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Abstract. We present a quantum electrodynamical model involving nitrogen-vacancy centers coupled to a whispering-gallery mode cavity. We consider two schemes to create the W state and Bell state, respectively. One scheme makes use of Raman transition with the cavity field virtually excited, and the other enables Bell state preparation and quantum information transfer by virtue of dark state evolution and adiabatic passage, which is tolerant to ambient noise and experimental parameter fluctuations. We justify our schemes by considering their experimental feasibility and challenge, using the currently available technology.
1. Introduction

The diamond nitrogen-vacancy (NV) center consisting of a substitutional nitrogen atom and an adjacent vacancy has attracted considerable attention after the first report of optically detected magnetic resonance on the single NV center in 1997 [1]. Owing to a sufficiently long electronic spin lifetime as well as the possibility of coherent manipulation at room temperature [2], the NV center is considered a promising building block for room temperature quantum computing in the future [3]–[5]. Since electronic spins can be well initialized and manipulated in an optical fashion, qubit readout and gating regarding the single-spin state have been achieved in individual NV centers [6]. By virtue of hyperfine coupling with the paramagnetic nuclei in the vicinity of the electron spin, i.e. $^{13}$C [7], $^{14}$N [8] and $^{15}$N [9], currently available techniques have demonstrated quantum information storage and retrieval between electronic and nuclear spins [7]. This technique also enables rapid and high-fidelity readout of quantum information from the electron spin [10]. However, the coherence between electron and nuclear spin qubits is restricted to the case of a few qubits owing to the limited number of nuclear spins individually addressable in frequency space [11, 12]. Hence, for scalability, it is necessary to develop methods for coupling distant NV centers.

We noticed a recent experiment to entangle a pair of separate NV centers within a diamond based on magnetic dipolar coupling [13]. But this idea is pretty hard for scalability. For distant NV centers with magnetic dipolar coupling unavailable, the best way for spin–spin entanglement seems to make use of parity projection by detecting the emitted photons relevant to different spin states [14]. However, the NV centers, although similar to the atomic cases, only allow linearly polarized radiation in the laser excitation, which makes coincident detection of emitted photons impossible. In addition, to effectively produce entanglement of the NV centers by parity projection of photons, we require that the ‘which-path’ information be removed due to interference after the photons go through the beam splitter. But the experimental reports so far have shown that 96% of the emitted photons reside in broad photon sidebands to the resonant zero phonon line (ZPL) at 637 nm even in the cryogenic situation [14]. This implies that most photons emitted from the NV centers could not effectively interfere in the beam splitter.
Alternatively, entanglement of separate NV centers could be achieved by coupling to the same cavity mode. In a recent publication [15], we proposed an idea to entangle more than two NV centers by one step of implementation, based on coupling to a microsphere cavity. The key idea of that proposal was the employment of the spin singlet state $^1A$ to encode a qubit. To our knowledge, however, this metastable $^1A$ state, although investigated from $C_{3v}$ group theory considerations [16] and other aspects, has not yet been fully understood [17]. Hence, most of the present work on quantum information processing with NV centers encodes qubits only in the sublevels of the ground state $^3A$.

We focus in this work on entangling distant NV centers without employing the state $^1A$. Specifically, we encode qubits in two of the ground state splittings, and the excited states are auxiliary with spontaneous emission effectively suppressed during our operation. The key point of our idea is to present a generalized Jaynes–Cummings model involving a quantized whispering-gallery mode (WGM) and $N$ identical NV centers. WGM microcavities are of typically dielectric rotational symmetry structures with WGMs traveling around the curved boundary and confined by continuous total internal reflection [18]. In particular, technological advances have made it possible to have strong light–matter coherent coupling in WGM resonators with a smaller mode volume $V_m$ and an extremely high quality factor $Q$ [19, 20]. Recent experimental progress in the nanocrystal microsphere system also provides experimental evidence for strong coupling between NV centers and the WGM of the silica microsphere [21], polystyrene microsphere [22] and gallium-phosphide microdisc [23], respectively. In addition, a very recent experiment has demonstrated the technique of deterministically coupling a single NV center to a photonic crystal cavity [24]. So far there has been much development on WGM cavities with forms such as the microtoroidal [25], microcylinders [26], microdiscs [27] and microspheres [28].

We will show how to generate $W$ state [29] and Bell states for these distant NV centers in such a composite nanocrystal–microsphere system. The main results of this work are twofold: firstly, by virtue of Raman transitions, we show the possibility, with virtual-photon-induced excitation in the large detuning case, to generate multipartite $W$ state with separate NV centers in different diamond nanocrystals, where the growth of the qubit number corresponds to the decrease of the operational time. Secondly, resorting to adiabatic passage technology [30, 31], we create Bell states of any pairs of qubits via dark-state evolution, which is robust to the cavity decay.

2. Entanglement generation by Raman configuration

2.1. Effective Hamiltonian

As sketched in figures 1(a) and (b), $N$ identical NV centers, respectively, in $N$ separate diamond nanocrystals can be strongly coupled to the WGM of a microsphere cavity or of a microtoroidal cavity. The NV center is a point defect in the diamond lattice, which consists of a nearest-neighbor pair of a nitrogen-atom impurity substituting a carbon atom and an adjacent carbon vacancy, as shown in figure 1(c). In the case of microspheres, WGM can be characterized by angular $l$, azimuthal $m$ and radial $s$ numbers. High values of $Q$ usually correspond to the modes with $l \gg 1$. Of greatest interest is the so-called fundamental WGM ($s = 1, l = m$), whose field is concentrated in the vicinity of the equatorial plane of the sphere. It is believed that these modes can be selectively excited by coupling to a tapered fiber [32]. Like in [15], our
Figure 1. (a) Schematic setup in the microsphere cavity case, where $N$ identical NV centers in diamond nanocrystals are equidistantly attached around the equator of a single fused-silica microsphere cavity. (b) Schematic setup in the microtoroidal cavity case, where $N$ identical NV centers in the vicinity of the microtoroidal cavity are able to interact with the WGM via the evanescent field. (c) Structure of the NV color center in the diamond lattice, consisting of a substitutional nitrogen (N) and a neighboring vacancy (V). (d) Level diagram for the $j$th NV center, where $\Delta_j$ and $\delta_j$ are detunings, $g_j$ is the coupling strength between the NV center and WGM, and $\Omega_j$ is the coupling strength between the NV center and the laser pulse. $D_{gs} = 2.88 \text{ GHz}$ is the zero-field splitting between the ground state sublevels $m_s = 0$ and $m_s = \pm 1$ ($m_s = \pm 1$ are degenerate at zero magnetic field due to $C_3$ symmetry) of the NV center. We encode qubits in the subspace spanned by the down-state $m_s = 0$ and the up-state $m_s = -1$.

Proposal is also based on recent experimental and theoretical progress, i.e. the possibility of $\Lambda$-type configuration of the optical transition in the NV center system [33] and the considerable enhancement of the ZPL by embedding the NV centers in some cavities [34]. But differently, the entanglement of the NV centers is achieved without involving the metastable $^1A$ state. In our case, we assume that each NV center located in a diamond nanocrystal is attached around the equator of a single fused-silica microsphere cavity [28] or microtoroidal cavity [25]. This composite nanocrystal–microsphere system takes advantage of the exceptional spin properties of NV centers as well as the ultrahigh quality factor $Q$ ($\geq 10^8$ even up to $10^{10}$), very small volume ($V_m \leq 100 \mu\text{m}^3$) and a simple fabrication technique of the cavity [35]. For convenience of description, we will mention below the WGM, but not relating the mode to any concrete cavity.

By combining laser pulses with carefully timed interaction with the WGM, one can model the NV center as a Λ-type three-level system as shown in figure 1(d), where the states $|3A, m_s = 0⟩$ and $|3A, m_s = −1⟩$ serve as the logical states $|0⟩$ and $|1⟩$ of the qubit, respectively, and the state $|3E, m_s = 0⟩$ is labeled by the state $|e⟩$. In our case, the WGM with frequency $ω_γ$ is far-off-resonant from the transition $|0⟩ ⇐⇒ |e⟩$ (with the frequency $ω_{e0}$), and the levels $|1⟩$ and $|e⟩$ (with transition frequency $ω_{e1}$) are coupled by a largely detuned laser with frequency $ω_γ$ and polarization $σ^+ [33, 36]$. The NV centers are fixed and separated by distance much larger than the wavelength of the WGM, interacting individually with laser beams. Hence, the direct coupling between NV centers is negligible. Assuming that the detuning $Δ_j$ is sufficiently larger than the coupling strength $g_j$ and $Ω_j$, the excited state $|e⟩$ can be adiabatically eliminated. Thus quantum logic gates and multipartite entangled states are available in the subspace spanned by $|0⟩$ and $|1⟩$. Using the rotating-wave approximation (RWA), the Hamiltonian in the interaction picture can be written in units of $\hbar = 1$ as [37]

$$H_i = \sum_{j=1}^{N} η_j[a^+ \sigma_j^- e^{-iδ_j t} + a \sigma_j^+ e^{iδ_j t}],$$

where $\sigma_j^+ = |1_j⟩⟨0_j|$, $\sigma_j^- = |0_j⟩⟨1_j|$ and $a^+ (a)$ is the creation (annihilation) operator of the WGM field. $η_j = g_j Ω_j (\frac{1}{Δ_j + δ_j} + \frac{1}{Δ_j})$ with $Δ_j = ω_{e1,j} - ω_{L,j}$ and $δ_j = ω_{e0,j} + ω_{L,j} - ω_{e1,j}$. For simplicity, we assume that the detuning $δ_j$ and the interaction term $η_j$ are identical for each qubit, that is, $η_j = η$ and $δ_j = δ$. In the case of $δ \gg η$, there is no energy exchange between the WGM and NV centers. If the quantized WGM is initially in the vacuum state, the effective Hamiltonian could be simplified as [38]

$$H_{eff} = γ \left[ \sum_{j=1}^{N} |1_j⟩⟨1_j| + \sum_{j,k=1,j\neq k}^{N} σ_j^+ σ_k^- \right],$$

where the first term corresponds to the dynamical energy shift regarding the level $|1⟩$, and the photon-dependent energy shift of the level $|0⟩$ is removed due to the vacuum state of the cavity. The rest of the terms in equation (2) denote the coupling between any pair of NV centers through the WGM, and $γ = |η^2/δ|$ is the effective coupling strength for the energy-conserving transition $|1_i0_j⟩ ⇐⇒ |0_i1_j⟩$.

2.2. Creation of W state

Let us first consider the creation of $N$-qubit W state for NV centers. If the first $(N − 1)$ NV centers are initially prepared in the state $|00 \cdots 0⟩_{1,2, \ldots, N−1}$ and only the $N$th NV center is initially prepared in the state $|1⟩_N$, one can easily obtain the following time-dependent state evolution:

$$|Ψ(t)⟩ = C_1 |00 \cdots 0⟩_{1,2, \ldots, N−1} |1⟩_N + C_2 |W⟩_{N−1} |0⟩_N,$$

with the coefficients $C_1 = (e^{-iNγ γt} + N − 1)/N$ and $C_2 = \sqrt{N−1}(e^{-iNγ γt} − 1)/N$. $|W⟩_{N−1} = (1/\sqrt{N−1}) |N−2, 1⟩$ is the generalized form of the $W$ state, which denotes the symmetric state involving $(N−2)$ zeros and 1 one. According to equation (3), the state of other $(N − 1)$ NV centers will surely collapse into the state $|W⟩_{N−1}$ in the case of the measurement on the $N$th NV center being $|0⟩_N$. As a result, we can find that the probability of obtaining the state $|W⟩_{N−1}$. 

Figure 2. The probability of obtaining the state $|W\rangle_{N-1}$ versus $\gamma t$.

is $|C_2|^2/(|C_1|^2 + |C_2|^2)$, which reaches maximum $P_{P_{\text{max}}} = 4(N-1)/[4(N-1) + (N-2)^2]$ at $t_N = (2k + 1)\pi/N \gamma$ with $k$ non-negative integers.

Figure 2 shows that the gating time $t_N$ is inversely proportional to the qubit number $N$, as is the maximal probability $P_{P_{\text{max}}}$. In this composite nanocrystal–microsphere system, the coupling between the NV center and WGM could reach $g_{\text{max}} = \Gamma_0 |\vec{E}(r)/E_{\text{max}}| \sqrt{V_a/V_m}$ [20], where $|\vec{E}(r)/E_{\text{max}}|$ is the normalized electric field strength at the location $r$, and $V_a = 3\lambda^2/4\pi \gamma$ denotes a characteristic interaction volume with $\lambda$ being the transition wavelength between the states $|e\rangle$ and $|0\rangle$, $\gamma$ the spontaneous decay rate of the excited state $|e\rangle$ and $c$ the speed of light. Using the values $\lambda = 637$ nm, $\gamma = 2\pi \times 83$ MHz [39], $V_m = 100 \mu$m$^3$ and $|\vec{E}(r)/E_{\text{max}}| = 1/6$, we have the maximal coupling $g_{\text{max}} \simeq 2\pi \times 1$ GHz, and the other experimental parameters can be adjusted as $\Omega_j = 2\pi \times 100$ MHz, $\Delta_j = 2\pi \times 10$ GHz and $\delta_j = 2\pi \times 100$ MHz. Provided $\eta_j = 2\pi \times 20$ MHz, we have $\gamma = |\eta_j^2/\delta_j| = 2\pi \times 4$ MHz, and the operation time $t_N$ is 0.0313, 0.0208 and 0.0156 $\mu$s in the case of $N = 4, 6$ and 8, respectively.

2.3. Estimate of decoherence

We now consider the influence due to decoherence, which results from the effective spontaneous emission from the states $|1\rangle$ to $|0\rangle$. Here we have neglected the WGM decay because the cavity decay rate could be $\kappa = \omega_{0}/Q = 2\pi \times 0.47$ MHz in the case of $Q = 10^9$, which is much smaller than the effective coupling rate $\gamma$. The characteristic spontaneous emission rate $\Gamma_{\text{eff}}$ regarding the states $|1\rangle$ and $|0\rangle$ could be estimated as $\Gamma_0 \Omega_j g_j/\Delta_j^2$ [40], where $\Gamma_0$ is the spontaneous decay rate of the excited state $|e\rangle$. Hence, the evolution of the system is described by the Lindblad equation [41]

$$\dot{\rho} = -i[H, \rho] + \Gamma_{\text{eff}}(2\sigma^- \rho \sigma^+ - \sigma^+ \sigma^- \rho - \rho \sigma^+ \sigma^-).$$

The spontaneous decay rates from $|e\rangle$ to $|1\rangle$ and from $|e\rangle$ to $|0\rangle$ are assumed to be equal.
Figure 3. (a) The fidelity of the state $|W\rangle_4$ versus $\gamma t$, where the blue, green and red curves correspond to $\Gamma_{\text{eff}} = \gamma / 50$, $\gamma / 100$ and $\gamma / 200$, respectively. (b) The fidelity of the state $|W\rangle_4$ versus $\Gamma_{\text{eff}} / \gamma$, where the gating time is $\pi / 4 \gamma$.

Figure 3 shows the fidelity of the state $|W\rangle_4$ when the spontaneous decay is considered. With the increase of $\Gamma_{\text{eff}}$, the fidelity decreases accordingly. However, our scheme can still achieve a high fidelity as long as the spontaneous decay is weak. In a realistic experiment, the situation would be more complicated than our consideration above. As a result, to carry out our scheme with high efficiency and high fidelity, we have to suppress the above-mentioned imperfect factors as much as we can.

3. Entanglement generation by adiabatic passage of dark states

3.1. Dark-state evolution

Under Raman resonance conditions between two Zeeman sublevels of the ground state, we focus on generating a Bell state with any pair of the NV centers (e.g. A and B) in the nanocrystal–microsphere system, via adiabatic passage of the dark states [42]. The adiabatic passage [43] is a useful and robust technique for quantum-state manipulation. It is a method of using two time-separated but partially overlapping pulses in the counterintuitive sequence to produce complete population transfer or an arbitrary coherent superposition between the initial and final states.

The difference from the above section is that the WGM and the laser pulses should be in resonance with the transitions $|0\rangle \leftrightarrow |e\rangle$ and $|1\rangle \leftrightarrow |e\rangle$, respectively, as shown in figure 4.

The interacting Hamiltonian, after RWA, has the form

$$H'_I = \sum_{j=A,B} [g_j a_j^\dagger |0_j\rangle \langle e_j| + \Omega_j(t) |e_j\rangle \langle 1_j| + H.c.],$$

(5)
To obtain the Bell state $|\beta\rangle$, we need to ensure that the coupling strength $g$ is much larger than the Rabi frequency of the laser pulse $\Omega_0$. Hence, the noteworthy features of our scheme are as follows: (i) in principle, the excited states of NV centers are negligibly populated due to the dark state dominant in the evolution, so the preparation of the Bell state is immune to spontaneous

\[ |D_f\rangle = \tilde{N}[\sqrt{2}\Omega_0(t)g_0|\text{Bell}\rangle_{AB}|0_c\rangle - \Omega_0^2(t)|0_A\rangle|0_B\rangle|1_c\rangle] \]  

with the Bell state $|\text{Bell}\rangle_{AB} = (|1_A\rangle|0_B\rangle + |0_A\rangle|1_B\rangle)/\sqrt{2}$. Our interest is in the components of the vacuum state $|0_c\rangle$ of the WGM, which as shown in equation (9) corresponds to the Bell state, and we have to project the WGM on the state $|0_c\rangle$. To obtain the Bell state $|\text{Bell}\rangle$ with high success rates, we need to ensure that the coupling strength $g_0$ is much larger than the Rabi frequency of the laser pulse $\Omega_0$. Hence, the noteworthy features of our scheme are as follows: (i) in principle, the excited states of NV centers are negligibly populated due to the dark state dominant in the evolution, so the preparation of the Bell state is immune to spontaneous

**Figure 4.** Level diagram for two NV centers $A$ and $B$, where $g_j$ ($j = A$ and $B$) are the coupling constants due to the WGM and $\Omega_j(t)$ are the Rabi frequencies relevant to laser pulses.
presents the numerical treatment for the QIT process: slowly changing the Rabi frequencies to meet the condition for transfer of quantum information could be achieved. Compared with the creation of the Bell pulse on NV center $B$ which can be coherently transferred to the NV center $A$, encoded on the NV center, transfer (QIT) between any pairs of NV centers, where the original quantum information is negligible during the operations. We will justify these points by numerics later.

Note that the method of dark-state evolution can also be applied to quantum information transfer (QIT) between any pairs of NV centers, where the the original quantum information is encoded on the NV center $A$ as $|\Psi_0\rangle = c_0|0_A\rangle + c_1|1_A\rangle$ (where $c_0$ and $c_1$ are arbitrary coefficients), which can be coherently transferred to the NV center $B$ (initially prepared in $|0_B\rangle$ state) via Raman transitions induced by a pair of time-delayed laser pulses. We can apply such a ‘counterintuitive’ pulse sequence from $\Omega_A(t)/\Omega_B(t) \ll 1$ to $\Omega_A(t)/\Omega_B(t) \gg 1$, namely, the pulse on NV center $B$ precedes the pulse on NV center $A$, which guarantees that adiabatic transfer of quantum information could be achieved. Compared with the creation of the Bell state, there is only a slight modification to step (iii) in the above-mentioned procedure, i.e. slowly changing the Rabi frequencies to meet the condition $\Omega_A \gg \Omega_B$, rather than $\Omega_A = \Omega_B$.

In addition, different from the case of preparing the Bell state, the initial state $|\Psi_0\rangle \langle 0_B|0_c\rangle$ will drive the system to undergo a different dark-state evolution involving two dark states $|D\rangle$ and $|D\rangle'$, where $|D\rangle' = |0_A\rangle|0_B\rangle|0_c\rangle$ is another dark state regarding the Hamiltonian in equation (5).

The QIT process could be briefly expressed as

$$|c_0\rangle|0_A\rangle + c_1|1_A\rangle \rightarrow |c_0\rangle|0_B\rangle + c_1|1_B\rangle \rangle |0_c\rangle.$$

(10)

### 3.2. Decay case

Without losing generality, we consider two NV centers with identical parameters and two laser pulses with Gaussian envelopes $\Omega_A(t) = \Omega_m e^{-(t-t_A)^2/\Delta \tau^2}$ and $\Omega_B(t) = \Omega_m e^{-(t-t_B)^2/\Delta \tau^2}$, where $\Omega_m$ is the maximal value of $\Omega_{(A)B}$ at the central time $\tau_j$ for the pulse $j \ (j = A, B)$, and $\Delta \tau$ is the laser beam waist. Figure 5 presents the numerical treatment for the QIT process: $|1_A\rangle|0_B\rangle|0_c\rangle \rightarrow |0_A\rangle|1_B\rangle|0_c\rangle$. We have compared the population of $|0_A\rangle|1_B\rangle|0_c\rangle$ in an ideal dark-state evolution described by equation (5) with that in the decay case. If no photon leakage really happens either from the excited state or from the cavity mode during the gating period, the system is governed by

$$H_{\text{decay}} = \sum_{j=A,B} \left[ g_j a^+ \langle 0_j |e_j \rangle + \Omega_j(t) \langle e_j |1_j \rangle + \text{H.c.} \right] - \frac{i}{2} a^+ a - \frac{\Gamma}{2} \sum_{j=A,B} |e_j \rangle \langle e_j |,$$

(11)

where $\kappa$ is the cavity decay rate, and $\Gamma$ is the spontaneous emission rate with respect to the excited state $|e\rangle$. We set the values of other experimental parameters as $g_A = g_B = g_0 = 2\pi \times 1$ GHz, $\Omega_m \simeq 2\pi \times 470$ MHz, $\tau_A = 6.8$ ns, $\tau_B = 5$ ns, $\Delta \tau = 1.8$ ns and $\kappa = \Gamma = g_0/10$, which fulfill the adiabatic conditions $\Omega_m \Delta \tau \gg 1$ and $g_j \Delta \tau \gg 1$ [44]. In our case, the time delay $(\tau_A - \tau_B)$ between two pulses $\Omega_A$ and $\Omega_B$ of the same step is chosen to be equal to $\Delta \tau$ for minimizing the non-adiabatic losses [45]. Moreover, the condition $g_j \gg \Omega_m$ guarantees that the cavity mode is negligibly populated during the interaction with the pulses, as demonstrated in figure 5.

We emphasize that the adiabatic passage method could make the QIT process robust against the fluctuations of experimental parameters, such as small variations of the peak Rabi frequencies $\Omega_m$ and of the WGM–NV coupling strength $g_0$. Once the adiabatic conditions $\Omega_m \Delta \tau \gg 1$, $g_j \Delta \tau \gg 1$ and $g_j \gg \Omega_m$ could be well satisfied, our scheme neither requires...
Figure 5. Numerical simulation exhibiting the populations versus $g_0 t$. Top: the solid curve denotes the population of $|1_A\rangle|0_B\rangle|0_c\rangle$. The dotted and dashed curves represent the population of $|0_A\rangle|1_B\rangle|0_c\rangle$ in the ideal and decay cases, respectively. The inset shows the Rabi frequencies $\Omega_A$ (solid line) and $\Omega_B$ (dashed line). Bottom: the solid, dotted and dashed curves represent the populations of $|0_A\rangle|0_B\rangle|1_c\rangle$, $|e_A\rangle|0_B\rangle|0_c\rangle$ and $|0_A\rangle|e_B\rangle|0_c\rangle$, respectively, where the parameters used are $\Omega_m \Delta \tau = 5$ and $g_0 \Delta \tau = 11$.

accurate manipulation of the intensities of the laser pulses nor demands precise control of the WGM–NV interaction. The only thing we need to do is keep the phase differences stable between the laser pulses $\Omega_A(t)$ and $\Omega_B(t)$, which is easily controllable experimentally. It should be mentioned that the fidelity of the Bell state $|\text{Bell}\rangle_{AB}$ depends only on the ratio of the WGM–NV coupling constants $g_A$ and $g_B$ after the laser pulses $\Omega_A(t)$ and $\Omega_B(t)$ are turned off. If $g_A$ and $g_B$ are not the same, the two NV centers will be prepared in the state $(g_A |1_A\rangle|0_B\rangle + g_B |0_A\rangle|1_B\rangle)/\sqrt{g_A^2 + g_B^2}$ with the fidelity $F = (g_A + g_B)^2/[2(g_A^2 + g_B^2)]$.

4. Feasibility and challenge

We examine the feasibility of our scheme and survey the relevant experimental parameters. As stated above, NV centers are well suited for cavity QED because they have a long electron spin relaxation time and their electronic states can be initialized, manipulated and measured...
through highly stable optical and microwave excitations at room temperature. Additionally, WGM microcavities exhibit exciting characteristics such as an extremely high quality factor $Q$, small mode volume $V_m$ and excellent scalability, which make it possible to achieve a high concentration of the optical field and relatively long photon confinement times. Experimentally, single NV centers strongly coupling to a WGM have been demonstrated in different kinds of microcavities [21]–[23], which are a great advance in WGM-based cavity QED research.

In the composite nanocrystal–microsphere system, we focus our attention on the dipole transition $|0\rangle \leftrightarrow |e\rangle$ with a ZPL at $\lambda = 637$ nm of the NV centers in a diamond nanocrystal. In the above-mentioned WGM–NV experiments, the NV centers actually interact with the evanescent field of the WGM. The evanescent field is of importance because it offers an effective way to exchange energy between the WGM and the external NV center. Thus, we assume that single NV centers are located near the microcavity surface in order for the WGM–NV coupling $g_{\text{max}}$ to reach $2\pi \times 1$ GHz [20] with the parameters in section 2.2. In the fused-silica microsphere cavity, the small radius of 10 $\mu$m could lead to a vacuum electric field of $150$ V cm$^{-1}$ at the sphere surface and to the $Q$ factor exceeding $10^{9}$, which imply the WGM decay rate to be $\kappa = \omega_{0}/Q = 2\pi \times 0.47$ MHz. Experimental studies have demonstrated $Q$ factors approaching $10^{10}$ in a silica WGM microsphere [35, 46], with values exceeding $10^{8}$ readily achievable over a broad range of cavity diameters and wavelengths. The strong coupling strongly depends on the critical photon number $n_0 = \Gamma^2_0/(2g_{\text{max}}^2)$ and the critical atom number $N_0 = 2\Gamma_0\kappa/(g_{\text{max}}^2)$, which give the number of photons required to saturate an NV-qubit and the number of NV-qubits required to have an appreciable effect on the WGM cavity transmission, respectively [20]. Based on these parameters, one can find that the strong coupling conditions $g_{\text{max}} \gg \kappa$, $\Gamma_0$ and $(n_0, N_0) \ll 1$ could be well satisfied in our scheme.

Nevertheless, in realistic experiments, WGMs are dominantly confined inside the microcavity body, and only the remaining energy of the WGM can be resident in the exterior evanescent field. Thus the coherent coupling strength cannot reach its maximum. That is why the maximal WGM–NV coupling in current experiments only reaches $2\pi \times 300$ MHz [23]. There is increasing interest in the development of alternative microcavity systems. We note that a recent work [47] has demonstrated an enhanced coherent interaction between the WGM and quantum dots using a kind of plasmonic WGM highly localized on the exterior surface of a metal-coated microrotoroidal. An alternative solution is using the silica microtoroid coated with a high-refractive-index (HRI) nanolayer [48]. The key idea is that this HRI nanolayer can compress the radial WGM field and move the WGM field in the coated microrotor to outside the silica surface.

Note that the preparation of entangled states and implementation of QIT via dark states are similar in spirit to the other atom [42] and atom-like systems [49]. However, we consider here a different system, and our proposal has several merits as given below. Firstly, the NV center in diamond is the only currently known viable solid-state qubit at room temperature, and the center’s highly localized bound states are well isolated from sources of decoherence. So the ground state can exhibit an extremely long spin coherence time of up to milliseconds, which is close to the regime needed for quantum error correction [50]. Secondly, the proposed QIT protocol has the potential for scalability because our QIT protocol does not require identical WGM–NV coupling strength, which implies that neither identical qubits nor exact placement of NV centers in cavities is needed. Thirdly, as the NV center nanocrystals are required to be attached along the equator of the microsphere with spacing bigger than the laser wavelength, individual addressing is not an obstacle in our scheme.
The above two schemes require different conditions in implementation. For example, in the first scheme the WGMs are detuned from the transitions in the NV centers, whereas the second scheme requires resonant coupling between the WGMs and the NV centers. If we employ the same WGM cavities to accomplish the schemes, the detuned and the resonant couplings regarding the WGM radiation could be achieved by changing the temperature of the system [21].

In present-day experiments, the electron spin relaxation time $T_1$ of diamond NV centers ranges from 6 ms at room temperature [11] to seconds at low temperature. In addition, the dephasing time $T_2 = 350 \mu s$ induced by the nearby nuclear-spin fluctuation has been reported [51]. A very recent experimental advance [52] with an isotopically pure diamond sample has demonstrated a longer $T_2$, i.e. $T_2 = 2$ ms. In our scheme, the operation times for preparing the four-qubit W state, the two-qubit Bell state and for accomplishing QIT are about 31.3, 6.786 and 8.0 ns, respectively. Hence, even at room temperature, up to $10^{3}–10^{4}$ gate operations are feasible under the present experimental conditions.

For more technical aspects, to make sure the NV centers are strongly and nearly equally coupled to the WGM cavity, we have to simultaneously attach separate NV centers around the equator of a single fused-silica microsphere resonator. The experimental difficulty lies in how to fix the nanocrystals appropriately with respect to the WGM. So far, strong coupling between a single NV center and the WGM of a silica microsphere [21] and a gallium-phosphide microdisc [23] and between two NV centers and a polystyrene microsphere [22] has been experimentally achieved. Hence, we wish that this would be soon extended to more NV centers. Further consideration would involve nuclear spins in the NV centers, which are more suitable to store quantum information due to a longer decoherence time. In this sense, we may also consider quantum computation with NV centers combining nuclear spins with electron spins, i.e. encoding the qubits in the nuclear spins and employing the electron spins as ancillas. The hyperfine interaction helps us to transfer the generated entanglement of the electron spins to the corresponding nuclear spins of NV centers [7].

5. Conclusion

In conclusion, we have proposed two schemes to prepare the W state and Bell state with separate NV centers in the diamond nanocrystal–microsphere system, respectively. In both these schemes, the cavity decay and the spontaneous emission from the excited states could be effectively suppressed. In particular, in the latter scheme with the adiabatic passage, the QIT is robust against experimental parameter fluctuation.

Our ideas could be applied to other cavity systems besides the WGM-type cavities. The number of entangled NV centers depends on the ratio of the size of the employed microcavity to the nanocrystal size. For more NV centers to be entangled, detection of emitted photons by parity projection would be necessary [14]. In this sense, we argue that this work has demonstrated a building block for a large-scale NV center system, which would be feasible in the near future.

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