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The mixing evolution and geometric properties of a passive scalar field in turbulent Rayleigh–Bénard convection

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Abstract. We report on measurements of a two-dimensional (2D) dye concentration field in turbulent Rayleigh–Bénard (RB) convection using the planar laser-induced fluorescence technique. The measurements were made in a vertical plane near the sidewall of a rectangular convection cell filled with water and with the Rayleigh number $Ra$ varying from $10^9$ to $10^{10}$, all at a fixed Prandtl number $Pr = 5.3$ and Schmidt number $Sc = 2100$. The mixing evolution and geometric properties of the measured passive scalar field are studied. It is shown that the mixing evolution of the passive scalar field in buoyancy-driven turbulence possesses certain features that are similar to those found in other types of turbulent flows, such as turbulent jets and grid-generated turbulence. These features include a power-law decrease in the mean concentration and exponential tails of single-point probability density function along the evolution path. Our results also show that the log-Poisson distribution is universal for the geometric measures of the passive scalar iso-contours, such as the perimeter, area, shape complexity and absolute value of local curvature; and the log-Poisson statistics may be used to model them. Regarding the influence of buoyancy on the turbulent scalar mixing, it is found that the buoyant scale, i.e. the Bolgiano scale $\ell_B$, cannot be identified from the measured geometric properties.
1. Introduction

Passive scalar mixing in turbulent flows is a common scene that one often encounters in nature and in many engineering applications. It is always accompanied by very complicated structures, such as convoluted filaments, sheets, surfaces and fluid interfaces. Such complex structures usually have a fractal geometry that can be quantified in terms of the shape complexity [1], fractal dimension [2, 3] and local curvature [4]. The shape complexity is a geometric measure of the complexity of an object’s shape and rougher objects have a larger shape complexity. Catrakis and Dimotakis [1] used the shape complexity to characterize the geometry of the iso-concentration contours in two-dimensional (2D) transverse slices of the passive scalar field of liquid-phase turbulent jets, and their results showed that the shape complexity of the iso-contours obeys a log-Poisson distribution. For the fractal dimension, which has been investigated extensively in the study of turbulence, two different types were identified. Constant dimensions were obtained in many experiments for various systems, where power-law and scale-free behaviors were observed [2, 3, 5–10]. In contrast, some studies revealed a scale-dependent fractal geometry, i.e. the fractal dimension is a function of the scale r [11–14].

There are a number of algorithms that have been used to calculate the values of such measures, including the most widely used box-counting method [2], the correlation method [15] and the perimeter–area method [2, 16]. For a given fractal system, not all of these algorithms will necessarily give the same result [3]. Another quantity that can be used to characterize the geometric properties of a 2D object is its local curvature. Gluckman et al [4] used the local curvature to characterize the geometry of both the iso-thermal and the iso-concentration surfaces in convective thermal turbulence.

There have been a lot of studies focusing on the behaviors of a passive scalar in turbulent flows; for more detailed elucidation of the problem, we refer the readers to the recent reviews by Sreenivasan [3, 17], Shraiman and Siggia [18], Warhaft [19], Falkovich et al [20], Sawford [21]...
and Dimotakis [22]. In the studies of passive scalar mixing, the measurements are made mostly in turbulent jets or in a grid-generated turbulence [7, 13, 23, 24]. Nevertheless, it may be buoyancy that often drives the turbulent flows in which real scalar mixing occurs, such as the motion of smoke from a cigarette and turbulent dispersion of contaminants in the atmosphere. As an important class of turbulent flows, buoyancy-driven turbulence differs from turbulent shear flows and grid-generated turbulence in many ways and hence may lead to different behaviors for the passive scalar mixing. Studying scalar mixing in such a type of turbulent flow is thus highly desirable and essential for one to understand the physical nature of a wide range of more complicated mixing problems occurring in nature. However, the number of studies on the passive scalar mixing in buoyancy-driven turbulence is rather limited.

Gluckman et al [4] studied the geometry of iso-concentration contours in a turbulent Rayleigh–Bénard (RB) convection system and found that the probability density function (PDF) of the local curvature of the iso-contours can be characterized approximately by a stretched exponential function with two asymmetric tails. They also found that these iso-contours have a limited fractal scaling range. However, due to the short duration of dye injection and the limited Rayleigh number $Ra$ ($10^7$ to $3 \times 10^8$, which is close to the soft-to-hard turbulence transition) of the experiment, their measured fractal dimension depends on time and on the selected concentration level. In previous work, we made a comparative study of the local active (temperature) and passive (dye concentration) scalar fluctuations in turbulent thermal convection [25]. Our results showed that the two scalars distribute differently, although they are advected by the same turbulent flow. Moreover, the active scalar was found to possess a higher level of intermittency. However, this early study focused mainly on the local properties of the scalar fields. As turbulent scalar mixing is always related to complicated 3D dynamic and geometric structures, the results obtained from single-point measurement are inevitably incomplete.

In the present paper, we report an experimental study of the 2D passive scalar mixing using the planar laser-induced fluorescence (PLIF) technique in a turbulent RB convection system, which has been shown to be a fruitful paradigm for the understanding of ubiquitous buoyancy-driven turbulent flows [26]–[32]. The fluorescence dye of Rhodamine 6G was injected continuously into the convection cell and its concentration was a passive scalar. The mixing evolution and the geometric properties of the passive scalar field were then studied.

The rest of this paper is organized as follows. We give a detailed description of the experimental setup and our measuring technique in section 2. Section 3 examines the evolution of passive scalar mixing, and the geometric properties of the passive scalar field are presented and analyzed in section 4. Conclusions from our findings are given in section 5.

2. The experimental setup, measuring technique and parameters

2.1. The convection cell and the dye-injection system

The shape of the convection cell is chosen based on the following considerations. Due to the azimuthal motion of the large-scale circulation (LSC) in convection cells with a cylindrical shape [33]–[36], the injected dye solution will meander with the LSC and cannot always pass through the measuring area, which would lead to a poor signal-to-noise ratio. Moreover, the curved sidewall will introduce distortions in the images viewed by the camera. In contrast, in a cube-shaped cell, the mean flow is confined to the diagonal plane of the cell [37, 38], which
means that the measuring area is only a projection of the flow field. Thus, a rectangular shape is chosen for our convection cell. The details of both the cell and the dye-injection system have been described previously [25]. Here, we give only their main features. The cell is a vertical rectangular box with its top and bottom plates made of 1.5-cm-thick copper and the sidewall composed of four 8-mm-thick Plexiglas plates. The cell’s length, width and height are 24.8 × 7.5 × 25.4 cm³, respectively. With this geometry, the symmetry of the system was broken and the LSC was found to be confined in the plane with the aspect ratio ≃1 [39]–[41]. Degassed and distilled water was used as the convecting fluid.

Figure 1 shows the schematic diagram of the dye-injection system. The dye solution of Rhodamine 6G was injected into the convection cell from a raised tank through a slit located in the bottom plate. The slit is parallel to the short edge of the cell and 1 cm from the sidewall. The length $d$ and the width of the slit are 2 cm and 1 mm, respectively, and therefore its area is very small compared to that of the bottom plate and it is not expected to introduce significant perturbation to the lower boundary layer. As we shall see in section 2.3, the perturbation of the dye injection to the lower boundary layer can indeed be neglected.

As the RB system is a closed system, a continuous injection of dye will increase the background concentration level in the convection cell and thus reduce the dynamical range of the fluorescence signal, as well as the signal-to-noise ratio. To solve this problem, we injected dilution water into the cell through the top plate from another raised tank to keep the background concentration constant [25]. In our present study, we chose the background concentration as $C'_{\text{back}} = 1.33 \times 10^{-7} \text{M}$ and the concentration of the injected dye solution as $C'_0 = 4.0 \times 10^{-7} \text{M}$. This somewhat large background will sacrifice the dynamic range of the experiment, but it can maintain a dynamic range that is independent of time. Because of the 14-bit CCD camera used (see section 2.2), the measured fluorescence signal would have a dynamic range that is wider than 12 bit despite the presence of this background. This should be enough for our present study.

The injection speeds of injected fluids are varied by changing the height difference between the raised tanks and the cell, and calculated from the ratio of the volume of the injected fluids per unit time to the area of the injection slit. For each measurement, the mean injection speeds of both the dye solution and the dilution water are approximately the same as that of the
Figure 2. Sample images of dye mixing and the coordinates used for 2D concentration field measurements. (a) Raw fluorescence image taken at $Ra = 1.2 \times 10^{10}$. (b) Normalized concentration field. (c) Extracted iso-concentration contours for the detection threshold $C_d = 0.1$. The origin of the coordinates is located at the center of the injection slit.

LSC, which was obtained from the particle image velocimetry measurements (e.g. $0.4 \text{ cm s}^{-1}$ at $Ra = 2 \times 10^9$ and $1 \text{ cm s}^{-1}$ at $Ra = 1.2 \times 10^{10}$, see [39]). A third tank (not shown in figure 1) was used to capture the out-flowing fluid and the same condition was applied for the speed of the out-flowing fluid.

2.2. The planar laser-induced fluorescence (PLIF) technique

The calibration and validation of the single-point laser-induced fluorescence technique in the turbulent RB system have been presented and discussed in our previous work [25]. In this paper, we extend this technique to the 2D situation, i.e. the technique of PLIF, which has been widely used and well documented previously [7]. Figure 2 shows sample images of dye mixing and the coordinates used for the PLIF measurements. When the dye solution was injected into the cell, a laser light sheet, which was generated by a laser light source and a set of cylindrical concave lens, was used to illuminate the flow. The thickness of the laser sheet is $\sim 0.5 \text{ mm}$. Because the absorption peak of the solution of Rhodamine 6G is about $520 \text{ nm}$ [25], we chose the $514.5 \text{ nm}$ green line from an argon ion laser (Coherent Innova 70) as the exciting light. For dye concentrations used in the present experiment, the resulting fluorescence intensity at one point is proportional to the local concentration. A cooled digital EMCCD camera with single-photon sensitivity (Andor iXon$^{\text{EM}}$ + 885) (see figure 1) and fitted with a Nikon 60 mm $f/2.8$ lens was used to take pictures of the concentration field. The camera has a spatial resolution of $1004 \times 1002$ pixels and a dynamic range of 14 bit. As the emission peak wavelength of Rhodamine 6G is about $560 \text{ nm}$ [25], a filter with a pass band from 550 to 570 nm was placed in front of the EMCCCD to block out the excitation laser light.
To eliminate the influence of such factors as the non-uniform intensity of the laser sheet, the camera dark noise and the details of the optical alignment in the captured images from CCD camera, the raw images were processed using a pixel-by-pixel calibration. Before each run, the noise images, $I_{\text{noise}}(x, z, t)$, were recorded briefly with no dye in the cell. Then, the dye solution was injected into the cell to increase the background concentration $C_{\text{back}}'$ to the preset value of $1.33 \times 10^{-7}$ M. When the injected dye solution had been well mixed by the turbulent flow and was distributed uniformly in the cell, we recorded the background images, $I_{\text{back}}(x, z, t)$, over a short period of time. After each run, the raw fluorescence image data, $I_{\text{raw}}(x, z, t)$, were calibrated and normalized using

$$C(x, z, t) = \frac{I_{\text{raw}}(x, z, t) - \langle I_{\text{back}}(x, z, t) \rangle}{\langle I_{\text{back}}(x, z, t) \rangle - \langle I_{\text{noise}}(x, z, t) \rangle} \times \frac{C_{\text{back}}'}{C_0' - C_{\text{back}}'},$$

where $C(x, z, t)$ is the normalized concentration obtained at time $t$ and at spatial point $(x, z)$ and $\langle \cdots \rangle$ represents a time average. Thus, $C(x, z, t)$ varies from 0 to 1, and $C(x, z, t) = 0$ for the background fluids and $C(x, z, t) = 1$ for the injected fluids. Figure 2(a) shows an example of a raw fluorescence image and the corresponding normalized image is shown in figure 2(b).

2.3. Experimental conditions and parameters

The dynamics of the system are determined by the Rayleigh number $Ra$ and the Prandtl number $Pr$, namely,

$$Ra = \frac{\alpha g \Delta T H^3}{\nu \kappa_T} \quad \text{and} \quad Pr = \frac{\nu}{\kappa_T},$$

where $\nu$ and $\kappa_T$ are the coefficients of kinematic viscosity and thermal diffusivity of the working fluid, respectively, and $H$ is the height of the cell. In addition, the Schmidt number $Sc$, defined as

$$Sc = \frac{\nu}{\kappa_C},$$

describes the passive scalar, where $\kappa_C$ is the molecular diffusivity of the fluorescence dye. There are two response parameters for the turbulent RB system: the Reynolds number $Re$ and the Nusselt number $Nu$, given by

$$Re = \bar{u} H / \nu,$$
$$Nu = Q H / \chi \Delta T,$$

where $\bar{u}$ is the mean velocity of the LSC, $Q$ is the heat–current density and $\chi$ is the thermal conductivity of the working fluid in the absence of convection. There are three typical length scales for the turbulent RB system: the Bolgiano scale $\ell_B$, the Kolmogorov scale $\eta$ and the Batchelor dissipation scale $\eta_B$. The global estimates of these length scales can be obtained from the following formulae [22], [42]–[45], in terms of the measured $Ra$, $Pr$, $Sc$ and $Nu$,

$$\ell_B = H Nu^{1/2} / (Ra Pr)^{1/4},$$
$$\eta = H Pr^{1/2} / (Ra Nu)^{1/4},$$
$$\eta_B = \eta Sc^{-1/2}.$$
Table 1. Experimental parameters, typical length scales and fractal dimensions. All length scales are in units of mm. \( Pr = 5.3 \) and \( Sc = 2100 \) for all four cases.

| Number | \( Ra \) (\( \times 10^9 \)) | \( Pe_C \) (\( \times 10^7 \)) | \( Re \) | \( Nu_0 \) | \( Nu_{dye} \) | \( \eta_B \) | \( \eta \) | \( \ell_B \) | \( \eta \) | \( \lambda \) | \( \frac{1}{|\kappa|} \) | \( d_y \) | \( d_B \) |
|--------|-----------------|-----------------|-------|--------|--------|---------|--------|--------|--------|--------|-------------|--------|--------|
| 1      | 1.2             | 1.25            | 5900  | 139.7  | 140.4  | 0.011   | 0.52   | 5.8    | 2.4    | 2.64   | 1.29        | 1.29   |
| 2      | 6.8             | 0.94            | 4500  | 117.4  | 116.8  | 0.014   | 0.63   | 6.2    | 2.6    | 2.48   | 1.29        | 1.28   |
| 3      | 3.7             | 0.69            | 3300  | 98.6   | 99.1   | 0.017   | 0.77   | 6.5    | 2.8    | 2.67   | 1.28        | 1.30   |
| 4      | 2.0             | 0.51            | 2400  | 82.1   | 82.3   | 0.021   | 0.94   | 6.9    | 3.1    | 2.17   | 1.31        | 1.30   |

Table 1 summarizes the experimental parameters and typical length scales of the present investigation. In our experiments, the mean temperature of the working fluid was kept at 31 °C and hence the Prandtl number \( Pr = 5.3 \) and the Schmidt number \( Sc = 2100. \) The experiments were done at four Rayleigh numbers, i.e. \( Ra = 2.0 \times 10^9, 3.7 \times 10^9, 6.8 \times 10^9 \) and \( 1.2 \times 10^{10}. \) The cell is tilted with a small angle of about 1° near the side of the dye injection slit in order that the flow direction of the LSC is locked \([46]–[48]\) and to ensure that the dye solution always moves upwards with the LSC after being injected into the cell. Previous flow visualizations \([30], [49]–[51]\) have shown that the spatial distributions of thermal plumes are inhomogeneous in the convection cell, i.e. they abound near the sidewall but hardly appear in the central region. Since buoyant forces are exerted on the fluid mainly via thermal plumes, the regions near the sidewall correspond to the buoyancy-dominant regions. As the objective of this paper is to study the turbulent scalar mixing driven by buoyancy, the experiments were thus performed near the sidewall (at 1 cm from the sidewall). Since the viscous boundary layer thickness near the sidewall is about 2–3 mm for the present \( Ra \) and \( Pr \) \([38]\), the chosen measuring area is outside the sidewall viscous boundary layer and inside the LSC. A total of 20 000 images of the passive scalar field were acquired for each \( Ra \) and at sampling rates from 2 to 5 frames per second (depending on \( Ra \)). The measuring region had an area of 7.2 × 16 cm², corresponding to 450 × 1000 pixels² for each normalized concentration image, and its coordinates were \(-1.8 \leq x/d \leq 1.8 \) and \( 0 \leq z/d \leq 8 \) (\( d = 2 \) cm is the length of the injection slit). Each pixel in the images corresponds to a square of side length 0.16 mm, a distance smaller than the smallest Kolmogorov scale in the study but 7–15 times larger (depending on \( Ra \)) than the Batchelor dissipation scale \( \eta_B \) (see table 1). Note that the geometric information at scales smaller than the measurement resolution cannot be revealed properly. However, the geometric information obtained at scales larger than the measurement resolution should be correct. Since the objective of the present work is to study the influence of buoyant forces on the behaviors of the turbulent scalar mixing and since buoyant forces only act on scales above the Bolgiano scale that is larger than the Kolmogrov scale, the present measurement resolution should be enough for our purpose. The Pécelt number for the passive scalar field, \( Pe_C = \bar{u}H/\kappa_C, \) (6) was much larger than 1 for all values of \( Ra \) (see table 1), suggesting that the mixing of the passive scalar field is dominated by the turbulent velocity field, rather than by its diffusion rate.

To check the perturbation of the dye and water injections to the boundary layers, we study the global heat transport of the system, which is expressed in terms of the Nusselt numbers \( Nu_0 \) and \( Nu_{dye}. \) Here, we use the subscript ‘0’ to denote no fluid injection and the subscript ‘dye’ to denote that the measurement is performed when both the dye solution and the
dilution water are injected into the cell. The global heat transport of the system is determined dominantly by the top and bottom boundary layers, because the boundary layers contribute the main resistance for the heat transfer through the cell. If the dye and water injections perturb the boundary layers dramatically, the measured $Nu_0$ and $Nu_{dye}$ should be different, while if the difference between $Nu_0$ and $Nu_{dye}$ is within measurement resolution, the perturbation of the injections to the boundary layers can be neglected. Comparing $Nu_{dye}$ with $Nu_0$ can thus indicate indirectly whether the boundary layers are perturbed dramatically by the injections or not. Figure 3(a) shows compensated $Nu_0/Ra^{1/3}$ (solid down-triangles) and $Nu_{dye}/Ra^{1/3}$ (open circles) as functions of $Ra$. One sees that $Nu_0$ agrees well with $Nu_{dye}$ for all four values of $Ra$. To examine this more quantitatively, we plot the relative difference between $Nu_0$ and $Nu_{dye}$, $(Nu_{dye} - Nu_0)/Nu_0$, in figure 3(b). It is seen that all data are within ±1%, which is within the measurement resolution of the $Nu$ measurement [52, 53]. We therefore conclude that the perturbation of the dye and water injections to the boundary layers can indeed be neglected.

3. Mixing evolution of passive scalar field

3.1. Spanwise mixing

After being injected into the convection cell, the passive scalar packets that meander in horizontal directions are advected upwards by the LSC. Under the action of the flow field, the scalar packets are stretched along the vertical direction and form a sheetlike topology (a linelike shape in the 2D measuring area). These sheet-like packets are then broken up into smaller disjointed sheets progressively and get mixed with the surrounding fluids. Figure 4 shows an example of the horizontal (transverse) profiles of the mean concentration $⟨C(x, z, t)⟩$ measured at different normalized downstream positions $z/d$ at $Ra = 3.7 \times 10^9$. Here the mean
concentration \(\langle C(x, z, t) \rangle\) was obtained by taking a time average of the measured normalized concentration \(C(x, z, t)\) at point \((x, z)\). Due to the linelike injection geometry, the profile has a flat part (with a value that is essentially the injection concentration) in the center when the scalar emerges initially in the convection cell. The flat part then expands and merges because of the mixing and dilution of the passive scalar packets, and the profile flattens out as the scalar is transported downstream by the turbulent flow.

An obvious feature one notices from figure 4 is that the position of the maximum value of the mean concentration along each horizontal profile shifts to the \(-x\)-direction as \(z/d\) increases. This is due to the fact that the direction of the LSC in the measuring plane is at a certain angle \(\theta_0\) with the vertical direction \([39, 40]\), rather than exactly along the injection centerline (\(z\)-direction). The nonzero \(\theta_0\) is probably a result of the LSC’s tendency to be locked along the diagonal of the plane, which corresponds to a more stable flow configuration as observed by Xia et al \([39]\). We therefore define the scalar center \(x_0(z)\) as

\[
x_0(z) = \frac{\int x\langle C(x, z, t) \rangle dx}{\int \langle C(x, z, t) \rangle dx}
\]

to describe the evolution path of the passive scalar distribution center.

Close to the injection source, the meandering motion of the scalar packets, which is a consequence of the turbulent large-scale displacements, is a significant mechanism that generates the concentration fluctuations \([24]\). When the downstream position \(z\) is larger than some critical value, the mixing evolution of the concentration field enters a self-similar regime. This self-similarity is twofold: both the profiles of the mean concentration and those of the standard deviation of concentration fluctuations can be well described by Gaussian curves. Figure 5(a) shows the horizontal profiles of the mean concentration \(\langle C(x, z, t) \rangle\) for different downstream positions of \(z/d\) measured at \(Ra = 3.7 \times 10^9\). Here, the profiles have been normalized by their own concentration values at the scalar center, \(\langle C(x_0(z), z, t) \rangle\), and the origins of the \(x\)-axis have been shifted to the scalar center \(x_0(z)\), which is different for different profiles, and the abscissas have been normalized by their respective full-width at half-maximum (FWHM) \(w_C\). One sees that after normalization the profiles collapse on top of each profile. 

**Figure 4.** Horizontal profiles of the mean concentration \(\langle C(x, z, t) \rangle\) at \(Ra = 3.7 \times 10^9\). \(z\) increases in the direction of the arrow.
Figure 5. Horizontal profiles of the mean concentration \( \langle C(x, z, t) \rangle \) (a) and the standard deviation of concentration fluctuations \( \sigma(x, z) \) (b) for three different downstream positions \( z/d \) at \( Ra = 3.7 \times 10^9 \). Here, \( \langle C(x, z, t) \rangle \) and \( \sigma(x, z) \) have been normalized by \( \langle C(x_0(z), z, t) \rangle \) and \( \sigma(x_0(z), z) \), respectively. The origins of the \( x \)-axis have been shifted to the scalar center \( x_0(z) \) and the abscissas have been normalized by the FWHM of each profile, \( w_C \) and \( w_\sigma \), respectively. The solid curves show the Gaussian fits.

other for both cases. In addition, all these profiles can be represented by a Gaussian function, which is shown as the solid curve in the figure. The profiles of the standard deviation (rms) of concentration fluctuations,

\[
\sigma(x, z) = \langle (C(x, z, t) - \langle C(x, z, t) \rangle)^2 \rangle^{1/2},
\]

are shown in figure 5(b), corresponding to the same data as shown in figure 5(a). Compared to the mean concentration profiles, the rms profiles are seen to be somewhat scattered. This is because, being a second order quantity, the rms requires a longer time average than the mean value for the same level of statistical accuracy. Nevertheless, one sees that, after a similar normalization process, all profiles again collapse on top of each other and they can also be described by a Gaussian distribution (shown as the solid curve). It is further found
that, with increasing $Ra$ this self-similar regime starts closer to the injection source and for $Ra = 1.2 \times 10^{10}$ it starts from $z \approx 0.7d$ and extends beyond the measuring area.

3.2. Streamwise mixing

As mentioned above, we use the scalar center $x_0(z)$, instead of the injection centerline, to describe the evolution path of the passive scalar mixing. Figure 6(a) shows the measured $x_0(z)/d$ as a function of $z/d$ at $Ra = 6.8 \times 10^{9}$ (the solid line). One sees that the line is at an angle of $\theta_0 \approx 4^\circ$ with the vertical direction, which is indicated by the dashed line for reference. We further find that the evolution paths for all other $Ra$ have a similar value of $\theta_0$ and thus the modification of the downstream positions induced by $\theta_0$ is small so that we may neglect this effect in the remainder of this paper.

Figure 6(b) shows the mean concentration $\langle C(x_0(z), z, t) \rangle$ along the evolution path of the scalar center $x_0(z)$ for all four values of $Ra$. One sees that the concentration field $\langle C(x_0(z), z, t) \rangle$ maintains a high, and constant, value close to the injection source (for example, for $Ra = 1.2 \times 10^{10}$, this corresponds to positions of $z \lesssim 0.2d$). In this range, the passive scalar packets have not been diluted and mixed by the background fluids and hence they do not possess any significant structures, i.e. the concentration of the scalar packets is nearly constant. Further downstream (e.g. $z \gtrsim 0.2d$ at $Ra = 1.2 \times 10^{10}$), the concentration field $\langle C(x_0(z), z, t) \rangle$ for all $Ra$ exhibits a power-law decrease with a roll-off rate of about $-1$, which is due to the mixing, dilution and meandering motion of the scalar packets. This roll-off rate reflects the outer scale of the scalar packets and it has been previously found that the roll-off rate of $-1$ applies to the linelike injection geometry [24], whereas a roll-off exponent of $-2$ applies to the axisymmetric injection geometry [13]. As the injection slit used in the present experiment corresponds to a linelike geometry, it is no surprise that a $-1$ roll-off rate is observed here. In addition, figure 6(b) shows that the onset of the power-law regime of $\langle C(x_0(z), z, t) \rangle$ also shifts downstream with decreasing $Ra$. 

Figure 6. (a) The normalized downstream position $z/d$ versus the normalized scalar center $x_0(z)/d$ at $Ra = 6.8 \times 10^{9}$. The dashed line shows the vertical direction and the injection centerline. (b) The downstream evolution of the mean concentration $\langle C(x_0(z), z, t) \rangle$ along the scalar center $x_0(z)$. Rayleigh number $Ra$ increases from top to bottom. The solid line shows a power-law relation with a roll-off rate $-1$ for reference.
The mixing evolution of the passive scalar field can also be characterized by the single-point PDF along the downstream path. Figure 7 shows the PDFs of $C(x_0(z), z, t)$ close to the injection source (at $z = 0.5d$ and $1.2d$) for $Ra = 3.7 \times 10^9$. Both PDFs exhibit a kind of bimodal distribution. Bimodal distributions have also been observed for the temperature field near a line source in grid turbulence [24], where temperature is a passive scalar. This double-peak PDF reflects the meandering motion of the scalar packets, i.e. the scalar center spends more time in the background medium or in the injected dye solution and less time in the mixed fluids, which is due to the fact that the injection solution has not been well mixed in this range. The left peak corresponds to the background concentration and the right one to the concentration of the injected solution. Both peaks may be represented by a Gaussian function (as shown by the two solid curves in figure 7). Along the evolution path, the injected fluorescence dye is progressively mixed and diluted by the surrounding turbulent medium and one sees that the right peak expands, merges and decays and finally disappears (e.g. for $z > 0.7d$ at $Ra = 1.2 \times 10^{10}$).

Figure 8(a) shows the PDFs $P(C)$ of $C(x_0(z), z, t)$, far away from the injection source for $z/d = 0.92, 3.95$ and $7.24$ ($Ra = 1.2 \times 10^{10}$). All these PDFs, which appear to be very different from those shown in figure 7, are composed of two parts: the left part, which corresponds to the background concentration with a Gaussian peak, and the right decreasing tail, which exhibits a self-preserving shape and can be described by an exponential function,

$$P(C) \sim e^{-\gamma C},$$

where $\gamma$ is a parameter that depends on the downstream position. Villermaux et al [54]–[56] argued that the exponential PDF shape results from the distribution of the number of cumulated stretchings experienced by the scalar sheets at a given instant of time, and that $\gamma$ should be proportional to the injection time (or downstream position) of the scalar packets in the medium and is a certain function of $Sc$ but independent of the Reynolds number, and they found

$$\gamma = \beta z/d \quad \text{with} \quad \beta = 1/0.12 \ln(5Sc).$$
Here, we plot $\gamma$ as a function of $z/d$ in figure 8(b). A clear linear range can be seen for all four values of $Ra$ investigated. The solid lines in figure 8(b) are the best linear fits to the corresponding data and their fitting slopes are $0.29 \pm 0.02$, $0.36 \pm 0.02$, $0.43 \pm 0.02$ and $0.48 \pm 0.02$ for $Ra = 2.0 \times 10^9$, $3.7 \times 10^9$, $6.8 \times 10^9$ and $1.2 \times 10^{10}$, respectively, which are all smaller than the value of $\beta = 1/0.12 \ln(5Sc) = 0.90$. The reason for this may be due to buoyancy effects. The inset of figure 8(b) shows the fitted slopes $\beta$ as a function of turbulent intensity, i.e. $Ra$. We find that a function

$$\beta = 2.5 \log_{10} \left( \frac{Ra}{Ra_0} \right) \quad \text{with} \quad Ra_0 = 1.4 \times 10^8$$

(11)

can be used to describe well the relation between $\beta$ and $Ra$ (the solid line in the figure). Here, $Ra_0$ indicates the minimum $Ra$ required for effective mixing. The result of $\beta$ increasing with $Ra$, obtained in the present investigation, reflects the increased mixing and dilution of the scalar by the turbulent flow.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure8.png}
\caption{(a) PDFs of $C(x_0(z), z, t)$ for $z/d = 0.92$, 3.95 and 7.24 at $Ra = 1.2 \times 10^{10}$. The dashed curve is a Gaussian fit to the left parts of all three PDFs. (b) The slope of the exponential tail of the PDF, $\gamma$, versus the normalized downstream positions $z/d$. $Ra = 2.0 \times 10^9$, $3.7 \times 10^9$, $6.8 \times 10^9$ and $1.2 \times 10^{10}$ from the bottom to the top. The lines show linear fits to the corresponding data and the fitting slopes $\beta$ are plotted as a function of $Ra$ in the inset.}
\end{figure}
4. Geometric properties of passive scalar iso-contours

In this section, we examine quantitatively the geometric properties of the passive scalar iso-contours, i.e. the shape complexity, fractal dimension and local curvature. To study the geometric properties of iso-concentration contours, a computer program was used first to identify the iso-concentration contours of the normalized fluoresce images, using the detection threshold $C_d$, in the range $2 < z/d < 8$, which is within the self-similar regime of the passive scalar mixing, and then to calculate the perimeter of, and area enclosed by, each contour. Over 1 million disjoint contours were identified from the total of 20,000 images (more than 50 contours per realization on average) for each $Ra$, which is sufficiently large for our present statistical analysis. Figure 2(c) shows an example of the identified contours for $C_d = 0.1$, corresponding to the raw fluorescence image in figure 2(a) and the normalized image in figure 2(b). Iso-contours generated from different thresholds share similar features with those shown in figure 2(c), and we find that iso-contours with high concentration are always embedded within iso-contours with low concentration and the iso-contours become more disconnected and isolated at increasing detection threshold. This observation agrees with the numerical simulations of homogeneous and isotropic turbulence [57] and the experimental investigation of turbulent jets [13]. Based on the extracted iso-contours, the shape complexity of iso-contours was obtained by applying both the perimeter-area and the box-counting methods. To calculate the curvature at a point $(x, z)$, we first fitted the $x$ and $z$ values of a contour separately as functions of the path length with second-order polynomials over segments of 11 points [4] and then used equation (20) (see section 4.3) to obtain the local curvature $\kappa(x, z)$. In the calculation of all geometric measures, only iso-contours with more than 11 points were used, and only curvatures whose absolute values were smaller than $6.25 \text{ mm}^{-1}$ were considered, corresponding to a radius of 0.16 mm, which is the smallest real size that can be identified in the experiments.

4.1. Shape complexity

In the 2D case, the shape complexity $\Omega_2$ is a dimensionless ratio between the perimeter $\Phi$ and the area $A$ of an object and is defined as [1]

$$\Omega_2 = \frac{\Phi}{2\sqrt{\pi A}}. \quad (12)$$

Here, the subscript ‘2’ refers to 2D. As a circle has the smallest perimeter $2\sqrt{\pi A}$ in all 2D objects with the same area $A$, the shape complexity $\Omega_2$ satisfies $1 \leq \Omega_2 < \infty$ and can be used to describe the departure of an object from the shape of a circle, and hence rougher objects always have larger values of $\Omega_2$.

Figure 9(a) shows the PDFs $P(\log_{10}(\Phi/d))$ of the logarithm of the obtained contour perimeter for all four values of $Ra$ and for $C_d = 0.1$, where the perimeter $\Phi$ has been normalized by the injection size $d$. Note that the PDFs have been shifted vertically for clarity and the arrows in the figure indicate the Bolgiano scale $\ell_B$ for the highest and lowest $Ra$ for reference. It is seen that, for the values of $\Phi$ that are smaller than 10 times the injection scale, all PDFs can be described by log-Poisson statistics,

$$P(\log_{10}(\Phi/d)) \sim 10^{-\nu_0 \log_{10}(\Phi/d)}. \quad (13)$$
The best fits to the data extend more than two decades from the smallest scale investigated (approximately four times the Kolmogorov scale $\eta$) to the scale of the order of the system size. It should be noted that the Poisson PDF of $\log_{10}(\Phi/d)$ is equivalent to a power-law PDF of $\Phi/d$, i.e.

$$P(\Phi/d) \sim (\Phi/d)^{-(\nu_{\Phi}+1)},$$

(14)

Figure 9. PDFs of the logarithms of (a) the normalized perimeter $\Phi/d$ and (b) the normalized area $A/S_{slit}$ of iso-concentration contours for $C_d=0.1$. $\triangledown$, $Ra = 2.0 \times 10^9$; $\Box$, $Ra = 3.7 \times 10^9$; $\triangle$, $Ra = 6.8 \times 10^9$; $\circ$, $Ra = 1.2 \times 10^{10}$. For clarity, each data set has been shifted up from its lower neighbor by a factor of 3. Solid lines are log-Poisson fits to the corresponding data. The arrows indicate the Bolgiano scale $\ell_B$ and $\ell_B^2$ for the highest and the lowest $Ra$ for reference. (c) The $Ra$ dependence of $\nu_{\Phi}$ and $\nu_A$. 

Figure 10. (a) PDFs of the logarithms of shape complexity $\Omega_2$ of iso-concentration contours for $C_d = 0.1$. $\nabla$, $Ra = 2.0 \times 10^9$; $\square$, $Ra = 3.7 \times 10^9$; $\Delta$, $Ra = 6.8 \times 10^9$; $\circ$, $Ra = 1.2 \times 10^{10}$. For clarity, each dataset has been shifted up from its lower neighbor by a factor of 3. Solid lines are log-Poisson fits to the corresponding data. (b) The mean values of shape complexity over all iso-contours, $\bar{\Omega}_2$ versus $Ra$.

since $P(\log_{10}(\Phi/d)) \sim (\Phi/d) P(\Phi/d)$, and thus indicates a scale-free behavior. The PDFs of the logarithm of the normalized area $A/S_{slit}$ enclosed by the iso-contours are plotted in figure 9(b); again the arrows indicate $\ell_B^2$ for reference. Here, $S_{slit}$ is the area of the injection slit. Similar log-Poisson PDFs, i.e.

$$P(\log_{10}(A/S_{slit})) \sim 10^{-\nu_A \log_{10}(A/S_{slit})}, \quad (15)$$

are observed for the range below 10 times the injection area, and the best fits to the data cover nearly three decades.

To study the influence of turbulent intensity on the distributions of iso-contour perimeter and area, we plot in figure 9(c) the exponents $\nu_\Phi$ and $\nu_A$ versus $Ra$. One sees that $\nu_\Phi$ and $\nu_A$ both increase with increasing $Ra$. This suggests that the probabilities of large-value perimeter and area become smaller as the flow becomes more turbulent and therefore indicates that with increasing $Ra$ the increased mixing can more easily fragment larger iso-contours into smaller ones.

Log-Poisson PDFs, i.e.

$$P(\log_{10} \Omega_2) \sim 10^{-\nu_2 \log_{10} \Omega_2}, \quad (16)$$

are also found for the shape complexity $\Omega_2$ of the iso-concentration contours for $C_d = 0.1$ and for all four values of $Ra$ investigated, which are shown in figure 10(a). The values of $\Omega_2$ span
more than one decade for all $Ra$, indicating that there are contours with a perimeter up to more than 10 times the equal-area circles. The best fits now cover almost the whole range of the data. Note that $\nu_{\Omega}$ can be used as a measure of the degree of scalar mixing since we have the relationship between the mean value of the shape complexity over all measured iso-contours, $\bar{\Omega}_{2}$, and $\nu_{\Omega}$ (for $\nu_{\Omega} > 1$), namely,

$$\bar{\Omega}_{2} = \frac{1}{\nu_{\Omega} - 1},$$

which implies that a larger value of $\nu_{\Omega}$ leads to a smaller value of $\bar{\Omega}_{2}$ and hence reflects stronger mixing. Figure 10(b) shows the $Ra$ dependence of mean shape complexity $\bar{\Omega}_{2}$. One sees that $\bar{\Omega}_{2}$ decreases as $Ra$ increases. The decrease in $\bar{\Omega}_{2}$ implies that the passive scalar iso-contours become smoother when the flow becomes more turbulent, again indicating the increased mixing and dilution with increasing $Ra$. It should be mentioned that the log-Poisson distribution of shape complexity has also been found for level-set islands and lakes in 2D slices of the scalar field of liquid-phase-jet turbulent flows [1]. However, analysis of the PDF of the islands/lakes size in the same turbulent flow system indicates a log-normal distribution at inner scales [11].

Recently, we performed a systematic experimental study of geometric properties of thermal plumes in turbulent Rayleigh–Bénard convection [70]. Our results show that, in contrast to the log-Poisson distributions of the passive scalar, the iso-contours of the active scalar, i.e. the boundaries of thermal plumes, in the same system obey the log-normal statistics.

### 4.2. Fractal dimension

Fractal dimension may be used as a measure of the roughness of surfaces or contours. Here, we use both the perimeter–area [2, 16] and the widely used box-counting (see, for example, [17]) methods to determine the fractal dimension of the passive scalar iso-concentration contours.

#### 4.2.1. Perimeter-area dimension

Lovejoy [16] used a power-law formula, i.e.

$$\Phi \sim \sqrt{A}^{d_P},$$

as proposed by Mandelbrot [12], to describe the relationship between the perimeter and the area of rain and cloud regions, where $d_P = 1.35$ is the contour dimension. Catrakis and Dimotakis [1] also applied equation (18) to fit the scatter plot of normalized perimeter versus area for the iso-concentration contours of a transverse passive scalar field in turbulent jets using Lovejoy’s value ($d_P = 1.35$).

Figure 11(a) shows the relation between our measured perimeter and area for all four values of $Ra$ and for $C_d = 0.1$. To reveal the overall trend of the relation clearly, we plot the normalized conditional average $\langle \Phi|A \rangle/d$ versus the normalized contour size $(A/S_{\text{slit}})^{1/2}$, instead of a direct scatter plot of the perimeter versus area as used in previous studies [1, 16]. One sees that $\langle \Phi|A \rangle$ increases monotonically with increasing contour size, as expected, for all $Ra$. We find that formula (18) can be used to fit the relation between $\langle \Phi|A \rangle/d$ and $(A/S_{\text{slit}})^{1/2}$ for all $Ra$ studied (the solid lines in figure 11(a)), while the power-law range becomes wider for higher $Ra$. For the highest $Ra$ ($= 1.2 \times 10^{10}$, open circles in figure 11), the fitting is found to cover nearly the whole data range. The fitted values of the contour dimension $d_P$ are found to vary between 1.28 and 1.31 (see table 1), which are close to Lovejoy’s value of 1.35, suggesting the existence of a certain universality among different systems. The fitted values of $P$ for the passive scalar
Figure 11. The logarithm of the normalized conditional average \( \langle \Phi | A \rangle / d \) (a) and of the compensated \( \langle \Phi | A \rangle / d \) versus the logarithm of the normalized size \( A / S_{slit}^{1/2} \) of the iso-concentration contours for \( C_d = 0.1 \). ▽, \( Ra = 2.0 \times 10^9 \); □, \( Ra = 3.7 \times 10^9 \); △, \( Ra = 6.8 \times 10^9 \); ○, \( Ra = 1.2 \times 10^{10} \). For clarity, in both (a) and (b), each dataset has been shifted up from its lower neighbor by a factor of 0.5. The arrows in (a) indicate the Bolgiano length scale \( \ell_B^2 \) for the highest and lowest Ra for reference.

are further found to be smaller than the value of 1.5 found for isothermal contours of thermal plumes (the active scalar) measured in the same system [70].

To make a detailed study of the scaling behavior of \( \langle \Phi | A \rangle / d \) with \( A / S_{slit}^{1/2} \), we show in figure 11(b) the compensated \( \langle \Phi | A \rangle / d \) versus \( A / S_{slit}^{d_P/2} \) as a function of the normalized contour size \( A / S_{slit}^{1/2} \). Here, \( d_P \) is obtained from the power-law fits in figure 11(a) (see table 1). It is seen that the compensated plots all have a plateau region with varying size. These plateaus indicate that the power-law relation equation (18) indeed holds over a certain range of contour size. One further sees that such a plateau range becomes wider with increasing \( Ra \), which could be a manifestation of the wider inertial range at larger \( Ra \).

In figure 12, we examine the threshold dependence of \( d_P \) for data taken at \( Ra = 3.7 \times 10^9 \) and \( 1.2 \times 10^{10} \). The dimensions shown in the figure were obtained from the fit to the perimeter–area relation within the power-law range using equation (18). The abscissa ranging
Figure 12. The fractal dimension $d_P$ as a function of the detection threshold $C_d$ chosen for the contour generation at $Ra = 3.7 \times 10^9$ (a) and $1.2 \times 10^{10}$ (b). The dashed lines mark the mean value of the plateau region for the corresponding data.

from 0 to 1 is our normalized measuring range. It is seen that $d_P$ for both $Ra$ does not vary much over a rather wide range of $C_d$. At $Ra = 3.7 \times 10^9$, this range is found to be from 0.1 to 0.5 and $d_P$ averaged within such a range is $1.29 \pm 0.02$, and at $Ra = 1.2 \times 10^{10}$ this range spans from 0.1 to 0.6 and $d_P$ averaged within such a range is $1.30 \pm 0.02$ (indicated as dashed lines in figure 12), implying that when applying the perimeter–area method the fractal dimension of the scalar iso-contours is essentially threshold-independent over such ranges.

4.2.2. Box-counting dimension. The box-counting method, among all algorithms, is widely used for determining the dimension of the fractal iso-contours. In the box-counting procedure for the 2D situation (see, for example, [3]), one covers the whole plane with square boxes of size $r$ and counts the number of boxes $N_2(r)$ containing the iso-contours. This procedure is repeated for various values of $r$ and the negative slope $d_B$ of log $N_2(r)$ versus log $r$ gives the dimension of the fractal iso-contours, i.e.

$$N_2(r) \sim r^{-d_B}.$$  \hfill (19)

Here, the subscript ‘2’ denotes the 2D embedding space for iso-contours.

The log–log plot in figure 13(a) shows the results obtained by applying the box-counting method to our normalized fluorescence images for all four values of $Ra$ and for $C_d = 0.1$. It is found that $N_2(r)$ decreases with $r/d$. Further analysis indicates that $N_2(r)$ also decreases with increasing detection threshold $C_d$ applied to the passive scalar field. All these trends agree with the numerical simulations of homogeneous and isotropic turbulence [57] and the experimental investigations of turbulent boundary-layers [14]. Over the range of $0.06 \lesssim r/d \lesssim 1$, which is within the inertial-convective regime, the constant scaling behavior can be seen for all datasets and the slopes of the best fits to the corresponding data using equation (19) yield a fractal dimension $d_B$ varying between 1.28 and 1.30 (see table 1). This dimension is surprisingly
Figure 13. (a) The number of square boxes \( N_2(r) \) of size \( r \) that cover completely the iso-contours for \( C_d = 0.1 \) versus the normalized box size \( r/d \) on a log–log scale. For reference, the arrows signify the Kolmogorov length scale \( \eta \) and the Bolgiano length scale \( \ell_B \) for the highest and lowest Ra and the solid lines mark the algebraic slope with \( r^{-1.3} \) in the log–log plot. (b) Compensated \( N_2(r)(r/d)^{d_B} \) versus \( r/d \). ▽, \( Ra = 2.0 \times 10^6 \); □, \( Ra = 3.7 \times 10^9 \); △, \( Ra = 6.8 \times 10^9 \); ○, \( Ra = 1.2 \times 10^{10} \). For clarity, in both (a) and (b), each dataset has been shifted down from its higher neighbor by a factor of 2.5.

the same, within experimental uncertainty, as that obtained from the perimeter–area method. However, this may be a mere coincidence since, as pointed out by Sreenivasan [3], these two different methods do not necessarily give the same dimension.

Figure 13(b) shows the compensated \( N_2(r)(r/d)^{d_B} \) as a function of \( r/d \). Here, \( d_B \) is obtained from the power-law fits over the range of \( 0.06 \lesssim r/d \lesssim 1 \) to the data in figure 13(a) and its values for different \( Ra \) are listed in table 1. One sees that there exists a plateau for all four values of \( Ra \), indicating the scale-free behavior between \( N_2(r) \) and \( r \). The range of the plateau is around \( 0.06 \lesssim r/d \lesssim 1 \) and is nearly independent of \( Ra \), which is in contrast to those obtained from the perimeter–area method.

Some previous studies have pointed out that the fractal dimension obtained from the box-counting method may depend on the detection threshold [9, 10, 13, 14, 58]. Here we chose several detection thresholds \( C_d \) to test whether this result applies to our system. Figure 14 shows the variation in the measured dimension \( d_B \) of the passive scalar field for different detection thresholds \( C_d \) at \( Ra = 3.7 \times 10^9 \) (figure 14(a)) and \( 1.2 \times 10^{10} \) (figure 14(b)) as two examples.
Here $d_P$ was obtained from the power-law fit to the data using equation (19) within the scaling range. It is seen that $d_P$ varies little over the range from 0.1 to 0.5 for $Ra = 3.7 \times 10^9$ and from 0.1 to 0.45 for $Ra = 1.2 \times 10^{10}$. The dashed lines in the figure mark the averaged $d_P$ within such plateau ranges. Again, our results reveal that the fractal dimension $d_P$ determined from the classical box-counting method is nearly threshold-independent over a certain range, but these ranges are somewhat smaller than those found in figure 12 by using the perimeter–area method. Note that Gluckman et al [4] also studied the fractal dimension of the iso-concentration contours of the passive scalar field in convective thermal turbulence using both the correlation and box-counting methods. However, due to both the limited $Ra$ ($10^7$ to $3 \times 10^8$, which is close to the soft-to-hard turbulence transition) and the short dye-injection duration of the experiment, the geometry of the iso-contours exhibits a limited range of approximate fractal scaling and their measured dimensions depend on both the injection time and the selected concentration threshold.

As mentioned in section 1, two different levels of fractal geometry within the inertial-convective regime have been identified via the classical box-counting method. Some early works (see [3] and references therein) observed a constant fractal dimension over the scales within the inertial-convective regime for the passive scalar iso-contours in turbulent flows. On the other hand, Catrakis and Dimotakis [11] suggested a scale-dependent fractal geometry for iso-contours, i.e. the fractal dimension depends on the spatial scale $r$, and such a scale-dependent fractal dimension was indeed observed for various turbulent systems (see, e.g., [12, 14]). Villermaux and coauthors [13, 59] further showed that, in addition to the scale size, the fractal dimension in turbulent jets also varies with time, injection scale and concentration threshold. It has been suggested that, to resolve these differences about the fractal property within the inertial-convective regime, studies at much higher Reynolds numbers would be required [57]. In our present study, however, $Re$ varies from 2400 to 5900 (see table 1), which is not high enough to settle this issue.

4.3. Local curvature

We now discuss the local curvature of the passive scalar iso-contours. The curvature at a point \((x, z)\) on a curve in a 2D plane is obtained by using the formula

\[
\kappa(x, z) = \frac{x''z' - x'z''}{(x'^2 + z'^2)^{3/2}},
\]

where the derivatives are with respect to the path length. Figure 15 shows the PDF of the local curvature, \(P(\kappa d)\), where the curvature has been non-dimensionalized by the injection size \(d\). In the figure, different line types represent data for different values of \(Ra\). We note that the results are robust and insensitive to the detection threshold \(C_d\) and the algorithm used to obtain the local curvature. The distributions are strongly non-Gaussian, and a remarkable feature of these PDFs is that the two extended tails of \(P(\kappa d)\) are not symmetric, with the positive tail having a larger probability than the negative one, especially for small \(Ra\). This asymmetry reflects the fact that the convex regions bounded by the scalar iso-contours (with positive curvature for our conventions) are more probable than the concave regions (with negative curvature). This feature was also observed by Gluckman et al [4] in a cubic cell at values of \(Ra\) close to the soft-to-hard turbulence transition. However, in the present case, we further found that the distribution of \(\kappa\) becomes more symmetric as \(Ra\) increases. To show this trend quantitatively, we examine the skewness of the local curvature, namely,

\[
S_\kappa = \frac{\langle (\kappa - \bar{\kappa})^3 \rangle}{\langle (\kappa - \bar{\kappa})^2 \rangle^{3/2}},
\]

where \(\bar{\kappa}\) is the mean value of \(\kappa\). The inset of figure 15 shows the \(Ra\) dependence of \(S_\kappa\). One sees that \(S_\kappa\) indeed decreases with increasing \(Ra\), indicating that the passive scalar mixing becomes more isotropic when the flow becomes more turbulent.

Gluckman et al [4] used a stretched exponential function, i.e.

\[
P(\kappa d) = A e^{a(\kappa d)^b},
\]

Figure 15. PDFs of the local curvature \(\kappa\), non-dimensionalized by the injection size \(d\), of the iso-concentration contours for \(C_d = 0.1\). Inset: the skewness of the local curvature, \(S_\kappa\), versus \(Ra\).
to characterize the distribution of the local curvature obtained in a cubic cell at values of $Ra$ close to the soft-to-hard turbulence transition. However, we find that equation (22) is not a good description for our present case, rather the relation

$$ P(|\kappa d|) \sim |\kappa d|^{-d_k} $$

(23)

provides a better choice. Figure 16(a) shows the PDFs of the absolute values of the local curvature, $|\kappa|$, for all four values of $Ra$ and for $C_d = 0.1$. To study the influence of the detection threshold, we also show the PDFs of $|\kappa|$ for three different $C_d$ (at $Ra = 1.2 \times 10^{10}$) in figure 16(b). Note that, as the spatial resolution of the present measurement is 0.16 mm, corresponding to a maximum resolvable absolute value of local curvature $|\kappa| = 125/d$ mm$^{-1}$, $|\kappa d| = 125$ is chosen as the cutoff value of $P(|\kappa d|)$. One sees that all these PDFs, for different $Ra$ and $C_d$, are similar to each other. But the distributions for large and small values of $|\kappa|$ exhibit distinctly different behaviors: for small-absolute-value curvatures, the PDFs are independent of $|\kappa|$, i.e. $d_k \simeq 0$, whereas for large-absolute-value curvatures, the PDFs decay with $d_k = -1.5$. Qualitatively, these may be understood as follows. After being injected into the

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*Figure 16. PDFs of the absolute value of the local curvature, $|\kappa|$, non-dimensionalized by the injection size $d$, of iso-concentration contours for (a) all four values of $Ra$ with $C_d = 0.1$ and (b) three detection thresholds at $Ra = 1.2 \times 10^{10}$. For clarity, each dataset has been shifted up from its lower neighbor by a factor of 5. The solid lines are reference $|\kappa|^{0}$ and $|\kappa|^{-1.5}$ power laws and the arrows indicate the Bolgiano scale $\ell_B$.**
cell, the dye packets are stretched and compressed by the turbulent flow. Along the stretching directions, the scalar concentration is approximately uniform, while along the compressive directions it varies rapidly. These cause the formation of a sheetlike topology of passive scalar packets [60]–[62]. Therefore, the points with large-absolute-value curvatures are located at the corners of the iso-contours, i.e. the ends of passive scalar sheets. In contrast, the points with small-absolute-value curvatures are located on the relatively smooth curves, which are largely aligned in the stretching directions. The PDFs shown in figure 16 imply that the distributions of the local curvatures, located at different positions of the passive scalar iso-contours, are determined by different mechanisms that are manifested by different scaling behaviors. Note that the transition occurs around $\kappa = 1/\ell_B$ (indicated by the arrows in figures 16(a) and (b)), suggesting that it may be buoyancy effects that cause the transition. It is interesting to point out that similar distributions of the local curvature were found for Lagrangian particle trajectories in recent numerical [63] and experimental [64] works, i.e. the scaling behavior with $d_\kappa = 1$ was found for small absolute values of $\kappa$, while $d_\kappa = -2.5$ was observed for large $|\kappa|$. In these works, the two power-law tails are related dynamically to the velocity and acceleration of Lagrangian particles, respectively, and can be reproduced from the uncorrelated Gaussian-distributed velocity and acceleration components [64].

4.4. The buoyant scale in the turbulent Rayleigh–Bénard (RB) system

In this subsection, we discuss the influence of buoyant forces on the geometric properties of the turbulent scalar mixing. Bolgiano [65] and Obukhov [66] have long proposed that there exists a characteristic length scale in buoyancy-driven turbulence, now commonly referred to as the Bolgiano scale $\ell_B$, above which buoyancy effects are significant (see, for example, [32]). In an earlier study, Ching et al [67] found that, above a certain scale, a significant correlation exists between the increment of the vertical velocity and that of the temperature, while the correlation is diminishingly small below that scale, suggesting that a buoyant scale indeed exists in turbulent thermal convection. Recently, we found that the cross-correlation coefficient between the increments of the temperature and the concentration exhibits similar behavior [25]. However, the global geometric measures of the passive scalar field presented in figures 9, 11 and 13 all show only a single power-law behavior and there is no transition around $\ell_B$. Therefore, no buoyant scale can be identified from the global measures of the geometric structures of the passive scalar field, such as perimeter and area. But this does not mean that the buoyant scale is absent in a turbulent RB system. In fact, the local geometric measure, i.e. the local curvature, may manifest the buoyancy effects. The arrows in figure 16 indicate the Bolgiano scale $\ell_B$ for reference. One sees that, although the distributions of $|\kappa|$ above and below $\ell_B$ both obey the log-Poisson statistics, they show scaling behaviors with different scaling exponents, i.e. the distribution of $|\kappa|$ is characterized by different values of $d_\kappa$ for $|\kappa| < \ell_B^{-1}$ and $|\kappa| > \ell_B^{-1}$. Taken together, these results suggest that a buoyant scale, i.e. the Bolgiano scale $\ell_B$, may be identified only from the local geometric measure of the passive scalar mixing in the turbulent RB system.

It should be mentioned that some previous studies have shown that the Taylor microscale $\lambda$ may play a significant role in the turbulent scalar mixing. For example, Miyauchi and Tanahashi [68] found that the coherent fine-scale structure of turbulence is directly related to $\lambda$. Wang and Peters [69] introduced a gradient trajectory method to identify finite-size spatial regions, called ‘dissipation elements’, which have a variable length that has the mean of the order of the Taylor scale $\lambda$. For our present study, we list several length scales in table 1,
where the Taylor microscale is estimated from \( \lambda = (15 \nu \sigma_u^2 / \bar{\epsilon})^{1/2} \) with \( \sigma_u \) being the root-mean-square value of the vertical velocity \([39]\) and \( \bar{\epsilon} \) the mean energy dissipation rate estimated from \( \bar{\epsilon} = \nu^3 / H^4 (Nu - 1) Ra Pr^{-2} \) \([27]\). The comparison among these scales shows that \( \lambda \) is approximately the same as the inverse mean absolute value of local curvature \( 1/|\kappa| \). This result suggests that the Taylor microscale \( \lambda \) may also contribute to the passive scalar mixing in the turbulent RB system. However, it is seen from table 1 that \( \ell_B \) is also of the order of \( 1/|\kappa| \), roughly a factor of 2 or 3 larger. Given that \( \ell_B \) and \( \lambda \) are both dimensional estimates, it is hard to say at the present stage which scale is more essential to the passive scalar mixing in buoyancy-driven turbulence. To answer this question, studies at a much higher Rayleigh numbers would be required.

5. Conclusions

We have experimentally examined in detail the mixing evolution and geometric properties of the passive scalar field in high-Schmidt-number buoyancy-driven turbulence. For mixing evolution, the scalar center is used to describe the evolution path of the passive scalar field and it is found that there is a self-similar region along the evolution path, within which the transverse profiles of both the mean concentration and concentration fluctuations can be characterized by Gaussian curves. In addition, along the evolution path and far from the injection source, the mean concentration is found to obey a power-law decay with a roll-off rate of \(-1\), which may be understood by the fact that a linelike injection geometry is used. The exponents of the decaying exponential tails of the single-point concentration PDFs are found to be proportional to the downstream positions with a slope proportional to the logarithm of \( Ra \). Our results also suggest that effective mixing exists only when the Rayleigh number \( Ra \) is above some threshold value \( Ra_0 \). To investigate its geometric properties, the iso-concentration contours of the 2D passive scalar field are extracted and studied. It is shown that the perimeter and shape complexity of the iso-contours, as well as the area they enclose, all obey log-Poisson statistics and the distributions of these quantities can reflect the increased mixing and stirring with increasing \( Ra \). By applying both the perimeter–area and the box-counting methods, the same fractal dimension, i.e. \( 1.29 \pm 0.02 \), for the scalar iso-contours is obtained. It is found that all these global geometric measures do not manifest the effect of buoyancy. The local curvature of the iso-contours \( \kappa \) is also studied. The distribution of \( \kappa \) is found to be strongly non-Gaussian and has two asymmetric extended tails. When the flow becomes more turbulent, the two tails become increasingly symmetric, indicating that the passive scalar mixing becomes more isotropic with increasing \( Ra \). It is further observed that the distribution of the absolute value of the local curvature \( |\kappa| \) consists of two log-Poisson tails and the inverse mean curvature \( 1/|\kappa| \) is found to be of the order of both \( \ell_B \) and the Taylor microscale. Our results would be useful in revealing the intrinsic features of the passive scalar mixing driven by buoyant forces.

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