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Warped five-dimensional models: phenomenological status and experimental prospects

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Abstract. Warped five-dimensional models, based on the original Randall–Sundrum geometry, have been extended beyond their initial purpose of resolving the gauge hierarchy problem. Over the past decade, various ingredients have been added to their basic structure in order to provide natural and predictive models of flavor and also to address existing constraints from precision data. In this paper, we examine the theoretical and experimental status of realistic models that accommodate current data, while addressing the hierarchy and flavor puzzles of the Standard Model. We also discuss the prospects for future discovery of the TeV-scale Kaluza–Klein states that are predicted to emerge in these models, and outline some of the challenges that the detection of such particles pose for experiments at the Large Hadron Collider.

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1. Introduction

The Standard Model (SM) of particle physics is remarkably successful in explaining a wide range of microscopic phenomena and has passed numerous tests over the past few decades. The only ingredient of this model that has yet to be discovered is the Higgs boson. It is the vacuum expectation value (vev) of this field that breaks the electroweak (EW) symmetry $SU(2)_L \times U(1)_Y$ down to $U(1)_{EM}$ of quantum electrodynamics and provides masses for elementary particles in the SM. However, this picture, though economical, leaves some intriguing questions unanswered. One obvious and central question is related to the patterns of the SM fermion masses (that, including the neutrinos, span 12 orders of magnitude) and mixing; this is the flavor puzzle, and is based on firm experimental evidence.

Another question, which is more theoretical in nature, arises when one considers the effect of quantum corrections on the Higgs vev. These corrections are quadratically sensitive to any physical mass scales that could emerge at energies above the weak scale $M_W \sim 100$ GeV. However, stringent requirements from precision data strongly suggest that new physics can only
appear at scales much larger than $M_W$. Besides, very high physical scales are well motivated in the context of a Grand Unified Theory (GUT) or theories of quantum gravity. Therefore, one is faced with the question of what stabilizes the Higgs potential well below such large scales. The severest version of this problem arises when the fundamental cut-off scale is assumed to be near the four-dimensional (4D) Planck mass $M_P \sim 10^{19}$ GeV; this is the hierarchy problem.

In this paper, we focus on a framework based on a model originally proposed by Randall and Sundrum [1] to address the hierarchy problem. The Randall–Sundrum (RS) model is based on a slice of AdS$_5$ spacetime between two flat 4D boundaries, often referred to as the ultraviolet (UV) (Planck) and infrared (IR) (TeV) branes. The branes are assumed to be separated by a distance of the order of the curvature radius of the background, which is in turn of the order of the 5D fundamental scale $M_5$. This model provides a natural resolution of the hierarchy by exponentially generating the weak scale from scales of the order of $M_5 \sim M_P$. To achieve this, the Higgs field is localized at the IR-brane and the warping along the fifth dimension redshifts the Planckian 5D Higgs vev down to the weak scale. The requisite brane separation was shown to be easily obtained in simple models [2], resulting in a stable geometry.

In the original RS model, all SM fields were localized on the IR-brane and the most distinct signature of this setup was the emergence of a spin-2 tower of Kaluza–Klein (KK) gravitons [3] at the TeV scale. However, it was soon realized that resolving the hierarchy only required the Higgs to be IR-localized [4] and other fields could propagate in the 5D bulk. The SM gauge sector [5, 6] and the fermions [7] were then promoted to 5D fields. It was shown that localization of fermion zero modes could be achieved using 5D mass terms [7], with the heavy fermions localized toward the Higgs and the light fields localized away from the Higgs, resulting in a natural and predictive model of flavor. In particular, given that 5D location sets the relevant physical mass scale in the RS background, light-flavor four-fermion operators are governed by large cut-off scales in warped flavor models [8], as required by precision data. This interesting feature allows one to address both the hierarchy and flavor puzzles within RS-type models, making them a very attractive model-building framework that can be experimentally tested at the weak scale.

While warped 5D models explain some of the important unresolved issues of the SM to a large extent, they are still subject to a degree of (fine-)tuning when confronted with precision EW [9] and flavor data [10]–[14]. The substance of this tension is traced back to the resolution of the hierarchy problem that would prefer the scale of new physics to be near $M_W$. This gives rise to an effective 4D cut-off scale in the TeV range, where all non-renormalizable interactions that are not forbidden by a symmetry will emerge, whereas precision data generally require the cut-off scale to be at $O(10)$ TeV or more. To eliminate such ‘little hierarchies’, the basic 5D SM structure of warped models have been augmented by various new ingredients, such as larger gauge symmetry groups [15, 16], in order to keep the scale of KK modes in the few-TeV range. This would remove the need for an inordinate amount of (fine-)tuning and also make the models more likely to be testable in various high-energy experiments.

Even though the most recent warped models of hierarchy and flavor can accommodate KK masses as low as 2–3 TeV [9], the discovery of the signature states quite generically poses a significant challenge. The basic reason for this originates in the way various states are localized in the bulk; light fermions are UV localized, whereas the KK modes and heavy fermions are IR localized. This suppresses the couplings of the KK modes to light flavors that are important initial (such as light quarks) and final (such as $\mu^\pm$) states in collider production and detection, respectively. A volume suppression also affects the couplings of the highly localized KK modes.

to gauge fields that are spread over the bulk. Hence, in models that explain the flavor problem we end with suppressed production of KK modes that eventually decay into heavy fields, such as $t\bar{t}$, requiring complicated event reconstructions. Also, the heavy final states are highly boosted due to the high mass of the KK parent. This makes the eventual decay of the heavy final states into di-jets difficult to distinguish from mono-jets, hence rendering quantum chromodynamics (QCD) background suppression challenging.

In the first half of this paper, we discuss the theoretical techniques that have been used to analyze the phenomenology of RS-type models. Although our results are of more general applicability, we focus on the most recent prototypical models that incorporate a realistic flavor structure (bulk SM gauge and fermion fields) and possibly allow for new physics within the reach of the Large Hadron Collider (LHC). We restrict ourselves to the original RS background and assume that there is a Higgs degree of freedom in the low-energy theory. We survey the phenomenological status of these realistic scenarios, thus providing a guide to the plausible values of parameters in viable models. Having determined the phenomenologically relevant range of parameters, we discuss the key collider signatures of warped 5D models in the second half of the paper. Our main focus is the models endowed with bulk symmetries that allow for the KK masses to be within the reach of the LHC, without the need for unnatural choices of parameters. Some of the experimental obstacles that these models pose, as well as proposals to overcome such problems, are outlined. We also briefly discuss the expected enhancement of clean signals in ‘truncated’ warped models whose UV cut-off is well below $M_p$, but still explain flavor. We conclude with a summary.

Before closing this introduction, we would like to emphasize that this paper is not meant to be an exhaustive or comprehensive review. Such an undertaking would require a much longer article due to the large amount of research performed on warped models in the past few years. In particular, important cosmological and gravitational effects that have been discussed in a number of interesting works lie outside the scope of our paper, but certainly deserve attention in separate or more comprehensive surveys of warped physics. As such, we have limited our survey to some of the most generic features of model building and phenomenology. To the extent possible, we have cited works that are directly or closely related to our discussions. However, unfortunately, many worthy papers have been left out. We hope that this paper presents a sufficiently inclusive account of the status of warped models to motivate the interested reader to pursue further references and delve more deeply into some of the subjects that we have (or have not) considered here.

**Part I. General tools and electroweak (EW) constraints**

In the first part of this paper, we describe the tools that have been used to study the physics of warped extra-dimensional scenarios. We put special emphasis on the explanation and comparison of the different techniques used. We focus on models that incorporate a custodial symmetry, and analyze the constraints from EW precision measurements. There are also important constraints from flavor data that, under the assumption of strict ‘anarchy’ (that the 5D flavor couplings present no structure at all and are all of the same order, including complex phases), can result in rather severe bounds on the new physics. These constraints depend on parameters different from those relevant for the EW precision constraints and can be evaded without significantly modifying the latter. Due to space constraints, we focus on the EW analysis, except for a brief section on flavor where we collect some of the recent references.
2. Methods in models with warped extra dimensions

We consider a 5D space-time with a warped metric, written in conformally flat coordinates as\footnote{Another commonly used form for the AdS metric is \( ds^2 = e^{-2ky} \eta_{\mu \nu} dx^\mu dx^\nu - dy^2 \), where \( k = 1/L_0 \) is the AdS curvature. The formul\ae\ in conformal coordinates can be transcribed into the ‘proper distance’ coordinates by setting \( z/L_0 = e^{ky}, \partial_z = a(z)^{-1} \partial_y \), and \( dy = a(z) dz \).}

\[
d s^2 = a(z)^2 (\eta_{\mu \nu} dx^\mu dx^\nu - dz^2), \quad a(z) = \frac{L_0}{z},
\]

where \( L_0 \leq z \leq L_1 \). Solving the hierarchy problem requires \( L_0 \approx M_p^{-1} \), and \( L_1 \approx (\text{TeV})^{-1} \), although other choices of scale have proved useful in addressing, e.g., the flavor problem \cite{17}.\footnote{Other gauge choices can also be useful \cite{18}. A gauge independent expansion can be obtained by choosing the KK modes of \( A_5 \) equal to \( \hat{\partial}_\mu f_\mu/\mu_\alpha \).}

2.1. Kaluza–Klein (KK) expansions

We start by collecting the main results for the KK wavefunctions associated with spin-2 \cite{3}, spin-1 \cite{5, 6} and spin-\( \frac{1}{2} \) \cite{7} fields propagating on the background of equation (1) (for a unified derivation, see \cite{8}). The KK wavefunctions are especially useful in studying the collider phenomenology of warped extra dimensions, to be discussed in the second part of this paper.

2.1.1. Gauge bosons. The 5D gauge action is

\[
S_{\text{gauge}} = - \frac{1}{4 g_5^2} \int d^5 x \sqrt{g} F^a_{\mu \nu} F^{a, \mu \nu} + S_{\text{gauge}}^{\text{UV}} + S_{\text{gauge}}^{\text{IR}},
\]

where \( M, N = 0, 1, 2, 3, z \) run over the 5D coordinates, \( g_5 \) is the 5D gauge coupling (with mass dimension \(-1/2\)), \( F^a_{\mu \nu} \) is the gauge field strength, \( a \) is a gauge index, and \( S_{\text{gauge}}^{\text{UV}}, S_{\text{gauge}}^{\text{IR}} \) contain possible brane-localized terms for the gauge fields, to be specified shortly. If the gauge field does not satisfy Dirichlet boundary conditions at both branes (see below) we can go to the unitary gauge, \( A_5 = 0 \), in which the KK decomposition reads

\[
A_\mu(x, z) = \sum_n A^n_\mu(x) f_n(z).
\]

The KK wavefunctions satisfy \( O(m_n, z) f_n(z) = 0 \), where \( m_n \) are the gauge KK masses and the differential operator in the coordinates of equation (1) is

\[
O(p, z) = \partial_\mu a(z) \partial_\mu + a(z) p^2.
\]

The solutions are written in terms of Bessel functions of the first and second kind, \( J_\alpha(x) \) and \( Y_\alpha(x) \), for which we shall use the shorthand notation

\[
J_\alpha^{0,1,z} = J_\alpha(m_n L_0), J_\alpha(m_n L_1), J_\alpha(m_n z),
\]

and similarly for \( Y_\alpha^{0,1,z} \). Then

\[
f_n(z) = A_n a(z)^{-1} \left[ J_\alpha^z + B_n Y_\alpha^z \right],
\]

where the overall constants \( A_n \) are determined by the orthonormality condition

\[
\int_{L_0}^{L_1} \! dz a(z) f_n(z) f_m(z) = L \delta_{nm},
\]
and we defined the volume factor
\[ L \equiv \int_{L_0}^{L_1} dz \ a(z) = L_0 \log \frac{L_1}{L_0}. \] (8)

The boundary conditions (b.c.s) take the form
\[ \left[ L_0 \partial_z - a(z) b_{UV} \right] f_n(z) \bigg|_{z=L_0} = 0, \] (9)
\[ \left[ L_0 \partial_z + a(z) b_{IR} \right] f_n(z) \bigg|_{z=L_1} = 0, \] (10)

where the \( b_i \) depend on the brane localized terms. For instance, in the presence of localized kinetic terms with coefficients \( r_i \) and localized mass terms \( m_i \),
\[ b_{UV,IR} \equiv a_0^{-2} \hat{r}_{UV,IR} \hat{m}_n^2 - \hat{m}_{UV,IR}, \] (11)

where \( a_0 \equiv a(L_0), a_1 \equiv a(L_1) \), and we defined dimensionless quantities \( \hat{m}_n = m_n L_0, \hat{r}_i = r_i / L_0 \) and \( \hat{m}_i = m_i L_0 \) for \( i = UV, IR \) (the natural scale of the dimensionful microscopic parameters is expected to be of order \( L_0 \)). The b.c. on the IR brane determines \( B_n = -J_n^{IR} / \hat{Y}_n^{IR} \), where
\[ \hat{J}_n^{UV,IR} \equiv \hat{m}_n J_n^{0,1} \pm a_0,1 b_{UV,IR} J_n^{0,1}, \] (12)

and an analogous definition for \( \hat{Y}_n^{UV,IR} \). The b.c. on the UV brane then leads to the eigenvalue equation
\[ \hat{J}_n^{UV} \hat{Y}_n^{IR} - \hat{Y}_n^{UV} \hat{J}_n^{IR} = 0, \] (13)

which determines the KK masses, \( m_n \).

The case with \( \hat{m}_{UV} = \hat{m}_{IR} = 0 \) leads to a zero-mass eigenvalue, \( m_0 = 0 \), corresponding to an unbroken 4D gauge symmetry. In the SM EW sector, an IR localized Higgs field leads to \( \hat{m}_{IR} \neq 0 \) and nonzero \( Z \) and \( W^\pm \) masses. The Higgsless limit [19] corresponds to \( \hat{m}_{IR} \rightarrow \infty \).

A case of interest in the models with custodial symmetry to be introduced in section 3.1 corresponds to Dirichlet boundary conditions on the UV brane, and can be obtained by taking \( \hat{m}_{UV} \rightarrow \infty \). In the following, for simplicity we will assume that the localized kinetic terms are sufficiently small to be neglected. However, it should be remarked that when \( \hat{r}_{IR} > 1 \) these can have important phenomenological consequences [20, 21]. If both b.c.s are Dirichlet, \( \hat{m}_{UV,IR} \rightarrow \infty \), a zero mode for \( A_5 \) remains in the spectrum [22]–[25]. This can have interesting consequences for Electroweak symmetry breaking (EWSB) [26, 27].

### 2.1.2. Fermions

The fermion action is
\[ S_{\text{fermion}} = \int d^5 x \sqrt{g} \left\{ \frac{1}{2} \bar{\Psi} e_A^M \Gamma^A D_M \Psi - \frac{i}{2} (D_M \Psi)^\dag \Gamma^0 e_A^M \Gamma^A \Psi - M \bar{\Psi} \Psi \right\} + S_{\text{fermion}}^{UV} + S_{\text{fermion}}^{IR}, \] (14)

where \( \Gamma^A = (\gamma^\mu, -i \gamma^5) \) are the flat space Dirac gamma matrices in 5D space, \( D_M \) is the covariant derivative with respect to the gauge symmetry as well as the general coordinate and local Lorentz transformations\(^9\), and \( S_{\text{fermion}}^{UV}, S_{\text{fermion}}^{IR} \) contain possible fermion-localized terms. It is convenient to express the 5D mass term in units of \( 1 / L_0 \), thus defining a dimensionless

\(^9\) For a diagonal metric of the form \( ds^2 = a(z)^2 \eta_{\mu\nu} dx^\mu dx^\nu + b(z)^2 dz^2 \) the spin connection in \( D_M \) cancels out in the fermion action, equation (14).

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Localized curvature (‘kinetic’) terms are simply obtained from equation (11) by the replacement $\psi_{L,R} \rightarrow (2 - c)\psi_{L,R}$. Equations (9) and (10), with $b_{UV,IR} \rightarrow \pm(2 - c) + b_{UV,IR}$, and $b_{IL}$ as in equation (11), but with $\hat{m}_i = 0$ and $r_i$ denoting the coefficients of possible fermion brane kinetic terms. Neglecting brane kinetic terms, the corresponding LH zero mode wavefunction is

$$f^0_L(z) = \sqrt{\frac{(1 - 2c)}{L_0 [(L_1/L_0)^{2 - 2c} - 1]}} \left( \frac{z}{L_0} \right)^{2 - c},$$

which corresponds to a chiral fermion exponentially localized toward the UV (IR) brane for $c > 1/2$ ($c < 1/2$), while the physical profile is flat for $c = 1/2$. The remaining constants for the massive LH modes in equation (17) are given by $B^n_i = -j_{c+1/2}^{IR}/\tilde{y}_{c+1/2}^{IR}$, where $\tilde{y}_{c+1/2}^{IR}$ are defined in equation (12), taking $b_i = b_i^L$. The profiles for the associated RH chiralities, $f^n_R(z)$, are simply obtained from equation (17) by the replacement $c + 1/2 \rightarrow c - 1/2$ (but with exactly the same constants $A^n_i$ and $B^n_i$ that appear in $f^n_L(z)$). The fermion KK masses are determined by equation (13), with $\alpha = c + 1/2$.

If instead the zero mode is right-handed, the corresponding profiles are obtained with the replacement $c \rightarrow -c$ everywhere in equations (17) and (18) (and exchanging the LH fields with the RH ones).

2.1.3. Gravitons. KK excitations of the graviton (spin-2 resonances) may give a striking signature of the extra-dimensional structure. We summarize briefly the relevant results for the graviton KK wavefunctions. The tensor fluctuations $h_{\mu\nu}$ are introduced by making $\eta_{\mu\nu} \rightarrow \eta_{\mu\nu} + h_{\mu\nu}(x,z)$ in equation (1). Starting from the Einstein–Hilbert action, linearizing in $h_{\mu\nu}$, and choosing the transverse-traceless gauge, $\partial^\mu h_{\mu\nu} = h^\mu_\mu = 0$, the KK expansion reads

$$h_{\mu\nu}(x,z) = \sum_n h^n_{\mu\nu}(x) f^n_\mu G(z),$$

where $(a(z)^{-3}\partial_z a(z)\partial_z + m^2_0)f^n_\mu G(z) = 0$, so that (see the definitions of equation (5))

$$f^n_\mu G(z) = A^n_\mu a(z)^{-2} \left[ J^\mu_z + B^n_\mu Y^0_z \right].$$

The normalization constants $A^n_\mu$ are obtained from equation (7) with $f_n(z) \rightarrow a(z)f^n_\mu G(z)$.

Localized curvature (‘kinetic’) terms [28] can be introduced through the functions of

that satisfy the homogeneous differential equation

\[
\left( D^2 + m^2 \right) \phi (z) = 0,
\]

where we use the notation for the Bessel functions of equations (2) and (12), with \( \alpha = 2 \).

2.2. Five-dimensional (5D) propagators: the gauge field case

Let us consider the Green’s function of the quadratic operator for a gauge boson. We add to the action (2) a gauge fixing term [18]

\[
L_{\text{gf}} = -\frac{1}{2\xi g_5^2} \left[ \partial_\mu A^\mu - \xi a(z) \right]^2,
\]

where \( \xi \) is the gauge fixing parameter. Working in mixed position/momentum space [29], the propagator takes the form

\[
-iP_p(z, z')P_{\mu\nu} - iP_{p/\sqrt{\zeta}}(z, z') \frac{P_{\mu} P_{\nu}}{p^2},
\]

where \( P_{\mu\nu} = \eta_{\mu\nu} - p\mu p\nu / p^2 \) is the transverse projector, and \( P_p(z, z') \) satisfies

\[
O(p, z) P_p(z, z') = \delta(z - z'),
\]

with \( O(p, z) \) given by equation (4). \( P_p(z, z') \) satisfies the same boundary conditions as the gauge field wavefunctions \( f_\mu(z) \), equations (9) and (10); for the inclusion of brane localized terms, see [21]. On the IR brane, we take \( m_{\text{IR}} = m \) in equation (11). For the UV b.c. we choose a simple Neumann or Dirichlet (i.e. \( b_{UV} = 0 \) or \( b_{UV} \to \infty \)), which imply, respectively, an unbroken or spontaneously broken gauge symmetry at the UV brane,

\[
\partial_z P^D_p(z, z') \big|_{z = L_0} = 0, \quad P^D_p(z, z') \big|_{z = L_0} = 0.
\]

The two solutions can be written in terms of two functions, denoted \( K_m(p, z) \) and \( S(p, z) \) [30], that satisfy the homogeneous differential equation

\[
O(p, z) K_m(p, z) = O(p, z) S(p, z) = 0,
\]

and boundary conditions

\[
K_m(p, L_0) = 1, \quad L_0 K'_m(p, L_1) + a_1 b_{IR} K_m(p, L_1) = 0,
\]

\[
S(p, L_0) = 0, \quad S'(p, L_0) = 1,
\]

where a prime denotes derivative with respect to \( z \). The Green’s functions read

\[
P^N_p(z, z') = \frac{K_m(p, z)}{K'_m(p, L_0)} - S(p, z) K_m(p, z),
\]

\[
P^D_p(z, z') = -S(p, z) K_m(p, z),
\]

where \( z_- \) and \( z_+ \) are the minimum and maximum of \( z \) and \( z' \), respectively. The explicit solutions for \( K_m \) and \( S \) are given in equations (A.1) and (A.4) of the appendix. The full propagators read

\[
P_p(z, z') = \frac{\pi}{2} \frac{z_- z_+}{L_0} \frac{\tilde{Y}_1^\text{UV} J_1^z - \tilde{Y}_1^\text{IR} Y_1^z \tilde{J}_1^z - \tilde{J}_1^\text{IR} Y_1^z}{\tilde{J}_1^\text{UV} \tilde{Y}_1^\text{IR} - \tilde{Y}_1^\text{UV} \tilde{J}_1^\text{IR}},
\]

where we use the notation for the Bessel functions of equations (5) and (12), with \( m_n \to p \), and \( b_{UV} = 0 \) for \( P^N_p(z, z') \), while \( b_{UV} \to \infty \) for \( P^D_p(z, z') \).
We have split the Neumann propagator in equation (28) into two pieces, one that vanishes at the UV brane and coincides exactly with the Dirichlet propagator, equation (29), and another that is the product of the holographic functions times the boundary field two point function, to be introduced in section 3.2. Alternatively, we can separate it into the contribution from the massless zero mode (before EWSB) and that of the massive modes

\[ P_p(z, z') = P_p^{(0)}(z, z') + \tilde{P}_p(z, z'), \]

where the Green’s function for the zero mode reads

\[ P_p^{(0)} = \frac{1}{p^2 L_0 \log(L_1/L_0)}, \]

and, for simplicity, we set \( r_{UV} = r_{IR} = 0 \) (see [20, 21, 31] for a thorough discussion of the phenomenological consequences of localized gauge kinetic terms). Propagator methods are also useful for resumming the effects of the fermion KK tower (see, e.g., [32]).

### 2.3. The holographic method

Let us consider a bulk gauge boson \( A_M \) with the following action:

\[ S_{\text{gauge}} = \int \frac{d^4 p}{(2\pi)^4} d\zeta \sqrt{g} \left\{ -\frac{1}{4 g_{5A}^2} (A_{MN})^2 + \frac{1}{2} v^2(z)(A_M)^2 \right\}, \]

where we allow for a bulk Higgs vev profile, \( v(z) \), with mass dimension 3/2.\(^1\) We go to a gauge with \( A_5 = 0 \) and assume that the gauge boson obeys the Neumann b.c.s on the UV brane. We write the 5D field as follows,

\[ A_\mu(p, z) = f_A(p, z) \tilde{A}_\mu(p), \]

with \( f_A(p, L_0) = 1 \) so that \( \tilde{A}_\mu(p) \) is the boundary value of the gauge field. The IR boundary condition is the same as for the 5D field. We assume the boundary field satisfies the equation of motion of a 4D gauge field, which implies the following equation for the holographic profile:

\[ O_\mu f_A(p, z) = 0, \]

where \( O_\mu = O(p, z) - a(z) \tilde{g}_{5A}^2 v^2(z) \), with \( O(p, z) \), as defined in equation (4). The effective action for the boundary field can be obtained at the tree level and to quadratic order in the fields by inserting the equations of motion back in the action and integrating over the extra dimension. The bulk action vanishes due to the equations of motion and the only remaining term is a boundary piece that reads

\[ S_{\text{bound.}} = -\frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} \tilde{A}_\mu \tilde{\Pi}_A \tilde{A}^\mu, \]

with the vacuum polarization function for the boundary field given by

\[ \tilde{\Pi}_A(p^2) = \frac{1}{2} \partial_\zeta f_A(p^2, z) \bigg|_{z=L_0}. \]

\(^{10}\) For EWSB localized on the IR brane, \( v^2(z) \equiv \frac{1}{2} \frac{1}{a(L_1)} \delta(z - L_1) \), which gives \( m = \frac{1}{2} g_{5A}^2 [v/a(L_1)]^2 \) in equations (25)–(27). We introduce the factors of 1/2 and 1/a(L_1) to match to the SU(2) \( \times U(1)_Y \) theory with \( v \sim 174 \text{ GeV} \).
Fermions can be treated in a similar way. The action for a bulk fermion is as in equation (14), but with an additional UV localized term [33]

$$\delta S_{\text{UV}} = \int_{\text{UV}} d^4x \sqrt{g_{\text{ind}}} \left( \pm \frac{1}{2} \right) \overline{\Psi} \Psi,$$

(38)

where $g_{\text{ind}}$ is the induced metric, and the factor of $+1/2 (-1/2)$ is determined by the requirement that the LH (RH) chirality be unconstrained on the UV brane. We assume the LH component satisfies the Neumann b.c.s on the UV brane and adopt an LH source description

$$\Psi_{L,R}(p, z) = f_{L,R}(p, z) \psi_{L,R}(p),$$

(39)

where $p \psi_{L,R}(p) = p \psi_{R,L}(p)$ and $f_{L,R}$ satisfy the equations

$$O_{-c} \bar{O}_{c} f_{L} = -p^2 f_{L}, \quad O_{c} f_{L} = pf_{R},$$

(40)

where $O_{c}$ is given in equation (16). The boundary conditions on the IR brane are as for the 5D field, while on the UV brane we impose

$$f_{L}(p, L_0) = 1.$$  

(41)

The boundary action for $\psi_L$ is computed by replacing the classical equations of motion and integrating over the extra dimension. Again, at the quadratic level there is no bulk contribution and the boundary contribution, equation (38), simply reads

$$S_{\text{bound.}} = \int \frac{d^4p}{(2\pi)^4} \bar{\psi}_L \frac{p}{p} \Sigma(p) \psi_L,$$

(42)

where the kinetic function for the boundary field is

$$\Sigma(p) = \frac{f_R(p, L_0)}{p},$$

(43)

where we use $\psi_R(p, L_0) = f_R(p, L_0) (\hat{p}/p) \psi_L$. Canonical normalization is obtained by the field redefinition $\bar{\psi}_L = \sqrt{\Sigma(p)} \psi_L$. In the limit of zero momentum we obtain $\bar{\psi}_L \rightarrow \psi_L / f_L^0(L_0)$, where $f_L^0(z)$ is the fermion zero-mode wavefunction, equation (18), and $\bar{\psi}_L$ has 4D mass dimension, $[\bar{\psi}_L] = 3/2$.

The explicit solutions for the gauge and fermion holographic profiles, $f_A(p, z)$ and $f_L(p, z)$, are given in equations (A.1) and (A.6) of the appendix, respectively.

2.4. Relation between the methods

The three methods reviewed in the previous sections are related to one another. The 5D propagator can be written in terms of the KK expansion as

$$P_p(z, z') = \sum_n \frac{f_n(z) f_n(z')}{p^2 - m_n^2}.$$

(44)

Similarly, the boundary kinetic function in the holographic method is given by the inverse of the (Neumann) boundary to boundary propagator

$$\partial_{z} f_A(p, L_0) = \frac{1}{P_p^N(L_0, L_0)} = K'_{m}(p, L_0),$$

(45)

where in the second equality we used equation (28) for the case where the EWSB mass $m$ is IR brane localized, together with the b.c.s (26) and (27). The holographic profile itself, $f_A(p, z)$, is
given by the amputated bulk to boundary propagator, where amputation means dividing by the boundary to boundary propagator

\[ f_A(p, z) = \frac{P_N^L(L_0, z)}{P_N^L(L_0, L_0)} = K_m(p, z). \tag{46} \]

In particular, both the holographic functions and the 5D propagator contain information on the whole spectrum, which can be extracted by evaluating the residue of the corresponding functions on-shell (they have poles at the corresponding masses of the physical particles). Equation (44) shows that the 5D propagators resum the contribution of the whole tower of KK modes and are therefore quite efficient when computing the indirect effects of the massive modes. The holographic method also resums the effect of the whole tower, although it does so in a different basis, which can be very useful in certain situations, as reviewed below.

3. The low-energy effective Lagrangian

3.1. Integrating out gauge boson KK modes with 5D propagators

We start our discussion of EW tests of models with warped extra dimensions by introducing the prototype of the realistic model \[15, 16\] and computing its low-energy effective Lagrangian. In the following sections, we show how to use different techniques to obtain the same effective Lagrangian and study the constraints on the model from EW precision data. The model has a bulk $SU(2)_L \times SU(2)_R \times U(1)_X$ gauge symmetry, broken by boundary conditions to the SM gauge symmetry $SU(2)_L \times U(1)_Y$ on the UV brane. Separating the gauge fields into zero modes and massive modes, the full covariant derivative can be written as

\[ D^\text{full}_\mu = D_\mu - i \left[ g_{5L} L^a_\mu T^a_L + g_{5R} R^b_\mu T^b_R + g'_5 Y B_\mu + g_5 Z'_\mu Q_X \right], \tag{47} \]

where $D_\mu$ represents the SM covariant derivative (in 4D momentum space) and we use tildes to denote the massive KK components of the 5D fields. Here, $a = 1, 2, 3, b = 1, 2, L^a_\mu$ and $R^b_\mu$ are the gauge fields corresponding to $SU(2)_L$ and $SU(2)_R$, respectively, and we define the hypercharge and $Z'$ gauge bosons as

\[ B_\mu = \frac{g_{5X} R^3_\mu + g_{5R} X_\mu}{\sqrt{g_{5R}^2 + g_{5X}^2}}, \quad Z'_\mu = \frac{g_{5R} R^3_\mu - g_{5X} X_\mu}{\sqrt{g_{5R}^2 + g_{5X}^2}}. \tag{48} \]

We denote with $g_{5L}, g_{5R}$ and $g_{5X}$ the 5D gauge couplings of the three bulk gauge groups. The corresponding couplings of the hypercharge and $Z'$ read

\[ g'_5 = \frac{g_{5R} g_{5X}}{\sqrt{g_{5R}^2 + g_{5X}^2}}, \quad g_{5Z'} = \sqrt{\frac{g_{5R}^2}{g_{5R}^2 + g_{5X}^2}}, \tag{49} \]

with charges

\[ Y = T^3_R + Q_X, \quad Q_{Z'} = \frac{g_{5R}^2 T^3_R - g_{5X}^2 Q_X}{g_{5R}^2 + g_{5X}^2}, \tag{50} \]

so that the electric charge is $Q = T^3_L + T^3_R + Q_X$.

Our aim in this section is to compute the low energy-effective Lagrangian for this model. We will do so at the tree level, assuming there is a light Higgs in the spectrum. The effective...
Lagrangian can be obtained at the tree level by solving the equations of motion of the heavy particles and replacing the solutions back into the Lagrangian. The 5D Lagrangian, omitting tensor indices, can be written as

\[ \mathcal{L}_5 = \mathcal{L}_{5 \text{SM}} - \frac{1}{2} \tilde{L}^a O L^a - \frac{1}{2} \tilde{R}^b O R^b - \frac{1}{2} \tilde{B} O \tilde{B} - \frac{1}{2} \tilde{Z} O \tilde{Z}' + \ldots, \]

where \( O = O(p, z) \) was defined in equation (4), and the dots denote interaction terms with more than one heavy field which, in the case of gauge bosons, do not give contributions to the effective Lagrangian at leading order. We have defined the effective currents

\[
\begin{align*}
J^a_{\mu} &= a^3 (f_h^0)^2 J^a_{\mu}, \\
J^Y_{\mu} &= a^3 (f_h^0)^2 J^Y_{\mu}, \\
J^{Z'}_{\mu} &= a^3 (f_h^0)^2 J^{Z'}_{\mu},
\end{align*}
\]

where \( f_h^0 \) is the wavefunction of the fermion zero modes, as defined in equation (18), the Higgs wavefunction \( f_h^0 \) is normalized as in equation (7), with \( f_h(z) \to L^{1/2} a(z) f_h^0(z) \), and the fermionic currents are

\[
\begin{align*}
J^{aL}_{\mu} &= \bar{\psi} Y^\mu T^{aL}_{\mu} \psi, \\
J^{Y}_{\mu} &= \bar{\psi} Y^\mu Y \psi, \\
J^{Z'}_{\mu} &= \bar{\psi} Y^\mu Q Z \psi,
\end{align*}
\]

while the Higgs currents are

\[
\begin{align*}
J^{aL}_{\mu} &= h^3 T^{aL}_{\mu} D_{\mu} h + \text{h.c.}, \\
J^{Y}_{\mu} &= h^3 Y i D_{\mu} h + \text{h.c.}, \\
J^{Z'}_{\mu} &= h^3 Q Z i D_{\mu} h + \text{h.c.}
\end{align*}
\]

We can now integrate out the heavy fields by replacing back into the Lagrangian the solution of the classical equations of motion, which can be written in terms of the Green’s function of the corresponding differential operator (see equations (28)–(32)) as

\[
\begin{align*}
\tilde{L}^a(z) &= g_{5L} \int_{L_0}^{L_1} dz' \tilde{P}^N_p (z, z') \tilde{J}^a_{\mu}, \\
\tilde{B}(z) &= g_{5L} \int_{L_0}^{L_1} dz' \tilde{P}^N_p (z, z') \tilde{J}^Y_{\mu},
\end{align*}
\]

for the fields that are unbroken on the UV brane, and

\[
\begin{align*}
\tilde{R}^b(z) &= g_{5R} \int_{L_0}^{L_1} dz' \tilde{P}^{D}_p (z, z') \tilde{J}^b_{\mu}, \\
\tilde{Z}'(z) &= g_{5Z'} \int_{L_0}^{L_1} dz' \tilde{P}^{D}_p (z, z') \tilde{J}^{Z'}_{\mu},
\end{align*}
\]

for those that vanish on the UV brane. The Neumann (with the zero mode subtracted) or Dirichlet propagator takes care of the corresponding boundary conditions on the UV brane. Also, the propagators are computed in the EW preserving vacuum (e.g. for an IR-localized Higgs, \( m = 0 \) in equation (30)). The resulting effective Lagrangian can be put into the standard basis of [34] by using the equations of motion of the SM fields. It reads

\[
L_{\text{eff}} = L_{\text{SM}} + a_h \mathcal{O}_h + \sum_{\psi_L} \alpha_{\psi_L} \mathcal{O}_{\psi_L} + \sum_{\psi} \alpha_{\psi} \mathcal{O}_{\psi} + \sum_{\psi_L, \psi' L} \alpha_{\psi_L, \psi' L} \mathcal{O}_{\psi_L, \psi' L} + \sum_{\psi, \psi'} \alpha_{\psi, \psi'} \mathcal{O}_{\psi, \psi'},
\]

\footnote{For instance, in models of Gauge–Higgs unification one has \( f_h^0(z) = \sqrt{2/[L_0(L_0^2/L_0^2 - 1)]} a(z)^{-2} \). For an IR brane localized Higgs: \( a^3(z,f_h(z)) = \delta(z - L_1) \).}
where $\psi_L$ stands for any of the SM LH doublets, $\psi$ for any of the SM fermion fields, the gauge fields in $L_{SM}$ are assumed to be canonically normalized, and the different operators are defined as follows ($\mathcal{O}_{WB}$ is not induced at the tree level on this basis).

- **Oblique operators**
  \[ \mathcal{O}_h = \left| h^\dagger D_\mu h^\mu \right|^2, \quad \mathcal{O}_{WB} = (h^\dagger \sigma^a h) W^a_{\mu\nu} B^{\mu\nu}. \]  
  (59)

- **Two-fermion operators**
  \[ \mathcal{O}_{h\psi}^\dagger = i(h^\dagger D_\mu h)(\bar{\psi} \gamma^\mu \psi) + \text{h.c.}, \quad \mathcal{O}^\dagger_{h\psi_L} = i(h^\dagger \sigma^a D_\mu h)(\bar{\psi}_L \gamma^\mu \sigma^a \psi_L) + \text{h.c.} \]  
  (60)

- **Four-fermion operators**
  \[ \mathcal{O}^\dagger_{\psi\psi'} = \frac{1}{1+\delta_{\psi\psi'}}(\bar{\psi} \gamma^\mu \psi)(\bar{\psi'} \gamma_\mu \psi'), \quad \mathcal{O}^\dagger_{\psi\psi'\psi_L} = \frac{1}{1+\delta_{\psi_L\psi'}^L}(\bar{\psi}_L \gamma^\mu \sigma^a \psi_L)(\bar{\psi'}_L \gamma_\mu \sigma^a \psi'_L). \]  
  (61)

The coefficients $\alpha_i$, which encode the dependence on the model parameters, read

\[ \alpha_h = \frac{\tilde{g}^2}{2}[\alpha^N - \alpha^D], \]  
(62)

\[ \alpha^\dagger_{h\psi_L} = \frac{\tilde{g}^2_{L,R}}{4} \beta^N_{\psi_L}, \]  
(63)

\[ \alpha^\dagger_{h\psi} = \frac{\tilde{g}^2}{2} Y_{\psi} \beta^N_{\psi} + \frac{\tilde{g}^2 R T^3_R(\psi) - \tilde{g}^2 Y_{\psi}}{2} \beta^D_{\psi}, \]  
(64)

\[ \alpha^\dagger_{\psi\psi'\psi_L} = \frac{\tilde{g}^2_{L,R}}{4} Y_{\psi\psi'}^N \psi_L, \]  
(65)

\[ \alpha^\dagger_{\psi\psi'} = \frac{\tilde{g}^2}{2} Y_{\psi} \gamma_{\psi\psi'}^N + [\tilde{g}^2 R T^3_R(\psi) - \tilde{g}^2 Y_{\psi}] [\tilde{g}^2 R T^3_R(\psi') - \tilde{g}^2 Y_{\psi'}] \frac{Y^D_{\psi'}}{\tilde{g}^2 R - \tilde{g}^2}. \]  
(66)

where $\tilde{g}^2_{L,R} = g^2_{L,R}/L$ and $\tilde{g}^2 = g^2_{S}/L$, with $L$ the volume factor (8), while $Y_{\psi}$ and $T^3_R(\psi)$ are the hypercharge and third component of $SU(2)_R$ isospin for the field $\psi$, respectively (for the SM fermions, $Y_u = 1/6$, $Y_d = 2/3$, $Y_e = -1/3$, $Y_l = -1/2$ and $Y_e = -1$). The parameters $\alpha^N,D$, $\beta^N,D$ and $\gamma^N,D$ are defined as

\[ \alpha^N,D = L \int_{L_0}^{L_1} dz \left| \tilde{a}^3(z) \right|^2 \tilde{P}^D(z, z') \tilde{a}^3(z') \right|^2, \]  
(67)

\[ \beta^D = L \int_{L_0}^{L_1} dz \left| \tilde{a}^4(z) \right|^2 \tilde{P}^D(z, z') \tilde{a}^4(z') \right|^2, \]  
(68)

\[ \gamma^D = L \int_{L_0}^{L_1} dz \left| \tilde{a}^4(z) \right|^2 \tilde{P}^D(z, z') \tilde{a}^4(z') \right|^2. \]  
(69)
Since we are interested in the contribution of dimension-6 operators, we can evaluate the 5D propagators at zero momentum,

$\tilde{P}_0^N(z, z') = \frac{z^2_c (1 + 2 \log(L_0/z_c)) + z^2_{<} (1 + 2 \log(L_1/z_{<})) - L^2_1/\log(L_1/L_0)}{4L_0 \log(L_1/L_0)}, \quad (70)$

$\tilde{P}_0^D(z, z') = \frac{L_0}{2} \left( 1 - \frac{z^2_{<}}{L^2_0} \right). \quad (71)$

Note that, although we have put a tilde on the Dirichlet propagator, there is no zero mode subtraction in that case.

The above results provide the dimension-6 effective Lagrangian for general models in warped extra dimensions with custodial symmetry and a light Higgs, after integration of the gauge boson heavy modes. Simple limits can be easily obtained from this general Lagrangian. For instance, if we consider that the bulk symmetry is just that of the SM, we obtain, by setting to zero the Dirichlet coefficients, the following effective Lagrangian,

$L_{\text{eff}} = L_{\text{SM}} + \frac{\tilde{g}^2}{2} \left[ \alpha^N J^a_{\mu} J^a_{\mu} + 2 \sum_{\psi, \psi'} \beta^N_{\psi, \psi'} J^a_{\mu} J^a_{\mu} + \gamma^N_{\psi, \psi'} J^a_{\mu} J^a_{\mu} J^a_{\mu} \right] 
+ \frac{\tilde{g}^2}{2} \left[ \alpha^N J^Y_{\mu} J^Y_{\mu} + 2 \sum_{\psi, \psi'} \beta^N_{\psi, \psi'} J^Y_{\mu} J^Y_{\mu} + \gamma^N_{\psi, \psi'} J^Y_{\mu} J^Y_{\mu} J^Y_{\mu} \right] + \cdots. \quad (72)$

Note that the Dirichlet terms involving fermions vanish in the limit of UV localized fermions. Thus, the above Lagrangian, with the replacement $\alpha^N \rightarrow \alpha^N - \alpha^D$, is also the effective Lagrangian of models with custodial symmetry and UV localized fermions.

In the case where the Higgs field is localized on the IR brane, $a^i(z)[f_h(z)]^2 = \delta(z - L_1)$, these coefficients are explicitly given by

$\alpha^N = \frac{L^2_1}{4} \left[ -2 \log \frac{L_1}{L_0} + 2 - \frac{1}{\log \frac{L_1}{L_0}} \right], \quad (73)$

$\beta^N_{\psi} = \frac{L^2_1}{4} \left[ 1 - \frac{1}{\log \frac{L_1}{L_0}} + g_2(c_{\psi}) \left( 1 - 2 \log \frac{L_1}{L_0} \right) - 2\tilde{g}_2(c_{\psi}) \right], \quad (74)$

where $c_{\psi}$ is the bulk mass parameter for the fermion $\psi$, and the auxiliary functions $g_n(c)$ and $\tilde{g}_n(c)$ are defined in equations (A.7) and (A.8) of the appendix. Finally, $\gamma^N$ is a complicated function of $c_{\psi}$ and $c_{\psi'}$. In the limit $c \rightarrow \infty$ (UV localized fermions), it simplifies to

$\gamma^N(c \rightarrow \infty) = -\frac{L^2_1}{4 \log(L_1/L_0)}. \quad (75)$

Also, for future reference, when the Higgs is localized on the IR brane

$\alpha^D = -\frac{L^2_1}{2} \log \frac{L_1}{L_0}, \quad \beta^D_{\psi} = -\frac{L^2_1}{2} \log \frac{L_1}{L_0} g_2(c_{\psi}). \quad (76)$

$^{12}$ The integration of fermion heavy modes can be performed trivially from the general results in [35], see for instance [36].
3.1.1. The universal case. An assumption that significantly simplifies the analysis of the EW precision test (EWPT) is that of universal new physics [37]. Models of universal new physics are those for which a combination of gauge bosons exists (so-called interpolating fields) \( \bar{W}^a \), with \( a = 1, 2, 3 \), and \( \bar{B} \) such that the only couplings of fermions (excluding Yukawa couplings) are of the form

\[
L_{\text{fermions}}^{\text{Univ.}} = \bar{W}_\mu^a J_f^{aL,\mu} + \bar{B}_\mu J_f^{Y,\mu} + \cdots ,
\]

(77)

where the dots denote kinetic and Yukawa terms for the fermions. In particular, the interpolating fields \( \bar{W}^a \) and \( \bar{B} \) are not canonically normalized. The SM fermionic currents are defined by (see equation (54))

\[
J_f^{aL,\mu} = \sum_\psi J_\psi^{aL,\mu} , \quad J_f^{Y,\mu} = \sum_\psi J_\psi^{Y,\mu} .
\]

(78)

In this case, all relevant contributions to the EWPT can be encoded in the vacuum polarizations of the interpolating fields (oblique corrections). Assuming unbroken QED, the quadratic Lagrangian for the interpolating fields, together with the gauge-fermion interactions, can be written as

\[
L_{\text{Oblique}} = -\mathcal{P}^{\mu\nu} \left[ \bar{W}_\mu^a \bar{\Pi}_{+-} (p^2) \bar{W}_\nu^a + \frac{1}{2} \bar{W}_\mu^a \bar{\Pi}_{33} (p^2) \bar{W}_\nu^a \right. \\
+ \frac{1}{2} \bar{B}_\mu \bar{\Pi}_{BB} (p^2) \bar{B}_\nu + \bar{W}_\mu^a \bar{\Pi}_{3B} (p^2) B_i \bigg] + L_{\text{fermions}}^{\text{Univ.}},
\]

(79)

where \( \mathcal{P}^{\mu\nu} \) is the transverse projector and we have defined \( \bar{W}^\pm \equiv (\bar{W}^1 \mp i \bar{W}^2)/\sqrt{2} \).

In the case that the coefficients \( \beta_\psi^N \equiv \beta \) and \( \gamma_\psi^N \equiv \gamma \) in equation (72) are independent of the fermion type, \( \psi \), the effective Lagrangian takes the form

\[
L_{\text{eff}} = L_{\text{SM}} + \frac{g^2}{2} \left[ \alpha J_h^{aL,\mu} J_{h,\mu} + 2 \beta J_h^{aL,\mu} J_f^{aL,\mu} + \gamma J_f^{aL,\mu} J_f^{aL,\mu} \right] \\
+ \frac{g_3^2}{2} \left[ \alpha J_h^{Y,\mu} J_{h,\mu} + 2 \beta J_f^{Y,\mu} J_f^{Y,\mu} + \gamma J_f^{Y,\mu} J_f^{Y,\mu} \right] + \cdots .
\]

(80)

This effective Lagrangian is not in the oblique form, since it includes corrections to the fermion gauge couplings and four-fermion interactions, proportional to \( \beta \) and \( \gamma \), respectively. The corrections are, however, universal, and can be written purely in terms of oblique corrections. This can be done in two equivalent ways; either by using the classical equations of motion of the SM fields, as determined by \( L_{\text{SM}} \), or by doing field redefinitions that eliminate the corrections involving fermions. The latter approach consists of performing a shift of the gauge fields, proportional to the fermion currents, that eliminates the four-fermion terms, followed by a gauge field rescaling that puts the fermion–gauge interactions into the form of equation (77). Replacing a Higgs vev \( \langle h \rangle = (0, v) \) in the Higgs currents (55), the field redefinitions are, to the first order in \( \alpha, \beta \) and \( \gamma \)

\[
W^a = \frac{1}{g} \bar{W}^a \left[ 1 - \frac{g^2 v^2}{2} \beta - \frac{\gamma}{2} \Pi_{aa}^{\text{SM}} \right] - \frac{g}{g_3} J_f^{aL,\mu} + \frac{1}{g^3} B \left[ \frac{g^2 v^2}{2} \beta - \frac{\gamma}{2} \Pi_{3B}^{\text{SM}} \right],
\]

(81)

\[
B = \frac{1}{g_3} \bar{B} \left[ 1 - \frac{g_3^2 v^2}{2} \beta - \frac{\gamma}{2} \Pi_{BB}^{\text{SM}} \right] - \frac{g}{g_3} J_f^{Y,\mu} + \frac{1}{g_3} \bar{W}^3 \left[ \frac{g^2 v^2}{2} \beta - \frac{\gamma}{2} \Pi_{3B}^{\text{SM}} \right],
\]

(82)

where the vacuum polarization functions in the SM limit are defined by

\[
\Pi_{aa}^{\text{SM}} = p^2 - \frac{g^2 v^2}{2}, \quad \Pi_{BB}^{\text{SM}} = p^2 - \frac{g_3^2 v^2}{2}, \quad \Pi_{3B}^{\text{SM}} = \frac{g^2 v^2}{2}.
\]

(83)

These field redefinitions lead to an oblique form for the effective Lagrangian written in terms of the interpolating fields $\bar{W}^u, \bar{B}$, as in equation (79), with the following vacuum polarization functions

$$
\bar{g}^2 \Pi_{aa} = \Pi_{aa}^{SM}(1 - \bar{g}^2 v^2 \beta - \gamma \Pi_{aa}^{SM}) - \frac{\bar{g}^2 v^4}{4} (\bar{g}^2 + \delta_a \bar{g}^2) \alpha + \delta_a \Pi_{3B}^{SM}(\bar{g} \bar{g}^2 v^2 \beta - \gamma \Pi_{3B}^{SM})
$$

$$
= - \frac{\bar{g}^2 v^2}{2} \left[ 1 - \left( \frac{\bar{g}^2}{\bar{g}^2 + \delta_a} \right) \hat{T}^{Op} \right] + \left( 1 - \hat{S}^{Op} \right) p^2 + \frac{2}{\bar{g}^2 v^2} W^{Op} p^4 + \ldots, \tag{84}
$$

$$
\bar{g}^2 \Pi_{BB} = \Pi_{BB}^{SM}(1 - \bar{g}^2 v^2 \beta - \gamma \Pi_{BB}^{SM}) - \frac{\bar{g}^2 v^4}{4} (\bar{g}^2 + \bar{g}^2) \alpha + \Pi_{3B}^{SM}(\bar{g} \bar{g}^2 \beta - \gamma \Pi_{3B}^{SM})
$$

$$
= - \frac{\bar{g}^2 v^2}{2} \left[ 1 - \left( \frac{\bar{g}^2}{\bar{g}^2 + 1} \right) \hat{T}^{Op} \right] + \left( 1 - \hat{S}^{Op} \right) p^2 + \frac{2}{\bar{g}^2 v^2} W^{Op} p^4 + \ldots, \tag{85}
$$

$$
\bar{g}^3 \Pi_{1} = \Pi_{3B}^{SM} \left[ 1 - \frac{v^2}{\bar{g}^2 + \bar{g}^2} \beta - \gamma \Pi_{3B}^{SM} \right] + \frac{\bar{g}^2 v^2}{2} \left[ \frac{v^2}{\bar{g}^2 + \bar{g}^2} \alpha + \beta \Pi_{3B}^{SM} \right]
$$

$$
= \frac{\bar{g}^2 v^2}{2} \left[ 1 - \left( \frac{\bar{g}^2}{\bar{g}^2 + 1} \right) \hat{T}^{Op} \right] + \frac{\bar{g}^2 \hat{S}^{Op} p^2 + \ldots, \tag{86}
$$

where $\Pi_{3B}^{SM} \equiv \Pi_{3B}^{SM} + \Pi_{BB}^{SM}$, and in the second equality of each definition expanded for small $p^2$ and the result written in terms of

$$
\hat{T}^{Op} \equiv \frac{\bar{g}^2 v^2}{2} (\alpha + 2 \beta - \gamma), \quad \hat{S}^{Op} \equiv \bar{g}^2 v^2 (\beta - \gamma), \quad W^{Op} \equiv - \frac{\bar{g}^2 v^2}{2} \gamma, \tag{87}
$$

which will be identified in section 4 as the oblique parameters of [37]. Recall that these same vacuum polarization functions also represent the effective Lagrangian of a custodially symmetric model with fermions localized on the UV brane if we make the replacement $\alpha \rightarrow \alpha^N - \alpha^D$.

### 3.2. The holographic method

In this section we rederive the effective Lagrangian of the model with custodial symmetry using the holographic method (for further discussions of holography see the review in [38]). We consider first the case of UV localized fermions, which is the situation in which the holographic method is the most efficient, as it produces with minimal effort the effective Lagrangian in the oblique form. In subsection 3.2.1, we discuss how the formalism changes when fermions are allowed to propagate in the bulk.

In the presence of a bulk EWSB vev, $v(z)$, the gauge part of the action reads

$$
S^{\text{Casst}}_{\text{gauge}} = \int \frac{d^4 p}{(2\pi)^4} d\sqrt{g} \left\{ -\frac{1}{4g^2_{SL}} (L^a_{M^N})^2 - \frac{1}{4g^2_{SR}} (R^a_{MN})^2 - \frac{1}{4g^2_{SX}} (X^a_{MN})^2 \\
+ \frac{1}{2} v(z)^2 (L^a_M - R^a_M)^2 \right\}
$$

$$
= \int \frac{d^4 p}{(2\pi)^4} d\sqrt{g} \left\{ -\frac{1}{4g^2_{SZ}} (V^a_{MN})^2 - \frac{1}{4g^2_{SZ}} (A^a_{MN})^2 - \frac{1}{4g^2_{SX}} (X^a_{MN})^2 + \frac{1}{2} v(z)^2 A^a_M \right\}
$$

$$
= -\frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} d\sqrt{g} \left\{ \bar{A}^a P A(p^2) \bar{A}^a + \bar{V}^a P V(p^2) \bar{V}^a + \bar{X} \Pi X(p^2) \bar{X} \right\}, \tag{88}
$$
where $g^2_{5Z} \equiv g^2_{5L} + g^2_{5R}$, and in the second equality we define the axial and vector combinations

$$A^a_M = L^a_M - R^a_M, \quad V^a_M = \frac{g^2_{5R}}{g^2_{5L}} L^a_M + \frac{g^2_{5L}}{g^2_{5R}} R^a_M. \tag{89}$$

In the last line we use the holographic method (indicated by the $\text{Hol}$ symbol) to obtain the effective action for the fields evaluated on the UV brane,

$$\bar{A}^a = A^a(z = L_0), \quad \bar{V}^a = V^a(z = L_0), \quad \bar{X} = X(z = L_0). \tag{90}$$

The corresponding vacuum polarizations read

$$\Pi_{A,V} = \frac{1}{g^2_{5Z}} \partial_z f_{A,V}(p^2, z = L_0), \quad \Pi_X = \frac{1}{g^2_{5X}} \partial_z f_X(p^2, z = L_0), \tag{91}$$

where $f_A$, $f_V$ and $f_X$ satisfy equation (35) with mass terms $M^2_A = g^2_{5Z} v^2$, $M^2_V = 0$ and $M^2_X = 0$, respectively. By assumption, the fermionic action is already localized on the UV brane. In particular, the coupling between the fermions and gauge bosons is universal, as in equation (77), with interpolating fields that are the 5D SM gauge boson fields evaluated at the UV brane. Replacing the boundary values of the axial, vector and $X$ fields in terms of the SM$^{13}$

$$\bar{A}^a = \bar{W}^a_L - \delta^{a3} \bar{B}, \quad \bar{V}^a = \frac{g^2_{5R}}{g^2_{5L}} \bar{W}^a_L + \delta^{a5} \frac{g_{5L}}{g_{5R}} \bar{B}, \quad \bar{X} = \bar{B}, \tag{92}$$

we obtain an effective Lagrangian in the oblique form (79), with vacuum polarizations

$$\bar{\Pi}_{aa} = \Pi_A + \frac{g^2_{5R}}{g^2_{5L}} \Pi_V, \tag{93}$$

$$\bar{\Pi}_{BB} = \Pi_A + \frac{g^2_{5L}}{g^2_{5R}} \Pi_V + \Pi_X, \tag{94}$$

$$\bar{\Pi}_{3B} = -\Pi_A + \Pi_V. \tag{95}$$

Assuming that $v(z)^2 = (v^2/2)[f^0_{hi}(z)]^2$, where the Higgs profile $f^0_{hi}(z)$ is normalized, as specified after equation (53), one can check that these vacuum polarization functions agree with those obtained using 5D propagators, equations (84)–(86), up to a redefinition of the vev, $v^2 \rightarrow (v^2[1 - \frac{1}{4}(g^2_{5L} + g^2_{5R})/v^2]/(L_0))^2 L_0$, when expanded to the same order in $v^2$ and with the replacement $\alpha \rightarrow \alpha^N - \alpha^D$ to take into account the modifications due to the custodial symmetry. Note also that in the holographic formalism the custodial symmetry is explicit from equation (93), i.e. a symmetry between the charged and neutral $W$s.

### 3.2.1. Holography with bulk fermions.

As we showed in the previous section, the holographic method is particularly suitable to compute the effective Lagrangian in the case of UV boundary localized fermions. The holographic method can still be used when the fermions propagate in the bulk, although obtaining the effective Lagrangian requires more work. In this section, we discuss the new steps one has to perform in the simple case of the SM bulk gauge symmetry, which can

---

$^{13}$ Note the difference with respect to equations (47) and (48) due to the non-canonical normalization of the fields in this section.
Figure 1. Vertex and four-fermion interaction contributions to the boundary action. The four-fermion interaction, with points at $z$ and $z'$, is to be computed with the Dirichlet propagator, equation (29).

be obtained from our custodial model by setting $R^b_M = 0, R^3_M = X_M = B_M$, and identifying the $U(1)_Y$ gauge coupling, $g'_5$, as in equation (49). The gauge part of the Lagrangian then reads

$$S_{\text{gauge}} = \int \frac{d^4 p}{(2\pi)^4} dz \sqrt{g} \left\{ -\frac{1}{4g_5^2}(W^b_{MN})^2 - \frac{1}{4g_5^2} [(A_{MN})^2 + (V_{MN})^2] + \frac{1}{2} v(z)^2 [(W^b_M)^2 + (A_M)^2] \right\}$$

where

$$\Pi_{W,A,V} = \Pi(g_5^2 v^2, g_5^2 v^2, 0),$$

and we indicate inside the parenthesis the masses to be used in equations (35) and (37). Now we have $g_5^2 \equiv g_5^2 + g_5^2$ and the following relation between the axial and vector fields and the SM neutral ones:

$$A_M = W^3_M - B_M, \quad V_M = g_5 \frac{g_5^2}{g_5} W^3_M + g_5 \frac{g_5^2}{g_5} B_M,$$

so that the vacuum polarization functions for the SM boundary fields read

$$\Pi_{+} = \Pi_W, \quad \Pi_{33} = \Pi_A + \Pi_V, \quad \Pi_{3B} = -\Pi_A + \Pi_V, \quad \Pi_{BB} = \Pi_A + \frac{g_5^2}{g_5^2} \Pi_V.$$ (99)

As in the previous section, these would be the vacuum polarization functions for the interpolating fields for UV localized fermion fields. When the fermion fields live in the bulk, however, these terms do not give the full contribution to the boundary effective Lagrangian. Indeed there are both vertex and four-fermion interaction corrections, as shown in figure 1. The left panel in the figure represents the vertex corrections. The contribution to the effective Lagrangian is obtained by matching the corresponding amputated three-point function in the full and effective theories. The diagram in the full theory requires bulk to boundary propagators for the gauge boson and fermions, and amputation means that the external legs are divided by the corresponding boundary to boundary propagators ($= g_5^2 / K_m'(p^2, L_0)$ for the case of an IR brane localized EWSB mass $m$, with $g_5$ and $m$ the parameters of the corresponding gauge boson; see

equations (35), (37) and (45). Assuming fermion localization universality,\(^\text{14}\) the resulting extra contribution to the boundary action reads

\[
S_{\text{Vertex}} = \int \frac{d^4 p}{(2\pi)^4} \left\{ \tilde{g}_W \tilde{W}^b \tilde{J}_f^{bL}\mu + \tilde{g}_A \tilde{A}_\mu \tilde{J}_z^\mu + s_\theta c_\theta \tilde{g}_V \tilde{V}_\mu \tilde{J}_Q^\mu \right\},
\]

where \(c_\theta = g_5/g_5z, \ s_\theta = g_5^2/g_5z, \ \tilde{J}_z^\mu = c_\theta^2 \tilde{J}_f^{3\mu} - s_\theta^2 \tilde{J}_f^\mu \) and \(\tilde{J}_Q^\mu = \tilde{J}_f^{3\mu} + \tilde{J}_f^\mu \), with \(\tilde{J}_f^{bL}\mu\) and \(\tilde{J}_f^\mu\), as defined in equations (78) and (54), but with \(\psi \rightarrow \tilde{\psi}\) representing the canonically normalized boundary fermion fields (see end of subsection 2.3). We also defined

\[
\tilde{g}_{W,V,A}(p^2) = \int_{L_0}^{L_1} dz \, a^4(z) [f^0_\psi(z)]^2 f_{W,V,A}(p^2, z),
\]

where \(f_{W,V,A}\) are the gauge boson ‘holographic’ profiles associated with equation (97). Since we are interested in the couplings of the fermion zero modes, we have evaluated the corresponding fermion profiles at \(p^2_\psi = 0\) in (102) (neglecting the small fermion masses from EWSB), so that the zero-mode wavefunctions, equation (18), appear inside the integral, and there is dependence on a single momentum scale, associated with the gauge boson.

Bulk fermions also imply that four-fermion interactions are generated in the boundary action, due to the diagram in the right panel of figure 1. Again the coefficient in the effective theory is obtained by matching the amputated four-point function in the full and effective theories. On the full theory side, we use the Neumann bulk-to-bulk propagator and integrate over both interaction points. On the effective theory side, we have to use the boundary-to-boundary propagator, using the couplings we have computed in equation (102). This latter term exactly coincides with the \(K(z)K(z')/K'(L_0)\) part of the full propagator, equation (28). The difference of the two therefore corresponds to computing the diagram on the right panel of figure 1 using the Dirichlet bulk-to-bulk propagator instead of the full one. The induced four-fermion operators are

\[
S_{4-\text{fermion}} = \int \frac{d^4 p}{(2\pi)^4} \left\{ \frac{g_5^2}{2L} \gamma^D_\mu \tilde{J}_f^{bL}\mu + \frac{g_5^2}{2L} \gamma^D_\mu \tilde{J}_z^\mu + s_\theta c_\theta g_5^2 \frac{g_5^2}{2L} \gamma^D_\mu \tilde{J}_Q^\mu \right\},
\]

where \(g_5^2\) is defined after equation (97), \(L\) is the volume factor, equation (8), and \(\gamma^D_{W,A,V}\) are defined in equation (69), with the Dirichlet propagator corresponding to \(W, A\) and \(V\) (i.e. gauge boson squared masses \(g_5^2, g_5^2v^2\) and 0, respectively). If we are interested in the contribution to the effective Lagrangian up to operators of dimension 6, we can evaluate the corresponding propagator at zero momentum and neglect EWSB effects, i.e. use equation (71). In that case, \(\gamma^D_W = \gamma^D_A = \gamma^D_V = \gamma^{\text{Hol}}\), and assuming universality we have

\[
\gamma^{\text{Hol}} = \frac{L_1^2 \log(L_1/L_0) (3 - 2c)(L_0/L_1)^2 - (2c - 1)^2 (L_0/L_1)^{4c - 2} + 8(1 - c)(L_0/L_1)^{2c - 1}}{4(2c^2 - 5c + 3) [1 - (L_0/L_1)^{2c - 1}]}.
\]

The effective Lagrangian given by equations (96), (101) and (103) is not in the oblique form, but can be rewritten in such a form due to the assumption of the universality of fermion localization. The simplest way to proceed is to first shift the gauge fields as

\[
\tilde{W}^b \rightarrow \tilde{W}^b - \frac{g_5^2}{2Lg_W} \gamma^D_\mu \tilde{J}_f^{bL}\mu, \quad \tilde{A} \rightarrow \tilde{A} - \frac{g_5^2}{2Lg_A} \gamma^D_\mu \tilde{J}_z^\mu, \quad \tilde{V} \rightarrow \tilde{V} - \frac{s_\theta c_\theta g_5^2}{2Lg_V} \gamma^D_\mu \tilde{J}_Q^\mu.
\]

\(^\text{14}\) Equations (96), (101) and (103) below are trivially generalized to the non-universal case by expressing the results in terms of the individual fermion currents of equations (78), and using the corresponding fermion profiles in equations (102) and (69).
so as to eliminate the four-fermion interactions (to first order in the $\gamma_i^{(D)}$), followed by a rescaling $W^b \rightarrow W^b/\tilde{g}_W$, $A \rightarrow A/\tilde{g}_A$ and $V \rightarrow V/\tilde{g}_V$, with

$$
\tilde{g}_i(p^2) = g_i(p^2) \left[ 1 + \frac{g_i^2}{2Lg_i^2(p^2)} \gamma_i \Pi_i(p^2) \right], \quad i = W, A, V,
$$

(105)

where $g_i^2 = g_5^2$ for $i = W$, $g_i^2 = g_5^2$ for $i = A, V$, $\tilde{g}_i$ are given in equation (102), and $\Pi_i$ are given in equation (97), for $i = W, A$ and $V$. Expressing the resulting Lagrangian in terms of $W^a$ and $B$, as in equation (98) one finds that the fermion couplings to the new gauge bosons are universal as in (77), while the vacuum polarizations are

$$
\tilde{\Pi}_{+-} = \frac{1}{g_W} \Pi_W, \quad \tilde{\Pi}_{33} = \frac{1}{g_A} \Pi_A + \frac{1}{g_V} \frac{g_5^2}{g_5} \Pi_V,
$$

(106)

$$
\tilde{\Pi}_{3B} = -\frac{1}{g_A} \Pi_A + \frac{1}{g_V} \frac{g_5^2}{g_5} \Pi_V, \quad \tilde{\Pi}_{BB} = \frac{1}{g_A} \Pi_A + \frac{1}{g_V} \frac{g_5^2}{g_5} \Pi_V,
$$

(107)

with $\Pi_{W,A,V}$, as defined in equation (97). Recall that all these quantities depend on $p^2$, including the $\tilde{g}_i$, even though we do not explicitly indicate so. Expanding in the EWSB masses to the corresponding order in $v^2$, one can check that these vacuum polarizations agree exactly with those obtained with the method of propagators. Note, however, that the above formulae hold for arbitrary $v$. Also, in the limit of UV localized fermions, $c \rightarrow +\infty$, one has $\gamma^{\text{Hol}} \rightarrow 0$ and $\tilde{g}_i \rightarrow 1$, so that equations (106)–(107) reduce to equations (99) and (100).

4. Electroweak precision tests (EWPT)

Very precise data from low-energy neutrino and electron scattering, LEPI and SLC data at the $Z$ pole, LEPII above the $Z$ pole and the Tevatron impose stringent constraints on any physics beyond the SM [39], commonly called EWPT. These constraints can be computed for each model of new physics by carefully considering the contribution to all the (pseudo)observables that constitute the EWPT. Under the assumption of linearly realized EWSB with a light Higgs and a large enough mass gap with the new physics, one can use the SM effective Lagrangian up to dimension 6 of [34] to easily constrain large classes of new models. Not all dimension 6 operators are relevant for EWPT. In general, operators that violate Charge conjugation-parity (CP) or flavor symmetries (except for the third family), operators that only involve quarks or gluons and operators that just renormalize the SM operators (i.e. terms of the form $h^7 h O^{\text{SM}}$, with $O^{\text{SM}}$ a dimension 4 operator that is already present in the SM Lagrangian) are irrelevant for EWPT. The relevant ones were classified in [40] (see also [41]) and their effects on the EWPT computed ([40] gives the $\chi^2$ as a function of the SM input parameters and the coefficients of the relevant dimension-6 operators). Using the results in [40] and our calculation of the effective Lagrangian for a general model with custodial symmetry, equations (58) and (62)–(71), one can obtain the constraints in any model of warped extra dimensions with a light Higgs. However, the prototype of the realistic warped model, with light fermions localized close to the UV brane, corresponds to universal new physics (except for the third generation) and the corresponding computation of EWPT can be more simply done by following [37].
4.1. Tree-level effects

As discussed in section 3.1.1, EW precision constraints can be easily implemented in models of universal new physics [37]. In this case, the relevant Lagrangian is given by equations (77) and (79). If we further assume that there is a mass gap with the new physics that allows us to reliably expand the vacuum polarizations as

$$\tilde{\Pi}(p^2) = \tilde{\Pi}(0) + p^2 \tilde{\Pi}'(0) + \frac{(p^2)^2}{2} \tilde{\Pi}''(0) + \cdots,$$

(108)

where the prime denotes derivative with respect to $p^2$ (higher derivative terms give contributions of mass dimension larger than 6), we can parameterize all relevant EWPT in terms of four oblique parameters

$$\hat{T} = 1 - \frac{\tilde{\Pi}_{13}(0)}{\tilde{\Pi}_{+-}(0)}, \quad W = \frac{g^2 M_W^2}{2} \tilde{\Pi}_{13}'(0),$$

(109)

$$\hat{S} = g^2 \tilde{\Pi}_{1B}(0), \quad Y = \frac{g^2 M_W^2}{2} \tilde{\Pi}_{BB}'(0).$$

(110)

Here, $g$, $g'$ and $M_W$ are fixed by the conditions

$$\frac{1}{g^2} = \tilde{\Pi}_{11}'(0), \quad \frac{1}{g'^2} = \tilde{\Pi}_{BB}'(0), \quad -\frac{M_W^2}{g^2} = \tilde{\Pi}_{+-}(0),$$

(111)

which, to the order we are interested in, can be identified with the experimentally measured weak gauge couplings and the $W$ mass. Some of these parameters are related to the Peskin–Takeuchi [43] parameters as $\alpha S = 4 s_W^2 \hat{S}$ and $\alpha T = \hat{T}$, where $\alpha$ is the electromagnetic fine structure constant.

If EWSB can be treated perturbatively, we can use the general results for the vacuum polarizations (84)–(86) to compute these oblique parameters. For arbitrary $v$ one can use either (93)–(95) for custodially symmetric models with UV localized fermions, or (106) and (107) for models without custodial symmetry, but arbitrary (though universal) fermion localization. In the case that EWSB is a perturbation, the normalization conditions, equation (111), give

$$\bar{g}^2 = g^2 \left[1 - \hat{S}^{\text{op}}\right], \quad \bar{g'}^2 = g'^2 \left[1 - \frac{g^2}{g'^2} \hat{S}^{\text{op}}\right], \quad \bar{v}^2 = v^2 \left[1 + \frac{g^2}{g'^2} \hat{T}^{\text{op}}\right],$$

(112)

where $\hat{T}^{\text{op}}$ and $\hat{S}^{\text{op}}$ are defined in equation (87). It is then straightforward to check that $\hat{T} = \hat{T}^{\text{op}}$, $\hat{S} = \hat{S}^{\text{op}}$ and $W = Y = W^{\text{op}}$. In the particular case that the fermions are UV localized, and in the absence of custodial symmetry, the oblique parameters are given explicitly by

$$\hat{T}_{\text{UV}} = \frac{g^2}{2 g^2} (m_W L_1)^2 \log(L_1/L_0), \quad W_{\text{UV}} = \frac{(m_W L_1)^2}{4 \log(L_1/L_0)},$$

(113)

$$\hat{S}_{\text{UV}} = \frac{(m_W L_1)^2}{2}, \quad Y_{\text{UV}} = \frac{(m_W L_1)^2}{4 \log(L_1/L_0)}.$$

(114)

Note that for UV localized fermions $\hat{T}$ is volume enhanced, $\hat{S}$ is neither enhanced nor suppressed and $W$ and $Y$ are volume suppressed. Thus, the most constrained experimentally

---

The Z-pole observables plus the $W$ mass depend only on the three linear combinations introduced in [21, 42]. These are $\alpha T_{\text{eff}} = \hat{T} - [W + (s_W^2/c_W^2)Y]$, $\alpha S_{\text{eff}} = 4 s_W^2 (\hat{S} - W - Y)$ and $\alpha U_{\text{eff}} = 4 s_W^2 (\hat{U} - W)$ [37].

is $\hat{T}$, followed by $\hat{S}$, while $W$ and $Y$ are very moderately constrained. In models with custodial symmetry and UV localized fermions, setting $\alpha \rightarrow \alpha^N - \alpha^D$ as discussed at the end of subsection 3.2, one finds $T^\text{Cust}_{UV} = 0$, so that the most stringent and robust constraints arise from $\hat{S}$. This is true only for UV localized fermion fields and is not maintained in general for bulk fermion fields.

Light fermions localized on the UV brane are a good approximation in models with a natural realization of flavor. However, the large mass of the top implies that neither of its chirality components can be too far from the IR brane and, in particular, that $b_L$ cannot be UV localized. The resulting corrections to the bottom couplings can be computed with identical results using any of the methods discussed in section 2. Here we use our general result for the effective Lagrangian of models with custodial symmetry, equation (58). The part of the effective Lagrangian that we are interested in reads

$$L_{\text{eff}} = L_{\text{SM}} + \alpha'_{hq} O'_{hq} + \alpha''_{hq} O''_{hq} + \cdots$$

$$= \frac{\bar{g}}{2c_W}Z,\bar{b}_L Y^\mu b_L \left[ -1 + \frac{2s^2_w}{3} - 2\bar{v}^2(\alpha'_{hq} + \alpha''_{hq}) \right] + \cdots. \quad (115)$$

Replacing the values of $\alpha'^{i,s}$ of equations (63) and (64), we obtain for the correction of the $Z\bar{b}_L b_L$ coupling

$$\frac{\delta g_{bl}}{g_{bl}} = \frac{2\bar{v}^2(\alpha'_{hq} + \alpha''_{hq})}{1 - 2s^2_w/3}$$

$$= -\bar{v}^2 \left[ \frac{\bar{g}^2_{LL} T^3_L - \bar{g}^2_{RR} T^3_R}{\bar{g}^2_{LL} T^3_L - \bar{g}^2_{RR} Y} \right] \left( \beta^N - \beta^D_{UV} \right). \quad (116)$$

where the quantum numbers and $\beta^{N,D}$ refer to $b_L$, and we explicitly subtract the global effect that we are parameterizing in terms of oblique corrections with the term $\beta^N_{UV}$ (we are assuming UV-localized light fermions). We separate the correction into two terms. The first term, proportional to $\beta^D$, corresponds to the coupling evaluated at zero momentum and, as was the case with $\hat{T}$, is volume enhanced (see equation (73)). The term proportional to $(\beta^N - \beta^D_{UV})$ is a correction that arises when the external gauge boson line is evaluated at $p^2 = m^2_Z$, and is neither (volume) enhanced nor suppressed (see (74) and (75)). This separation is clear if one computes the coupling using the holographic method and the expansion in equation (A.9).

The volume enhanced term exactly vanishes if we take either $T^3_L = T^3_R = 0$ or $T^3_L = T^3_R$ and $\bar{g}_L = \bar{g}_R$. The vanishing of the coupling at zero momentum is guaranteed by a subgroup of the custodial symmetry, as first discussed in [16], and applies to any fermion satisfying $T^3_R = T^3_L = 0$ or $T^3_R = T^3_L$ and $g_L = g_R$. However, the on-shell coupling does not vanish, reading for $T^3_L = T^3_R$ and $g_L = g_R$

$$\frac{\delta g_{bl}}{g_{bl}} = -\frac{\bar{v}^2}{1 - 2s^2_w/3} \left[ \frac{\bar{g}^2_{LL} T^3_L - \bar{g}^2_{RR} Y}{\bar{g}^2_{LL} T^3_L - \bar{g}^2_{RR} Y} \right] \left( \beta^N - \beta^D - \beta^N_{UV} \right). \quad (117)$$

Assuming a boundary Higgs, we obtain using equations (73), (74) and (76),

$$\frac{\delta g_{bl}}{g_{bl}} = -\frac{\bar{v}^2 L^2_1}{4(1 - 2s^2_w/3)} \left[ \frac{\bar{g}^2_{LL} T^3_L - \bar{g}^2_{RR} Y}{\bar{g}^2_{LL} T^3_L - \bar{g}^2_{RR} Y} \right] \left[ g_2(c_{bl}) - 2\bar{g}_2(c_{bl}) \right]. \quad (118)$$

where $g_2(c)$ and $\bar{g}_2(c)$ are defined in equations (A.7) and (A.8) of the appendix.
4.2. Loop effects

So far we have concentrated on tree-level effects. Due to the summation over the KK towers, loop effects can generically be expected to be relevant. In generic theories, such effects are UV sensitive and can only be estimated, e.g. based on NDA in higher dimensions [44]. However, in theories with custodial symmetry [15] (and custodial protection of some fermion couplings to the $Z$ [16]) some of the EW (pseudo)observables, namely $T$ and $\delta g_{h_L}$, are calculable, since the symmetries forbid the associated counterterms. Given that the corresponding tree-level contributions are small, it is pertinent to assess more carefully the importance of such loop effects.

The one-loop contribution to the $T$-parameter due to the top KK tower was computed in certain scenarios in [45] (see also [46]). In particular, it was observed that when the fermions are assigned to representations (which involve bi-doublets under $SU(2)_L \times SU(2)_R$) that protect the $g_{h_L}$ coupling from large tree-level contributions, typically the contribution to $T$ from the full tower associated with the top quark decreases with respect to the SM one (i.e. the massive KK tower gives a negative $\Delta T$). This can be understood as a consequence of the quantum number assignments, plus the boundary conditions necessary to preserve the custodial symmetry on the IR brane (and requiring also that the zero-mode sector be the SM one). Given that there is a sizable positive tree-level contribution to the $S$ parameter, an EW fit based on the oblique parameters can put quite severe bounds on the scale of new physics in these scenarios. Interestingly, there are well-defined regions of parameter space where some of the top KK excitations become light and make $\Delta T$ positive, thus also allowing for lighter KK gauge excitations [45]. This can have important implications for the phenomenology of fermion KK modes, as well as for that of the KK gauge bosons, which can decay into the light KK fermions with significant branching fractions (as studied in the context of a Gauge–Higgs unification scenario in [47]). We should remark that the one-loop contribution to $T$ due to gauge fields has not been computed, although it is expected to be smaller than the one from the top sector since the KK gluons are heavier and their couplings are controlled by the weak gauge couplings, as opposed to the top Yukawa coupling.

Also, the one-loop contributions to $\delta g_{h_L}$ were computed in [9]. These tend to be small, but not necessarily negligible. As we see in the next section, such loop contributions can be important in relaxing the constraints from EW precision measurements on the scenarios we consider.

4.3. Summary of EW constraints in models with custodial symmetry

With the low-energy effective Lagrangian presented in the previous sections one can assess the indirect constraints from precision measurements on models with custodial symmetry in warped spaces, under the assumption that there exists a light Higgs degree of freedom. In the general case that the fermions propagate in the bulk, one should perform a global fit to the EW observables based on the dimension-6 Lagrangian of equation (58). Such an analysis was performed in [9]. It was found that, depending on the SM fermion $SU(2)_R$ quantum number assignments, the bounds from the EW constraints on the gauge boson masses are typically in the 2.5–3.5 TeV range (neglecting brane kinetic terms), although in certain models they could be as low as 1.5 TeV.

Due to the $T^3_R$ charges in equations (64) and (66), the low-energy effects of the heavy physics are in general not universal, even when all fermions share the same localization profiles.

However, the RS interpretation of the flavor structure as arising from the ‘anarchy’ of the Yukawa couplings (see section 5 below) requires that the light fermion families be localized close to the UV brane, in which case the low-energy corrections become of the universal type (the effects of the massive $SU(2)_R$ gauge bosons become exponentially suppressed and the c-dependence of equations (68) and (69) disappears). It is then simpler to use the fits based on the oblique parameters, equations (109) and (110). The dependence on the model parameters is given in equation (87). Furthermore, for UV localized fermions, $W$ and $Y$ are volume suppressed and can be neglected. However, in general, the corrections to the $Zb_L b_L$ coupling, which at the tree level are given by equation (116), need to be taken into account. When $g_L = g_R$ and $T_L^3 = T_R^3$ the tree-level contributions to $\delta g_{b_L}$ are relatively small due to a custodial protection [16], but there can be non-negligible loop-level effects. Thus, a fit to $S$, $T$ and $\delta g_{b_L}$ is typically appropriate.

Using the code of [40], we obtain the $1\sigma (\Delta \chi^2 = 1)$ intervals

$$S = -0.03 \pm 0.09, \quad T = 0.03 \pm 0.09, \quad \frac{\delta g_{b_L}}{g_{b_L}} = (-0.4 \pm 1.4) \times 10^{-3}. \quad (119)$$

Here we have used the combined CDF and DØ top mass measurement of March 2009 (using up to 3.6 fb$^{-1}$ of data per experiment), $m_t = 173 \pm 1.3$ GeV $c^2$ [48], as well as the combined CDF/DØ $W$ mass measurement of August 2008, $M_W = 80.432 \pm 0.039$ GeV $c^2$ [49]. We include the Z-pole observables, the low-energy measurements (except for NuTeV) and LEPII data. We also use a Higgs mass of $m_h = 117$ GeV.

In models without custodial protection, the main constraint comes from $T = \tilde{T}/\alpha$. Requiring $T_{\text{tree}} < 0.21$ (at 2$\sigma$), assuming a light Higgs as in the fit of equation (119), and using equation (113), gives a strong lower bound of $1/L_1 \sim 4.4$ TeV (hence gauge boson masses $M_{KK} > 2.45/L_1 \approx 11$ TeV). Such a constraint can be somewhat relaxed for a heavier Higgs (and for sizable gauge and/or fermion brane kinetic terms [21]). The situation is dramatically improved in models with custodial protection, since $T_{\text{tree}}$ vanishes in this case (for UV localized fermions). A bound based on $S < 0.15$ (at 2$\sigma$) alone results in $1/L_1 \sim 1.5$ TeV ($M_{KK} > 3.7$ TeV). However, falling inside the 95% C.L. ellipsoid ($\Delta \chi^2 = 7.81$ for the three parameters $S$, $T$ and $\delta g_{b_L}$) can allow for $S \sim 0.23$ (if $T \sim 0.25$ and $\delta g_{b_L}/g_{b_L} \sim -0.8 \times 10^{-3}$). In this case, and in the absence of additional contributions to $S$, one can have $1/L_1 \sim 1.25$ TeV ($M_{KK} > 3$ TeV). As mentioned in subsection 4.2, loop level contributions to $T$ and $\delta g_{b_L}$ can be relevant (and depend on additional model parameters, although typically $\Delta T_{\text{loop}}$ and $\delta g_{b_L}^{\text{loop}}$ are correlated). Requiring that these additional contributions optimize the EW fit usually selects well-defined regions of parameter space (e.g. localization of the third family quarks, with important consequences for their KK spectrum, which can include fermion states lighter than the gauge KK modes).

In summary, we see that models with custodial protection and an implementation of the flavor structure through fermion localization can be consistent with gauge boson KK excitation of about 3 TeV. EW constraints place similar bounds on models of Gauge–Higgs unification [27, 46], where an interesting connection between EWSB and EWPT arises, via the appearance of relatively light KK fermion states [50, 51]. We take these scales as a guide to the study of the collider phenomenology in the second part of this paper.

Some of the above bounds could be somewhat relaxed in the presence of moderate IR brane kinetic terms. Also, playing with the localization parameters of LH versus RH fermions it may be possible to lower the scale of new physics somewhat more [9]. Another possibility
compatible with a lower scale of new physics is to consider a different background, e.g. soft-wall models (see, for instance, [52]–[56]).

Higgsless models [19], which have not been covered in this paper due to constraints on scope, require a much lower scale that can be made compatible with EWPT at the tree level by localizing the light fermions close to $c = 1/2$ [57], for which all corrections become small, and imposing the custodial protection of the $T$ parameter and the $Z\bar{b}b$ coupling [58] (for a review, see [59] or section 3 of [60]).

5. A few remarks on flavor (constraints)

As remarked on in the introduction, the RS framework with bulk fields provides a theory of flavor. The main idea [7] arises from the $c$-dependence of the fermion zero-mode wavefunctions (18); for $c > 1/2$, the profiles are localized near the UV brane, and therefore the overlap with an IR-localized Higgs field (so as to address the hierarchy problem) is exponentially suppressed. This allows one to easily generate hierarchical effective 4D Yukawa couplings, even if the microscopic 5D Yukawa couplings exhibit no special structure, an assumption usually called ‘anarchy’ (in other scenarios, such as gauge–Higgs unification, the fermion–Higgs interactions depend on localized fermion mixing mass parameters to which the anarchy assumption can be applied). In warped spaces, with their associated $z$-dependence of the cut-off scale, (light) fermion localization near the UV brane also provides an effective cut-off on non-renormalizable flavor-violating operators that is sufficiently high to significantly suppress their effects, even with the new KK physics entering at a few TeV.

The consequences of ‘anarchy’ for flavor physics were considered originally in [10]–[14], and more recently in [61]–[69]. The largest effects arise at the tree level from flavor changing KK gluon couplings. Although most of the predictions are consistent with current flavor constraints for the scales determined by the EWPT—and often lead to interesting expectations for indirectly observing the new physics in the near future—there are some observables that can impose more severe bounds. Most notably, CP violation in the kaon system can lead to a bound of $O(10)$ TeV on the KK scale [61].

Thus, a strict application of the anarchy assumption can result in a KK scale beyond the reach of the LHC. This may suggest some nontrivial structure (for instance, flavor symmetries), see, e.g., [70]–[78], or perhaps that these models exhibit a moderate amount of fine tuning (hopefully in the flavor sector, so that discovering the new physics at the LHC remains a possibility). Given the model dependence of such conclusions, we take the more robust bounds from EW precision measurements as a starting point to summarize the LHC collider phenomenology of warped scenarios.

Part II. Collider phenomenology of warped models

In our presentation of the collider phenomenology of warped models, we use a different notation than that of the preceding discussion. This is largely done to match the conventions used in several of the papers that are cited in the following. In our notation $k = 1/L_0$ and the size of the extra dimension is denoted by $\pi r_c$. Also, note that, in most of what follows, the LHC center of mass energy $\sqrt{s} = 14$ TeV is assumed, unless otherwise specified. Obviously, given

\[\text{16} \] In what follows, we focus on KK signals of compactification. However, an embedding of the RS model in string theory (UV completion) could result in the appearance of additional signals; see, for example, [79, 80].
the parton-distribution-function (PDF) dependence of cross sections for production of TeV-scale KK modes, many of the conclusions presented below have to be revisited for smaller values of $\sqrt{s}$.

6. KK gravitons

We begin our discussion of KK phenomenology with the graviton sector, since KK gravitons were the first tower of states whose collider phenomenology was studied [3] and constitute a key and common signature of warped models. As noted previously, in the original RS model the entire SM content was assumed to be confined to the IR-brane. The most distinctive signature of this setup was then the tower of graviton KK modes. These states would appear as a series of spin-2 resonances close to the weak scale, which sets the scale of their masses and couplings to the SM. Specifically, the KK graviton masses are given by

$$m_G^G = x_G^G(kr_c \pi),$$

where $x_\alpha = 3.83, 7.02, \ldots$ for $n = 1, 2, \ldots$, are given by the roots of $J_1$ Bessel function, to a very good approximation [3]; the zero mode for $n = 0$ is the massless 4D graviton. The interactions of the graviton KK modes with the SM fields are given by

$$L = -\frac{1}{\bar{M}_P} T^{\alpha\beta}(x) h^{(0)}_{\alpha\beta}(x) - \frac{1}{\Lambda_\pi} T^{\alpha\beta}(x) \sum_{n=1}^{\infty} h^{(n)}_{\alpha\beta}(x),$$

with $\bar{M}_P \sim 10^{18}$ GeV the reduced Planck mass, $T_{\alpha\beta}$ the SM energy-momentum tensor, $h^{(n)}_{\alpha\beta}(x)$ KK modes of the graviton, and $\Lambda_\pi \equiv e^{-kr_c \pi} \bar{M}_P$ [3].

The best direct limits on the original RS model are from the Tevatron experiments. We present recent limits from the CDF [81] and D0 [82] experiments in figures 2 and 3. The graviton KK phenomenology of the original RS model can be described by the KK mass and the ratio $k/M_P$. Calculations in this background are reliable as long as $k/M_5$ is not $O(1)$ or larger (this is a classical argument that can be somewhat modified if we consider quantum effects [83]). For this reason, $k/M_5 \lesssim 1$ is often considered in phenomenological and experimental studies, as seen, for example, in these figures. We see that the direct bounds on the mass of the lightest KK graviton range over 300–900 GeV for $0.01 \lesssim k/M_P \lesssim 0.1$. The LHC reach for the first graviton KK mode in this model was calculated by the ATLAS [84] and CMS [85] collaborations and is roughly 3.5 TeV, with 100 fb$^{-1}$ of integrated luminosity.

Once we extend the RS setup to provide a model of flavor, the above conclusions regarding the graviton KK phenomenology at colliders change drastically. The current Tevatron bounds do not take these effects into account; however, we generally expect the direct bounds to become much weaker. This is due to the fact that, in warped flavor models, KK graviton couplings to UV-localized light fermions become highly suppressed [86], roughly like their Yukawa couplings [83], and couplings to gauge fields (in units of $1/\bar{M}_P$) [86]

$$C_{\alpha\beta}^{\Lambda_\pi} = e^{-kr_c \pi} \frac{2 \left[ 1 - J_0 (x_G^G) \right]}{kr_c \pi \left( x_G^G \right)^2 |J_2 (x_G^G)|},$$

where $J_{0,2}$ denote Bessel functions, become volume $(kr_c \pi)$ suppressed. Hence, as alluded to before, important production and decay channels become either inaccessible or suppressed. Given that a TeV-scale graviton KK tower is the most generic prediction of the RS model,
largely independent of various model building assumptions, it is worth reconsidering its phenomenology within warped flavor models.

The reach of the LHC experiments for the lightest graviton KK mode was accordingly reexamined in [83, 87]. With the light fermions basically decoupled, the dominant production channel at the LHC is through gluon fusion, which is suppressed by $\frac{k_r^c}{\pi}$. As for the dominant decay channels, generically we expect the right-handed top quark and the Higgs sector, including the longitudinal gauge fields $Z_L$ and $W_L^\pm$, to be the only important final states, due to their large overlap with the graviton KK wavefunction. With these assumptions, the partial

Quite often (however not as a rule [45]), it is the right-handed top quark that is the most IR-localized SM fermion, given that the doublet $(t, b)_L$ is subject to precision bounds on $b$-quark couplings, as noted in the preceding discussion of precision constraints.

**Figure 2.** CDF limits (95% confidence level) on the product of cross section and dimuon branching fraction, for the lightest KK graviton in the original RS model. Theoretical cross sections and expected limits from simulated experiments (SE) are also shown. Reprinted from [81] with permission from the American Physical Society. Copyright 2009 by the American Physical Society.

**Figure 3.** D0 bounds on the lightest KK graviton mass, as a function of $k / \bar{M}_P$, in the original RS model. Reprinted from [82] with permission from the American Physical Society. Copyright 2008 by the American Physical Society.
widths of the first KK graviton into pairs of $t_R$, the Higgs boson $h$, $Z_L$, and $W_L^\pm$ are given, respectively, by [83, 87]

$$\frac{\Gamma_{t_R}}{3N_c} = \Gamma_h = \Gamma_{Z_L} = \frac{\Gamma_{W_L}}{2} = \frac{(\bar{c} x_1^G)^2 m_1^G}{960\pi},$$

(123)

where $N_c = 3$ is the number of colors in QCD, $\bar{c} \equiv k / M_R$ (our barred notation for this parameter differs from that of [83]), to avoid confusion with the bulk fermion mass parameter $c$, discussed before), and final states have been treated as massless, which is a good approximation due to the much larger expected mass of the KK mode. Also, the decay widths to longitudinal polarizations have been estimated using the equivalence theorem (which is valid up to $M_R^2/(m_1^G)^2$ corrections) to relate these widths to that of the physical Higgs. Here, we note that two-body decay into final states that include a KK mode and a heavy SM mode are (i) volume suppressed, when the relevant zero mode has an approximately flat profile, or (ii) kinematically forbidden (due to a large KK fermion mass), or else (iii) suppressed by a power of $(H)$. If we assume that $t_R$ is highly IR-brane-localized, (ii) applies to an otherwise potentially important $G_1 \rightarrow t_R t_R$.

Given the above considerations, Fitzpatrick et al [87] focused on the $t_R t_R$ final state to estimate the LHC reach for the lightest KK graviton. One of the issues that affects the utility of this channel is the efficiency for top identification, made more challenging because of the collimated decay products of highly boosted tops. We discuss this issue in greater detail in later sections. The narrow width of the KK graviton, under the above assumptions, can be helpful in limiting the background in this channel. However, one has to account for smearing of the measured resonance mass.

In [87], the background was taken to be all $t \bar{t}$ with invariant mass within 3% of $m_1^G$, corresponding to a typical smearing $E \times 3\%$ for the ATLAS experiment. The top identification efficiency was treated as uncertain in this work and the reach, given by $S/\sqrt{B} = 5$, assuming 100 fb$^{-1}$ of integrated luminosity, was considered for three values of efficiency, 1, 10 and 100%, as presented in figure 4. These results suggest that the reach for $G_1$ may be expected to be 1.5–2 TeV, depending on top identification efficiency.

Given that the longitudinal gauge bosons $Z_L$ and $W_L^\pm$ are manifestations of the IR-localized Higgs sector Goldstone modes, their couplings to the graviton KK modes are substantial, as deduced from equation (123). In particular, the process $pp \rightarrow G_1 \rightarrow Z_L Z_L$, with both $Z$s decaying into $\ell^\pm = e^\pm, \mu^\pm$, provides a clean signal, unencumbered by complicated event reconstructions. In the absence of prompt di-lepton and di-photon channels, this process provides a ‘golden mode’ for KK graviton discovery. Based on these considerations, Agashe et al [83] examined the possibility of searching for warped gravitons in the $Z_L Z_L$ decay channel.

In [83], it was determined that the vector boson fusion cross section for KK graviton production is roughly an order of magnitude smaller than the gluon fusion channel considered above. The dominant SM background to $ZZ$ production though comes from $q \bar{q}$ initial states. Hence, one expects forward–backward cuts on pseudo-rapidity $\eta$ to be efficient in reducing the background, mediated via $t/\mu$ channels, while not affecting the signal a great deal; this was shown to be case in [83], typically yielding $S/B$ significantly larger than unity. Note that due to negligible initial state $q \bar{q}$ coupling to $G_1$, the signal and background do not interfere to a good approximation.

The reducible SM background to the above KK graviton signal depends on the decay modes of the $Z$s. For hadronic decays of both $Z$s, the four-jet QCD background was deemed too large to allow this channel to be of use. Even the case with one $Z$ decaying hadronically...
and the other decaying into $\ell^\pm$ poses a serious challenge. This is due to the large boost of the $Z$s (similar to $t$s in the previous discussion) which makes the opening angle of signal di-jets of the order of $M_Z/\text{TeV} \sim 0.1$ [83], while typical cone size for jet reconstruction was taken to be $O(0.4)$ (as quoted from [88]). This makes the relevant SM background come from $Z + j$, which was calculated to be an order of magnitude larger over the resonance width. This problem of jet-merging in the decays of highly boosted particles is a generic challenge in searches for TeV scale resonances that preferentially decay into heavy SM final states, and will be encountered in our later discussions of other KK modes.

Based on the clean 4$\ell$ final state, the signal and background were calculated over the KK graviton width $\Gamma_G$, as shown in figure 5. We can see from the plots that the $\eta$-cut significantly improves $S/\sqrt{B}$; the dependence of background on $\bar{c}$ comes from the dependence $\Gamma_G \propto \bar{c}^2$ that sets the integration interval. Naive dimensional analysis suggests that higher curvature terms are suppressed by $\Lambda \sim 24^{1/3} \pi M_5$ [83], which is why values of $\bar{c} > 1$ were considered. Agashe et al [83] concluded that for $\bar{c} \lesssim 2$ (for reliable calculations), the LHC reach ($S/\sqrt{B} > 4$) for $G_1$ is about 2 (3) TeV with 300 (3000) fb$^{-1}$.

Figure 4. The reach ($S/\sqrt{B} = 5$) for the lightest KK graviton $G_1$ with 100 fb$^{-1}$ of integrated luminosity and top identification efficiency 100, 10 and 1%, clockwise from the top. With increasing $v_{tR}$ (corresponding to $-c_{tR}$ in the notation of part I) the profile of $t_R$ is more IR localized; $M_4L$ corresponds to $1/\bar{c}$ in our notation. Reprinted from [87] with permission from SISSA.
Another potentially important channel for KK graviton discovery is via the $W^\pm L$ final state, discussed in [89]. These authors note that the KK graviton has a branching fraction into the $W^\pm$ that is twice that of the $Z_L Z_L$ final state and that the branching fraction of the $W^\pm$ into a leptonic final state is about 11% compared to 3% for the $Z$. However, when the highly boosted $W$s decay into leptons, the neutrinos will be mostly back-to-back and the missing energy information will be lost [83, 89]. In addition, the hadronic decay of one of the $W$s will again be collimated and be subject to a large $W^+ j$ background [83, 89]. The results of [89] suggest that the reach for the KK graviton in the $W^\pm_L$ channel with $300 \text{ fb}^{-1}$ will not be significantly above 2 TeV, unless analysis techniques are developed to suppress the $W^+ j$ background.

If a graviton KK mode is detected at the LHC it will be important to establish its spin. Antipin and Soni [90] considered whether this is feasible, in models that yield realistic flavor using bulk fermions, and concluded that, with 1000 $\text{ fb}^{-1}$, the KK graviton spin for masses up to $\sim 2$ TeV may be confirmed. The authors of [85] and [91]–[93] also considered this question, but in the original RS model with the entire SM on the IR brane, where $e^\pm$ and $\mu^\pm$ decay modes are accessible. Using the ‘center-edge asymmetry’ in the angular distribution of the final state leptons, Osland et al [92] found it feasible to identify the graviton spin at the $2\sigma$ level, for $\bar{c} = 0.1$, up to a mass of 3.2 TeV, with 100 $\text{ fb}^{-1}$. However, we again note that the original RS model is subject to stringent bounds on cut-off-scale operators which, in the absence of tuning, would require $m_G^1 \gg 1$ TeV.

Simple warped models predict that the lightest graviton is $3.83/2.45 \simeq 1.56$ times heavier than the lightest KK gauge boson. Given the preceding discussion of precision data, it is then expected that $m_G^1 \gtrsim 4$ TeV, corresponding to the yellow regions in figure 5. Therefore, the above analyses suggest that, in generic models, discovering the warped KK graviton at the LHC is quite challenging, at design or perhaps even upgraded (SLHC [94, 95]) luminosity.

7. KK gluons

The KK gluons $g_n$ generally offer the best reach for discovery at the LHC, as indicated by the results of [96, 97] that we summarize below; we do not present the details of the analysis in
these works and refer the interested reader to these references for more information. First of all, the level-1 KK gauge fields are generally expected to be the lightest such excitations, within the known (SM+gravity) sectors; the KK masses for a gauge field $A$ are given by equation (120), with $x^G_n \rightarrow x^A_n \approx 2.45, 5.57, \ldots$. Secondly, the $SU(3)_c$ coupling constant $g_s$ of the SM gluon is larger than those of other SM gauge fields. This leads to a larger production cross section, which is proportional to $g_s^2$, for the KK gluons. However, in realistic models of flavor, the couplings of $g_1$ to light flavors are diminished compared to its SM counterpart ($g_0$, the gluon), suppressing production through light quarks at colliders; the couplings to the gluon of the SM are zero by orthonormality of KK modes. On the other hand, the dominant branching fraction is set by coupling to IR-localized top quarks, which is enhanced compared to the SM gauge coupling. 

The orthogonality of KK modes. On the other hand, the dominant branching fraction is set by coupling to IR-localized top quarks, which is enhanced compared to the SM gauge coupling. This leads to a large width for KK gluons, which is enhanced compared to the SM gauge coupling. 

The work of Agashe et al. [96] focused on a setup in which $Q^3 = (t, b)_L$ was quasi IR-localized, $t_R$ was basically on the IR-brane, and all other fermions $f$ were UV-localized. Let $\xi \equiv \sqrt{\log(M_p/\text{TeV})} \approx \sqrt{k \tau_c} \sim 5$. Then the couplings of the lightest KK gauge field $A^{(1)}$ are given by the following approximate relations [96]

$$
\frac{g^{fA^{(1)}}}{g_{SM}} \approx \frac{1}{\xi}, \quad \frac{g^{Q^3Q^3A^{(1)}}}{g_{SM}} \approx 1, \quad \frac{g^{f\bar{f}A^{(1)}}}{g_{SM}} \approx \xi, \quad \frac{g^{A^{(0)}A^{(0)}A^{(1)}}}{g_{SM}} \approx 0, \quad \frac{g^{HHA^{(1)}}}{g_{SM}} \approx \xi, \quad (124)
$$

where $g_{SM}$ denotes a generic SM gauge coupling, and $A^{(0)}$ is a gauge field zero mode. Note that $H$ includes both the physical Higgs ($h$) and unphysical Higgs, i.e. longitudinal W/Z by the equivalence theorem (the derivative involved in this coupling is similar for RS and SM cases and hence is not shown for simplicity). For the case of KK gluons, the last coupling in equation (124) does not exist and the fourth equality is exact, since $SU(3)_c$ is unbroken. Here, the effects of EWSB, expected to be small, are ignored. We note that, in alternative models that possess a custodial symmetry to protect the coupling $Zb\bar{b}$, one can arrange for the couplings of $Q^3$ and $t_R$ to be interchanged, or for both to have intermediate values [16].

At the LHC, the dominant $g_1$ production is through the $u\bar{u}$ and $d\bar{d}$ initial states. However, the background $t\bar{t}$ is mainly from $gg$ fusion and is more forward-peaked. Hence, a $p_T$ cut suppresses the background more than the signal. In [96], the preferred reconstruction mode was $tt \rightarrow b\bar{b}jj\ell\nu$. In order to minimize the impact of PDF uncertainties, this work focused on the differential cross section as a function of the $t\bar{t}$ invariant mass $m_{t\bar{t}}$ and looked for a ‘bump’ in this distribution; due to the large width of the KK gluon a sharp resonance is not expected. Particle and parton level results from [96] are shown in the left panel of figure 6. With a total efficiency of order 1%, including $b$-jet-tagging (20%) and cut (about 20%) efficiencies, as well as the branching fraction to the final state (0.3), it was concluded that $S/B = 11.0 \ (4.2)$ and $S/B = 2.0 \ (1.6)$ for $m_{t\bar{t}} = 3 \ (4)$ TeV are achievable for 100 fb$^{-1}$ at the LHC [96]. It is then concluded that the LHC reach for the KK gluon $g_1$ is $m_{t\bar{t}} \sim 4$ TeV.

It is also noted in [96] that the enhanced coupling of $t_R$ (in this setup) to KK gluons provides an important handle on the signal since, whereas in the warped model the decay of the KK mode is largely dominated by $t_R$, the SM background is dominated by QCD, which is left–right symmetric. Here, due to the large boost of the top quark, its chirality is preserved and can be deduced from its decay products; leptons tend to be forward (backward)-emitted, in the rest frame of $t_{R(L)}$, relative to the direction of the top quark boost. Defining

$$
P_{LR} = 2 \times \frac{N_+ - N_-}{N_+ + N_-}, \quad (125)
$$

where $N_{\pm}$ is the number of forward/backward positrons in the above sense, the right panel of figure 6 shows a distinct asymmetry which can be correlated with bumps in the invariant mass distribution. Note that in the above setup, the much smaller SM asymmetry, mostly from the left-handed weak coupling, has the opposite sign in the signal region [96].

An independent study regarding the LHC reach for $g_1$ was performed in [97], using assumptions about the fermion profiles similar to that discussed above. This resulted in nearly the same level of suppression in the production, dominance of the $t_R$ branching fraction and total width. Here, as in [96], it was recognized that, for reasonable values of $m_{g_1}^{\pm}$, the highly boosted final state tops will have collimated decay products. Hence, the usual techniques of top identification will not be applicable and new methods have to be developed to gain control over the background.

In [97], the effects of potential methods for background suppression, generally requiring a detailed knowledge of jet morphology and detector simulations, were parameterized by a range of signal dilution factors. In figure 7, the $t\bar{t}$ mass distribution for $m_{g_1}^{\pm} = 2 (5) \text{ TeV}$ is presented in the left (right) panel. Here, the two-object $t\bar{t}$ final states are subject to QCD di-jets and $b\bar{b}$ backgrounds that resemble the signal. The various curves in each panel represent different levels of fake rate and suggest that signal extraction is possible if a background rejection factor of about 10 is achieved. Lillie et al [97] conclude that, based on their analysis, given 100 fb$^{-1}$, a conservative LHC reach for $g_1$ is about 5 TeV. Note that this is a somewhat more optimistic conclusion than that reached above. The authors of [97] attribute this to the different method, based on strong cuts, adopted in [96] to identify hadronic tops that could result in a more limited reach. Here, we note that without a more detailed study based on jet structure and realistic simulations of background rejection, it is difficult to determine how conservative this extended reach is. Given that realistic values of KK gluon mass lie above 2–3 TeV, we see that in either case the LHC can potentially access interesting values of $m_{g_1}^{\pm}$. Identifying boosted tops are also studied in [98, 99].

The spin of the KK gluon provides another handle on the signal. The prospects for establishing the $1 + \cos^2 \theta$ behavior of the signal top angular distribution was examined in [97].
Figure 7. $t\bar{t}$ invariant mass distribution for signal plus some background, given different values of the top-tagging fake rate, for $m_t^R = 2(5)$ TeV in the left (right) panel.

While this distribution has a characteristic forward-peaked structure, the background is also strongly forward-peaked. Strong $p_T$ cuts on reconstructed tops or tagged top-jets could largely suppress the background and enhance $S/B$. However, we note that a good grasp of top-tagging efficiency and background rejection is needed before a firm conclusion can be made. This work also considered the possibility of establishing the chiral structure of the gluon KK coupling, in a fashion similar to that of the preceding discussion, concluding that it is feasible to do so for $m_t^R$ up to 2–3 TeV, above which lepton isolation cannot be effectively achieved, making the requisite measurements more challenging.

Djouadi et al [100] consider $t\bar{t}$ final state resulting from the production of the gauge KKs. The observables are dominated by the KK gluon, since its cross section is much bigger than the EW KKs. Similar to [96], they show that the forward–backward asymmetry of the top quark can be significantly altered from the SM prediction. Moreover, Djouadi et al [101] explain the 2σ deviation in the $t\bar{t}$ forward–backward asymmetry at the Tevatron by a 3 TeV KK gluon contribution, while the long-standing 3σ discrepancy in the $b$-quark forward–backward asymmetry is improved by the EW gauge KK contributions [102].

Associated production. Guchait et al [103] and Djouadi et al [100] consider the associated production of the KK gluon with either $t\bar{t}$ or $b\bar{b}$, with the KK gluon decaying to $t\bar{t}$. Taking into account irreducible backgrounds, they find that the reach at the LHC with 100 fb$^{-1}$ is about 3 TeV in this channel. They point out that optimizing the cuts might lead to a better reach, although dealing with combinatorics for the four-top final-state is no trivial matter.

New decay channels. As mentioned in Part I, in models with custodial and $Zbb$ coupling protection, precision constraints seem to favor regions of parameter space where some of the KK excitations of the top quark are relatively light. The KK gluon can then decay into a pair of KK top ($t^1$) states with a significant branching fraction, and become a relatively broad resonance due to its large coupling to $t^1$ [47].

Before closing this section, we note that, during initial phases, the LHC may operate at or below $\sqrt{s} = 10$ TeV. Given that the KK gluon is a promising mode of discovery for warped scenarios, here we would like to provide an estimate of the LHC reach for this state at the lower center of mass energy. As a check for our estimate, first we consider the reach at

---

\[
\sqrt{s} = 14 \text{ TeV}. \text{ Consistent with the aforementioned results of [96], we find for } m_t^V = 3 \text{ TeV that the luminosity required at the LHC for a 5 } \sigma \text{ significance at } \sqrt{s} = 14 \text{ TeV is about } 25 \text{ fb}^{-1}, \text{ for cuts and efficiencies given therein. For } \sqrt{s} = 10 \text{ TeV, using a Monte Carlo simulation, we find that the luminosity required for 5 } \sigma \text{ discovery increases to about } 115 \text{ fb}^{-1} \text{ for similar cuts and efficiencies.}
\]

8. **EW KK states (KK \( W \) and \( Z \))**

We restrict our discussion here to models where the EW gauge group in the bulk is taken to be \( SU(2)_L \otimes SU(2)_R \otimes U(1)_X \), with hypercharge being a linear combination of \( U(1)_R \) and \( U(1)_X \). The extra \( SU(2)_R \) (relative to the SM) ensures suppression of the contributions to the EWPT (specifically the \( T \) parameter) [15]. The detailed theory and LHC phenomenology of the neutral EW states are presented in [104], and that of the charged EW states in [105] from which we will summarize the main aspects of their results. For just the SM gauge group in the bulk, see [106] for a discussion of the AdS/CFT correspondence and LHC signatures.

In these theories, there are three neutral EW gauge bosons denoted as \( W^3_L \), \( W^3_R \) and \( X \), and two charged gauge bosons denoted as \( W^\pm_L \) and \( W^\pm_R \). The \( SU(2)_R \otimes U(1)_X \rightarrow U(1)_Y \) symmetry breaking by b.c.s leaves one combination of zero-modes in \( (W^3_R, X) \) massless (the hypercharge gauge boson \( B \)), while rendering the orthogonal combination \( (Z_X) \) massive. In the charged sector, this breaking leaves the \( W^\pm_R \) without a zero mode. The SM Higgs doublet is promoted to a bi-doublet of \( SU(2)_L \otimes SU(2)_R \) with zero \( U(1)_X \) charge and, as in the SM, is responsible for \( SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{EM} \) symmetry breaking by the Higgs VEV. This leaves one combination of zero modes in \( (W^3_R, B) \) massless (the photon \( A \)), while making the orthogonal combination \( (Z) \) massive, and in the charged sector making the \( W^+_L \) massive.

The bulk gauge fields can be expanded as a tower of KK states. In each of these neutral and charged tower states, we restrict to the zero and 1st KK modes only. The zero mode is the SM, and we denote the 1st level KK neutral states by \( A_1, Z_1 \) and \( Z_{X1} \), and the charged by \( W^+_{L1} \) and \( W^+_{R1} \).

EWSB mixes these states and the resulting mass eigenstates are denoted by the neutral \( \tilde{A}_1, \tilde{Z}_1 \) and \( \tilde{Z}_{X1} \), and the charged \( \tilde{W}_{L1} \) and \( \tilde{W}_{R1} \). We also refer to the heavy neutral mass eigenstates collectively as \( \tilde{Z}' \), and the charged as \( W' \). Note that the (EW preserving) KK masses for the first KK states (for both the neutral and charged KK states) are quite degenerate, such that the EWSB mixing (mass)\(^2\) term is larger than KK (mass)\(^2\) splitting for \( m_{KK} \lesssim 3.5 \text{ TeV} \). Hence, for the interesting range of KK masses, we expect large mixing between \( Z_1 \) and \( Z_{X1} \) in the neutral sector (the \( A_1 \) does not mix), with the heavy mass eigenstates roughly 50–50 admixtures of \( Z_1 \) and \( Z_{X1} \) and a small admixture of \( Z(0) \). The lightest mass eigenstate is of course identified as the SM \( Z \) boson and is mostly the \( Z(0) \). Similarly, in the charged sector, the two heavy mass eigenstates are a large mixture of \( W^+_{R1} \) and \( W^+_{L1} \), with a small admixture of \( W^{(0)+} \). Although the mixings between the first and zero levels are small, it is important to keep these effects in \( Z' \) and \( W' \) decays to SM gauge bosons, since they lead to \( O(1) \) effects when the small mixings are overcome by the enhancement due to the longitudinal polarization of the energetic \( Z \) and \( W \) in the final state (as expected from the equivalence theorem).

**Couplings.** As already mentioned, warped models can naturally explain the SM fermion mass hierarchy. In such models, as shown schematically in equation (124), ignoring small EWSB effects, the light SM fermions have a small coupling to all KKs (including graviton) based simply on the overlaps of the corresponding profiles, while the top quark and Higgs
have a large coupling to the KKs. The exact couplings including EWSB effects are presented in [104, 105].

*Phenomenology.* Here we summarize the main results of [104, 105], which studied the KK $Z$ and $W$ comprehensively. The results presented below are for the following choices of fermion bulk mass parameters: $c_{Q_L} = 0.4$, $c_{t_R} = 0$, and all the other $cs > 0.5$, and for $\xi = \sqrt{k_\pi r_c} = 5.83$.

The $Zb\bar{b}$ coupling can be protected [16] against excessive corrections by making the third generation quarks bi-doublets under $SU(2)_L \otimes SU(2)_R$, by including extra non-SM fermions to complete the representation. The $W'$ can decay to some of these extra non-SM fermions, which is taken into account in computing the BRs in [105], but as a first-step the analysis is based on SM final states only.

The total widths of the KK $Z$ and $W$ are typically about 5–10% of their mass. For reasons already mentioned, they predominantly decay into heavier fermions (top) and to the Higgs (including $W_L$ and $Z_L$, the longitudinal modes). The main decay channels are $Z' \rightarrow t\bar{t}$, $W_LW_L$, $Z_Lh$ and $W' \rightarrow t\bar{b}$, $W_LZ_L$, $W_Lh$. Although the $t\bar{t}$ BR can be large, the KK gluon is degenerate with the EW states and has a much larger cross section into the $t\bar{t}$ channel, rendering this channel not very useful as a probe of the EW KKs.

*LHC signatures.* We summarize here the LHC signatures analyzed in [104, 105]. The total cross section for $pp \rightarrow Z'$ and $W'$ at the LHC with $\sqrt{s} = 14$ TeV are shown in figure 8 as a function of its mass. Drell–Yan-type production is dominant, with the vector boson fusion channel about an order of magnitude smaller. We consider next the dominant decay modes, their signatures at the LHC and the reach for their discovery.

Owing to the large mass of the KK $Z$ and $W$, the SM final states they decay into are significantly boosted, resulting in their decay products being highly collimated in the lab-frame.

![Figure 8.](http://www.njp.org/)
Figure 9. Jet-mass distributions for the $W \rightarrow jj$ versus QCD jet for a cone size of 0.4, with and without $E$, $\eta$ and $\phi$ smearing (left), and for $t$ versus $b$-jet for a cone size of 1.0 (right). Both are for a 2 TeV KK gauge boson. Reprinted from [104, 105], respectively with permission from the American Physical Society. Copyright 2007 and 2009 by the American Physical Society.

For example, for the semi-leptonic $Z' \rightarrow WW$, the presently typical jet reconstruction cone size of $\Delta R = 0.4$ will cause the two jets from the $W$ decay to probably be reconstructed as a single jet (albeit a fat jet), impeding our ability to reconstruct a $W$ mass peak. This means that we would pick up an SM QCD jet background which is typically large, and would require special consideration to keep it from overwhelming the signal. In order to discriminate the merged jets of the $W$ from a QCD jet, one can use the jet mass, which is the combined invariant mass of the vector sum of four-momenta of all hadrons making up the jet, as shown in figure 9 (left). Techniques to discern QCD jets are also studied in [107, 108]. Although the two jets from $W \rightarrow jj$ are severely overlapping in the hadronic calorimeter, it may be possible to utilize the better granularity of the electromagnetic calorimeter and the tracker to resolve ‘sub-jets’ [109] and obtain a reasonable discriminating power against QCD. Similarly, in the $W' \rightarrow t\bar{t}b$ channel, $t\bar{t}$ production can become a source of background since a highly collimated top can resemble a $b$-jet. A top and a $b$-jet can again be discriminated against by using the jet-mass variable as shown in figure 9 (right).

Kinematic cuts can be applied to maximize the signal and suppress the background. We refer the reader to [104, 105] for distributions of various kinematic variables for many of the dominant final states and cuts based on them. We summarize, in table 1, the LHC reach after suitable cuts for the $Z'$ and $W'$ found in those studies, where the number of events is small, Poisson statistics is used to find the significance and the equivalent Gaussian significance is quoted. We see that upwards of 300 fb$^{-1}$ is needed to probe a 3 TeV KK $Z$ or $W$ state.

In what we have discussed so far, owing to the flavor connection, the BRs into experimentally clean leptonic channels are quite small (about $10^{-3}$), rendering them useless, and one had to work with more complicated final states. However, if warped models are taken as a theory of flavor alone (and not generating the gauge-hierarchy), the UV scale can be lowered to as low as $O(10^3)$ TeV [17] (a more detailed study of flavor constraints in [110] finds the minimum UV cut-off to be several $10^3$ TeV) instead of the usual Planck scale. Such ‘truncated’ models result in $Z'$ leptonic BRs that are big enough to lead to a very good significance at the LHC, in the di-lepton channel [17, 111]. A big leptonic cross section due to the $Z'$ exchange is also found in [112], where a model is presented in which the left-handed light quarks and leptons are peaked toward the IR brane, giving much larger light fermion couplings to
Table 1. Summary of the best channels (from [104, 105]) for the $Z'$ (top-table), and, $W'$ (bottom-table), giving the luminosity and significance for the mass shown. For the $W' \rightarrow t \bar{b}$ channel the numbers without (and with) the reducible $t \bar{t}$ background are shown.

<table>
<thead>
<tr>
<th>$Z'$ channel</th>
<th>$M_{Z'}$ (TeV)</th>
<th>$L$ (fb$^{-1}$)</th>
<th>$S/B$</th>
<th>Significance ($\sigma$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W W \rightarrow \ell \nu j j$</td>
<td>3</td>
<td>1000</td>
<td>0.2</td>
<td>4.6</td>
</tr>
<tr>
<td>$m_h = 120$: $Z h \rightarrow \ell \ell b \bar{b}$</td>
<td>3</td>
<td>1000</td>
<td>2</td>
<td>5.7</td>
</tr>
<tr>
<td>$m_h = 150$: $Z h \rightarrow (jj)(jj)\ell\nu$</td>
<td>3</td>
<td>300</td>
<td>1.2</td>
<td>4.7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$W'$ channel</th>
<th>$M_{W'}$ (TeV)</th>
<th>$L$ (fb$^{-1}$)</th>
<th>$S/B$</th>
<th>Significance ($\sigma$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t b \rightarrow \ell v b \bar{b}$</td>
<td>2</td>
<td>1000</td>
<td>0.4 (0.2)</td>
<td>3.4 (2.5)</td>
</tr>
<tr>
<td>$Z W \rightarrow \ell \ell \ell \nu$</td>
<td>3</td>
<td>1000</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>$m_h = 120$: $W h \rightarrow \ell v \tilde{b} \tilde{b}$</td>
<td>3</td>
<td>300</td>
<td>2.4</td>
<td>6.2</td>
</tr>
<tr>
<td>$m_h = 150$: $W h \rightarrow (jj)\ell\nu(jj)$</td>
<td>3</td>
<td>300</td>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

the $Z'$. However, this comes at the price of requiring the bulk mass $c_L$ parameters for the three generations to be highly degenerate in order to keep FCNCs under control.

9. KK fermions

The presence of KK excitations of SM fermions is a generic prediction of warped models with bulk fermions, and their discovery would be a 'smoking gun' signature. For example, see [113] on how the KK fermion spectrum correlates with the SM fermion masses. However, Davoudiasl et al [114] point out that discovering these modes would probably be possible only in a future collider and not at the (even upgraded) LHC. This is due to the small single-production cross section and their large mass, as they are heavier than gauge KK modes that are constrained by precision EW measurements. For a model independent analysis at the Tevatron of heavy vector-like fermions with significant mixings with SM fermions, see [115].

As explained earlier, in order to obtain custodial protection of $Z b \bar{b}$ coupling, the third generation quarks can be extended to be bi-doublets under $SU(2)_L \otimes SU(2)_R$, which then necessitates the introduction of extra non-SM fermions to complete the representation [16]. The presence of these non-SM fermions is model dependent, but some of them could be fairly light (100s of GeV) [15, 116] (see [117] for sample numerical values). Several works have considered the discovery of these light non-SM fermions at the LHC. In models with custodial protection of $Z b \bar{b}$, the $t_R$ could be in either a singlet of $SU(2)_L \otimes SU(2)_R$, or be in $(1, 3) \oplus (3, 1)$. In the latter case, one of the custodial partners of the $t_R$, denoted $\tilde{b}_R$, can be fairly light, and Dennis et al [118] consider its LHC signatures by looking at 4-$W$ events. With 10 fb$^{-1}$, they show that the peak in the di-jet mass distribution stands out above the background. Contino and Servant [119] consider the single and pair-production of the custodial partner $T_{5/3}$ (of the SM left-handed quark doublet) at the LHC. In figure 10, we show from [119] the cross-sections as a function of its mass. They show that in the same-sign di-lepton channel, discovery at the LHC could come with less than 100 pb$^{-1}$ (20 fb$^{-1}$) of integrated luminosity for a mass of 500 GeV (1 TeV). See also [120] for a study of LHC signatures of vector-like quarks with these quantum numbers in single and multi-lepton channels. In models with custodial protection, a light top
Figure 10. Production cross sections at the LHC for $T_{5/3}$ as a function of its mass. The dashed line refers to pair-production; the solid and the two dotted curves refer to single production for the three values of the coupling. Reproduced from [119] with permission from SISSA.

KK mode $t^l$ plays an important role in relaxing the constraints from EWPT [9, 45, 50]. Taking into account the $s$-channel KK gluon exchange, which interferes constructively with the gluon diagram, this mode may be detected via pair production up to a mass of 1.5 TeV, at the LHC with about 300 fb$^{-1}$ [47].

Bouchart and Moreau [121] include the effects of mixing between SM fermions with KK excited fermions and mixing in the gauge sector and show that this leads to a better fit to EW observables compared to the SM, including explaining the discrepancy in $A_{FB}^{b}$. The effects from fermion mixing in $A_{FB}^{b}$ are in addition to the contribution from the KK gluon already discussed in section 7. They find, in this class of models, that quite large values of the Higgs mass (about 500 GeV) still give acceptable EW fits.

10. The radion

The radion is a scalar field associated with fluctuations in the size of the extra dimension. The mass of the radion is dependent on the mechanism that stabilizes the size of the extra dimension. This was first achieved in a simple model with a bulk scalar (with its own dynamics and constraints) in [2], where it can be shown that, generically, the radion may be expected to be the lightest new state in an RS-type setup [122]. This stabilization mechanism was further elaborated in [123, 124]; for an alternative mechanism based on Casimir energy associated with a compact dimension see, for example, [125] (an earlier attempt can be found in [126]). The radion interactions with SM fields, being of 5D gravitational nature, arise through operators of dimension-5 and higher characterized by a scale $\Lambda \sim \text{TeV}$ [122]. For the most part, the radion couplings are similar to Higgs couplings. The radion mass is expected to be a few tens to hundreds of GeV, to have escaped detection at LEP and to be consistent with precision EW data [124, 127]. This also implies that no observable deviations from Newton’s law in torsion balance experiments are expected [128]. However, the radion field can mix with the Higgs boson after EWSB, which involves another parameter, the coefficient of the curvature-scalar term [129].

LHC signatures of the radion ($r$) have been analyzed both in the case where SM fields are IR localized (original RS model), and that where the SM is in the bulk, and the search methodology usually parallels Higgs searches. Similar to the Higgs, the main production channel is $gg \rightarrow r$, induced at the loop level in the original RS model, given for example in [129]. They find that the ratio of the radion significance (in $gg \rightarrow r$ followed by $r \rightarrow \gamma\gamma$ and $r \rightarrow ZZ \rightarrow 4\ell$) to the corresponding SM Higgs significance ($R_S$) at the LHC can vary from about ten to a hundredth as $\Lambda$ varies from 500 GeV to 20 TeV (for the coefficient of the curvature-scalar term set to zero).

For the case of the SM in the bulk there is a tree-level coupling of the radion to massless gauge bosons (including the gluon), as pointed out in [130], and the radion couplings to all SM fields including loop-induced couplings can be found in [131]. $R_S$ from [131] is shown in figure 11 and its variation is similar to that of the original RS case. Here, the curvature-scalar term is assumed to be zero. The $\gamma\gamma$ channel (left) and $ZZ \rightarrow 4\ell$ (right) are shown both for the SM in the bulk (solid lines) and SM fields IR localized (dashed lines). The dependence on $\Lambda$ is also shown. Toharia [132] shows that the radion BR into $\gamma\gamma$ can be quite dramatically enhanced in the bulk SM case for nonzero curvature–scalar coupling.

Azatov et al [133] explore an FCNC observable mediated by the radion $r \rightarrow tc$ at the LHC and find that an interesting region of parameter space can be probed with 300 fb$^{-1}$. For a related process, Agashe and Contino [68] have shown that the Higgs FCNC $BR(h \rightarrow tc)$ can be about $5 \times 10^{-3}$ in warped models. They also find that $BR(t \rightarrow hc)$ can be about $10^{-4}$. These effects can be looked for at the LHC.

11. Conclusions

We began this paper with a detailed presentation of various techniques that are useful in analyzing the physics of theories with extra dimensions, particularly warped extra dimensions. These are the KK description (useful for discussing the collider phenomenology) and techniques that allow us to resum the low-energy effects of the new physics (propagator and holographic

Figure 11. The ratio of the radion significance to the SM Higgs significance in the $\gamma\gamma$ (left) and $ZZ \rightarrow 4\ell$ (right) channels at the LHC. The solid lines are for the case of the SM fields in the bulk, while the dashed are for IR-localized SM fields. Reprinted from [131] with permission from the American Physical Society. Copyright 2007 by the American Physical Society.
methods), as well as their relations. An important application is to the determination of indirect bounds on the scale of new physics from EW precision constraints. Models with bulk gauge and fermion fields (that allow addressing the flavor puzzle), together with a custodial symmetry, can be consistent with precision measurements with gauge KK excitation around 3 TeV. Additional flavor constraints (not reviewed here) can result in stronger bounds, although these can be consistent with the above-KK scale with a moderate amount of fine tuning, or with additional flavor structure.

In surveying the collider phenomenology of warped 5D models, we largely focused on their KK signals, both in the original RS model (with the SM content on the IR-brane) and in models that can explain the flavor puzzle within a 5D version of the SM. In these latter models, suppressed couplings to the SM zero modes make the collider discovery of the warped resonances significantly more difficult. In the simplest models, the KK gluons have the best prospects for discovery, up to masses of about 4 TeV, but the KK gravitons, a distinct feature of warped models, would probably lie outside the reach of the LHC experiments. Hence, the upcoming LHC experiments can probe interesting regions of parameters in realistic warped hierarchy/flavor models. Improved analysis techniques, such as those relevant to the hadronic decays of heavy boosted particles, can generally enhance the discovery prospects for the warped KK modes.

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Appendix. Low-energy expansions

We collect here some useful results for the functions involved in the calculation of EWPT in models in AdS$_5$. We will always consider $L_0 \ll L_1$ and expand to leading order in $L_0/L_1$. We also define the dimensionless parameter $\hat{m} \equiv mL_0$ (where $m$ is the mass appearing in the IR boundary conditions).

The function $K_m(p, z)$, defined by equations (25) and (26), which corresponds to the gauge boson holographic profile, is explicitly given by

$$K_m(p, z) = a(z)^{-1} \frac{J_1^c + BY^c_1}{J_0^c + BY^0_1},$$  \hspace{1cm} (A.1)

where $B = -\hat{J}_1^\text{IR}/\hat{Y}_1^\text{IR}$, and we use the definitions of the Bessel functions, equations (5) and (12), but with $m_n \to p$. The vacuum polarization for the boundary field in the holographic method is

$$K_m'(p, L_0) = -L_0\hat{m} \left[ 1 - \frac{\hat{m}}{2} \right] + p^2L_0 \left[ \log \frac{L_1}{L_0} - \frac{\hat{m}}{2} \right] + \frac{p^4 L_0 L_1^2}{2} \left[ 1 - \frac{5}{8} \hat{m} \right] + \cdots.$$  \hspace{1cm} (A.2)
For an IR-localized Higgs, it is sufficient to consider the bulk profile for $z \gg L_0$. Expanding for small momenta and small $\hat{m}$ one finds

$$K_m(p, z) = 1 - \frac{z^2}{2L_1^2} \hat{m} \left( 1 - \frac{\hat{m}}{2} \right) + p^2 \frac{z^2}{4} \left[ 1 + 2 \log \frac{L_1}{z} + \hat{m} \left( \frac{z^2}{4L_1^2} - 1 \right) \right] + \frac{p^4}{2} \frac{z^2 L_1^2}{32} \left[ 8 - \frac{z^2}{L_1^2} \left( 5 + 4 \log \frac{L_1}{z} \right) + \hat{m} \left( -5 + \frac{2z^2}{L_1^2} - \frac{z^4}{6L_1^4} \right) \right] + \cdots. \quad (A.3)$$

Similarly, the function $S(p, z)$, defined by equations (25) and (27), which is independent of $\hat{m}$, is explicitly given by

$$S(p, z) = -\frac{\pi z}{2} \left( Y_1^0 J_1^z - J_1^0 Y_1^z \right), \quad (A.4)$$

again with $m_n \to p$ in the definitions (5). It can be expanded for small $p$ as

$$S(p, z) = \frac{L_0}{2} \left( \frac{z^2}{L_0^2} - 1 \right) + p^2 L_0^3 \left[ \frac{1}{16} \left( 1 - \frac{z^4}{L_0^2} \right) + \frac{z^2}{4L_0^2} \log \frac{z}{L_0} \right] + \cdots. \quad (A.5)$$

For completeness, we also record the fermion holographic profile (see equations (40) and (41))

$$f_L(p, z) = a(z)^{-5/2} \frac{J^{c+1/2}_c + B_f Y^{c+1/2}_c}{J^{c+1/2}_c + B_f Y^{c+1/2}_c}, \quad (A.6)$$

where $B_f = -\tilde{f}^{IR}_{c+1/2}/\tilde{Y}^{IR}_{c+1/2}$ and $m_n \to p$ in the definitions (5) and (12). The comments about IR-localized terms made after equation (17) apply. The RH profile can be obtained from $pf_R(p, z) = O_c f_L(p, z)$, where $O_c$ was defined in equation (16).

Finally, in the main text we also use the auxiliary functions

$$g_n(c) = \int_{L_0}^{L_1} dz \left( \frac{z}{L_1} \right)^n a(z)^4 \left[ f^0_L(z) \right]^2 = \left( \frac{L_0}{L_1} \right)^n \frac{2c - 1}{2c - n - 1} \frac{1 - (L_0/L_1)^{2c-n-1}}{1 - (L_0/L_1)^{2c-1}}, \quad (A.7)$$

$$\tilde{g}_n(c) = \int_{L_0}^{L_1} dz \log \left( \frac{z}{L_1} \right) \left( \frac{z}{L_1} \right)^n a(z)^4 \left[ f^0_L(z) \right]^2 = \left( \frac{L_0}{L_1} \right)^n \frac{2c - 1}{(2c - n - 1)^2} \frac{1 - (2c - n - 1) \log(L_1/L_0) - (L_0/L_1)^{2c-n-1}}{1 - (L_0/L_1)^{2c-1}}. \quad (A.8)$$

Note that as $c \to +\infty$, $g_n(c) \to (L_0/L_1)^n$ and $\tilde{g}_n(c) \to (L_0/L_1)^n \ln L_0/L_1$, which are exponentially small for positive $n$. Expanding the functions of equation (102) for small $p^2$ using equation (A.3), one obtains

$$\tilde{g}_l(p^2) = \tilde{g}_l(0) + p^2 \tilde{g}_l(0) + \frac{1}{2} p^4 \tilde{g}_l(0) + \cdots, \quad (A.9)$$
where
\[ \tilde{g}_I(0) = 1 - \frac{1}{2} \hat{m} \left( 1 - \frac{1}{2} \hat{m} g_2 \right), \]  
(A.10)
\[ \tilde{g}_I'(0) = \frac{L_1^2}{4} \left[ g_2 - 2 \hat{g}_2 + \hat{m} \left( \frac{1}{4} g_4 - g_2 \right) \right], \]  
(A.11)
\[ \tilde{g}_I''(0) = \frac{L_1^4}{32} \left[ 8 g_2 - 5 g_4 + 4 \hat{g}_4 + \hat{m} \left( -5 g_2 + 2 g_4 - \frac{1}{6} g_6 \right) \right], \]  
(A.12)
and \( \hat{m} \) corresponds to \( \hat{m} \), 0 and \( \hat{m}/c^2 \) for \( f = W, V, A \), respectively.

References

Peskin M E and Takeuchi T 1990 Phys. Rev. Lett. 65 964
Grojean C 2007 C. R. Physique 8 1068


Brooijmans G 2007 Talk presented at the Workshop on Possible Parity Restoration at High Energy (Beijing (China) 11–12 June 2007)


