Physical and geometrical properties of thermal plumes in turbulent Rayleigh–Bénard convection

To cite this article: Quan Zhou and Ke-Qing Xia 2010 New J. Phys. 12 075006

View the article online for updates and enhancements.

Related content
- The mixing evolution and geometric properties of a passive scalar field in turbulent Rayleigh–Bénard convection
  Quan Zhou and Ke-Qing Xia
- Experimental techniques for turbulent Taylor–Couette flow and Rayleigh–Bénard convection
  Chao Sun and Quan Zhou
- Scaling laws in turbulent Rayleigh–Bénard convection under different geometry
  Hao Song and Penger Tong

Recent citations
- Statistical characterization of thermal plumes in turbulent thermal convection
  Sheng-Qi Zhou et al
- Flow patterns in inclined-layer turbulent convection
  Wei Qiang and Hui Cao
- Simultaneous measurement of temperature and velocity fields in convective air flows
  Daniel Schmeling et al
Physical and geometrical properties of thermal plumes in turbulent Rayleigh–Bénard convection

Quan Zhou\textsuperscript{1,2} and Ke-Qing Xia\textsuperscript{1,3}

\textsuperscript{1} Department of Physics, The Chinese University of Hong Kong, Shatin, Hong Kong, People’s Republic of China
\textsuperscript{2} Shanghai Institute of Applied Mathematics and Mechanics, Shanghai University, Shanghai 200072, People’s Republic of China
E-mail: kxia@phy.cuhk.edu.hk

Received 11 February 2010
Published 8 July 2010
Online at http://www.njp.org/
doi:10.1088/1367-2630/12/7/075006

Abstract. We present a systematic experimental study of geometric and physical properties of thermal plumes in turbulent Rayleigh–Bénard convection using the thermo-chromic-liquid-crystal (TLC) technique. The experiments were performed in three water-filled cylindrical convection cells with aspect ratios 2, 1 and 0.5 and over the Rayleigh number range $5 \times 10^7 \leq Ra \leq 10^{11}$. TLC thermal images of horizontal plane cuts at various depths below the top plate were acquired. Three-dimensional (3D) images of thermal plumes were then reconstructed from the 2D slices of the temperature field. The results show that the often-called sheetlike plumes are really 1D structures and may be called rodlike plumes. We find that the number densities for both sheetlike/rodlike and mushroomlike plumes have power-law dependence on $Ra$ with scaling exponents of $\sim 0.3$, which is close to that between the Nusselt number $Nu$ and $Ra$. This result suggests that it is the plume number that primarily determines the scaling exponent of the $Nu–Ra$ scaling relation. The evolution of the aspect ratio of sheetlike/rodlike plumes reveals that, as $Ra$ increases, the plume geometry changes from more-elongated to less-elongated. Our study of the plume area fraction (fraction of coverage over the surface of the plate) further reveals that the increase in plume numbers with $Ra$ mainly comes from an increase in plume emission, rather than fragmentation of plumes. In addition, the area, perimeter and shape complexity of the 2D horizontal cuts of sheetlike/rodlike plumes were studied and all are found to obey log-normal distributions.

\textsuperscript{3} Author to whom any correspondence should be addressed.
1. Introduction

As an important class of turbulent flows, turbulent thermal convection occurs ubiquitously in nature. One can find it in the planets and stars, in the Earth’s mantle and outer core, and in the atmosphere and oceans. In addition, turbulent thermal convection plays a crucial role in heat transport and mass mixing in many engineering applications. Turbulent Rayleigh–Bénard (RB) convection, a fluid layer sandwiched between two parallel plates and heated from below, has long been used as a model system to study natural convection [1]–[3]. Two of the important issues in the study of turbulent RB convection are heat transport and coherent structures such as thermal plumes. In the first one, one tries to understand how heat transported upwards across the fluid layer, which is characterized by the Nusselt number $Nu = J/(\chi \Delta / H)$, depends on the turbulent intensity, which is characterized by the Rayleigh number $Ra = \alpha g \Delta H^3/\nu \kappa$. Here, $J$ is the temperature current density across the fluid layer with height $H$ and with an applied temperature difference $\Delta$, $g$ is the gravitational acceleration, and $\alpha$, $\nu$, $\chi$, and $\kappa$ are, respectively, the volume expansion coefficient, kinematic viscosity, thermal conductivity, and thermal diffusivity of the convecting fluid. In many theories and experiments, it is often assumed that there is a simple power-law relation between $Nu$ and $Ra$, i.e. $Nu \sim Ra^\beta$. In the second issue, one wants to understand the statistical and geometrical properties of thermal plumes, which are localized thermal structures. They have been shown to play a key role in many natural phenomena and engineering applications, such as in mantle convection, where mantle plumes just below Earth’s crust are responsible for the formation of volcanoes (see, for example, [4, 5]), and in nuclear explosions and stellar convection where plumes dominate both the dynamics and the energy transport (see, for example, [6]). The two issues are not independent of each other. It has recently been shown that, for a turbulent RB system, most heat is carried and transported by thermal plumes [7]–[9]. Accordingly, it remains a challenge to establish a quantitative relationship between thermal plumes and heat transport. Although a
theoretical effort [10] has recently focused on this connection, few such experimental studies have been made.

Thermal plumes generated by a localized hot/cold spot have a well-organized structure, which consists of a mushroom cap with a sharp temperature gradient and a stem that is relatively diffuse [11, 12]. Plumes with such a structure are often referred to as mushroomlike plumes, which are also observed in turbulent RBC when viewed from the side [7], [13]–[18]. By extracting plumes from temperature time series measured locally near the cell sidewall, Zhou and Xia [19] showed that the size of mushroomlike plumes obey log–normal statistics. However, the morphology of the plumes is totally different when one observes them from above (or below): the thermal plume is extended in one horizontal direction but concentrated in the orthogonal horizontal direction [13, 14], [20]–[25]. Plumes with such a structure are often assumed to have significant vertical extent and thus are called sheetlike plumes. Puthenveettil and Arakeri [23] studied near-wall structures in turbulent natural convection driven by concentration differences across a membrane, and they found that the plume spacings show a common log–normal probability density function (PDF). Zhou et al [24] further revealed that both the area and the heat content of sheetlike plumes are log–normal distributed. Shishkina and Wagner [26] investigated quantitatively geometric properties of sheetlike plumes using direct numerical simulations. However, systematic experimental studies of geometrical structures of sheetlike plumes are still lacking. Recently, Funfschilling et al [25] suggested that, when viewed from above, thermal plumes near the two plates should be better referred to as linelike plumes, as the vertical extent of these structures does not seem to be established and it appears more likely that they are one-dimensional (1D) excitations in the marginally stable boundary layers.

As two different configurations of thermal plumes coexist simultaneously in the turbulent RB system, it is natural to ask how sheetlike and mushroomlike plumes transform from each other. By using the thermochromic-liquid-crystal (TLC) technique, Zhou et al [24] showed, in a cylindrical cell with unity aspect ratio (Γ = D/H with D as the inner diameter of the convection cell), that hot fluids (plumes) move upwards, impinging on the top plate from below, then spread horizontally along the top plate and form waves or sheetlike plumes. As they travel horizontally along the plate, these sheetlike plumes collide with each other or with the sidewall, convolute and form swirls. As these swirls are cooler than the bulk fluid, they spiral away from the plate, form mushroomlike plumes, merge and cluster together. The detachment of these mushroomlike plumes associated with strong vertical vorticity close to the thermal boundary layer was also observed in a recent numerical Lagrangian convection with free-slip boundary conditions [27]. By the symmetry of the system, the same process is expected for the morphological evolution of thermal plumes occurring near the bottom plate. However, in [24] the morphological evolution was visualized only at a single Rayleigh number (Ra = 2.0 × 10^9), and it is not clear whether this process is universal for much higher Ra or for cells with different aspect ratios. The understanding of this evolution process is of great importance, as the characterization of coherent structures is essential to the understanding of turbulent flows in many systems.

In the present paper, we report new experiments regarding the temperature and velocity fields measured at varying depths from the top plate over the Rayleigh number range 5 × 10^7 ≤ Ra ≤ 10^11 and the Prandtl number range 4.1 ≤ Pr = ν/κ ≤ 5.4, and in three water-filled cylindrical sapphire cells with aspect ratios 0.5, 1 and 2. We present quantitative results on the relationship between plume number density and Nu, and results that relate the evolution of the plume morphology to heat transport. The remainder of the paper is organized as follows.
Figure 1. A schematic drawing of the convection cell used in the experiments. The top and bottom plates are made of sapphire so that the flow in a horizontal plane can be visualized and captured. A: cooling chamber cover, B: top sapphire plate, C: plexiglas sidewall, D: bottom sapphire plate, E: heating chamber cover, F: thermistors, G: nozzles for cooling or heating water, H: stainless steel rings as part of the heating and cooling chambers, I: nozzle for transferring fluid into the cell, J: nozzle for letting air out of the cell, K: square-shaped glass jacket.

We give a detailed description of the experimental setup and data analysis method in section 2. In section 3, we study the $z$- and $Ra$-dependence of plume number and discuss the relationship between thermal plumes and the $Nu$–$Ra$ scaling. Section 4 presents a study of the geometric properties of sheetlike plumes, which are mainly based on data from the aspect ratio 1 cell. We summarize our findings and conclude in section 5.

2. Experimental setup and procedures

2.1. The convection cell

The experiments were carried out in three cylindrical sapphire cells [24, 28]. A schematic diagram of the cells is shown in figure 1. To capture and study the horizontal temperature and velocity fields from the top, two sapphire discs (Almaz Optics, Inc) with diameter 19.5 cm and thickness 5.0 mm were chosen as the top and bottom plates for their good thermal conductivity (35.1 W mK$^{-1}$ at 300 K) compared with other transparent materials. Two chambers, constructed from stainless steel rings $H$ and plexiglas discs $A$ and $E$, are used to heat the bottom plate and cool the top plate. Each chamber is connected to a separate refrigerated circulator by four nozzles. Water flows into the chamber by two nozzles in two opposite direction and leaves from the other two nozzles perpendicular to the inlets. The sidewall of the cell is a vertical tube made of plexiglas (C) with inner diameter $D = 18.5$ cm and wall thickness 8 mm, respectively. The separations between the top and bottom plates $H$ are $9.3$, $18.5$ and $37$ cm so that the aspect ratios of the cells are respectively $\Gamma = 2$, $1$ and $1/2$. Draining ($I$) and filling ($J$) tubes are fitted on the plexiglas tube at a distance of $1.5$ cm from the bottom and top plates, respectively. A square-shaped jacket ($K$) made of flat glass plates and filled with water is fitted to the outside of the
sidewall [18], which greatly reduces the spatial variation in the intensity of the white lightsheet caused by the curvature of the cylindrical sidewall. Four rubber O-rings (not shown in the figure) are placed between the two sapphire plates and the steel rings and between the rings and the two plexiglas discs to avoid fluid leakage. Four thermistors $F$ (model 44031, Omega Engineering Inc) are used to measure the temperature difference between the two sapphire plates. To keep good contact between the plates and the thermistors, the thermistors are wrapped in a heat transfer compound (HTC10s from Electrolube Limited). It is found that the measured relative temperature difference between the two thermistors in the same plate is less than 3% of that across the convection cell for both plates and for all $Ra$ investigated. This indicates a uniform distribution of the temperature across the horizontal plates.

2.2. Liquid crystal measurements and the experimental parameters

The visualization technique employing TLC particles has been widely used and documented in fluid visualization experiments and was used in the present work to visualize the temperature and velocity fields in horizontal fluid layers of varying depth from the cell’s top plate. Two types of TLC microspheres (Hallcrest, Ltd) were used in the experiments. One type (model R29C4W) was used for low-$\Delta$ experiments with the mean bulk temperature $T_0 = 30^\circ C$ and $Pr = 5.4$, and the other type (model R40C5W) was used for high-$\Delta$ measurements with $T_0 = 42^\circ C$ and $Pr = 4.1$. Both these particles have a mean diameter of 50 $\mu m$ and density of 1.03–1.05 g cm$^{-3}$, and were suspended in the convection fluid in very low concentrations (about 0.01% by weight), at which the influence of TLC particles on the fluid can be neglected. The peak wavelength of light scattered by these particles changes from red to green and then to blue within a temperature window of 4 $^\circ C$ from about 29–33 $^\circ C$ for R29C4W, and of 5 $^\circ C$ from about 40–45 $^\circ C$ for R40C5W. Table 1 summarizes the experimental parameters. In the experiments, the mean bulk temperature was set to 30 $^\circ C$ for R29C4W ($Pr = 5.4$) and to 42 $^\circ C$ for R40C5W ($Pr = 4.1$), so that the background fluid appears blue and the red and green regions correspond to cold fluid, i.e. cold plumes. Figure 2 shows a schematic diagram of the optical setup for the experiments. A halogen photo optic lamp (S) with a power of 650 W was used as the light source. One concave mirror $L1$ and two condensing lenses $L2$ and $L3$ were used to collect the light from $S$ and focus it onto the central section of the cell. A horizontal sheet of white light, generated by a diverging cylindrical lens $L4$ and then projected onto an adjustable slit, was passed through the cell parallel to the top plate. The thickness of the lightsheet inside the cell is approximately

<table>
<thead>
<tr>
<th>$\Gamma = 2$</th>
<th>$\Gamma = 1$</th>
<th>$\Gamma = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Ra$</td>
<td>$Pr$</td>
<td>$\Delta$ (K)</td>
</tr>
<tr>
<td>$5.7 \times 10^7$</td>
<td>5.4</td>
<td>2.6</td>
</tr>
<tr>
<td>$1.0 \times 10^8$</td>
<td>5.4</td>
<td>5.0</td>
</tr>
<tr>
<td>$2.0 \times 10^8$</td>
<td>5.4</td>
<td>10.0</td>
</tr>
<tr>
<td>$3.0 \times 10^8$</td>
<td>5.4</td>
<td>15.1</td>
</tr>
<tr>
<td>$4.4 \times 10^8$</td>
<td>4.1</td>
<td>13.7</td>
</tr>
<tr>
<td>$6.5 \times 10^8$</td>
<td>4.1</td>
<td>20.2</td>
</tr>
<tr>
<td></td>
<td>$1.0 \times 10^{10}$</td>
<td>4.1</td>
</tr>
</tbody>
</table>

Figure 2. The optical setup for thermochromatic liquid crystal visualization and measurements: $S$: halogen lamp, $L_1$: concave mirror, $L_2$ and $L_3$: condensing lenses, $L_4$: diverging cylindrical lens.

Figure 3. An example of the thermochromatic liquid crystal image and the identification and extraction of sheetlike plumes. Image of TLC microspheres with the camera exposure time of 0.02 s taken at 2 mm from the top plate at $Ra = 1.1 \times 10^9$. The black lines in the figure mark the contours of the 22 extracted sheetlike plumes.

3 mm. A Nikon D1X camera, with a resolution of $2000 \times 1312$ pixels and 24 bit dynamic range, was placed on top of the cell to take photographs of the TLC microspheres. With short camera exposure time (0.02 s), the captured photographs give the instantaneous temperature field, and with long camera exposure time (0.77 s) in addition they show the trajectories of the particles. Note that in our present study all TLC photographs were taken in the bulk regime where the mean temperature does not depend on position.

To study thermal plume properties quantitatively, we counted plume numbers and extracted sheetlike plumes. For sheetlike plumes, we took 150 to 300 consecutive images (at $z = 2$ mm and with camera exposure time of 0.02 s) at 30–60 s intervals for a given $Ra$, so that two successive images in each sequence are statistically independent. Figure 3 shows an example
Figure 4. Evolution of sheetlike plumes with the Rayleigh number. Images of TLC microspheres with the camera exposure time of 0.02 s taken at 2 mm from the top plate at (a) $Ra = 1.1 \times 10^9$, (b) $3.0 \times 10^9$ and (c) $1.0 \times 10^{10}$. (d), (e) and (f), which correspond to (a), (b) and (c), respectively, are extracted plumes with background in the original images removed. Note that (a) is the same as figure 3.

of a TLC image and the identified sheetlike plumes. A cold sheetlike plume is extracted by first manually drawing a contour around its perimeter (see the black lines in figure 3) and then using software to collect all pixels enclosed by the contour. A program is then used to calculate the perimeter $P$ and the area $A$ of each extracted plume. To ensure that plumes are identified correctly, the operator uses knowledge gained from viewing movies of plume motions. A total of 3000–6000 plumes are identified from the 150–300 images, which are not large numbers but should be indicative of the statistical properties. Figures 4(d)–(f) show three examples of extracted plumes with backgrounds removed, which correspond to images of TLC microspheres in figures 4(a)–(c), respectively, for three different values of $Ra$ obtained from the unity-aspect-ratio cell. For mushroomlike plumes, we took sequences of images at a varying depth $z$ from the top plate and with a camera exposure time of 0.77 s at 30 s intervals for each $Ra$. Each sequence consists of 200 images for a given depth. A cold mushroomlike plume was identified as an object with nonzero vorticity and with temperature (color) much lower than that of the background fluid [24] (see, e.g., figure 5(b)). As mushroomlike plumes are partly entangled together, they cannot be separated completely. Nevertheless, we can identify them individually as swirls with cold temperature and thus count their numbers. The cold sheetlike plumes were also identified and counted from these images, as objects with a linelike shape and lower temperature.
3. Plume number statistics

3.1. Morphological evolution of thermal plumes

The process of morphological evolution from sheetlike to mushroomlike plumes has been revealed by Zhou et al [24] in a cylindrical cell. Here, the same process was also observed for all values of $Ra$ and $\Gamma$ investigated in the experiment. Figure 5(a), taken at 2 mm from the top plate, shows how this evolution comes about. One sees that, near the top plate, the motion of TLC microspheres appears to emanate from certain regions or ‘sources’ with bluish color, suggesting that hot fluids (plumes) are moving upwards, impinging on and spreading horizontally along the top plate. Along the particle traces, the color turns from blue to green and red, implying that the wave fronts are cooled down gradually by the top plate (and the top thermal boundary layer) as they spread. As they travel along the plate’s surface, sheetlike plumes collide with each other or with the sidewall. As different plumes carry momenta in different directions, they merge, convolute and form swirls (hence generating vorticity). As these swirls are cooler than the bulk fluid, they spiral away from the plate, and then merge and cluster together (see, e.g, figure 5(b))

We note that, given the symmetry of the system, the same morphological evolution of thermal plumes should occur near the bottom plate. The physical picture of this evolution process is illustrated in figure 6.

3.2. Depth-dependent properties

Because of the collision and convolution of sheetlike plumes and the transformation from sheetlike to mushroomlike ones, it is expected that the number of sheetlike plumes should decrease from the top plate, while that of mushroomlike ones should increase, and this is indeed the case, as shown in figure 7. Figures 7(a), (c) and (e) show the mean numbers of (cold) sheetlike plumes $N_{\text{pl}}^\text{sheet}$ versus the normalized height $z/\delta_{\text{th}}$ obtained from the cells with

3 See the movie on the plume morphological evolution at http://www.phy.cuhk.edu.hk/turbulence/plume_movie.htm.
Figure 6. The process of morphological evolution from sheetlike to mushroom plumes.

Figure 7. Mean numbers of (a), (c) and (e) (cold) sheetlike plumes $N_{\text{sheet}}$ as functions of the distance $z$ normalized by the thermal boundary layer thickness $\delta_{th}$ and (b), (d) and (f) (cold) mushroomlike plumes $N_{\text{mush}}$ as functions of the distance $z$ normalized by the cell height $H$ for all measured $Ra$. (a) and (b) for $\Gamma = 2$, (c) and (d) for $\Gamma = 1$ and (e) and (f) for $\Gamma = 0.5$. With increasing $Ra$, the thermal boundary layer thickness $\delta_{th}$ decreases from 0.016$H$ to 0.002$H$ [29].
Figure 8. Mean numbers of (cold) sheetlike plumes $N_{\text{sheet}}^\text{pl}$ as a function of the scaled distance $z/\delta_{\text{th}}$ in a semi-log plot. Solid lines are exponential fittings to the data.

Figure 9. (a) $1/b_t$ and (b) $z_C/\delta_{\text{th}}$ versus $Ra$ for $\Gamma = 2$ (up-triangles), 1 (circles) and 0.5 (down-triangles). The dashed line in (b) marks the mean value of $z_C$ for $Ra > 3 \times 10^8$.

$\Gamma = 2, 1$ and 0.5, respectively. (Here, $\delta_{\text{th}}$ is the thickness of the thermal boundary layer, which was measured in [29, 30].) One sees that $N_{\text{sheet}}^\text{pl}$ decays quickly away from the top plate and the behavior is similar for all $Ra$ studied. Beyond the depth $z \approx 15\delta_{\text{th}}$, one can hardly identify any sheetlike plumes from the acquired images. Note that, in addition to transforming into mushroomlike plumes, the rapid reduction of $N_{\text{sheet}}^\text{pl}$ with depth could also be attributed to the limited vertical extent of sheetlike plumes themselves, i.e. plumes whose vertical extents are shorter than a certain height $h$ could not be captured by the images that are taken farther than $h$ from the top plate. We further note that the $z$-dependence of plume numbers $N_{\text{sheet}}^\text{pl}$ may be well described by a decreasing exponential function, i.e. $N_{\text{sheet}}^\text{pl} = N_0 e^{-b_t z/\delta_{\text{th}}}$, with fitting parameters $N_0$ and $b_t$ for nearly all measured $Ra$, three examples of which are plotted in figure 8. Hence, $1/b_t$ may be used as a typical dimensionless vertical extent of sheetlike plumes, i.e. the quantity $h$ mentioned above. Figure 9(a) shows $1/b_t$ as a function of $Ra$. It is seen that $1/b_t$ increases with $Ra$ for the $\Gamma = 2$ and 0.5 cells, while it remains nearly constant for the $\Gamma = 1$ cell. We cannot judge the significance of the different behaviors of $1/b_t$ varying with $Ra$ in cells with different $\Gamma$. However, $1/b_t$ varies only in the range of a few times the thickness of the thermal boundary layer ($2 < 1/b_t < 5$). This suggests that the vertical extent of the so-called sheetlike plumes does not have enough spatial extension in the vertical direction to form sheets.
For mushroomlike plumes, the mean numbers of (cold) mushroomlike plumes \(N_{\text{pl}}^{\text{mush}}\) as a function of the scaled depth \(z/H\) is shown in figures 7(b), (d) and (f). With increasing \(z\), \(N_{\text{pl}}^{\text{mush}}\) for all \(Ra\) first increases rapidly, then decreases and finally remains approximately constant for positions \(z > 0.2H\). When \(N_{\text{pl}}^{\text{mush}}\) first increases with \(z\), it crosses over with \(N_{\text{pl}}^{\text{sheet}}\) at some crossover-depth \(z_c\). The normalized crossover-depths \(z_c\) are shown in figure 9(b). It is seen that \(z_c \approx 2.5\delta_0\), when \(Ra \leq 3 \times 10^8\). For \(Ra > 3 \times 10^8\), although the data points seem to be somewhat scattered, they exhibit no clear dependence on \(Ra\) and have a mean value of \(\sim 4.84\delta_0\). This implies that the transformation occurs within a region whose height is only associated with the thermal boundary layer thickness. If one takes this value as the typical vertical extent of sheetlike plumes, these results again suggest that the vertical extent of what are called sheetlike plumes is only of the order of several \(\delta_0\), not much larger than it, and thus not large enough to have a sheetlike shape. This will be further discussed in section 4.4. After \(N_{\text{pl}}^{\text{mush}}\) attains its maximum value around \(z = z_p\), it then drops sharply beyond the depth \(z \approx 0.2H\), because of the mixing, merging and clustering of mushroomlike plumes. As found by Zhou et al [24], this region corresponds to the region of full width at half maximum of vertical vorticity profile and can be used as a quantitative definition and measure of the mixing zone [19, 32]. It is further found that the peak position in the profiles of \(N_{\text{pl}}^{\text{mush}}, z_p\), has no obvious \(Ra\)-dependence for each \(\Gamma\), but decreases slightly from 0.08\(H\) for \(\Gamma = 2\) to 0.03\(H\) for \(\Gamma = 0.5\).

### 3.3. The Rayleigh number dependency

We next focus on the \(Ra\)-dependence of plume number density. For sheetlike plumes, the number was counted just outside the thermal boundary layer and the counted number, \(N_{\text{pl}}^{\text{sheet}}\), for each cell can be well described by a power-law \(N_{\text{pl}}^{\text{sheet}}/(\pi D^2/4) = \alpha_s(\Gamma) Ra^{\beta_s(\Gamma)}\), with two fitting parameters \(\alpha_s\) and \(\beta_s\) as functions of \(\Gamma\). Here, the fitted values of \(\alpha_s\) and \(\beta_s\) for all three aspect ratios are listed in table 2. It is seen that \(\alpha_s(\Gamma)\) varies with the cell’s aspect ratio \(\Gamma\) while \(\beta_s(\Gamma)\) is approximately the same for all three cells. As the range of \(Ra\) is limited for each aspect ratio, we shifted the plume number densities for the \(\Gamma = 2\) and 0.5 cells to agree with data for the \(\Gamma = 1\) cell in their respective overlapping range of \(Ra\). We then fitted a single power law to the combined data set from the three cells that span almost three decades of \(Ra\). Figure 10 shows the shifted numbers, normalized by the area of the top plate \(\pi D^2/4\), as well as the plume number from the \(\Gamma = 1\) cell as dark-green symbols. It is seen that a single power law

\[
N_{\text{pl}}^{\text{sheet}}/(\pi D^2/4) = \alpha_s Ra^{\beta_s} \quad \text{with} \quad \alpha_s = 1.4 \quad \text{and} \quad \beta_s = 0.29 \pm 0.03
\]  

(1)

can be used to well describe the data from all three cells.

For mushroomlike plumes, one sees from figures 7(b), (d) and (f) that there is a peak on the height-profile of the mushroomlike plume number \(N_{\text{pl}}^{\text{mush}}\) for all three cells and for all measured

<table>
<thead>
<tr>
<th>(\Gamma)</th>
<th>(\alpha_s)</th>
<th>(\beta_s)</th>
<th>(\alpha_m)</th>
<th>(\beta_m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>1.2</td>
<td>0.29 ± 0.03</td>
<td>0.45</td>
<td>0.31 ± 0.03</td>
</tr>
<tr>
<td>1</td>
<td>1.5</td>
<td>0.28 ± 0.03</td>
<td>0.37</td>
<td>0.33 ± 0.03</td>
</tr>
<tr>
<td>2</td>
<td>1.6</td>
<td>0.30 ± 0.03</td>
<td>1.86</td>
<td>0.28 ± 0.03</td>
</tr>
</tbody>
</table>
Ra. We use the value at the peak of the height-profile of the mushroomlike plume number to examine the Ra-dependence of $N_{\text{pl}}^\text{mush}$. It is found that a power-law relation $N_{\text{pl}}^\text{mush}/(\pi D^2/4) = \alpha_m(\Gamma) Ra^{-\beta_m(\Gamma)}$ can also be used to describe the Ra-dependence of $N_{\text{pl}}^\text{mush}$. Again, the fitted $\beta_m(\Gamma)$ is nearly $\Gamma$-independent (see table 2). Figure 10 shows the Ra-dependence of $N_{\text{pl}}^\text{mush}/(\pi D^2/4)$ (red symbols). Here, the $\Gamma = 2$ and 0.5 data have also been shifted to agree with the $\Gamma = 1$ in their respective overlap regions of Ra. It is seen that $N_{\text{pl}}^\text{mush}$ shows a similar trend with Ra as $N_{\text{pl}}^\text{sheet}$. Again, the relationship between $N_{\text{pl}}^\text{mush}$ and $Ra$ can be well represented by power-law fits:

$$N_{\text{pl}}^\text{mush}/(\pi D^2/4) = \alpha_m Ra^{-\beta_m} \quad \text{with} \quad \alpha_m = 0.41 \quad \text{and} \quad \beta_m = 0.32 \pm 0.03. \quad (2)$$

To make a quantitative connection between thermal plumes and the heat flux, we use the previous experimental finding from both Eulerian and Lagrangian measurements that the heat flux are mainly carried by thermal plumes [7, 9]. As mushroomlike plumes are evolved morphologically from sheetlike plumes [24], the total heat flux carried by either type should be the same, i.e.

$$Nu \simeq N_{\text{pl}}^\text{sheet} F_{\text{pl}}^\text{sheet} \simeq N_{\text{pl}}^\text{mush} F_{\text{pl}}^\text{mush} \sim Ra^{\delta}. \quad (3)$$

Here, $F_{\text{pl}}^\text{sheet}$ and $F_{\text{pl}}^\text{mush}$ are the mean heat flux carried by individual sheetlike and mushroomlike plumes, respectively, and $\delta \simeq 0.3 \pm 0.02$ in the Ra and Pr ranges of the experiment [32, 33]. Recall that in the Ra-dependence of $N_{\text{pl}}^\text{sheet}$ (equation (1)) and $(N_{\text{pl}}^\text{mush})_{\text{peak}}$ (equation (2)) discussed above, one sees that the scaling exponents of the $Nu-Ra$ relation and of the Ra-dependence of plume numbers are the same within experimental uncertainty. This indicates that the heat flux transported by individual plumes is nearly independent of the turbulent intensity, i.e. Ra, and hence the $Nu-Ra$ scaling relation is determined primarily by the number of thermal plumes.
4. Geometric properties of sheetlike plumes

4.1. Aspect ratio of sheetlike plumes

A striking feature that one can observe from figure 4 is that, as the Rayleigh number is increased, the morphology or geometry of sheetlike plumes change from a more elongated shape to less elongated and more fragmented. The extracted horizontal cuts of sheetlike plumes may be characterized by a typical length $l$ and a typical width $w$. With increasing $Ra$, the length of sheetlike plumes seems to decrease, while the plume’s width seems to increase. To describe these geometric properties quantitatively, we define the aspect ratio of sheetlike plumes, $\gamma_{pl}^{\text{sheet}}$, as

$$\gamma_{pl}^{\text{sheet}} = \frac{l}{w},$$

with

$$P = 2(l + w),$$

$$A = lw,$$

$$l \geq w. \quad (4)$$

Figure 11 shows the $Ra$-dependence of the mean aspect ratio of sheetlike plumes $\langle \gamma_{pl}^{\text{sheet}} \rangle$ in a log–log plot. It is seen that $\langle \gamma_{pl}^{\text{sheet}} \rangle$ decreases with increasing $Ra$ and the relation can be described by a power-law with a scaling exponent $-0.23$. We note that $\langle \gamma_{pl}^{\text{sheet}} \rangle$ can also be described by a logarithmic function of $Ra$, and we cannot definitively conclude which one, between power law and logarithm, is a better choice. Figures 12(a) and (b) show $Ra$-dependence of (a) the mean normalized area $\langle 4A/\pi D^2 \rangle$ and (b) the mean normalized perimeter $\langle P/D \rangle$ of sheetlike plumes. Again, these two quantities are found to decrease with increasing $Ra$. All these behaviors are well illustrated in figure 4, and suggest that, when the flow becomes more turbulent, the increased mixing, collision and convolution can more easily fragment large sheetlike plumes into smaller ones.

Figure 13 shows the plume area fraction $f_{pl}$, which is defined as the ratio between the average of the total plume area contained in each TLC photograph and the area of the cell top plate, as a function of the Rayleigh number in a log–log plot. It is seen that $f_{pl}$ increases with increasing $Ra$ and a power-law function with a scaling exponent 0.23 can be used to describe well the relationship between $f_{pl}$ and $Ra$. Using the shadowgraph technique, Funfschilling et al studied $Ra$-dependence of the plume area fraction [25]. Although the scaling range is limited,
they found a power-law scaling of $f_{pl}$ with $Ra$. The fitted scaling exponent is found to be around 2, which is significantly different from the present value of 0.23. It should be noted that sheetlike plumes were identified using a temperature-gradient threshold in [25], while a temperature threshold was chosen to extract sheetlike plumes for our present study. Therefore, different scaling behaviors for the area fraction may come from different definitions of sheetlike plumes. Note also that our present scaling exponent was obtained for $f_{pl} \gtrsim 0.1$, while that of [25] was obtained for $f_{pl} \lesssim 0.1$. Therefore, the different fitting ranges could be the reason for the different observed scaling exponents. In fact, figure 17 of [25] also reveals that the slope of $f_{pl}$ versus $Ra$ for $f_{pl} \gtrsim 0.1$ is much flatter than that for $f_{pl} \lesssim 0.1$, which is qualitatively consistent with the present results.

Next, we examine the relationship between plume perimeter and area. Figure 14(a) shows a scatter plot of the normalized perimeter $P/D$ and the normalized size $(4A/\pi D^2)^{1/2}$ of sheetlike plumes, which contains a total of 6071 sheetlike plumes, extracted from a sequence of 260 images at $Ra = 3.0 \times 10^9$. Here, the perimeter and area are normalized by the diameter and the area of the conducting plate, respectively. Although the data look somewhat scattered, one sees that the perimeter increases with area and all data points can be fitted by a power-law

---

**Figure 12.** $Ra$-dependence of (a) the mean normalized area $\langle 4A/\pi D^2 \rangle$ and (b) the mean normalized perimeter $\langle P/D \rangle$ of sheetlike plumes.

**Figure 13.** The area fraction $f_{pl}$ of sheetlike plumes (area coverage of plumes over the top plate) as a function of $Ra$ in a log–log plot. The solid line represents a power-law fit to the data.
Figure 14. (a) Scatter plot of the normalized perimeter $P/D$ versus the normalized size $(4A/\pi D^2)^{1/2}$ of sheetlike plumes at $Ra = 3.0 \times 10^9$. (b) The conditional average $\langle P|A \rangle/D$ on the normalized size $(4A/\pi D^2)^{1/2}$ for the same data as (a). Inset is the plot of fractal dimensions of sheetlike plumes’ boundary $d_A$ versus $Ra$. Solid lines in both (a) and (b) are power-law fits.

To better explore this feature, we study the conditional average $\langle P|A \rangle/D$, i.e. the average of perimeter of the extracted plumes possessing a particular volume $A$. In figure 14(b), we plot the normalized conditional average of $P$ versus the normalized plume area. In the figure, a good scaling range can be seen and the solid line is a power-law fit $(P/D) = 28.1 \times (4A/\pi D^2)^{1.50 \pm 0.02}$. We find that such scaling behavior exists for all $Ra$ investigated. The inset of figure 14(b) shows the $Ra$-dependence of $d_A$. One sees that these $d_A$ have a mean value of 1.50, but decrease from 1.53 to 1.46 when $Ra$ increases from $6.7 \times 10^8$ to $6.3 \times 10^9$. These measured values of $d_A$ are further found to be larger than the value of 1.29 found for isoconcentration contours of passive scalars measured in the same system [35].

Mathematically, as introduced by Mandelbrot [36] and Lovejoy [37], $d_A$ is the fractal dimension of the boundary of sheetlike plumes and satisfies $1 \leq d_A < 2$. Fractal dimensions in turbulence have been widely studied (see, for example, [38]). In the present case, our results seem to suggest that the boundary of sheetlike plumes is fractal with the dimension of around 1.50 and the slight decrease in $d_A$ may suggest that the shape of sheetlike plumes becomes smoother when the flow becomes more turbulent, as a result of the increased mixing. However, the extracted sheetlike plumes, as shown in figure 4, do not look like fractal objects. To understand this and to see if applying the machinery of fractal analysis to the geometric properties of plume can shed some light on the problem, we study the shape complexity of the plumes.

4.2. Shape complexity of sheetlike plumes

The geometric complexity of an object can be characterized by its shape complexity, which is a dimensionless ratio between its area and volume. The shape complexity was previously used to study the shape of isocontours for passive scalar fields in turbulence [34]. We introduce it

$$\frac{P}{D} \sim (\sqrt[1.50]{\frac{4A}{\pi D^2}})^{d_A} \quad \text{with} \quad d_A = 1.50.$$
here to describe the shape of sheetlike plumes. For a 2D closed contour, the shape complexity is defined as

$$\Omega_2 = \frac{P}{2\sqrt{\pi A}},$$

where $P$ and $A$ are the perimeter of the contour and area enclosed by the contour, respectively, and the subscript 2 refers to 2D. As a circle has the minimum perimeter $2\sqrt{\pi A}$ among all 2D objects with the same area $A$, the shape complexity $\Omega_2$ satisfies $1 \leq \Omega_2 < \infty$ and can be used to describe the departure of an object from the shape of a circle. For a fractal object, a larger $\Omega_2$ implies a rougher contour (or surface) of the object. For a non-fractal object, a larger $\Omega_2$ signifies that the object is less like a circle (e.g. it is more elongated).

Figure 15(a) shows a scatter plot of the shape complexity $\Omega_2$ and the normalized dimension (or size) $(4A/\pi D^2)^{1/2}$ of sheetlike plumes, obtained from the same data set as those in figure 14. It is seen that the mean trend of the relation between the size and shape complexity can be captured by an increasing function, i.e. the plume with larger size has a higher probability of possessing a larger $\Omega_2$. To illustrate this mean trend more clearly, the conditional average $\langle \Omega_2 | A \rangle$ on the normalized size $(4A/\pi D^2)^{1/2}$ is plotted in figure 15(b). The solid line in the figure is a power-law fit, i.e.

$$\Omega_2 \sim (\sqrt[4]{4A/\pi D^2})^{d_\Omega} \quad \text{with} \quad d_\Omega = 0.50 \pm 0.01$$

and the reasonable scaling range can be found. Here, $d_\Omega \simeq d_A - 1$ is an immediate consequence of the definition of shape complexity (equation (6)) together with the power-law relation (equation (5)). The inset of figure 15(b) shows the $Ra$-dependence of $d_\Omega$. One sees that $d_\Omega$ decreases slightly with increasing $Ra$, which may be understood from the decrease in $d_A$.

Figure 16(a) shows the $Ra$-dependence of the mean shape complexity $\langle \Omega_2 \rangle$. It is seen that $\langle \Omega_2 \rangle$ decreases with increasing $Ra$. This suggests that, because of the increased mixing, collision and convolution of sheetlike plumes, the shape of sheetlike plumes becomes closer to the shape of a circle when the flow becomes more turbulent. This also implies the decreases of plume’s aspect ratio with increasing $Ra$. Indeed, with the definitions of $\gamma_{pl}^{\text{sheet}}$ (equation (4)) and
Figure 16. (a) Ra-dependence of the mean shape complexity $\langle \Omega_2 \rangle$ of sheetlike plumes. (b) The ratio between $\langle \Omega_2 \rangle$ and $\frac{1}{\sqrt{\pi}}\left( \frac{1}{\sqrt{\gamma_{\text{pl}}}} + \frac{1}{\sqrt{\gamma_{\text{sheet}}}} \right)$, $C_R$ versus $Ra$.

$\Omega_2$ (equation (7)), one can obtain the relation between of $\Omega_2$ and $\gamma_{\text{sheet}}$, i.e.

$$\Omega_2 = \frac{1}{\sqrt{\pi}} \left( \frac{1}{\sqrt{\gamma_{\text{pl}}}} + \frac{1}{\sqrt{\gamma_{\text{sheet}}}} \right),$$

which implies that $\Omega_2$ decreases with decreasing $\gamma_{\text{sheet}}$ when $\gamma_{\text{sheet}} \geq 1$. Furthermore, $\Omega_2$ is proportional to $\gamma_{\text{pl}}$ when $\gamma_{\text{sheet}} \gg 1$. Accordingly, the decrease in $\langle \Omega_2 \rangle$, shown in figure 16(a), may be understood as the result of the change in aspect ratio of the sheetlike plumes, i.e. the decrease in $\langle \gamma_{\text{pl}} \rangle$ (see figure 11), as the $\gamma_{\text{sheet}}$ is much larger than 1 for most situations. To see this more clearly, we study the ratio $C_R$ between $\langle \Omega_2 \rangle$ and $\frac{1}{\sqrt{\pi}}\left( \frac{1}{\sqrt{\gamma_{\text{pl}}}} + \frac{1}{\sqrt{\gamma_{\text{sheet}}}} \right)$. Note that equation (8) is valid only for an individual plume, while $C_R$ is in an average sense. Figure 16(b) shows $C_R$ as a function of $Ra$. It is seen that $C_R \approx 1$ for nearly all $Ra$ investigated. This result clearly demonstrates that the decrease in $\Omega_2$ is caused by the decrease in the plume’s aspect ratio, rather than by a decreased roughness in the case of a fractal object, which the sheetlike plumes are not.

4.3. Distributions of geometric measures of sheetlike plumes

In a previous study, Zhou et al [24] found that the area of sheetlike plumes obeys log–normal statistics. Here, we further investigate distributions of other geometric measures of sheetlike plumes. Figures 17(a)–(f) show the measured PDFs of normalized geometric measures, i.e. $4A/\pi D^2$, $P/D$, $\Omega_2$, $l/D$, $w/\delta_{\text{th}}$ and $\gamma_{\text{sheet}}$, of sheetlike plumes. It is seen that all these quantities have the same distribution, i.e. the log–normal distribution. The same distributions were also found for all of these quantities at other $Ra$ investigated in our experiments. Together with the log–normal distributions found for mushroomlike plumes [19], these findings suggest that the log–normal distribution is universal for thermal plumes and log–normal statistics may be used to model them, at least in turbulent RBC. In addition, the log–normal statistics of thermal plumes are different from those of passive scalars measured in the same system, which is found to obey log-Poisson statistics [35].
4.4. Three-dimensional (3D) structures of sheetlike plumes

Finally, we study the 3D structures of sheetlike plumes. The results obtained in section 3.2 have suggested that the vertical extent of what are called sheetlike plumes is only of the order of the thermal boundary layer thickness, and hence is not large enough to form a sheet. To see this more clearly, we used a tomographic reconstruction technique to construct the 3D image of thermal plumes from sequences of 2D images acquired near the top plate of the cell. To achieve this, the convection cell was placed on a translational stage and, as the cell traversed continuously at a speed of $\sim 1$ cm s$^{-1}$, a series of photographs of TLC microspheres were recorded by a Nikon D3 camera (3CCD, with a resolution of $2397 \times 1591$ pixels) operating at 11 frames s$^{-1}$. As the speed of the horizontal motion of the sheet-like plumes is about $0.4$ cm s$^{-1}$, the sequences of the horizontal slices of the plumes may be regarded as taken at approximately the same time. In post-experiment analysis, 2D horizontal cuts of cold plumes were extracted from the images taken in each run and a MATLAB script was used to reconstruct 3D thermal plumes from these extracted 2D cuts. Figure 18(a) shows an example of the reconstruction of 3D thermal plumes near the top plate. From the figure, one can see how mushroomlike plumes are formed by the convolution (or spiraling) of sheetlike plumes. Figure 18(b), which is an enlarged region of the image, shows this more clearly. Another noteworthy feature is that most plumes have a 1D structure, rather than a sheetlike shape. This geometric feature may be illustrated more clearly by figure 18(c), which shows the reconstruction of an enlarged individual 3D plume. It is seen that both the height and the width of this plume extend only to a few millimeters, while its length extends to a few centimeters. This is the most direct evidence so far which shows that thermal plumes near the conducting plates are only 1D structures, with the horizontal length being much...
Figure 18. (a) 3D sheetlike/rodlike thermal plumes obtained near the top plate by the tomographic reconstruction technique ($Ra = 2.0 \times 10^9$, $\Gamma = 1$). (b) An enlarged region of (a), showing the morphological transformation of rodlike plumes into mushroomlike ones through convolution/spiraling. (c) An individual rodlike plume.

larger than their horizontal width and vertical extent, which has been suggested previously [25]. Therefore, we may hereafter call thermal structures near the conducting plates rodlike plumes rather than sheetlike plumes. Future investigations will be focused on the geometric properties of 3D structures of both rodlike and mushroomlike thermal plumes.

5. Conclusions

In this paper, we have presented a detailed experimental study of the temperature and velocity fields in turbulent RB convection using the thermochromic-liquid-crystal (TLC) visualization
The number statistics and geometric properties of sheetlike/rodlike thermal plumes were investigated. Major findings are summarized as follows:

1. When observed from above, the previously named sheetlike thermal plumes near the top plate seem to be only 1D structures and should be called rodlike plumes hereafter. These plumes evolve morphologically, i.e. convolute or spiral, into mushroomlike plumes. The width ($\ell$) of the region near the plates within which these rodlike plumes exist and evolve morphologically is associated only with the thermal boundary thickness (i.e. $\ell \sim \delta_{th}$).

2. The numbers of rodlike and mushroomlike plumes, $N_{\text{sheet pl}}$ and $N_{\text{mush pl}}$, are both found to scale as $Ra^{0.3}$. This finding suggests that the total amount of heat flow is dominated only by the number of thermal (rodlike or mushroomlike) plumes and the normalized mean heat content carried by each type of plume is approximately independent of $Ra$.

3. As the turbulent intensity is increased, large rodlike plumes are more easily fragmented into smaller ones and hence their aspect ratios decrease.

4. Although rodlike plumes do not look like fractal objects, power-law relations can be used to characterize the relationship between their perimeter and area, and between their size and shape complexity.

5. Geometric measures of thermal plumes are found to have a universal distribution, i.e. the log–normal distribution. Thus, log–normal statistics may be used to model thermal plumes. In contrast, for the mixing of passive scalars measured in the same system, it is found that the same quantities obey log-Poisson statistics [35].

6. The $Ra$-dependence of the plume area density ($f_{pl} \sim Ra^{0.23}$) suggests that the plume number increase with $Ra$ ($N \sim Ra^{0.3}$) can be attributed largely to the increased emission of plumes (as $Ra$ is increased), rather than as a result of fragmentation of plumes.

Finally, the preliminary results presented in the last section suggest that future studies of thermal plumes should focus on the characterization of their 3D properties.

Acknowledgments

We gratefully acknowledge support of this work by the Research Grants Council of Hong Kong SAR (Nos CUHK403806 and 403807) (KQX) and by the Natural Science Foundation of Shanghai (no. 09ZR1411200), ‘Chen Guang’ project (no. 09CG41), and RFDP of Ministry of Education of China (no. 20093108120007) (QZ).

References


[18] Xi H D, Lam S and Xia K Q 2004 From laminar plumes to organized flows: the onset of large-scale circulation in turbulent thermal convection *J. Fluid Mech.* **503** 47–56
[28] Xi H D, Zhou Q and Xia K Q 2006 Azimuthal motion of the mean wind in turbulent thermal convection *Phys. Rev. E* **73** 056312
[29] Lui S L 1997 Experimental investigation of the temperature field in turbulent convection *MPhil Thesis* The Chinese University of Hong Kong


[35] Zhou Q and Xia K Q 2010 Mixing evolution and geometric properties of passive scalar field in high-Schmidt-number buoyancy-driven turbulence New J. Phys. submitted


[37] Lovejoy S 1982 Area-perimeter relation for rain and cloud areas Science 216 185–7

[38] Sreenivasan K R 1991 Fractals and multifractals in fluid turbulence Annu. Rev. Fluid Mech. 23 539–600