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The role of quantum fluctuations in a system with strong fields

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Abstract. In this paper, we study how quantum fluctuations modify the quantum evolution of an initially classical field theory. We consider a scalar ϕ^4 theory coupled to an external source as a toy model for the color glass condensate description of the early time dynamics of heavy-ion collisions. We demonstrate that quantum fluctuations considerably modify the time evolution driving the system to evolve in accordance with ideal hydrodynamics. We attempt to understand the mechanism behind this relaxation to ideal hydrodynamics by using modified initial spectra and studying the particle content of the theory.

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1. Introduction

One of the outstanding theoretical problems in heavy-ion physics is a first principles understanding of the isotropization and thermalization of the matter produced in collision. The fact that the system is nearly thermal and isotropic at early times has been deduced from hydrodynamic model fits [1]–[6] to the measured spectra and elliptic flow [7]–[10]. Estimates of the relaxation time range from $\tau_{relax} \sim 0.5$ to 2 fm [3], which is hard to accommodate within a simple picture of interacting quasi-particles.

However, a quasi-particle description is not crucial to thermalization, and in this paper we will demonstrate that an initially strong classical field undergoing quantum evolution may evolve in accordance with ideal hydrodynamics. We will show that the presence of secular divergences (modes whose occupation number grows with time) becomes semi-classical on relatively short time scales and must be resumed to all orders in a standard perturbative expansion.

While this paper focuses on scalar ϕ^4 theory, it is suggested that similar mechanisms may be at work within the framework of the color glass condensate (CGC) description [11]–[18] of high-energy nuclei. Work on the application of the techniques shown here to the case of classical Yang–Mills is currently in progress [19]. The goal of the present paper is to understand the role secular divergences play in modifying the time evolution of the classical field. We demonstrate that the quantum evolution of a scalar field evolves in accordance with ideal hydrodynamics. Finally, we speculate on the mechanism behind this relaxation by looking at modified spectra and the time evolution of the number density.

This paper is largely based on the first written on this topic [20]; for more details of the resummation scheme, see [20]. It is worth pointing out that the resummation of secular divergences is qualitatively similar to the resummation of leading logarithms $(g^2 \ln(1/x_{1,2}))^n$ of the incoming partons' momentum fractions required for the computation of inclusive quantities at leading log order [21]–[23]. These results have proven to be valuable for a quantitative understanding of the near side angular correlations observed in nucleus–nucleus [24, 25] and proton–proton [26, 27] collisions. We would like to point out that a considerable amount of work has been done on theories similar to ϕ^4 in the context of reheating after inflation [28]–[30]. In addition, considerable progress has been made on the thermalization problem in the context of heavy-ion collisions through the use of *N*-particle irreducible effective actions [32]–[37].

2. The model

The CGC inspired scalar theory model has the Lagrangian

$$\mathcal{L} = \frac{1}{2} (\partial_{\mu} \phi) (\partial^{\mu} \phi) - V(\phi) + J\phi, \qquad (1)$$

where the interaction potential is

$$V(\phi) = \frac{g^2}{4!}\phi^4 \tag{2}$$

and J is an external source that mimics the large x-color charges of the incoming nuclei. Since the external current vanishes after the collision takes place, we take our source to be non-vanishing for $x^0 < 0$ only,

$$J(x) \sim \theta(-x^0) \frac{Q^3}{g}.$$
(3)

The role of the external source is to initialize a classical field (having occupation number $\sim 1/g^2$) that evolves solely via their self-interactions at $x^0 \ge 0$. This source term also brings an external scale to the problem. Since we assume that the source is turned on adiabatically from $x^0 \rightarrow -\infty$, we can consider the evolution of the classical field as an initial value problem at $x^0 = 0$ with $\phi(x^0 = 0) \sim Q/g$ and $\dot{\phi}(x^0 = 0) = 0$.

Throughout this paper we use the same model parameters. To avoid confusion we now state these parameters once and for all. For both the homogeneous and non-homogeneous systems, we take $\phi_0 = 12$, $\dot{\phi}_0 = 0$ and g = 0.5 (a very weak coupling considering the factor of 4! in front of the potential). For the case of the three-dimensional (3D) simulations, we employ a 12³ lattice with a volume of 12³. This lattice size will have a momentum cutoff of $k_{\text{max}} \approx 5.44$. For these model parameters the resonance mode exists between $3 \sim k_{\text{res}} \sim 3.22$ at t = 0. The zero mode has an effective mass $m^2 \equiv (g\phi_0)^2/2 = 18$ and a period of the oscillator of $T \approx 3$.

3. The homogeneous system and homogeneous fluctuations

In this section, we consider a classical background field that is homogeneous in all space and undergoes quantum evolution with space-independent fluctuations (i.e. zero-mode fluctuations). While highly unrealistic, this simple toy model will allow us to see how the mechanism of phase decoherence leads to ideal hydrodynamic evolution. The Lagrangian for a uniform non-expanding scalar theory is

$$\mathcal{L} = \frac{1}{2}\dot{\phi}^2 - V(\phi),\tag{4}$$

where $\dot{\phi} = d\phi/dt$. The classical evolution can be found in closed form. Since the energy $\mathcal{H} = \dot{\phi}^2/2 + V(\phi)$ remains constant throughout the evolution, we can write

$$\frac{1}{2}\dot{\phi}^2 = E_0 - V(\phi),$$
 (5)

where E_0 is the initial energy of the system, which is determined by the initial condition of our classical field

$$E_0 = \frac{1}{2}\dot{\phi}_0^2 + \frac{g^2}{4!}\phi_0^4,\tag{6}$$

where $\phi_0 = \phi(t = t_0)$. Equation (5) can be integrated to obtain

$$t - t_0 = \frac{1}{\sqrt{2}} \int_{\phi_0}^{\phi(t)} \frac{\mathrm{d}\psi}{\sqrt{E_0 - V(\psi)}}.$$
(7)

At this point it will be useful to introduce some notation. Let us define

$$\epsilon^2 \equiv g^2/4!,\tag{8}$$

and make the change of variables $\sqrt{\epsilon}\psi = -E_0^{1/4}\cos\theta$. We are also free to set $t_0 = 0$ for the non-expanding case and we find

$$t = \frac{1}{2\sqrt{\epsilon}E_0^{1/4}} \int_{\theta_0}^{\theta(t)} \frac{d\phi}{\sqrt{1 - \frac{1}{2}\sin^2\phi}},$$
(9)

where

$$\theta(t) = \cos^{-1} \left(\frac{\sqrt{\epsilon}\phi(t)}{E_0^{1/4}} \right),$$

$$\theta_0 = \cos^{-1} \left(\frac{\sqrt{\epsilon}\phi_0}{E_0^{1/4}} \right).$$
(10)

The above integral equation can be solved for $\phi(t)$ in terms of the Jacobi elliptic function of the first kind having elliptic modulus 1/2.

$$\phi(t) = \frac{E_0^{1/4}}{\sqrt{\epsilon}} \operatorname{cn}_{1/2} \left[2\sqrt{\epsilon} E_0^{1/4} t - F_{1/2}(\theta_0) \right], \tag{11}$$

where $F_{1/2}(\theta_0)$ is the incomplete elliptic integral of the first kind of modulus 1/2. The above result is periodic with period

$$T = \frac{2}{\sqrt{\epsilon}E_0^{1/4}}K(1/2),$$
(12)

where $K(1/2) \approx 1.85407$ is the complete elliptic integral of the first kind. Note that the period of oscillations depends on the initial conditions through E_0 . The fact that the period of oscillations depends on the initial condition is a signature of nonlinear evolution and is crucial for phase decoherence.

It is worth noting that to a very good approximation (within about 15%) the above expression for ϕ can be approximated by

$$\phi(t) \approx \frac{E_0^{1/4}}{\sqrt{\epsilon}} \cos\left[\frac{2\pi}{T}(t-\xi)\right],\tag{13}$$

where ξ is a phase set by the initial conditions

$$\xi = \frac{\theta_0}{2\sqrt{\epsilon}E_0^{1/4}}.$$
(14)

3.1. Stress–energy tensor

With an analytic expression for $\phi(t)$ available, we can now find analytic expressions for the stress-energy tensor as well. For the homogeneous non-expanding system, there are two independent components of the stress-energy tensor:

$$T^{00} = \frac{1}{2}\dot{\phi}^{2} + V(\phi),$$

$$T^{ij} = \delta^{ij} \left(\frac{1}{2}\dot{\phi}^{2} - V(\phi)\right),$$
(15)

with all other components vanishing. Using the expressions derived in the previous section we find

$$T^{00} = E_0,$$

$$T^{ij} = E_0 \left[1 - 2 \operatorname{cn}_{1/2}^4 \left(2 \sqrt{\epsilon} E_0^{1/4} t - F_{1/2}(\theta_0) \right) \right]$$

$$\approx E_0 \left[1 - 2 \cos^4 \left(\frac{2\pi}{T} (t - \xi) \right) \right].$$
(16)



Figure 1. T^{00} and T^{11} for a uniform non-expanding ϕ^4 theory. The dashed line uses the approximation for T^{11} explained in the text.

As an example, in figure 1 we plot T^{00} and T^{11} as a function of time. The figure shows that there is good agreement between the true solution and the approximate form of T^{11} . Clearly, this LO result does not have a well-defined equation of state.

3.2. Spectrum of fluctuations

We now want to superimpose quantum fluctuations on top of our classical background field. In this section, we will consider the following toy model for the spectrum of fluctuations:

$$F(a, \dot{a}) = \delta(\dot{a}) \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{a^2}{2\sigma^2}\right),\tag{17}$$

where σ characterizes the variance of the zero-mode fluctuations. For this toy model, we will treat σ as a free parameter. It will be computed from first principles later. We should stress that this is a highly unrealistic model since we are ignoring any quantum fluctuation that is non-homogeneous in space.

The expectation value of an inclusive operator (such as the stress-energy tensor) is defined as

$$\langle \mathcal{O} \rangle = \int_{-\infty}^{+\infty} \mathrm{d}a \, \mathrm{d}\dot{a} \, F(a, \dot{a}) \mathcal{O}_{\mathrm{LO}}(\phi_0 + a, \dot{\phi}_0 + \dot{a}), \tag{18}$$

where $\mathcal{O}_{LO}(\phi_0 + a, \dot{\phi}_0 + \dot{a})$ is the operator of interest computed at leading order with initial conditions shifted by a and \dot{a} . For this particular choice of fluctuations and using our approximate solutions for $\phi(t)$ found in the previous section, the integrals over a and \dot{a} can be done analytically. The result is shown in figure 2 for $\sigma = 0.4$. The analytic expression is not very enlightening. It essentially consists of a number of terms having oscillations at different frequencies, which die off exponentially at different rates. But it is instructive to pull out the one term that dies off most slowly. Its envelope is given by

$$\sim \mathrm{e}^{-2c^2g^2\sigma^2t^2},\tag{19}$$



Figure 2. $T^{00}/3$ and T^{11} for a uniform non-expanding ϕ^4 theory after averaging over a Gaussian distribution of fluctuations in ϕ_0 having $\sigma = 0.4$. The dashed curve shows the envelope, as given in the text.

where we have defined the constant $c \equiv \frac{\pi}{K(1/2)\sqrt{4!}} \approx 0.3459$. We can now identify a relaxation time

$$\tau_{\rm relax} = \frac{1}{\sqrt{2}cg\sigma} \approx \frac{2}{g\sigma}.$$
(20)

While this is a very unrealistic model, it is nice that the above result could be derived analytically. In the example shown in figure 2, the fluctuations are completely absent after $2 \times \tau_{\text{relax}}$. In other words, by $t \sim 4/(g\sigma) \approx 20$ the system has a well-defined equation of state $(\epsilon = 3p)$ and evolves in accordance with ideal hydrodynamics.

The mechanism behind the relaxation of the pressure is quantum decoherence, which we now explain. Each initial condition in our ensemble average is shifted by a random Gaussian variable $(\phi_0 \rightarrow \phi_0 + a)$ and this corresponds to a shift in the initial energy E_0 of the system. The time evolution of each individual system is periodic with a slightly different period of oscillation, as given by equation (12). When performing the ensemble average the differing periods of each system result in a phase decoherence forcing the pressure to relax to its equilibrium value. Let us stress that this will not occur in a ϕ^2 theory. In this case, the period of oscillation will *not* depend on the initial condition.

4. Non-homogeneous fluctuations

In the previous section, we showed how a homogeneous system undergoing zero-mode fluctuations relaxes to a system evolving according to ideal hydrodynamics. Although the previous case is of pedagogical interest since it shows simply how the decoherence of the quantum field leads to relaxation of the pressure, it is highly unrealistic in that it does not include space-dependent fluctuations.

We now consider the same model in 3D including the space-dependent fluctuations as predicted from quantum field theory. To motivate the need for the resummation, we first discuss the case of linearized perturbations.



Figure 3. Evolution of a linear perturbation on top of the background field. Left: linear growth of the $\mathbf{k} = 0$ mode. Middle: exponential growth of a resonant mode $\mathbf{k}_{res} = 0.5165g\phi_0$. Right: typical behavior of a stable perturbation $\mathbf{k} = 2\mathbf{k}_{res}$.

4.1. Linear perturbations

In this section, we now consider how a linearized perturbation evolves on top of the homogeneous background field. We decompose the background field into a homogeneous part $\phi_{k=0}$ and a small field perturbation a(x). The equation of motion for the Fourier transform of our field perturbation a(x) is

$$\ddot{a}_{\pm \mathbf{k}} + \left[\mathbf{k}^2 + V''(\phi_{\mathbf{k}=\mathbf{0}})\right] a_{\pm \mathbf{k}} = 0.$$
(21)

In the above expression, $\phi_{k=0}$ is the zero mode solution given by equation (11). We now numerically solve equation (21) in order to investigate how linear perturbations evolve on top of the background field. In figure 3, we show how the amplitude of three **k** modes evolve when given an initial amplitude of $a_k(t=0) = 0.1$. The first plot shows the zero mode whose amplitude grows linearly with time. The second mode is taken from within the resonance band and clearly grows exponentially with time. The third mode shows the typical behavior of a high momentum mode (here shown for $k = 2k_{res}$). Whereas the high momentum modes can be treated perturbatively as their amplitude does not grow with time, the lower **k** modes lead to secular divergences. Clearly, at times when gt (for modes outside the resonance band) or $ge^{\mu t}$ (for resonance modes) becomes of $\mathcal{O}(1)$, a resummation becomes necessary.

5. Results from the full fluctuation spectrum

5.1. Initial condition

As is clear from the previous discussion, quantum fluctuations on top of the homogeneous background field will play an important role in the resulting dynamics. The spectra of these quantum fluctuations are derived from first principles. In this case, the classical field ϕ and its conjugate momentum $\pi \equiv \partial \mathcal{L}/\partial \dot{\phi} = \dot{\phi}$ are promoted to quantum operators $\hat{\phi}$ and $\hat{\pi}$ obeying the equal time commutation relations

$$\left[\hat{\boldsymbol{\phi}}(\mathbf{x}), \,\hat{\boldsymbol{\pi}}\left(\mathbf{y}\right)\right] = \mathrm{i}\delta^{3}(\mathbf{x} - \mathbf{y}).$$
(22)

The field operators can be rewritten in terms of creation and annihilation operators

$$\hat{\phi}(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{\mathrm{d}^3 k}{\sqrt{2\omega_{\mathbf{k}}}} \left[\hat{a}_{\mathbf{k}}^{\dagger} \mathrm{e}^{\mathrm{i}k_{\mu}x^{\mu}} + \hat{a}_{\mathbf{k}} \mathrm{e}^{-\mathrm{i}k_{\mu}x^{\mu}} \right]$$
(23)

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obeying

$$\left[\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{p}}^{\dagger}\right] = \delta^{3}(\mathbf{k} - \mathbf{p}).$$
⁽²⁴⁾

Using the above mode decomposition, one can easily show that the two-point correlation function in a homogeneous background field takes the form

$$\langle \hat{\phi}(\mathbf{x})\hat{\phi}(\mathbf{y})\rangle = \frac{1}{(2\pi)^3} \int \frac{\mathrm{d}^3 k}{2\omega_{\mathbf{k}}} \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})},\tag{25}$$

$$\langle \hat{\pi} \left(\mathbf{x} \right) \hat{\pi} \left(\mathbf{y} \right) \rangle = \frac{1}{(2\pi)^3} \int \frac{\mathrm{d}^3 k}{2} \omega_{\mathbf{k}} \mathrm{e}^{\mathrm{i}\mathbf{k} \cdot \left(\mathbf{x} - \mathbf{y} \right)},\tag{26}$$

where $\omega_{\mathbf{k}}^2 = \mathbf{k}^2 + m^2$. Our semi-classical simulation will therefore consist of a Gaussian random field having the power spectrum

$$\mathcal{P}_{\phi}(\mathbf{k}) = \frac{1}{2(2\pi)^{3}\omega_{\mathbf{k}}}$$
(27)

superimposed on top of the homogeneous background field. The power spectrum as written above is UV divergent and this is regulated by the lattice spacing. If we impose a momentum cutoff Λ the energy density will contain terms that behave parametrically as Q^4/g^2 , $Q^2\Lambda^2$ and Λ^4 . The Λ^4 is a pure vacuum contribution and can be computed by performing simulations with the source J turned off, which can then be subtracted from the corresponding result. The $Q^2\Lambda^2$ term is not renormalizable in the usual sense since it mixes diagrams having an arbitrarily high number of loops. In practice, what is done is to choose a cutoff that is sufficiently large in order to encompass the relevant physics ($\Lambda \gtrsim m$) but small enough to keep the cutoff-dependent terms negligible with respect to the classical contribution ($\Lambda \ll Q/\sqrt{g}$).

5.2. Results

Figure 4 shows the pressure and energy density $(\epsilon/3)$ as a function of time with an ensemble average of 1000 simulations. The main conclusion of this paper is that the ensemble averaged pressure relaxes towards $\epsilon/3$ and therefore has a well-defined equation of state and evolves in accordance with ideal hydrodynamics. We should stress that even though there exists a well-defined (i.e. time-independent) equation of state, the system is not necessarily in thermal equilibrium, as we will show. This rapid establishment of an equation of state known as *prethermalization* has been studied in the context of a linear σ -model using the 2PI effective action in [38]. In the present paper, we found similar conclusions; there can be the rapid establishment of an equation of state via phase decoherence regardless of whether scattering processes can thermalize the system.

It is apparent from figure 4 that the time evolution evolves in two stages. First, in the window $0 \le t \le 50$ the amplitude of the pressure oscillations decreases very quickly to moderate values. Then, from a time $t \sim 50$ onwards, there is a slight rebound and a gradual approach towards complete relaxation.

To try to interpret this result and understand the role of different excitations, we perform additional calculations using a modified spectrum of fluctuations. Even though these modified spectra will result in incorrect quantum expectation values, the results may be useful in understanding the role of different fluctuations. In figure 5, we show the resulting pressure after an ensemble average of 250 configurations for various initial spectra, which we now





Figure 4. Time evolution of the pressure averaged over an ensemble of 1000 configurations using the spectrum of fluctuations as given from the quantum field theory.

discuss. In one case (upper left figure) we omit quantum fluctuations of the zero mode. In a second case (upper right figure), we omit any initial fluctuation within the resonance band. The lower two figures show spectra that omit any initial fluctuation having k < 4.4 (lower left) and k > 2 (lower right). Of course, once the time evolution begins, there is nothing stopping self-interactions from causing excitations to scatter into these initially unoccupied modes. Not including the resonance modes in the initial spectrum of fluctuations (as done in the top right part of figure 5) is different from the analysis of [20], where the lattice cutoff was chosen to be below the resonance band. In the latter case, the resonance modes can never become occupied, which was found to significantly modify the evolution of the pressure.

There is much that one can infer from figure 5. The upper two figures comprise results missing a small fraction of the initial spectrum. In the first case, we neglected initial fluctuations of the zero mode, while in the second case we neglected the very few modes that sit within the narrow resonance band. In both these cases, the results are qualitatively similar to the results using the full spectrum shown in figure 4. In the case of the zero mode, it is not surprising that its absence does not affect the result. One can estimate the relaxation time from the zero mode alone based on the previous section, where we showed in equation (20) that the relaxation time in the homogeneous case is inversely proportional to the standard deviation of the Gaussian fluctuations as given by the power spectrum. In this case,

$$\tau_{\rm relax} \approx \frac{2}{g\sqrt{\mathcal{P}_{\phi}(\mathbf{k}=0)}} \approx 180 \tag{28}$$

for g = 0.5. Clearly, this is a much longer relaxation time than observed in the full 3D simulation. In the case of the absent resonance modes, one can see by looking at the occupation numbers that the occupied modes are very quick to scatter and perturb the initially unoccupied resonance band.

What is more interesting is when we neglect a large portion of the initial spectra. The lower left plot shows the ensemble averaged pressure with a spectrum including the intermediate



Figure 5. Evolution of the ensemble averaged pressure for various *incorrect* spectra of fluctuations. Each case consists of an ensemble average over 250 configurations. Top left: omitting the zero mode. Top right: omitting modes within the resonance band. Bottom left: omitting modes with k < 4.4. Bottom right: omitting modes with k > 2.

momentum modes (basically we include modes higher than the resonance band). The evolution from $0 \le t \le 50$ is remarkably similar to the result using the full spectrum. In the lower right plot we have used a spectrum consisting of only low momentum modes. In this case, we no longer have the rapid relaxation at $t \sim 50$ but instead have a gradual relaxation that extends to $t \sim 150$. Based on this analysis we can understand the two-stage relaxation observed when using the full spectrum. The first relaxation in the period $0 \le t \le 50$ is clearly controlled in some manner by quantum fluctuations above the resonance band, while the second (more gradual) stage of relaxation, taking place for $t \ge 50$, is controlled by modes below the resonance band. The time scale for relaxation due to the low momentum modes is of the order of that estimated in equation (28) for the zero mode. Of course this interpretation is only qualitative. Selfinteractions immediately cause modes that are initially unoccupied to become occupied, and the result becomes a complex interplay between many modes that cannot be understood simply by studying the linear evolution of individual quanta.

It is interesting to note that the time scale for prethermalization in the 3D simulation has the same order of magnitude (it relaxes about 3–4 times faster) as the crude estimate of

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Figure 6. Evolution of the number density at various times. The curves at t = 160 and t = 200 are almost indistinguishable from the spectra at t = 240.

equation (28) found for the homogeneous case. If we take the estimate from equation (28) seriously, we see a faster relaxation with increasing coupling constant g. The power spectrum entering equation (28) is determined by the quantum field theory in question. Modes with lower k will have the largest fluctuations. In our case $\mathcal{P}_{\phi}(\mathbf{k} = 0) \sim 1/(g\phi_0)$. Since $\phi_0 \sim 1/g$, the amplitude of the quantum fluctuations is always $\mathcal{O}(1)$.

It is clear that as we vary g, the energy density of the system, which is of order $g^2 \phi_0^4$, varies as well. To see the parametric behavior of the relaxation time on g at a fixed energy density, we instead take $\phi_0 \sim 1/g^{1/2}$. In this case, we find that

$$\tau_{\rm relax} \sim \frac{1}{g\sqrt{\mathcal{P}_{\phi}(\mathbf{k}=0)}} \sim \frac{1}{g^{3/4}},\tag{29}$$

which is consistent with the $1/g^{2/3}$ behavior extracted from the 3D simulation of [20].

In figure 6, we show the number density defined by

$$n_{\mathbf{k}} \equiv \langle 0|\hat{a}_{\mathbf{k}}^{\dagger}\hat{a}_{\mathbf{k}}|0\rangle = \frac{1}{2} \left(\omega_{\mathbf{k}}|\phi_{\mathbf{k}}|^2 + \frac{|\dot{\phi}_{\mathbf{k}}|^2}{\omega_{\mathbf{k}}} \right) - \frac{1}{2}$$
(30)

at various times along the evolution. The initial condition is such that the number density is zero, $n_k = 0$, except for the zero mode that is highly occupied. As the system evolves one sees the appearance of peaks. Whether these peaks correspond to resonance modes is not clear. Even though we know the location of the resonance band at $t \approx 0$ the effective mass of the background field changes with time, thereby changing the location of the resonance band with time. Of course, larger lattice simulations will be needed to reinforce these statements. At later times, when the system has fully relaxed, the number density is smooth with a power-law fall off.

In figure 7, we show the final number density. In this case, we have plotted the spectra at the discrete values allowed by our grid. The solid curve is a fit to $n_{\mathbf{k}} \sim \omega_{\mathbf{k}}^{-s}$ with s = 1.45. It clearly does not fall as $1/\omega_{\mathbf{k}}$ as one would expect from classical thermal equilibrium. Interesting future work will examine the late time behavior of the particle number to see whether it scales according to Kolmogorov turbulence [31, 39].

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Figure 7. Final number density versus momentum. The solid curve is the best fit to $n_{\mathbf{k}} \sim \omega_{\mathbf{k}}^{-s}$ with s = 1.45.

6. Conclusions

In conclusion, it is apparent that quantum fluctuations modify the evolution of a classical scalar theory to the point where it evolves in accordance with ideal hydrodynamics. We have attempted to understand this behavior by using modified spectra of fluctuations and by studying the particle content of the theory. We observe that there is a two-stage relaxation process; the rapid early time relaxation is somehow controlled by modes of intermediate momentum (at and above the resonance band) followed by a longer, more gradual relaxation that is controlled by the lower momentum (near zero) modes. The methods used for the scalar field can presumably be extended to the case of gauge theories.

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