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Spin diffusion in Fermi gases

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Spin diffusion in Fermi gases

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Abstract. We examine spin diffusion in a two-component homogeneous Fermi gas in the normal phase. Using a variational approach, analytical results are presented for the spin diffusion coefficient and the related spin relaxation time as a function of temperature and interaction strength. For low temperatures, strong correlation effects are included through the Landau parameters, which we extract from Monte Carlo results. We show that the spin diffusion coefficient has a minimum for a temperature somewhat below the Fermi temperature with a value that approaches the quantum limit $\sim \hbar/m$ in the unitarity regime, where m is the particle mass. Finally, we derive a value for the low-temperature shear viscosity in the normal phase from the Landau parameters.

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1. Introduction

Cold atomic gases provide a unique possibility for studying many-body physics in a controlled way, as one can tune properties such as the strength and sign of the interaction as well as the population in various internal states. Fermi gases in the unitarity regime characterized by a diverging scattering length $|a| \rightarrow \infty$ are being intensively investigated. They represent a universal realization of a strongly interacting Fermi gas in the sense that their properties are independent of the details of the interaction. Thermodynamic as well as microscopic equilibrium quantities have been measured in a series of impressive experiments [1]–[6]. For high temperatures, one can calculate their properties in a controlled way using the virial expansion [7]–[9]. For low temperatures, the system is strongly interacting and poses a challenging problem. Quantum Monte Carlo calculations are therefore at present the most reliable method for extracting their equilibrium properties [10]–[12].

There is increasing focus on the transport properties of these gases. As opposed to thermodynamic quantities, strong interactions can change transport coefficients by orders of magnitude, making them attractive to study. The shear viscosity η of a Fermi gas has been extracted from the damping of collective modes [13] as well as expansion experiments [14]. One can calculate the viscosity accurately for a Fermi gas in the unitarity limit for high temperatures [15]. Furthermore, for $T \ll T_c$ with T_c being the critical temperature for superfluidity, the contribution to the viscosity from phonons has been calculated for superfluid ^4He [16] and for a superfluid Fermi gas in the unitarity regime [17]. These two results agree if the sound velocity of a Fermi gas is inserted in the expression for ^4He , showing that the microscopic origin of superfluidity is irrelevant [18]. At intermediate temperatures, it is very challenging to calculate η from a microscopic theory, since many effects have to be considered including particle self-energies [19], pairing and vertex corrections [18, 20]. Using methods from string theory, it has been shown that the viscosity of a class of strongly interacting many-body systems obeys the bound $\eta/s > \hbar/4\pi k_B$, where s is the entropy density [21], and it has been conjectured that this bound holds for all fluids [22]. This has made the study of the viscosity of cold atomic Fermi gases in the unitarity limit directly relevant to other fields including quark-gluon and high-energy physics [23].

Very recently, measurements of spin transport properties of a two-component Fermi gas have been reported [24]. Inspired by this, we analyze in this paper the spin diffusion coefficient D of a two-component homogeneous Fermi gas in the normal phase. Using a variational approach, we calculate D for both weak and strong interactions. For high temperatures, our calculations are analogous to the ones leading to the accurate result for shear viscosity [15]. In the low temperature regime where the system is strongly correlated, we use Fermi liquid theory to calculate D . The strong coupling effects are contained in the Landau parameters which are extracted from Monte Carlo calculations. In this way, we expect to obtain reliable results for D even in the unitarity limit. It follows from our high and low T results that the spin diffusion coefficient exhibits a minimum for a temperature below T_F . In the unitarity limit, we find that the minimum is $D \sim \hbar/m$. Since D in general decreases with increasing coupling strength, this value can be interpreted as the quantum limit for how small D can become in atomic gases, when interactions are as strong as quantum mechanics allows. In analogy with the intriguing conjecture of a global minimum bound for η , it would be very interesting to compare this quantum minimum for D with experimental results as well as with D for other strongly interacting systems. We also calculate

the closely related spin relaxation time, and briefly discuss how η can be extracted from the Landau parameters.

2. Spin diffusion in the hydrodynamic regime

We consider a gas of fermions of mass m in two internal states, which we denote by spin $\sigma = \uparrow, \downarrow$. In equilibrium, the densities of the two components are equal, $n_\uparrow = n_\downarrow = n/2 = k_F^3/6\pi^2$, with n being the total density. We shall focus on the non-equilibrium situation with a spatially varying magnetization $n_\uparrow(\mathbf{r}) - n_\downarrow(\mathbf{r})$. The corresponding spin current $\mathbf{j} = \mathbf{j}_\uparrow - \mathbf{j}_\downarrow$ is in the hydrodynamic regime given by Fick's law

$$\mathbf{j} = -D\nabla(n_\uparrow - n_\downarrow), \quad (1)$$

where D is the spin diffusion coefficient. The main goal of this paper is to calculate D as a function of the coupling strength and temperature.

In vacuum, the interaction between atoms with spins $\uparrow$ and $\downarrow$ is given by the cross section

$$\frac{d\sigma_{\text{sc}}^{\uparrow\downarrow}}{d\Omega} = \frac{a^2}{1 + p_r^2 a^2}, \quad (2)$$

where a is the s-wave scattering length and p_r the relative momentum. There are many-body corrections to (2) but they are small for $T \gg T_F$ with T_F being the Fermi temperature of the gas, even in the unitarity limit [15, 25]. For the densities and temperatures of interest, we can ignore the bare interaction between atoms with equal spin. For high temperature, the typical scattering momenta scale as $\sim \sqrt{mk_B T}$ and it follows from (2) that the gas is weakly interacting irrespective of the value of a .

For low T , the gas is strongly interacting in the unitarity regime $|a| \rightarrow \infty$ and there is at present no quantitatively reliable microscopic theory for calculating its transport properties. Assuming that the gas is in the normal phase for $T \ll T_F$, we can, however, use the Fermi liquid theory. Combined with the relevant Landau parameters extracted from non-perturbative Monte Carlo calculations, we can in this way derive reliable results for D in the strongly interacting normal phase [26, 27]. Since the gas is superfluid for $T < T_c$ and T_c is predicted to be a sizable fraction of T_F in the unitarity limit [12], it is not obvious that one can fulfill the criterion $T_c < T \ll T_F$ for a normal Fermi liquid. However, we will show that the minimum of D seems to be located for $T > T_c$ where pairing is not relevant. Also, one can in fact quench the superfluid order for $T < T_c$ by rotating the gas [28].

2.1. The Landau–Boltzmann equation

We proceed using kinetic theory for high T and Fermi liquid theory for low T to describe the spin dynamics of the gas. In the hydrodynamic regime, the typical length scale of the dynamics is much longer than the mean free path. The non-equilibrium distribution function is then close to a local hydrodynamic form, i.e. $f_\sigma(\mathbf{r}, \mathbf{p}) \simeq 1/\{\exp[\epsilon_p - \mu_\sigma(\mathbf{r})] + 1\}$ with ϵ_p being the quasi-particle energy and $\mu_\uparrow(\mathbf{r})$ and $\mu_\downarrow(\mathbf{r})$ spatially varying chemical potentials corresponding to the magnetization $n_\uparrow(\mathbf{r}) - n_\downarrow(\mathbf{r})$. Plugging the local equilibrium distributions into the left side of the steady state linearized Landau–Boltzmann equation for the two-spin components and taking the difference yields [26, 29]

$$\beta \mathbf{v}_p \cdot \nabla(\mu_\uparrow - \mu_\downarrow) = -\frac{I_\uparrow - I_\downarrow}{f^0(1 - f^0)}, \quad (3)$$

with $\mathbf{v}_p = \nabla_p \epsilon_p$ being the velocity and f^0 the distribution function in equilibrium. The difference of the linearized collision integrals is

$$I_{\uparrow} - I_{\downarrow} = \int \frac{d^3 p_2}{(2\pi)^3} d\Omega \frac{|\mathbf{p} - \mathbf{p}_2|}{m^*} \frac{d\sigma_{sc}^{\uparrow\downarrow}}{d\Omega} (\Psi - \Psi_2 - \Psi_3 + \Psi_4) f^0 f_2^0 (1 - f_3^0) (1 - f_4^0), \quad (4)$$

with m^* being the effective mass. We have written $\delta f_{\uparrow}(\mathbf{r}, \mathbf{p}) - \delta f_{\downarrow}(\mathbf{r}, \mathbf{p}) = f^0(\mathbf{r}, \mathbf{p})[1 - f^0(\mathbf{r}, \mathbf{p})]\Psi_{\sigma}(\mathbf{r}, \mathbf{p})$ with δf_{σ} being the deviation of the distribution function away from local equilibrium. The differential cross section for the scattering between particles with opposite spins is $d\sigma_{sc}^{\uparrow\downarrow}/d\Omega$ with Ω being the solid angle between the outgoing and ingoing relative momenta, $\mathbf{p}_r' = (\mathbf{p}_4 - \mathbf{p}_3)/2$ and $\mathbf{p}_r = (\mathbf{p}_2 - \mathbf{p})/2$. There is also a term in (4) coming from the induced interaction between parallel spin quasi-particles which we have not written explicitly, since it does not contribute to the variational results for D given below. In the following, we take the magnetization to vary in the z -direction. The spin current is then given by

$$j_z = \int \frac{d^3 p}{(2\pi)^3} (\delta f_{\uparrow} - \delta f_{\downarrow}) v_z. \quad (5)$$

2.2. A variational expression for D

The Landau–Boltzmann equation (3) can be written as $\kappa = H[\Phi]$ with $H = (I_{\uparrow} - I_{\downarrow})/f^0(1 - f^0)$ and $\kappa = -\beta v_z \partial_z (\mu_{\uparrow} - \mu_{\downarrow})$. Likewise, the spin current (5) can be written as $-\langle \kappa \Psi \rangle k_B T / \partial_z (\mu_{\uparrow} - \mu_{\downarrow})$ with the definition $\langle A \rangle = \int d^3 p f^0(1 - f^0) A / (2\pi)^3$. Using this, one can in analogy with the case of charge current [29, 30] derive a variational bound for the spin current given by

$$j_z \geq - \frac{k_B T}{\partial_z (\mu_{\uparrow} - \mu_{\downarrow})} \frac{\langle U \kappa \rangle^2}{\langle U H[U] \rangle}. \quad (6)$$

Here, U is a trial function for the deviation function Ψ . By expanding U in polynomials, it has been shown that using the driving term in the Landau–Boltzmann equation as an ansatz, i.e. $U \propto \kappa$, yields very accurate results for the viscosity η [19]. We will therefore use the ansatz $U = p_z \propto \kappa$ appropriate for spin diffusion in the following. This gives

$$D = \frac{\beta}{m^{*2}} \frac{1}{\chi_s} \frac{\langle p_z^2 \rangle^2}{\langle p_z H[p_z] \rangle} \quad (7)$$

as our variational expression for the spin diffusion coefficient with $\chi_s = \partial(n_{\uparrow} - n_{\downarrow})/\partial(\mu_{\uparrow} - \mu_{\downarrow})$ being the spin susceptibility. Equation (7) serves as the starting point for our results concerning spin diffusion. It is in fact the use of the ansatz $U = p_z$ combined with momentum conservation which gives that it is only the scattering between opposite spins that enters in the variational expression for D .

3. High temperature limit

We now calculate D in the classical limit $T \gg T_F$. Using $f^0 = \exp[-\beta(p^2/2m - \mu)] \ll 1$ with μ being the chemical potential in equilibrium, the integrals in (7) can be performed and we

obtain

$$D = \frac{3\sqrt{\pi}}{8} \frac{\sqrt{k_B T}}{\sqrt{mn}\bar{\sigma}_{sc}^{\uparrow\downarrow}} = \frac{9\pi^{3/2}\hbar}{32\sqrt{2}m} \times \begin{cases} \frac{1}{(k_F a)^2} \left(\frac{T}{T_F}\right)^{1/2} & \text{for } T \ll T_a \text{ (weak coupling)} \\ \left(\frac{T}{T_F}\right)^{3/2} & \text{for } T \gg T_a \text{ (unitarity limit)} \end{cases}. \quad (8)$$

Here,

$$\bar{\sigma}_{sc}^{\uparrow\downarrow} = 4\pi a^2 \int_0^\infty dx \frac{x^5 e^{-x^2}}{1 + x^2 T/T_a}, \quad (9)$$

where $k_B T_a = \hbar^2/ma^2$ is the thermal average of the scattering cross section weighted with p^3 . We have $\bar{\sigma}_{sc}^{\uparrow\downarrow} = 4\pi a^2$ and $\bar{\sigma}_{sc}^{\uparrow\downarrow} = 2\pi\hbar^2/mk_B T$ in the weak coupling and unitarity limits, respectively. To derive (8), we have used $\chi_s = n/2k_B T$ for the spin susceptibility in the classical limit. The temperature dependence in (8) can be understood as follows: The spin diffusion coefficient scales as $D \sim l_{mf}v$ with $l_{mf} = 2/n\sigma_{sc}$ being the mean free path and v a typical velocity. In the classical regime, $l_{mf} \sim 1/na^2$ for weak coupling and $l_{mf} \sim mk_B T/\hbar^2 n$ in the unitarity limit. Since $v \sim \sqrt{k_B T/m}$, we recover the $T^{1/2}$ and $T^{3/2}$ scaling in (8). The temperature scaling is analogous to what is found for the shear viscosity [15]. The result (8) for a gas in the classical limit $T \gg T_F$ has also recently been derived in [24].

4. Low temperature limit

For $T \ll T_F$, the quasi-particle scattering takes place around the Fermi surface. The collision integral can then be reduced to

$$\langle p_z H[p_z] \rangle = \frac{m^{*2} k_F^3}{18\pi^2} (k_B T)^3 I_{\text{angle}}, \quad (10)$$

where

$$I_{\text{angle}} = \int_0^\pi d\theta \frac{\sin\theta(1 - \cos\theta)}{\cos(\theta/2)} \int_0^{2\pi} d\phi \frac{d\sigma_{sc}^{\uparrow\downarrow}}{d\Omega} (1 - \cos\phi) \quad (11)$$

is the cross section averaged over the Fermi surface. Here, θ is the angle between the incoming scattering momenta $\mathbf{p}$ and $\mathbf{p}_2$ and ϕ is the angle between the relative momenta $\mathbf{p}_r$ and $\mathbf{p}'_r$. The cross section at the Fermi surface can be written as

$$\frac{d\sigma^{\uparrow\downarrow}(\theta, \phi)}{d\Omega} = \frac{\pi^2}{16k_F^2} [N(0)\mathcal{T}^{\uparrow\downarrow}(\theta, \phi)]^2, \quad (12)$$

where $\mathcal{T}^{\uparrow\downarrow}$ is the scattering matrix for opposite spins and $N(0) = m^*k_F/\pi^2$ is the density of states at the Fermi level. The scattering matrix can be parameterized in terms of the Landau parameters F_l^i as [26]

$$N(0)\mathcal{T}^{\uparrow\downarrow}(\theta, \phi) = \frac{1}{2} \sum_l [A_l^s - 3A_l^a + (A_l^s + A_l^a) \cos\phi] P_l(\cos\theta), \quad (13)$$

where $P_l(x)$ are the Legendre polynomials and

$$A_l^i = \frac{F_l^i}{1 + F_l^i/(2l+1)}. \quad (14)$$

In (14), the symmetry of the triplet part of the scattering is taken into account by the $\cos \phi$ factor [31]. We now assume that it is sufficient to use the $l = 0$ Landau parameters. Using equations (12) and (13) in (11) and solving the resultant trigonometric integrals yields, when plugged into (7),

$$D = \frac{24\hbar}{\pi^3 m} \frac{1 + F_0^a}{C_1^2 + C_3^2/2 - C_1 C_3} \left(\frac{T_F}{T} \right)^2$$

$$= \frac{3\hbar}{8\pi m} \left(\frac{T_F}{T} \right)^2 \times \begin{cases} \frac{1}{(k_F a)^2} & \text{for } k_F a \ll 1 \text{ (weak coupling),} \\ 2.6 & \text{for } k_F a \gg 1 \text{ (unitarity limit).} \end{cases} \quad (15)$$

We have defined $C_1 = A_0^s - 3A_0^a$ and $C_3 = A_0^s + A_0^a$ and used $\langle p_z^2 \rangle = k_B T m^* k_F^3 / 6\pi^2$, and $\chi_s = N(0)/2(1 + F_0^a)$ for $T/T_F \ll 1$ to derive (15). Also, $m^* = m$ when $F_1^s = 0$. For $k_F |a| \ll 1$, one has $F_0^s = 2k_F a / \pi$ and $F_0^a = -2k_F a / \pi$, which gives the weak coupling result in (15). The spin diffusion coefficient increases as T^{-2} for low T . This is the usual effect of Fermi blocking of the scattering which enables the quasi-particles to travel, further thereby increasing the spin current.

To obtain the numerical value in (15) for $k_F |a| \gg 1$, we have used values for the Landau parameters extracted from Monte Carlo calculations for a strongly interacting two-component Fermi gas: $F_0^s \simeq -0.44$ and $F_0^a \simeq 2$ [10, 32], giving $A_0^s \simeq -0.79$ and $A_0^a \simeq 0.7$. Since $A_0^s + A_0^a \simeq -0.1$ almost fulfills the sum rule $\sum_l (A_l^s + A_l^a) = 0$, it seems that it is reasonable only to use the $l = 0$ Landau parameters. Note that the interaction between opposite spins (13) is attractive for these Landau parameters: $N(0)\mathcal{T}^{\uparrow\downarrow} \simeq -1.4 - 0.06 \cos \phi$. This reflects the tendency for pairing in the gas, which suppresses the spin susceptibility by the factor $1/(1 + F_0^a) \simeq 1/3$. Since it is the chemical potentials and not the magnetization $n_\uparrow - n_\downarrow$ which drive the Landau–Boltzmann equation (3), the spin diffusion coefficient is increased by the same factor.

5. Minimum of the spin diffusion coefficient

In figures 1 and 2, we summarize our results by plotting D as a function of T and $1/k_F |a|$. In figure 1(a), we plot D in units of $D_{\text{high}}/(k_F a)^2$ with $D_{\text{high}} = 9\pi^{3/2}\hbar/32\sqrt{2}m$ as a function of T in the weak coupling regime. The low and high T curves are given by the weak coupling limits of (8) and (15). Likewise, we plot in figure 1(b) $D(T)/D_{\text{high}}$ in the unitarity limit as given by (8) and (15). Since $D(T)$ will interpolate between these two limits, these results show that $D(T)$ will exhibit a minimum. Figure 1 indicates that in the unitarity limit, the minimum is for T somewhat below T_F but above $T_c \sim 0.2T_F$, where pairing effects have to be included. The scale of the minimum value is $D_{\text{min}} \sim 9\pi^{3/2}\hbar/32\sqrt{2}m \simeq 1.1\hbar/m$. Since D decreases with increasing coupling strength, this result can be regarded as the quantum limit of D when the interactions are as strong as quantum mechanics allows.

In figure 2, we plot D as a function of $1/k_F |a|$ in the high temperature limit (a) and in the low temperature limit (b) using the units $D_{\text{high}} \times (T/T_F)^{1/2}$ and $3\hbar/8\pi m \times (T_F/T)^2 = D_{\text{low}} \times (T_F/T)^2$, respectively. The weak and strong coupling results are again obtained from (8) and (15), and $D(1/k_F |a|)$ will interpolate between these two limits. We see that D decreases with increasing coupling strength $k_F |a|$ and that it eventually flattens out in the limit $k_F |a| \rightarrow \infty$.

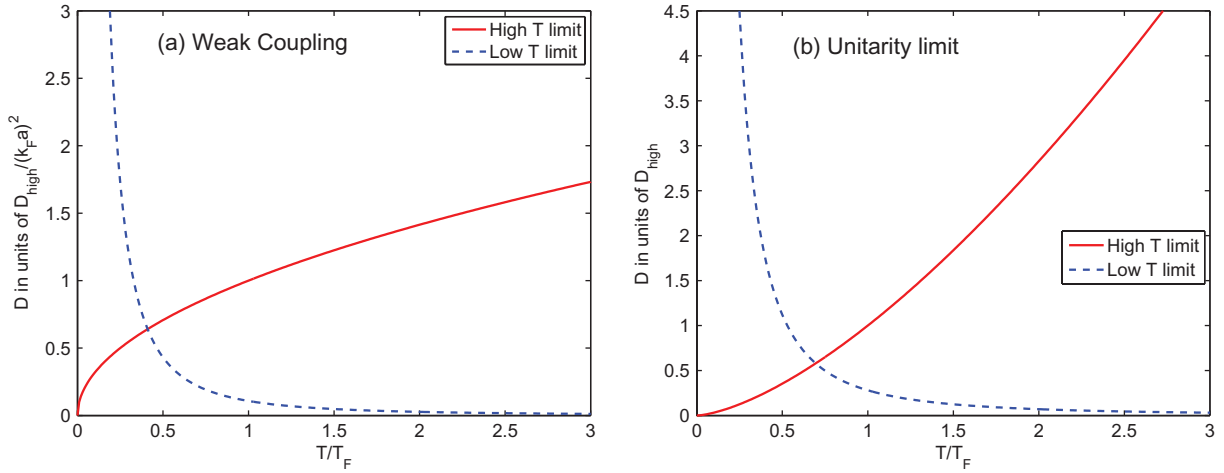


Figure 1. The spin diffusion coefficient as a function of T in (a) the weak and (b) the strong coupling limit.

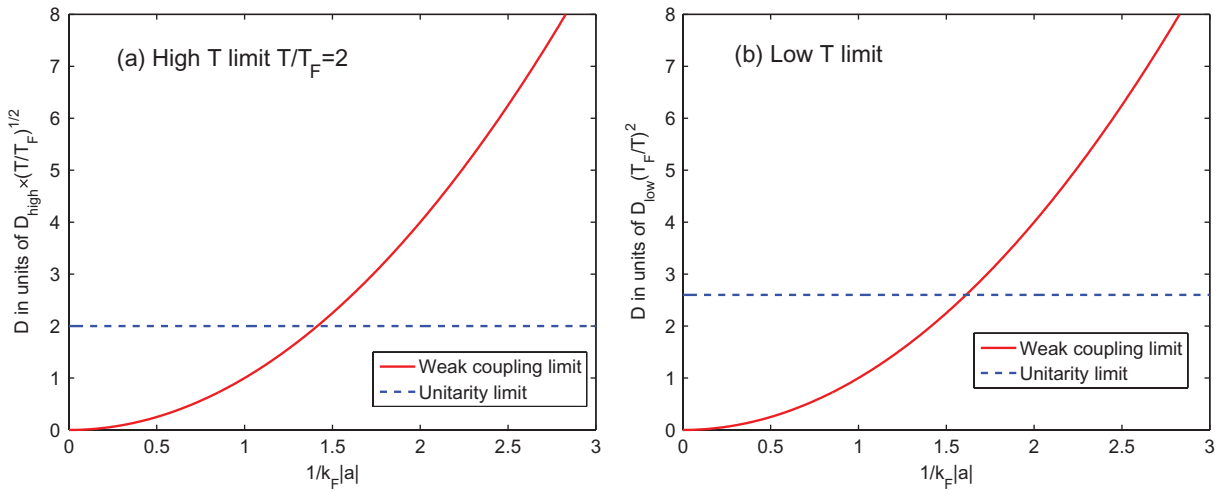


Figure 2. The spin diffusion coefficient as a function of $1/k_F|a|$ in (a) the high T and (b) the low T limit.

6. Spin relaxation time

From our results, we now derive an expression for the spin relaxation time that gives the typical time between scattering events for spin dynamics. A suitable definition of τ_D can be obtained from the relaxation time approximation, i.e. by assuming $I_\uparrow - I_\downarrow = -(\delta f_\uparrow - \delta f_\downarrow)/\tau_D$. Using this, the linearized Landau–Boltzmann equation (3) is easily solved for $\delta f_\uparrow - \delta f_\downarrow$. Plugging into (5) then gives the usual expression $D = \frac{1}{3}(1 + F_0^a)v_F^2\tau_D$ for $T \ll T_F$ with v_F being the Fermi velocity. We define the spin diffusion time τ_D by comparing this to (15), which gives

$$\tau_D = \frac{9\hbar}{16\pi k_B T_F} \left(\frac{T_F}{T} \right)^2 \times \begin{cases} \frac{1}{(k_F a)^2} & \text{for } k_F a \ll 1 \text{ (weak coupling),} \\ 0.9 & \text{for } k_F a \gg 1 \text{ (unitarity limit).} \end{cases} \quad (16)$$

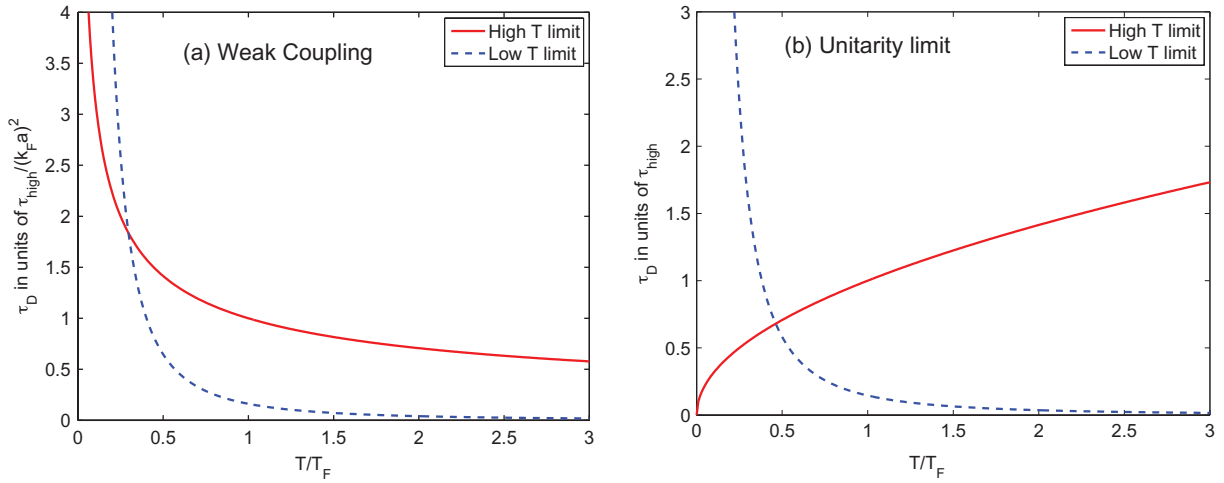


Figure 3. The spin relaxation time as a function of T in (a) the weak and (b) the strong coupling limit.

for $T \ll T_F$. Likewise, the relaxation time approximation yields $D = k_B T \tau_D / m$ for $T \gg T_F$. On comparison with (8), this gives

$$\tau_D = \frac{9\pi^{3/2}\hbar}{32\sqrt{2}k_B T_F} \times \begin{cases} \frac{1}{(k_F a)^2} \left(\frac{T}{T_F}\right)^{-1/2} & \text{for } T \ll T_a \text{ (weak coupling)} \\ \left(\frac{T}{T_F}\right)^{1/2} & \text{for } T \gg T_a \text{ (unitarity limit)} \end{cases}, \quad (17)$$

for $T \gg T_F$. In figure 3, we plot the high and low T limits spin relaxation times as given by (16) and (17). We have used the units $\tau_{\text{high}}/(k_F a)^2$ for the weak coupling case and τ_{high} in the unitarity limit with $\tau_{\text{high}} = 9\pi^{3/2}\hbar/32\sqrt{2}k_B T_F$. Again, we see that the spin relaxation time exhibits a minimum for $T < T_F$ in the unitarity limit. However, τ_D increases monotonically with decreasing T in the weak coupling regime.

The spin relaxation time is useful for estimating the nature of the spin dynamics in a particular trapped atomic gas experiment: when $\omega\tau_D \ll 1$ with ω being the relevant trapping frequency, the spin dynamics is hydrodynamic, whereas the spin dynamics is collisionless for $\omega\tau_D \gg 1$.

7. Viscosity for low T

For completeness, we briefly outline how one can obtain the viscosity η and the viscous relaxation time τ_η for $T/T_F \ll 1$ for a normal gas from the Landau parameters. The variational approach for calculating η is explained in [30]. For $T/T_F \ll 1$, the viscosity is determined by the angular average of the cross section over the Fermi surface as in (10) and (11). For the viscosity however, it is the full scattering cross section $d\sigma_{\text{sc}}^{\uparrow\downarrow}/d\Omega + \frac{1}{2}d\sigma_{\text{sc}}^{\uparrow\uparrow}/d\Omega$ that enters. Here $d\sigma_{\text{sc}}^{\uparrow\uparrow}/d\Omega$ describes the induced interaction between parallel spins, which cannot be ignored in general, even though the bare interaction (2) is only between opposite spins. One has, in analogy

with (12),

$$\frac{d\sigma^{\uparrow\uparrow}(\theta, \phi)}{d\Omega} = \frac{\pi^2}{16k_F^2} [N(0)\mathcal{T}^{\uparrow\uparrow}(\theta, \phi)]^2, \quad (18)$$

with

$$N(0)\mathcal{T}^{\uparrow\uparrow}(\theta, \phi) = \sum_l (A_l^s + A_l^a) P_l(\cos \phi) \cos \phi. \quad (19)$$

Taking only the $l=0$ Landau parameters, we obtain $N(0)\mathcal{T}^{\uparrow\uparrow}(\theta, \phi) = (A_0^s + A_0^a) \cos \phi = -0.12 \cos \phi$. This means that the induced interaction between parallel spins is attractive but much weaker than the interaction between different spins. Following steps analogous to the spin diffusion case described above, we obtain

$$\eta = \frac{24}{\pi^3} \frac{T_F^2}{T^2} \frac{1}{C_1^2 + (3/4)C_3^2} n\hbar = 0.1 \left(\frac{T_F}{T} \right)^2 n\hbar. \quad (20)$$

for $T \ll T_F$. Using the relaxation time result, $\eta = n \frac{k_F^2}{m^*} \tau_\eta / 5$, to define the viscous relaxation time τ_η , we obtain

$$\tau_\eta = \frac{60}{\pi^3} \frac{T_F^2}{T^2} \frac{1}{C_1^2 + (3/4)C_3^2} \frac{\hbar}{k_B T_F} = 0.2 \left(\frac{T_F}{T} \right)^2 \frac{\hbar}{k_B T_F}. \quad (21)$$

Comparing (21) with (16), we see that the two relaxation times are qualitatively the same as expected. They therefore yield the same prediction for the crossover between hydrodynamic and collisionless behavior.

8. Conclusions

Using a variational approach, we analyzed the spin diffusion coefficient and the spin relaxation time for a two-component homogeneous Fermi gas in the hydrodynamic limit. We derived analytical results in the high and low temperature regimes including strong coupling effects through Landau parameters extracted from Monte Carlo calculations. Our results indicate that the spin diffusion coefficient exhibits a minimum for a temperature below T_F but above T_c , with a value that scales as $\sim \hbar/m$ in the unitarity regime. It would be very interesting to compare this result with the value of the spin diffusion coefficient in other strongly interacting systems. Also, one should analyze the effects of pairing on spin diffusion in atomic gases. New experimental insight is particularly relevant due to the lack of a controllable method for calculating the minimum value of D quantitatively in the strong coupling limit. We finally provided expressions for shear viscosity in terms of the Landau parameters.

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