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Low loss negative refraction metamaterial using a close arrangement of split-ring resonator arrays

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Abstract. In this work, we realize a low loss negative refraction using indefinite metamaterial of a close arrangement of split-ring resonator (SRR) arrays. A beam shift experiment has been performed to verify the negative refraction and demonstrate the low loss property. The enhanced magnetic resonance between two neighbouring SRRs broadens the resonance range and produces a frequency range with low loss. Transmission properties of the metamaterial are also analysed to characterize the negative refraction around the resonance frequency in detail.

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1. Introduction

Metamaterials with a negative refraction index give rise to many exotic properties [1, 2], such as negative refraction, and support the propagation of evanescent modes, which may lead to an important application: the hyperlens [3]. A well-established route for constructing NIM structures is based on Veselago’s theory [4] of left-handed materials (LHM), simultaneous negative permittivity ($\varepsilon$) and magnetic permeability ($\mu$) with different types of metamaterials. However, the negative permittivity and permeability produced by electromagnetic resonance may bring about a very high loss [5, 6]. Recently, an alternative approach to NIMs using an indefinite material brought a new perspective to the design of metamaterials that does not involve the use of electromagnetic resonance [7, 8]. Furthermore, researchers have also shown that, in the long wavelength limit, the hyperbolic dispersion relation of a three-dimensional (3D) indefinite medium can be valid even for evanescent modes, allowing for application as a superlens [9–13].

So far, almost all indefinite materials produce a negative group refraction index by using strong anisotropy of permittivity, whose dielectric tensor elements (considered along the principal axis) are not all of the same sign [13, 14]. Arrays of metallic poles and multilayered structures have been used to realize anisotropic permittivity, such as silver nanowires in AAO [15] and multilayered semiconductor films [16]. For microwave applications, however, these designs are still of high loss, because of the large imaginary part of permittivity in the Drude model. According to the symmetry of Maxwell’s equations, anisotropic permeability can also produce a negative refraction by forming a hyperbolic equifrequency contour (EFC) [17]. The split-ring resonator (SRR) is the most common design to achieve a negative permeability by magnetic resonance when the H field penetrates the SRR loop [5, 6, 18]. Usually, the magnetic resonance brings a high loss, which makes the refraction wave greatly attenuated [17]. Yet the two neighbouring SRRs are close enough to each other; the strong coupled effect will broaden the resonance range, which may supply a part of the frequency where the loss is rather low. Here, we report on a low loss negative refraction metamaterial with anisotropic permeability using a close arrangement of single SRR arrays. We performed the beam shifting experiment described in [19] to demonstrate the ability for negative refraction and low loss. Moreover, transmission properties were also analysed at different frequency ranges in the resonance band.

2. Theory

Consider the single-SRR arrays shown in figure 1; the permeability is anisotropic for a TM (the $H$ field polarized in the $x$–$z$-plane) electromagnetic wave and can be described by the tensor below,

$$\mu = \mu_0 \begin{pmatrix} \mu_{xx} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$  \hspace{1cm} (1)

When the size of the SRR unit is much smaller than the wavelength of the incident wave, the SRRs can be treated as a homogenous medium according to the effective medium
Figure 1. The composition and typical size of the anisotropic bulk metamaterial and its scheme of negative refraction. (a) The bulk is made of periodic SRR arrays on a substrate of Rogers 5880. Teflon slab and SRRs arrays are arranged along the \( x \)-axis alternately. (b) The hyperbolic EFC of the metamaterial. \( S_i \) and \( k_i \) are the Poynting vector and the wave vector of the incident wave, respectively. The components of \( k_i \) and \( k_r \) along the interface are equal to each other: \( k_i = k_r = K \). The refracted wave vector \( k_r \) and Poynting vector \( S_r \) can be determined by satisfying Maxwell’s theorem.

theory. Consequently, the \( x \) component \( \mu_{xx} \) can be well approximated by the modified Lorentzian-oscillator function [5, 20, 21],

\[
\mu_{xx} = 1 - \frac{A_m \omega^2}{\omega^2 - \omega_m^2 + i\omega \gamma_m},
\]

where \( A_m \) is the oscillator amplitude, \( \omega_m \) is the resonance centre frequency and \( \gamma_m \) is the damping constant. For a TM wave, the wave vector \( k \) is incident on the metamaterial sample at an angle of \( \theta_i \). Working from Maxwell’s equations, we can summarize the dispersion relationship as

\[
\frac{k_x^2}{\mu_{zz}} + \frac{k_z^2}{\mu_{xx}} = \frac{\omega^2}{c^2 \varepsilon}.
\]

According to equation (2), we can obtain a negative \( \mu_{xx} \) but a positive \( \mu_{zz} \) in the resonance frequency region. And this anisotropic permeability, which results in a hyperbolic EFC, enables the TM wave to refract negatively in the SRR array, as shown in figure 1(b).

3. Experiment

The metamaterial sample is composed of periodic arrays of Teflon slab and single SRRs, etched by lithography techniques on a 0.254-mm-thick Rogers 5880 substrate. Figure 1(a) shows the composition of the sample and the size of the SRR. There were 37 units (50 mm) in the
Figure 2. (a) Photograph of the SRR arrays and (b) schematic diagram of the beam shift experiment. The bulk metamaterial is placed in a plane waveguide with a microwave absorber along the two sides. An electromagnetic wave propagates along the $x$-axis and impinges on the metamaterial sample with an angle of $30^\circ$. A microwave detector moves along the $z$-axis on the output port to determine the centre of the outgoing beam. The SRRs in the bulk are parallel to the $y$-axis.

$x$-direction of the sample and 50 units (250 mm) in the $z$-direction. Each unit with a height of 10 mm and a width of 5 mm contained two SRRs. Figure 2(a) is the actual picture of the SRR arrays sample. Figure 2(b) depicts the layout of the beam shifting experiment. A rectangular bulk was placed in a parallel plate waveguide (PPW) with absorbers along both sides. The SRR loop in the bulk was parallel to the $y$-axis. An electromagnetic wave beam was fed into the PPW and was incident on the bulk with an angle of $30^\circ$. A microwave detector was moved along the $z$-axis on the output side of the PPW to detect the location of the outgoing beam and its power after transmitting through the bulk at certain frequencies where only the first mode (equivalent to a TEM wave) can propagate. The origin point of the detector is defined as the location of a microwave transmitted through a bulk medium with an infinite refraction index. Thus, a negative displacement ($D$) means the outgoing wave experienced a negative refraction, while a positive $D$ means a positive refraction. The measurements were carried out by a network analyser, HP 8720ES. A Teflon sample with a positive refraction index ($\varepsilon = 2.1 + 0.001i$) was also measured as a contrast to the results of the metamaterial.

4. Results and discussion

Figure 3 shows the experimental results obtained at the X band for a Teflon bulk and a bulk SRR array metamaterial. The horizontal axis represents the location of the detector in mm and the vertical axis represents the frequency in GHz. The power detected at the output side is depicted by the contour map in dBm by using different colours. As shown in figure 3(a), the result for Teflon shows a positive refraction with a positive beam shift in the whole frequency range of 8–12 GHz. In contrast, in figure 3(b), there is a bandstop around 10 GHz. The frequency band above the bandstop or below 9 GHz is a bandpass with a positive refraction. The frequency band below the bandstop but above 9 GHz is a bandpass (about 500 MHz) with a negative refraction.
The peak value of the power detected from the wave transmitted through the metamaterial sample is about $-15$ dBm, almost the same as that detected on the Teflon sample. This means that the loss of the SRR array is rather small. This low loss negative refraction band can be attributed to the strong coupled interaction between two neighbouring SRRs. This behaviour can be understood from the $H$ field distribution obtained by HFSS 11 (commercial software based on the finite-element method), illustrated in figure 4. In our design, the distance between two SRRs is $t_1 + t_2$ (about 1.35 mm). Such a short distance enhances the conductance between the two SRRs, which may decrease the resonance centre frequency $\omega_{m0}$ with respect to a structure with large distance. On the other hand, the $H$ field penetrates through both SRRs at the magnetic resonance (figure 4(a)), which intensifies the coupling effect between SRRs and raises the magnetic plasma frequency $\omega_{mp}$. Consequently, the resonance bandwidth $\omega_{mp} - \omega_{m0}$ is broadened.

**4.1. Low loss negative refraction**

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**Figure 3.** Measured transmitted power as a function of frequency and of the location of the outgoing beam for (a) the Teflon bulk and (b) the metamaterial.

**Figure 4.** $H$ field distribution for SRRs with the typical size shown in figure 1 with two different distances between them. (a) The distance is just 1.35 mm ($t_1 + t_2$) and the coupling effects enhance the magnetic resonance. (b) The distance is so large ($c = 5$ mm) that there is almost no coupling between SRRs.
Figure 5. (a) Permeability of a single SRR with a $5 \times 5 \times 1.35$ mm lattice, computed by the $S$-parameter retrieval method. The distance between two neighbouring SRRs is 1.35 mm, which broadens the resonance region greatly. Loss in range A is very low, which can be applied to realize negative refraction effectively. The absolute value of permeability in range B is less than one, which may result in a forbidden band because of the total reflection. (b) The scheme of the EFC of the metamaterial at the B band in panel (a). (c) The EFC with $\omega$ around $\omega_{mp}$; the refraction angle is almost 90°. The refractive Poynting vectors $S_r$ in (b) and (c) have to be perpendicular to the EFC. With the frequency approach to $\omega_{mp}$, the hyperbolic EFC becomes slender and the refraction angle increases until total reflection occurs.

The $S$-parameters retrieval method has been used to compute $\mu_{xx}$ around the resonance band in such a case [22], as shown in figure 5. The bandwidth of $\mu_{xx} < 0$ is broadened to almost 1.5 GHz, where there is a region (the A band in figure 5) in which the imaginary part of the permeability is rather small. That is why the metamaterial is of low loss. For a usual design, the lattice is cubic with a constant value of $c = 5$ mm, and each SRR owns a magnetic resonance, respectively (figure 4(b)). Thus, the resonance region is narrow with a very large imaginary part, which may result in a high loss.

4.2. The transmission property of the metamaterial

As shown in figure 3(b), there is a negative refraction bandpass, a positive refraction bandpass and a bandstop. At the frequency range far from the magnetic resonance, the permeability is normal and nearly isotropic, and then we can obtain a positive refraction at these bandpasses. At the resonance region, the permeability is a tensor, as described by equation (1), which may result in a negative refraction due to the anisotropy. However, there is a bandstop around the magnetic plasma frequency $\omega_{mp}$. That is because the absolute value of $\mu_{xx}$ around $\omega_{mp}$ is less than one, according to equation (2). As a result, the wave is transmitted from an optically denser medium to an optically thinner medium. With the frequency approach to $\omega_{mp}$, the hyperbolic EFC becomes more and more slender (figures 5(b) and (c)). As a result, the output location
Figure 6. (a) Magnification of the area around the peak value in figure 3. The arrow indicates the moving direction of the peak values with the frequency approaching $\omega_{mp}$. (b) The negative displacement with $\omega$ approaching $\omega_p$.

would move in the negative direction (indicated by the arrow in figure 6(a)) further. In the meantime, the critical angle of the bulk decreases. Total reflection does not occur until the critical angle decreases to 30°. Therefore, there is a bandstop formed by total reflection around $\omega_{mp}$, as shown in figure 3. Figure 6(b) shows the relative displacement for negative refraction with $\omega$ approaching $\omega_p$, corresponding to figure 6(a).

5. Conclusion

In conclusion, we have fabricated a negative refraction metamaterial with a wide NR band and low loss. The beam shifting experiment has been used to verify the negative refraction and obtain the operating bandwidth of 500 MHz. The peak power after the negative refraction can reach $-15$ dBm, similar to the results for Teflon. A short distance between the SRRs enhances the coupling effect of the magnetic resonance, which can broaden the resonance band with low loss. The transmission properties were also analysed and it was shown that there is a bandstop around the magnetic plasma frequency induced by the total reflection. This work proposes another way to realize negative refraction with low loss at microwave frequencies, and it supplies useful information for the fabrication of a superlens by using an indefinite metamaterial in the THz range or an even higher frequency range.

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