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# Onset of self-steepening of intense laser pulses in plasmas

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**Abstract.** The self-steepening of laser pulses with intensities in excess of  $10^{18}$  W cm<sup>-2</sup> and with typical durations shorter than 30 fs propagating in underdense plasmas is examined by resorting to the framework of photon kinetics. Thresholds for self-steepening at the back/front of short laser pulses are determined, along with the self-steepening rates, and the connection between self-steepening, self-compression and frequency chirps is established. Our results are illustrated with particle-in-cell simulations, revealing the key physical mechanisms associated with the longitudinal laser dynamics, critical for the propagation of intense laser pulses in underdense plasmas.

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#### 1. Introduction

When a laser pulse propagates through a nonlinear optical medium, it changes the refractive index according to its intensity. As this occurs, the peak and the edges of the laser pulse travel with different velocities leading to an asymmetric evolution of the laser pulse profile, i.e. self-steepening. Equivalently, as it steepens, the laser pulse frequencies are blue-shifted and red-shifted by different amounts, which also leads to asymmetric frequency spectra. Self-steepening can thus be examined by investigating the laser pulse intensity profile, electromagnetic (EM) field profile, or frequency spectra. In this work, the role of self-steepening in the laser intensity profile is examined in detail.

Self-steepening is a key phenomenon in nonlinear optics [1], and has been thoroughly examined in several media (such as liquids, crystals, glasses [2, 3] or fiber optics [4]) where it leads to asymmetric self-compression of the incident laser pulse and to the formation of optical shocks [1]. However, the nature and the role of self-steepening in the interaction of intense ( $\geq 10^{18} \text{ W cm}^{-2}$ ) and short ( $\leq 30 \text{ fs}$ ) lasers with transparent/underdense plasmas is still rather unexplored in conditions that are relevant for state-of-the-art experiments [5]–[8, 10].

The understanding of self-steepening in plasmas is important from a fundamental point of view, and also as an important mechanism occurring in several plasma-based applications [10]–[15]. On the one hand, self-steepening leads to distortions of the incident laser intensity profile, for instance, in experiments with the purpose of shortening the laser pulse [10]–[12]. On the other hand, enhancing self-steepening at the front of the laser is essential for the generation of single-cycle laser pulses [11, 12]. In addition, as efforts are being made in order to guarantee laser self-guiding in laser wakefield accelerators [5], and provided that the wake is properly loaded [16], the optimal acceleration structures are mainly determined by the evolution of the front of the pulse and thus by self-steepening.

Analytical models have described self-steepening based on the transverse evolution of the laser pulse. Since the transverse focusing rates are larger at the center than at the front of the laser, because the laser power is locally higher in the center than in the edges, the laser pulse longitudinal intensity profile becomes naturally asymmetric, and self-steepened [17]. This process was investigated in the long pulse limit and in the weakly relativistic regime, predicting self-steepening at the laser pulse front. However, self-steepening is mostly dominated by purely one-dimensional (1D) effects, which were not considered previously [18], for instance, in self-guided or matched propagation scenarios in plasma channels.

In this paper, we leverage on photon kinetic theory [19]–[21] to examine the longitudinal laser dynamics in plasmas. Photon kinetics explores the equivalence between the wave equation for EM waves, and the kinetic photon transport equation [20], [22]–[25]. The simplicity of the photon kinetic approach unraveled central results in nonlinear optics [19], and in white light parametric instabilities in plasmas [26, 27]. Photon-in-cell codes, which exploit the particle-like dynamics of the laser pulse photons, also provided novel insights in several laser–plasma interaction scenarios [20, 28]. Here, we use photon kinetics to describe the early laser pulse self-steepening and self-compression in plasmas, for arbitrary laser pulse intensities and durations. Novel regimes, where self-steepening occurs either at the front or at the back of the laser, are identified and verified with particle-in-cell (PIC) simulations [29, 30]. It is shown that the onset of self-steepening at the front of the laser occurs for mildly relativistic regimes. A comparison between the relative importance of self-steepening and relativistic self-focusing [5] is also provided.

#### 2. Photon kinetic model for intense laser-plasma interactions

In photon kinetics, the laser pulse is represented by a distribution function of quasi-particles  $\mathcal{N}(\mathbf{k}, \mathbf{r}, t)$  [19]–[21]. The early self-steepening of a laser pulse can be examined using the 1D photon distribution function, which, in underdense plasmas, where the ratio of the photon frequency to the plasma electron frequency is  $\omega \gg \omega_p$ , is given by [20, 21], [23]–[25]

$$\frac{\partial \mathcal{N}}{\partial t} + \frac{\mathrm{d}z}{\mathrm{d}t}\frac{\partial \mathcal{N}}{\partial z} + \frac{\mathrm{d}k}{\mathrm{d}t}\frac{\partial \mathcal{N}}{\partial k} = \mathcal{O}\left(\frac{\omega_{\mathrm{p}}^2}{\omega^2}\right),\tag{1}$$

where t is the propagation time normalized to the inverse of the plasma frequency  $\omega_p^{-1}$ , z is the propagation distance normalized to  $k_p^{-1} = c/\omega_p$ , and k is the wavenumber of the photons in the z-direction normalized to  $k_p$ . The right-hand side of equation (1) represents corrections to the geometric optics approximation that can be readily neglected for  $\omega \gg \omega_p$ . In addition, this analytical framework is valid independently of the laser pulse duration and intensity, as long as the envelope approximation can be employed. The initial photon distribution function of a Gaussian laser pulse with normalized envelope of the vector potential  $a \equiv q A/(m_e c^2) =$  $a_0 \exp(-z^2/L_0^2)$  is given by  $\mathcal{N}_0 = \sqrt{\pi}/(8\sqrt{2})\omega_0 L_0 a_0^2 \exp\{-\left[2z^2/L_0^2 + (k-k_0)^2 L_0^2/2\right]\}$ , where  $k_0$  and  $\omega_0$  are the central laser wavenumber and frequency, respectively, and  $L_0$  is the initial laser pulse length. The photon density is given by  $n_{\rm ph}(z, t) = \int \mathcal{N}_0(z, k, t) dk / \int \mathcal{N}_0(z, k, t) dk dz$ (in this paper, all integrations extend from  $-\infty$  to  $+\infty$ ) [20, 21]. For  $\omega \gg \omega_{\rm p}$ , equation (1) describes the photon number (or classical wave action) conservation. Thus, formally, the solution of equation (1) is  $\mathcal{N}(z, k, t) = \mathcal{N}_0[z_i(z, k, t), k_i(z, k, t), t_i]$ , where  $z_i, k_i$  and  $t_i$  are the initial position, wavenumber and propagation time of each photon. The phase-space trajectory of each photon, (z(t), k(t)), is determined according to the equations for the characteristics of equation (1), which correspond to the 1D ray-tracing equations (RTEs)  $\dot{z} = k/\omega$  and  $\dot{k} = -(1/2\omega)\partial_z \bar{n}$ , where  $\dot{q} = d_t(q)$  with q being a generic quantity, and  $\bar{n} = n/(n_0\gamma)$  is the normalized plasma density, n is the local plasma density,  $n_0$  is the background plasma density and  $\gamma$  is the relativistic mass factor. The photon frequency and the wavenumber are related by the dispersion relation for a forward propagating EM wave in an underdense plasma,  $\omega(k) = [k^2 + \bar{n}]^{1/2}$ . As long as the laser pulse duration and central frequency are such that the envelope approximation can be employed, the laser dynamics is mainly determined by photons with frequency  $\omega \simeq \omega_0$  (and wavenumber  $k \simeq k_0$ ). Thus, the RTEs can be expanded in Taylor series around  $\Delta k = k_0 - k$ . In the co-moving frame variables  $(\xi = z - v_{glinear}t, \tau = t)$ , where  $v_{glinear} \simeq 1 - 1/2k_0^2$ , and the resulting RTEs become

$$\dot{\xi}(\tau) = \frac{1 - \bar{n}}{2k_0^2} + \frac{\bar{n}\Delta k}{k_0^3} + \mathcal{O}\left(k_0^{-4}\right),\tag{2}$$

$$\dot{\Delta k}(\tau) = -\frac{\bar{n}'}{2k_0} + \frac{\bar{n}'\Delta k}{2k_0^2} + \mathcal{O}\left(k_0^{-3}\right),\tag{3}$$

where  $\bar{n}' \equiv \partial \bar{n} / \partial \xi$ .

Equations (2) and (3) describe the phase-space trajectory of a photon initially at  $(\xi_i, \Delta k_i)$ . Equation (2) determines the group velocity of each photon. The first term on the right-hand side of equation (2) leads to corrections of the laser linear group velocity and to self-steepening [17]. The second term contributes to self-steepening if  $L_0 \leq \lambda_p$  (i.e. if  $\bar{n}$  is asymmetric with respect

to the centroid of the laser), and is responsible for pulse self-compression in the limit of long pulses [18]. Equation (3) describes the blue/red shifts of the photons due to the gradients of  $\bar{n}$ .

The ratio  $\alpha$  between the first and the second terms of equation (2),  $\alpha \sim \phi k_0 / \Delta k \simeq \phi k_0 L_0$ , where  $\phi = (\bar{n} - 1) / \bar{n}$  is the electrostatic potential [31], and  $\Delta k \simeq L_0^{-1}$  can be used to examine the range of validity of the previous models [17, 18]. The results in [18] are valid for  $\alpha \ll 1$ , while the results in [17] are valid in the limit  $\alpha \gg 1$ , which corresponds to lower plasma densities and/or stronger plasma responses. However, for photons located at  $\xi \sim L_0/2$ —which are the most relevant ones for the evolution of the front of the laser—and for state-of-the-art lasers with a duration of 10 fs, and intensities exceeding  $10^{19}$  W cm<sup>-2</sup>, propagating in plasmas with  $10^{17}$ – $10^{18}$  cm<sup>-3</sup>,  $\alpha \simeq 1$ . Thus, the inclusion of both terms of equation (2) is required to examine self-steepening and the global laser pulse dynamics in conditions which are directly relevant for state-of-the-art experiments with short laser pulses.

#### 3. Onset of self-steepening and optical shock formation

The trajectories in phase space for early times are determined using a polynomial approximation for  $\xi$  and  $\Delta k$  up to  $\mathcal{O}(\tau^2)$ . In order to retrieve the early longitudinal evolution of the laser the solutions of equations (2) and (3), ( $\xi(\xi_i, \Delta k_i, \tau), \Delta k(\xi_i, \Delta k_i, \tau)$ ) are inverted to yield ( $\xi_i(\xi, \Delta k, \tau), \Delta k_i(\xi, \Delta k, \tau)$ ) up to  $\mathcal{O}(\tau^2)$ , and then inserted in  $\mathcal{N}$ . In the variables ( $\xi, \Delta k, \tau$ ), the photon density is then  $n_{\text{ph}}(\xi, \tau) = \int \mathcal{NJ} d\Delta k / \int \mathcal{N} d\Delta k d\xi$ , where  $\mathcal{J} = \partial(\xi_i, \Delta k_i) / \partial(\xi, \Delta k)$ . Expansion of  $n_{\text{ph}}$  up to  $\mathcal{O}(\tau^2)$  yields

$$n_{\rm ph} \propto e^{-2\xi^2/L_0^2} \left[ 1 + \frac{A_{\rm vg}\tau}{2L_0^2 k_0^2} + \left( P_{\rm disp} + P_{\rm sc} + B_{\rm vg} \right) \left( \frac{\tau}{2L_0^2 k_0^2} \right)^2 \right],\tag{4}$$

where  $A_{vg} = 4\xi(1-\bar{n}) + L_0^2\bar{n}'$  is the dominant contribution for the early propagation in the limit  $\alpha \gg 1$ . The term  $P_{\text{disp}} = -\left[2/(k_0^2L_0^2)\right] \left[L_0^2 - 4\xi^2\right] \left[k_0^2L_0^2(\bar{n}-1)^2 + 4\bar{n}^2\right]$  is a dispersive term,  $P_{\text{sc}} = -\left[2\xi/k_0^2\right] \left[12\bar{n} + k_0^2L_0^2(5\bar{n}-3)\right]\bar{n}' + \left[L_0^2/(2k_0^2)\right] \left[4\bar{n} + k_0^2L_0^2(3\bar{n}-1)\right]\bar{n}''$  is a self-compression term which, in combination with  $P_{\text{disp}}$ , recovers the known self-compression theory in the limit  $\alpha \ll 1$  [18] and  $B_{vg} = \left[L_0^2/(2k_0^2)\right] \left[4 + 3k_0^2L_0^2\right]\bar{n}'^2$ .

Equation (4) provides a global description of self-compression, self-amplification and selfsteepening. It predicts self-steepening either at the front or at the back of the laser, for arbitrary laser intensities and durations in the limits  $k_0 \gg 1$  and  $\Delta k/k_0 \ll 1$ .

The key features associated with the asymmetric evolution of the laser can be retrieved by determining the skewness of the distribution in equation (4). The skewness,  $\gamma_{skew} = \int n_{ph}(\xi, \tau)(\xi - \xi_c)^3 d\xi/L(\tau)^3$ , quantifies the asymmetry of a distribution [32], where  $\xi_c = \int n_{ph}(\xi, \tau)\xi d\xi / \int n_{ph}(\xi, \tau)d\xi$  is the laser pulse centroid and  $L(\tau)^2 = \int n_{ph}(\xi, \tau)(\xi - \xi_c)^2 d\xi / \int n_{ph}(\xi, \tau)d\xi$  is the square of the laser pulse length. For a laser moving to the right, self-steepening at the back (front) of the pulse leads to  $\gamma_{skew} > 0$  ( $\gamma_{skew} < 0$ ). Considering terms  $\mathcal{O}(\tau)$  in equation (4), and expanding  $A_{vg}$  up to  $\mathcal{O}(\xi^3)$  reveals that the direction of the early self-steepening of a short laser pulse is  $\langle \Delta k \rangle = \int n_{ph} \Delta k d\Delta k / \int n_{ph} d\Delta k \propto \bar{n}'(\xi = 0)$ , which means that if the back of the laser is down-shifted (up-shifted) with respect to the front, self-steepening occurs at the back (front) of the laser pulse. An illustration of self-steepening at the back of the pulse is shown in figure 1 with results from a 3D PIC simulation in QuickPIC [29].

The onset of self-steepening can be determined analytically. The quasi-static plasma response for a linearly polarized laser pulse is given by  $\phi'' = -(1/2)[1 - (1 + a(\xi)^2/2)/(1 + \phi)^2]$ .



**Figure 1.** Results from a 3D PIC simulation showing the evolution of a laser in a pre-formed parabolic plasma channel. (a) Initial laser profile at  $\tau = \tau_i$ . (b) Laser profile at later stages of the propagation ( $\tau > \tau_i$ ), showing self-steepening at the back of the laser with  $L^+ > L^-$ .

In the weakly relativistic regime,  $\phi_{WR} \simeq (1/4) \int_{\xi}^{\infty} a^2(x) \sin[x - \xi] dx$ , which can be determined for a Gaussian laser profile provided that  $L_0 \ll \lambda_p$  [31], by expanding  $\sin[x - \xi]$  for  $x \ll \lambda_p$ . The onset of self-steepening  $\gamma_{skew} < 0$  can then be written as the threshold condition  $\kappa^t$ :

$$a_0 L_0 > \kappa^t = 4/\sqrt{\pi - 1} \simeq 2.7. \tag{5}$$

In order to illustrate self-steepening at the front/back of the laser we have designed a set of 1D PIC simulations in OSIRIS [30] according to the threshold condition using  $a_0L_0 = 1.7 < \kappa^t$  and  $a_0L_0 = 17 > \kappa^t$ . The length of the simulation window, which moves at the speed of the light, is 25  $c/\omega_p$  long, and is divided into 4000 cells, with 200 particles per cell. The results comparing the laser pulse length, peak vector potential, and skewness evolution between the





**Figure 2.** Evolution of the laser length  $L^2$ , peak normalized vector potential  $a^2$  and skewness  $\gamma_{skew}$ , given by 1D PIC simulations (markers) and the analytical model using the numerical plasma response given by the quasi-static equations (solid lines). The results shown in (a) were obtained using  $a_0 = 1.0$ ,  $L_0 = 1.7$  and  $k_0 = 20$ , whereas in (b),  $a_0 = 10$ ,  $L_0 = 1.7$  and  $k_0 = 20$  were used.

simulations and equation (4) are shown in figure 2. The results from our model are obtained by inserting in equation (4) the numerical solution of the quasi-static plasma response. The square of the vector potential of the laser is estimated according to  $a^2(\xi, \tau) = n_{\rm ph}(\xi, \tau)/\langle \omega(\xi, \tau) \rangle$ , where  $\langle \omega(\xi, \tau) \rangle = \int \omega \mathcal{N} dk / \int \mathcal{N} dk$  is the average local frequency of the laser. In figure 2(a),  $a_0 L_0 < \kappa^t$  and therefore the skewness is positive and the laser beam is compressed while  $a^2$  increases. In figure 2(b),  $a_0 L_0 > \kappa^t$ , and thus the blowout of electrons, is strong enough to cause self-steepening at the front of the laser ( $\gamma_{\rm skew} < 0$ ).

Although the full laser dynamics results from an interplay between the transverse and longitudinal motion of the laser pulse photons, our model, given by equation (4), can also give insights into conditions that are directly relevant for experiments. State-of-the-art experiments typically use laser pulses with initial peak intensities  $a_0 = 1$ , with a central wavelength  $\lambda_0 = 800$  nm, and typical duration of 30 fs. In addition, typically,  $n_0 \simeq 4 \times 10^{18}$  cm<sup>-3</sup>, for which  $k_0/k_p = 20$ . In this situation, equation (4) predicts that the laser pulse self-compresses at a rate  $d(L^2/L_0^2)/d\tau \simeq -0.06k_p^2/k_0^2$ . Thus, at each propagation mm the laser becomes 7.25 fs shorter, a result that is consistent with recent experimental observations [10]. At the same time, the skewness of the laser decreases at a rate  $d\gamma_{\rm sk}/d\tau \simeq -0.3k_p^2/k_0^2$ , which indicates that the front of state-of-the-art laser pulses self-steepens. This is also consistent with the numerical modeling of the seminal experiments on the generation of quasi-monoenergetic electron bunches in the laser wakefield accelerator [33]–[36].

It is also interesting to compare the results from equation (4) in conditions that are directly relevant for the generation of single-cycle laser pulses [11]. To this end, a laser with  $a_0 = 4$ , with  $L_0 = \pi$  and with  $k_0/k_p = 5$  was considered. The early self-compression rates are  $d(L^2/L_0^2)/d\tau \simeq -0.12k_p^2/k_0^2$ . Therefore, a single-cycle laser pulse is formed for  $\tau \leq k_0^2/0.12 \simeq 200$ , consistent with the previous results [11]. The skewness of the laser pulse intensity profile also decreases as  $d\gamma_{sk}/d\tau \simeq -0.6k_p^2/k_0^2$ , which indicates that the self-compression mechanism is closely associated with the compression of the front edge of the laser pulse, and was already observed in the simulations in [11].

The early longitudinal dynamics of the laser beam can be physically illustrated by examining the trajectories of single photons. The distance between any two photons of the laser located initially at  $\xi_1$  and  $\xi_2$  is  $D = D_0 + \Delta v_g \tau + O(\tau^2)$ , where  $D_0 = \xi_1 - \xi_2$  and  $\Delta v_g = v_{g1} - v_{g2}$  is the difference between the group velocity of each photon. According to equations (2) and (3),  $D = D_0 + [\bar{n}(\xi_2) - \bar{n}(\xi_1)](\tau/2k_0^2) + O(\tau^2)$ . For a linearly polarized long laser pulse, in the weakly relativistic regime, such that  $\bar{n} \equiv \bar{n}_{WR} = 1 - (a_0^2/4) \exp(-2\xi^2/L_0^2)$  [31], the distance between two photons at the front (back) of the laser,  $L^+(L^-)$  (cf figure 1), initially separated by half the pulse length and located at  $\xi_1 = L_0/2$  and  $\xi_2 = 0$  ( $\xi_1 = 0$  and  $\xi_2 = -L_0/2$ ), is

$$L^{\pm} = \frac{L_0}{2} \mp \frac{\sqrt{e} - 1}{8\sqrt{e}} \frac{a_0^2 \tau}{k_0^2}.$$
 (6)

This indicates that the front of the pulse compresses while the back stretches, leading to the steepening at the front. In addition, an optical shock can occur at the laser pulse front when  $L^+ = 0$ , or equivalently

$$\tau^{+} = \frac{4\sqrt{e}}{\sqrt{e} - 1} \frac{L_0 k_0^2}{a_0^2} \simeq 10.2 \frac{L_0 k_0^2}{a_0^2} \tag{7}$$

in qualitative agreement with [17]. If the plasma response associated with a linearly polarized and short  $(L_0 \ll \lambda_p)$  laser pulse is used, both self-steepening and optical shock formation can occur at the front or at the back of the laser. The condition for self-steepening at the front is  $L^+/L^- < 1$ , or equivalently, and using equation (6),  $a_0L_0 > 2.9 \simeq \kappa^t$ . Moreover, the onset of optical shock formation at the front  $(\tau^+)$  and at the back  $(\tau^-)$  of the laser occurs after

$$\tau^{+} \simeq \frac{k_{0}^{2}}{a_{0}^{2}L_{0}} \left( 40.4 + 2.9a_{0}^{2}L_{0}^{2} + 0.033a_{0}^{4}L_{0}^{4} \right)$$
(8)

and

$$\tau^{-} \simeq \frac{k_0^2}{a_0^2 L_0} \left( 18.6 + 4.3a_0^2 L_0^2 + 0.2a_0^4 L_0^4 \right).$$
<sup>(9)</sup>

In equations (8) and (9), the numerical factors correspond to approximations of the constants which are functions of  $\pi$ , and e, and cumbersome roots, which are thus avoided. It is straightforward to extend these arguments to describe self-compression by including terms of the order of  $\mathcal{O}(\tau^2)$  in the photon trajectories  $\xi_{1,2}$ . In the light of a similar interpretation for self-focusing [37], this suggests that self-steepening can be viewed as a longitudinal asymmetric self-focusing.

A rigorous quantification of self-steepening is required for practical purposes. According to equation (4), and using  $\bar{n}_{WR}$ ,  $\xi_c = a_0^2 \tau / (8\sqrt{2}k_0^2)$  in the weakly relativistic regime. This result

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**Figure 3.** Map of the early skewness rates  $\partial_{\tau} \gamma_{skew}$ . Squares mark the region where  $\gamma_{skew} = 0$ , while circles mark the region where  $\gamma_{skew}$  is minimum. The black dashed line and the solid gray line are fits to the conditions where  $\gamma_{skew} = 0$  and where  $\gamma_{skew}$  is minimum, respectively.

matches the nonlinear relativistic shifts to the laser linear group velocity of [37] including finite pulse length effects. Inserting  $\xi_c$  into the expression for the skewness then yields

$$\gamma_{\rm skew} = -\frac{3\sqrt{\pi}a_0^2\tau}{16k_0^2} < 0. \tag{10}$$

If  $L_0 \leq \lambda_p$ , the early rates for the skewness evolution can only be retrieved by coupling equation (4) with the numerical quasi-static plasma response. The results are presented in figure 3. We observe that for early times  $(\tau/k_0^2 \ll 1)$ , the rate depends on  $k_0^2$  only (cf  $A_{\rm vg}$ term in equation (4)). The parameters that correspond to  $\gamma_{skew} = 0$  were fitted to a Laurent series yielding  $a_0L_0 = 2.6 + 2.5L_0^2 - 1.6L_0^3 + O(L_0^4)$ ,  $L_0 < L_m$  with  $L_m = 1.9$ , corresponding to a mildly relativistic regime where  $\phi \lesssim 1$ . We note that the condition  $a_0 L_0 > \kappa^t$  closely matches the fit in the limit  $L_0 \ll c/\omega_p$ . If  $L_0 > L_m$ , the skewness is negative for any initial  $a_0$  because  $\gamma_{\rm skew} \propto -\bar{n}''(\xi=0)$  can become negative (regardless of the laser initial intensity) solely by increasing the laser pulse length. For a sinusoidal wake,  $\bar{n}''(\xi = 0) > 0$  if  $L_0 \gtrsim \lambda_p/4 \simeq L_m$ . State-of-the-art laser wakefield acceleration experiments typically use lasers with  $L_0 \sim \lambda_p/2$ and self-steepening should occur at the laser pulse front. However, it is possible to conceive experimental scenarios where the opposite can be observed (for instance, by using shorter laser pulses). Examining the dependence of the rate with  $a_0$ , keeping the laser pulse length fixed, shows that increasing  $a_0$  leads to higher self-steepening rates at the front of the laser pulse up to  $a_0L_0 < 16.7 - 0.122L_0^2 + 0.34L_0^3 - 0.08L_0^4 + O(L_0^4)$ . From then on the initial rates for  $\gamma_{skew}$ decrease. This occurs because for such high  $a_0$ , the blowout of the plasma electrons is so strong [5] that most of the laser effectively travels in a vacuum-like region.

The global dynamics of the laser pulse is also affected by the transverse dynamics associated with self-focusing. The relative importance between self-focusing and self-steepening can be illustrated in the case of a linearly polarized long laser pulse, in the weakly relativistic regime. In this case, and using  $\bar{n}_{\rm WR}$ , the evolution of the pulse front length is  $L^+ = L_0 - (\sqrt{2} - 2)a_0^2\tau/(8\sqrt{\pi}k_0^2) + \mathcal{O}(\tau^2)$ . The pulse waist evolves with  $W = W_0 + 4\tau^2(1 - P/P_c)/k_0^2W_0^4 + \mathcal{O}(\tau^3)$ , where  $P/P_c = a_0^2W_0^2/32$  is the ratio between the power of the laser and the critical power for self-focusing. Thus, unlike self-focusing, compression of the front of the laser pulse will occur, regardless of the laser power and intensity. In addition, for early propagation, the longitudinal compression rate  $\dot{L}^+/L_0$  is higher than the transverse focusing rates  $\dot{W}^2/W_0^2$ , as long as

$$\tau < \frac{(2 - \sqrt{2})(P/P_{\rm c})W_0^3}{4\sqrt{\pi}(P/P_{\rm c} - 1)L_0}.$$
(11)

Moreover, in matched propagation regimes  $(P = P_c)$ , ideally suited for plasma-based accelerators, all the dynamics of the laser occurs in the longitudinal direction.

#### 4. Conclusions

In conclusion, we have examined for the first time early laser pulse longitudinal evolution with the inclusion of self-steepening and self-compression for initial arbitrary laser intensities and pulse durations, as long as the envelope approximation is valid. Conditions and early rates for the onset of self-steepening and optical shock formation at the front/back of the pulse were derived. Our results show that experiments can be designed such that longitudinal modulations are enhanced in order to increase the plasma wave amplitude, to facilitate self-injection and to further increase the energy gain of electron beams. Furthermore, in this work, we have identified the conditions for self-steepening to occur both at the front and at the back of the laser pulse, and that can also be used to identify new regimes to laser pulse amplification beyond the limits of state-of-the-art laser technology via optical shock formation.

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