tiny amount of noise is nearly the same as for no noise at all. But the effects of KS are quite different: again, global order is rapidly lost, even though the flow velocity is very small. Thus, the addition of spatially correlated flow to the model appears to be a nearly singular perturbation: *any* amount of flow, no matter how small, leads to a breakdown in the global stability of the flocks.

Before moving on, let us discuss briefly a case intermediate between random, uncorrelated noise and KS: the case of a flock perturbed with a *single* Fourier mode. For this case, the detailed behavior unsurprisingly depends on the spatial scale of this mode. If the wavelength of the mode is much larger than the relevant scales of the particles (such as the interaction radius r_{int} or the typical inter-particle distance $n_0^{-1/3}$), the direction of the flock may vary slowly over time but the global polarization is unaffected by the flow. If the wavelength is very small, it behaves just like random noise, since the flow experienced by each particle is essentially uncorrelated.
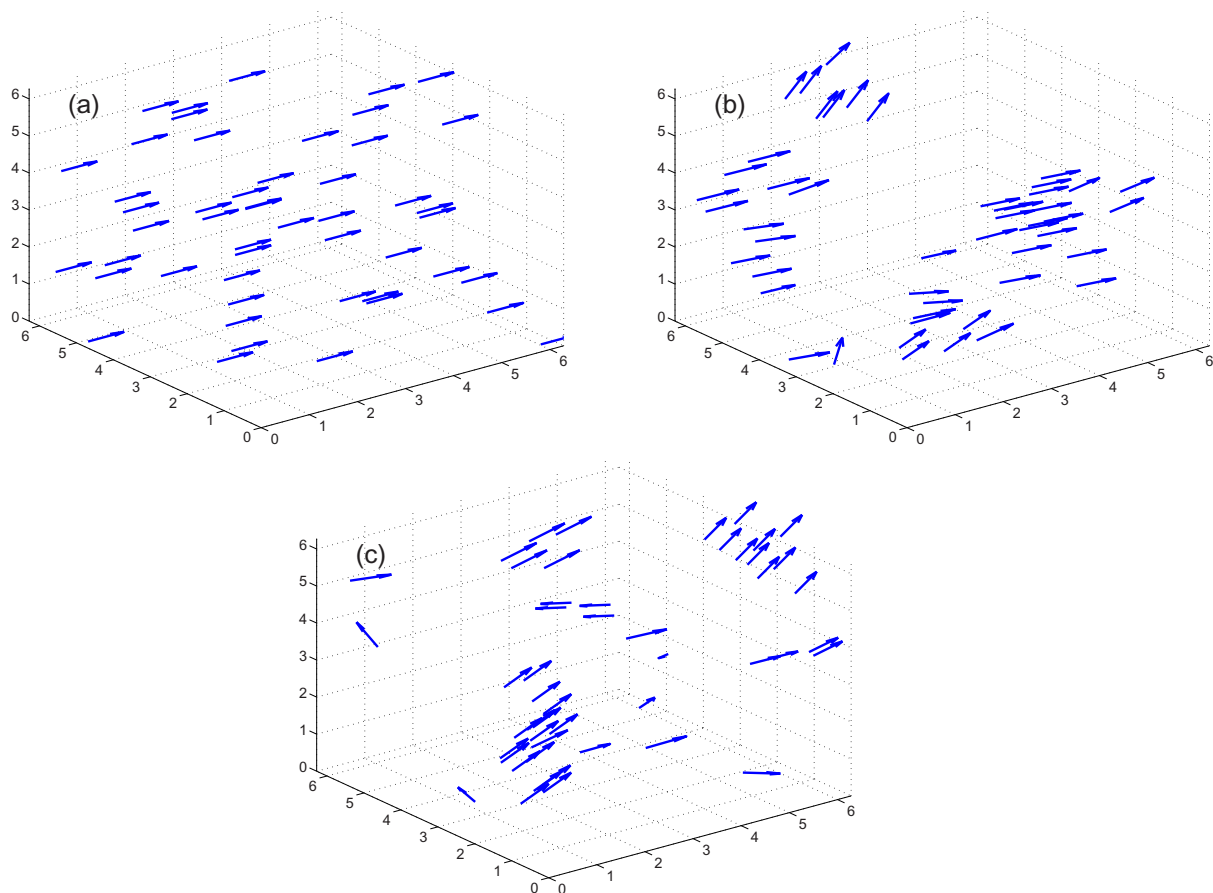
Figure 2. Particle positions and velocities (shown by the arrows) for $v_{\rm f}/v_{\rm p} = 10^{-4}$ and $\zeta = 2.4$ at three different times: (a) $t = 0$, (b) $t = 66$ and (c) $t = 154$. Although the particles began in an ordered state (a), they quickly break into smaller clusters.

If the wavelength is of the same order of magnitude as the relevant particle length scales, we find some suppression of the global order for a transient period. As time increases, however, the global order of the flock returns to the unperturbed case as the inter-particle spacing dynamically adjusts and the flock finds an equilibrium size. We only find an appreciable, time-independent effect of the flow on the global order when the simulation is both dense with modes in the relevant range of scales and contains larger and smaller scales.

3.2. Local order: dynamical clusters

As shown above, even small amounts of flow can destroy the global ordering of our model flocks. But a large population of particles may be more complex than can be captured by a simple global order parameter: the population may have *local* order as well. To understand how the flow changes the local structure, we must look at the flocks in more detail. When we do so, it becomes immediately clear that the flow field does not completely destroy the flocking behavior; rather, smaller sub-flocks are preserved.

In figure 2, we show an example of this effect for the case where $v_{\rm f}/v_{\rm p} \approx 10^{-4}$ and $\zeta = 2.4$ (deep in the stable flocking regime for the unperturbed model). We initially aligned all the
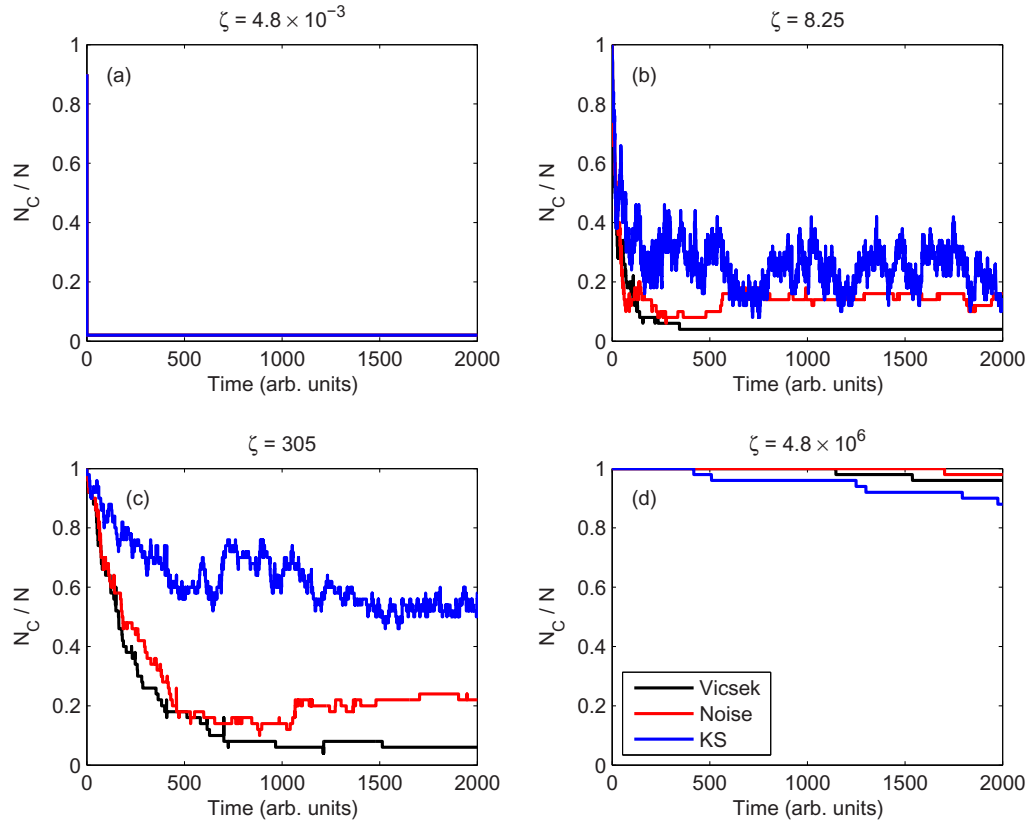
Figure 3. Number of identified clusters N_C scaled by the total number of particles N as a function of time for four different values of ζ. In all cases, the particles were initiated with random orientations, and $v_f/v_p \approx 10^{-4}$. When r_{int} is relatively large and ζ is small (a), the particles quickly form a single flock. But as r_{int} decreases, the particles form smaller clusters that form and break dynamically. At very small r_{int} (d), the particles are nearly all independent.

particles at the beginning of the simulation. But as time evolved, the global order broke down, and only smaller units were left. Within each of these sub-flocks, the ordering remained high; we observed, however, that the size and number of these sub-flocks fluctuated over time.

To quantify this behavior, we implemented an algorithm to identify these sub-flocks automatically. Similar to how dynamical clusters have been identified in bacterial colonies [13], we defined distinct clusters as groups of particles that were both close enough in distance (so that no particle was more than r_{int} away from the rest of the cluster) and in orientation (so that no particle's velocity vector was oriented more than 0.15 radians away from the rest) [14].

In figure 3, we show the variation with time of the number of clusters N_C, scaled by the number of particles N, which gives the maximum possible value of N_C. We show data for $v_f/v_p \approx 10^{-4}$ and four different values of ζ. In each case, we started the particles with random orientations, so that $N_C(t=0) = N$. For the strong interaction case (where r_{int} is large and ζ is small), the flock quickly orders into a single cluster and remains as a single flock indefinitely. But as r_{int} shrinks, the steady-state number of clusters grows, though the fluctuations may be large.
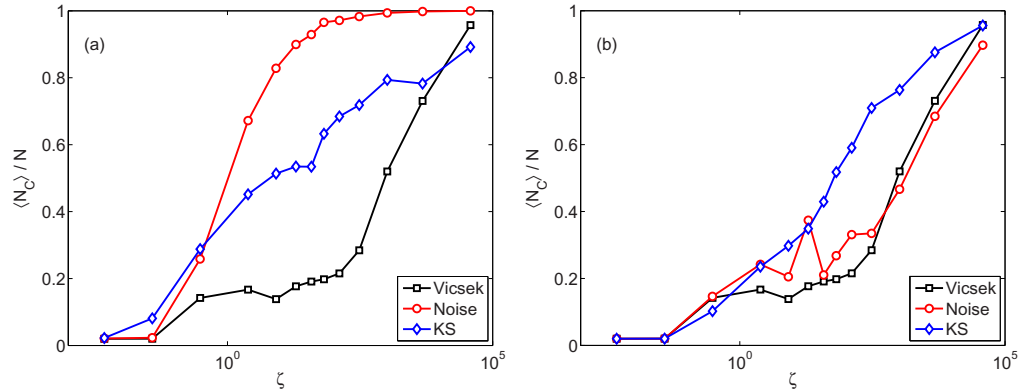
Figure 4. The average number of clusters $\langle N_C \rangle$ (computed after an initial transient) scaled by the number of particles in the simulation N as a function of ζ for (a) $v_f/v_p \approx 0.1$ and (b) $v_f/v_p \approx 10^{-4}$, the two cases shown in figure 1. As $\langle N_C \rangle$ approaches N, the particles become independent of each other and local order is lost.

The case of random, uncorrelated noise is somewhat similar for this value of v_f/v_p: the addition of noise also forms some sub-flocks for large values of ζ, although as can be seen in figure 3 in general the behavior is close to that of the unperturbed case. But for larger values of v_f/v_p, things are very different. As we showed in figure 1, large values of either noise or flow lead to a breakdown of global order, while global order is retained for small values of noise but is broken for flow. In figure 4, we show the average number of clusters (after an initial transient) for the same two cases shown in figure 1. For the case of $v_f/v_p \approx 0.1$ (figure 4(a)), where global order is quickly lost for both noise and flow, random noise also leads to a rapid breakdown of *local* order, since the number of identified sub-flocks rapidly approaches the number of particles in the simulation, indicating that the particles are essentially all independent. The KS case is qualitatively different: even though global order is lost, some local order is still retained even at high flow speeds. In contrast, at low flow speeds, shown in figure 4(b), the average number of clusters for noise and for the unperturbed case are nearly the same, while more sub-flocks are observed for the KS case. In general, then, our data suggest that spatiotemporally correlated flow promotes local ordering over a wide range of flow conditions, even though it suppresses global ordering. For random noise, on the other hand, local and global ordering behavior very similarly.

3.3. Fragmentation conditions

We have established that the imposition of complex flow suppresses the global ordering of the particle aggregation by breaking the flock into distinct clusters. But how does this occur? To look for the mechanism by which large flocks are split by the flow, we studied all the flock fragmentation events we observed. Although determining necessary and sufficient conditions under which flocks always break is very difficult, a general picture emerges from the data.

When a cluster is broken by the flow, the number of individuals it contains must shrink. There are two ways for this to happen: either single particles (or potentially small groups, if the number density is high enough) might be pulled off from the cluster boundaries, or the flock
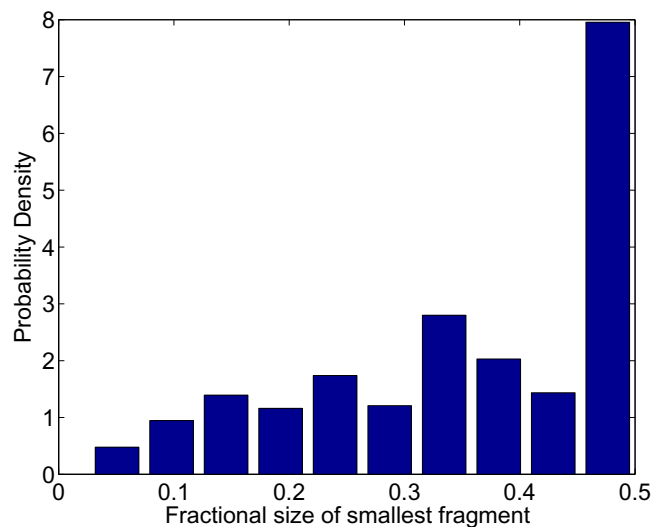
Figure 5. Probability density function of the fractional size of broken clusters (that is, the number of particles in a piece that broke off of a parent clusters divided by the number of particles in the parent cluster) for the case of $\zeta = 2.4$ and $v_f/v_p \approx 10^{-4}$. Data are only shown for broken clusters that are no larger than half of the parent cluster.

may split internally into two large fragments. Of these two scenarios, our data suggests the latter: clusters fragment not by losing particles from their periphery, but rather by breaking into large chunks. To demonstrate this, we plot in figure 5 the probability density function of the relative size (in units of the original, unbroken cluster size) of the fragment broken off when a cluster splits, for the case of $v_f/v_p = 10^{-4}$ and $\zeta = 2.4$. It is clear that it is not only very small pieces that disassociate from their parent flocks.

Two criteria appear to be necessary for this process to occur: splay in the velocity field, and fluctuations in the local number density inside the cluster. We show these conditions schematically in figure 6, and note that both will occur in our system: a complex flow field such as that produced by KS will have strong velocity gradients, and large fluctuations in number density are well known to occur in flocking models [15]. The first condition means that different parts of the flock must feel coherent flow velocities that tend to push them in different directions. For weak flows, however, this condition is not enough, since the orientational interaction is still strong. But if a given particle sees an anisotropic distribution of neighbors, as is sketched in figure 6, with two neighbors on one side of the velocity–field separatrix but only one on the other, it can switch its direction of motion. This behavior depends crucially on the existence of spatial and temporal correlations in the velocity field; this mechanism would not apply in the case of random noise.

4. Summary and conclusions

We have simulated the behavior of a simple model of flocking in the presence of a turbulent-like flow field produced by a KS. We find, surprisingly, that even very small levels of background flow can destabilize flocks, leading to a breakdown of the expected global order. Local order,
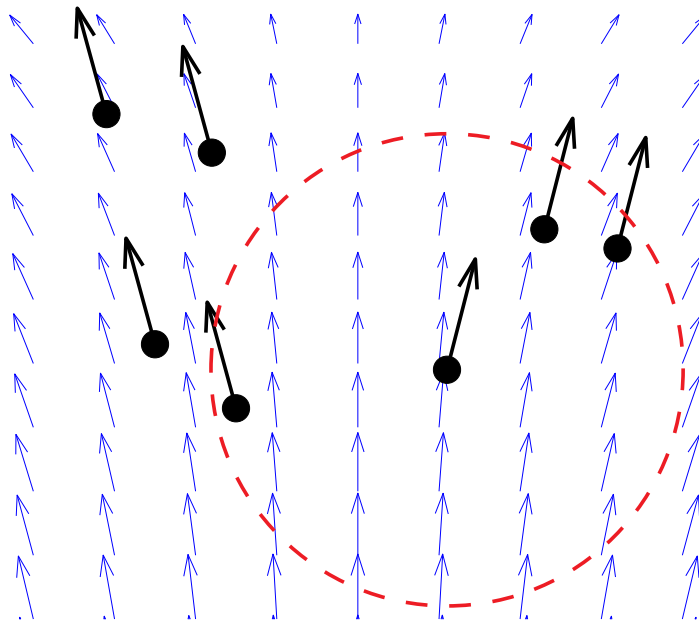
Figure 6. Sketch of the conditions under which clusters will break. The velocity field is shown with blue arrows; in this case, there is a separatrix in the middle of the sketch. Particles (and their associated velocity vectors) are shown in black. The red circle shows the interaction volume for the particle at its center; in this case, it interacts with two particles on one side of the separatrix and only one on the other, leading to a splitting of the cluster into two smaller units.

however, is preserved, as the overall particle aggregation is dynamically broken into small but highly ordered clusters. We identify two conditions that appear to be required for clusters to break: splay in the velocity field and a nonuniform distribution of particles. Our results suggest that models of collective animal motion are extremely fragile against spatiotemporally correlated fluctuations, and that such realistic flow fields must be considered when designing models that aim to capture actual biological systems.

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