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Axions as dark matter particles

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Abstract. We overview the current status of axions as dark matter. The axion is the pseudo-Nambu–Goldstone boson which arises from the Peccei–Quinn solution to the strong CP problem. Additionally, cold axion populations that can contribute to the dark matter of the universe will be generated via this mechanism. After reviewing these topics, we focus on constraints from the laboratory, astrophysics and cosmology. We discuss the current status of experimental searches and the consequences of the distribution of dark matter axions in the galactic halo for these searches. The axion remains an excellent candidate for the dark matter and future experiments, particularly the Axion Dark Matter eXperiment (ADMX), will cover a large fraction of the axion parameter space.

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1. Introduction

Despite our knowledge of dark matter’s properties, what it consists of is still a mystery. The standard model of particle physics does not contain a particle that qualifies as dark matter. Extensions to the standard model do, however, provide viable particle candidates. The axion, the pseudo-Nambu–Goldstone boson of the Peccei–Quinn (PQ) solution to the strong CP problem [1]–[4], is a strongly motivated particle candidate. We review the strong CP problem and its resulting axion in section 2.

In the early universe, cold axion populations arise from vacuum realignment [5]–[7] and string and wall decay [8]–[20]. Which mechanisms contribute depends on whether the PQ symmetry breaks before or after inflation. These cold axions were never in thermal equilibrium with the rest of the universe and could provide the missing dark matter. The cosmological production of cold axions is outlined in section 3.

Current constraints on the axion parameter space, from astrophysics, cosmology and experiments, are reviewed in section 4. The signal observed in direct detection experiments depends on the phase-space distribution of dark matter axions. In section 5, we discuss possible structure for dark matter in our galactic halo and touch on the implications for detection. Experimental searches and their current status are discussed in section 6. Finally, the current status of the quantum chromodynamics (QCD) axion is summarized in section 7.
This is a brief review, designed to overview the current status of axions as dark matter. For more advanced details, we refer the reader to the extensive literature (e.g. reviews can be found in [21]–[25]).

2. The strong CP problem and the axion

2.1. Strong CP problem

The strong CP problem arises from the non-Abelian nature of the QCD gauge symmetry, or colour symmetry. Non-Abelian gauge potentials have disjoint sectors that cannot be transformed continuously into one another. Each of these vacuum configurations can be labelled by an integer, the topological winding number, \( n \). Quantum tunnelling occurs between vacua. Consequently, the gauge invariant QCD vacuum state is a superposition of these states, i.e.

\[
|\theta\rangle = \sum_n e^{-in\theta} |n\rangle.
\]

(1)

The angle \( \theta \) is a parameter that describes the QCD vacuum state, \( |\theta\rangle \).

In the massless quark limit, QCD possesses a classical chiral symmetry. However, this symmetry is not present in the full quantum theory due to the Adler–Bell–Jackiw anomaly [26, 27]. In the full quantum theory, including quark masses, the physics of QCD remains unchanged under the following transformations of the quark fields, \( q_i \), quark masses, \( m_i \) and vacuum parameter, \( \theta \):

\[
q_i \rightarrow e^{i\alpha_i \gamma_5 / 2} q_i,
\]

(2)

\[
m_i \rightarrow e^{-i\alpha_i} m_i,
\]

(3)

\[
\theta \rightarrow \theta - \sum_{i=1}^{N} \alpha_i,
\]

(4)

where the \( \alpha_i \) are phases and \( \gamma_5 \) is the usual product of gamma matrices.

The transformations of equation (2) through equation (4) can be used to move phases between the quark masses and \( \theta \). The angle \( \theta \) is not invariant and thus not an observable. Under these transformations, the observable physics of QCD remains the same; however, they do not represent a symmetry of the QCD theory, due to the change in \( \theta \). However, the quantity,

\[
\bar{\theta} = \theta - \text{arg det} \mathcal{M} = \theta - \text{arg}(m_1 m_2 \cdots m_N),
\]

(5)

where \( \mathcal{M} \) is the quark mass matrix, is invariant and thus observable.

The presence of \( \theta \) in QCD violates the discrete symmetries P and CP. However, CP violation has not been observed in QCD. An electric dipole moment for the neutron is the most easily observed consequence of QCD, or strong, CP violation. \( \theta \) results in a neutron electric dipole moment of [21]–[24]

\[
|d_n| \sim 10^{-16} \bar{\theta} e \text{ cm},
\]

(6)

where \( e \) is the electric charge. The current experimental limit is [28]

\[
|d_n| < 6.3 \times 10^{-26} e \text{ cm}
\]

(7)

and thus, \( |\bar{\theta}| \lesssim 10^{-9} \). There is no natural reason to expect \( \bar{\theta} \) to be this small. CP violation occurs in the standard model by allowing the quark masses to be complex and thus the natural value
of $\theta$ is expected to be of order one. This is the strong CP problem, i.e. the question of why the angle $\bar{\theta}$ should be nearly zero, despite the presence of CP violation in the standard model.

The PQ solution [1, 2] to this problem results in an axion [3, 4]. While other solutions to the strong CP problem have been proposed, the presence of the axion in the PQ solution makes it the most interesting when searching for the dark matter of the universe. Thus, we focus on this solution only in the following.

2.2. The PQ solution

The axion is the pseudo-Nambu–Goldstone boson from the PQ solution to the strong CP problem [1]–[4]. In the PQ solution, $\theta$ is promoted from a parameter to a dynamical variable. This variable relaxes to the minimum of its potential and hence is small.

To implement the PQ mechanism, a global symmetry, $U(1)_{PQ}$, is introduced. This symmetry possesses a colour anomaly and is spontaneously broken. The axion is the resulting Nambu–Goldstone boson and its field, $a$, can be redefined to absorb the parameter $\bar{\theta}$. While initially massless, non-perturbative effects, which make QCD $\bar{\theta}$ dependent, also result in a potential for the axion. This potential causes the axion to acquire a mass and relax to the CP conserving minimum, solving the strong CP problem.

As there are no degrees of freedom available for the axion in the standard model, new fields must be added to realize the PQ solution. In the original, Peccei–Quinn–Weinberg–Wilczek (PQWW) axion model, an extra Higgs doublet was used. We review this model to demonstrate the PQ mechanism.

Assume that one of the two Higgs doublets in the model, $\phi_u$, couples to up-type quarks with strengths $y^u_i$ and the other, $\phi_d$, couples to down-type quarks with strengths $y^d_i$, where $i$ gives the variety of up- or down-type quark. We label the up- and down-type quarks $u_i$ and $d_i$, respectively (rather than $q_i$, as in the previous section). With a total of $N$ quarks, there are $N/2$ up-type quarks and down-type quarks. The leptons may acquire mass via Yukawa couplings to either of the Higgs doublets or to a third Higgs doublet. We ignore this complication here and simply examine the couplings to quarks.

The quarks acquire their masses from the expectation values of the neutral components of the Higgs, $\phi^0_u$ and $\phi^0_d$. The mass generating couplings are

$$L_m = y^u_i u^*_L i^\dagger \phi^0_u u_R + y^d_i d^*_L i^\dagger \phi^0_d d_R + h.c.$$  \hspace{1cm} (8)

Peccei and Quinn chose the Higgs potential to be

$$V(\phi_u, \phi_d) = -\mu^2_u \phi^*_u \phi_u - \mu^2_d \phi^*_d \phi_d + \sum_{i,j} a_{ij} \phi^*_i \phi_j + \sum_{i,j} b_{ij} \phi^*_i \phi_j \phi^*_i \phi_j,$$  \hspace{1cm} (9)

where the matrices $(a_{ij})$ and $(b_{ij})$ are real and symmetric and the sum is over the two types of Higgs fields. With this choice of potential, the full Lagrangian has a global $U_{PQ}(1)$ invariance,

$$\phi_u \rightarrow e^{i2\alpha_u} \phi_u,$$ \hspace{1cm} (10)

$$\phi_d \rightarrow e^{i2\alpha_d} \phi_d,$$ \hspace{1cm} (11)

$$u_i \rightarrow e^{-iu_i\gamma^5} u_i,$$ \hspace{1cm} (12)

$$d_i \rightarrow e^{-id_i\gamma^5} d_i,$$ \hspace{1cm} (13)

$$\bar{\theta} \rightarrow \bar{\theta} - N(\alpha_u + \alpha_d).$$ \hspace{1cm} (14)
When the electroweak symmetry breaks, the neutral Higgs components acquire vacuum expectation values:

\[ \langle \phi^0_u \rangle = v_u e^{i \theta_u / v_u}, \quad \text{(15)} \]

\[ \langle \phi^0_d \rangle = v_d e^{i \theta_d / v_d}. \quad \text{(16)} \]

One linear combination of the Nambu–Goldstone fields, \( P_u \) and \( P_d \), is the longitudinal component of the Z-boson,

\[ h = \cos \beta_v P_u - \sin \beta_v P_d. \quad \text{(17)} \]

The orthogonal combination is the axion field,

\[ a = \sin \beta_v P_u + \cos \beta_v P_d. \quad \text{(18)} \]

Using equations (15)–(18) with equation (8), the axion couplings to quarks arise from

\[ -L_m = m_u u_i u_i^\dagger e^{i (\sin \beta_v / v_u) a} u_R i e^{i (\cos \beta_v / v_u) a} d_R i + \text{h.c.}, \quad \text{(19)} \]

where \( m_u = y_u v_u u_i \) and \( m_d = y_d v_d d_i \). The axion field dependence can be moved from the mass terms using the transformations of equations (12)–(14). Direct couplings between the axion and quarks will remain in the Lagrangian, through the quark kinetic term. Defining \( v = \sqrt{v_u^2 + v_d^2} \), the corresponding change in \( \theta \) is

\[ \theta \to \theta - N (v_u/v_d + v_d/v_u) a / v, \quad \text{(20)} \]

which can be absorbed by a redefinition of the axion field.

Non-perturbative QCD effects explicitly break the PQ symmetry, but do not become important until confinement occurs. These effects give the axion field a potential and when significant, the field relaxes to the CP conserving minimum. Hence the PQ mechanism, which replaces \( \theta \) with the dynamical axion field, solves the strong CP problem.

Under the PQWW scheme, the axion mass is tied to the electroweak symmetry breaking scale, resulting in a mass of the order of 100 keV. This heavy PQWW axion has been ruled out by observation, as discussed in section 4.1. This does not, however, eliminate the possibility of solving the strong CP problem with an axion. In the following section, we discuss viable axion models.

2.3. Axion models

‘Invisible’ axion models, named so for their extremely weak couplings, are still possible. In an invisible axion model, the PQ symmetry is decoupled from the electroweak scale and is spontaneously broken at a much higher temperature, decreasing the axion mass and coupling strength. Two benchmark, invisible axion models exist: the Kim–Shifman–Vainshtein–Zakharov (KSVZ) [29, 30] and Dine–Fischler–Srednicki–Zhitnitsky (DFSZ) [31, 32] models. In both these models, an axion with permissable mass and couplings results.

In the KSVZ model, the only Higgs doublet is that of the standard model. The axion is introduced as the phase of an additional electroweak singlet scalar field. The known quarks cannot directly couple to such a field, as this would lead to unreasonably large quark masses. Instead, the scalar is coupled to an additional heavy quark, also an electroweak singlet. The axion couplings are then induced by the interactions of the heavy quark with the other fields.
The DFSZ model has two Higgs doublets, as in the PQWW model, and an additional electroweak singlet scalar. It is the electroweak singlet that acquires a vacuum expectation value at the PQ symmetry-breaking scale. The scalar does not couple directly to quarks and leptons, but via its interactions with the two Higgs doublets.

PQ symmetries also occur naturally in string theory, via string compactifications, and are always broken by some type of instanton. While this could be expected to make the axion an outstanding dark matter candidate, string models favour a value of the PQ scale that is much higher than that allowed by cosmology (see the discussion in 3). A review of the current situation can be found in [33]. As discussed in [33], it is difficult to push the PQ scale far below $1.1 \times 10^{16}$ GeV and easier to instead increase its value. Anthropic arguments can be made in favour of such a high value of $f_a$ (see e.g. [34]–[36] and references therein) and recent discussions of limits on the ‘anthropic’ axion window can be found in [37, 38].

Generically, axion couplings to other particles are inversely proportional to $f_a$; however, the exact strength of these couplings are model dependent. For example, the coupling between axions and photons can be written as

$$\mathcal{L}_{a\gamma\gamma} = g_{a\gamma\gamma} a E \cdot B,$$

where $E$ and $B$ are the electromagnetic field components. The coupling constant is

$$g_{a\gamma\gamma} = \frac{g_{\gamma\gamma}}{f_a},$$

where $\alpha$ is the electromagnetic fine structure constant, $10^9$ GeV $\lesssim f_a \lesssim 10^{12}$ GeV is the axion decay constant, of the order of the PQ scale, and $g_{\gamma\gamma}$ is a constant containing the model dependence. Explicitly,

$$g_{\gamma\gamma} = \frac{1}{2} \left( \frac{E}{N} - \frac{2(4 + z)}{3(1 + z)} \right),$$

where $z$ is the ratio of the up and down quark masses, $N$, the axion colour anomaly and $E$, the axion electromagnetic anomaly [39]. The term containing the ratios of light quark masses is approximately equal to 1.95. The model dependence arises through the ratio, $E/N$. In grand-unifiable models, $E$ and $N$ are related and $E/N = 8/3$. The DFSZ axion model falls into this category and in this case, $g_{\gamma\gamma} = 0.36$. For a KSVZ axion, $E = 0$ and $g_{\gamma\gamma} = -0.97$.

It is possible for an axion to solve the strong CP problem, as shown by the existence of the KSVZ and DFSZ axion models. While significant for that alone, the axion also provides an interesting candidate for the cold dark matter of the universe.

3. Cosmological production of axions

3.1. Properties of axion dark matter

Axions satisfy the two criteria necessary for cold dark matter: (i) a non-relativistic population of axions could be present in our universe in sufficient quantities to provide the required dark matter energy density and (ii) they are effectively collisionless, i.e. the only significant long-range interactions are gravitational.

Despite having a very small mass [21]–[24],

$$m_a \simeq 6 \times 10^{-6} \text{ eV} \left( \frac{10^{12} \text{ GeV}}{f_a} \right),$$

axion dark matter is non-relativistic, as cold populations are produced out of equilibrium. There are three mechanisms via which cold axions are produced: vacuum realignment [5]–[7], string decay [8]–[18] and domain wall decay [18]–[20]. In this section, we discuss the history of the axion field as the universe expands and cools to see how and when axions are produced. We also review vacuum realignment production in detail, as there will always be a contribution to the cold axion populations from this mechanism and, as discussed below, it may provide the only contribution. A complete description of the cold axion populations can be found in [40].

3.2. Topological axion production

There are two important scales in dark matter axion production. The first is the temperature at which the PQ symmetry breaks, $T_{\text{PQ}}$. Which of the three mechanisms contributes significantly to the cold axion population depends on whether this temperature is greater or less than the inflationary reheating temperature, $T_R$. The second scale is the temperature at which the axion mass, arising from non-perturbative QCD effects, becomes significant. At high temperatures, the QCD effects are not significant and the axion mass is negligible [41]. The axion mass becomes important at a critical time, $t_1$, when $m_\text{a} t_1 \sim 1$ [5]–[7]. The temperature of the universe at $t_1$ is $T_1 \simeq 1$ GeV.

The PQ symmetry is unbroken at early times and temperatures greater than $T_{\text{PQ}}$. At $T_{\text{PQ}}$, it breaks spontaneously and the axion field, proportional to the phase of the complex scalar field acquiring a vacuum expectation value, may have any value. The phase varies continuously, changing by order one from one horizon to the next. Axion strings appear as topological defects.

If $T_{\text{PQ}} > T_R$, the axion field is homogenized over vast distances and the string density is diluted by inflation, to a point where it is extremely unlikely that our visible universe contains any axion strings. In the case $T_{\text{PQ}} < T_R$, the axion field is not homogenized and strings radiate cold, massless axions until non-perturbative QCD effects become significant at temperature, $T_1$. Agreement has not been reached on the expected spectrum of axions from string radiation and there are two possibilities. Either strings oscillate many times before they completely decay and axion production is strongly peaked around a dominant mode [8, 9], [11]–[16] or much more rapid decay occurs, producing a spectrum inversely proportional to momentum [10, 42]. Rapid decay produces $\sim 70$ times less axions than slow string decay, leading to different cosmological bounds on the axion mass (see section 4.2).

When the universe cools to $T_1$, the axion strings become the boundaries of $N$ domain walls. For $N = 1$, the walls rapidly radiate cold axions and decay (domain wall decay). If $N > 1$, the domain wall problem occurs [43] because the vacuum is multiply degenerate and there is at least one domain wall per horizon. These walls will end up dominating the energy density and cause the universe to expand as $S \propto t^2$, where $S$ is the scale factor. Although other solutions to the domain wall problem have been proposed [18], we assume here that $N = 1$ or $T_{\text{PQ}} > T_R$.

Thus, if $T_{\text{PQ}} < T_R$, string and wall decay contribute to the axion energy density. If $T_R < T_{\text{PQ}}$, and the axion string density is diluted by inflation, these mechanisms do not contribute significantly to the density of cold axions. Then, only vacuum realignment will contribute a significant amount.

3.3. Vacuum realignment mechanism

Cold axions will be produced by vacuum realignment, independent of $T_R$. Details of this method are discussed below, but the general mechanism is as follows. At $T_{\text{PQ}}$, the axion field amplitude
may have any value. If $T_{PQ} > T_R$, homogenization will occur due to inflation and the axion field will be single valued over our visible universe. Non-perturbative QCD effects cause a potential for the axion field. When these effects become significant, the axion field will begin to oscillate in its potential. These oscillations do not decay and contribute to the local energy density as non-relativistic matter. Thus, a cold axion population results from vacuum realignment, regardless of the inflationary reheating temperature.

To illustrate vacuum realignment, consider a toy axion model with one complex scalar field, $\phi(x)$, in addition to the standard model fields. The potential for $\phi(x)$ in our toy model is

$$V(\phi) = \frac{\lambda}{4}(|\phi|^2 - v_a^2)^2. \quad (25)$$

When the universe cools to $T_{PQ} \sim v_a$, $\phi$ acquires a vacuum expectation value,

$$\langle \phi \rangle = v_a \exp(i\theta(x)). \quad (26)$$

The relationship between the axion field, $a(x)$, and $\theta(x)$ is

$$a(x) \equiv v_a \theta(x). \quad (27)$$

The axion decay constant in this model is

$$f_a \equiv \frac{v_a}{N}. \quad (28)$$

For the following discussion, we set $N = 1$.

At $T \sim \Lambda$, where $\Lambda$ is the confinement scale, non-perturbative QCD effects give the axion a mass. An effective potential,

$$\tilde{V}(\theta) = m_a^2(T) f_a^2 \left(1 - \cos \theta \right) \quad (29)$$

is produced. The axion acquires mass, $m_a$, due to the curvature of the potential at the minimum. This mass is temperature, and thus time dependent due to the temperature dependence of the potential [41]. The temperature dependence can be derived [5]–[7] from non-perturbative QCD effects [41]:

$$m_a(T) \simeq 0.2 m_a \left( \frac{\Lambda}{f_a} \right)^4, \quad (30)$$

where $m_a$ is the axion mass at low temperatures, given by equation (24).

In a Friedmann–Robertson–Walker universe, the equation of motion for $\theta$ is

$$\ddot{\theta} + 3 H(t) \dot{\theta} - \frac{1}{S^2(t)} \nabla^2 \theta + m_a^2(T(t)) \sin(\theta) = 0, \quad (31)$$

where $S(t)$ is the scale factor and $H(t)$, the Hubble constant, at time $t$. Near $\theta = 0$, $\sin(\theta) \simeq \theta$.

We first consider the case, $T_{PQ} > T_R$. Under these circumstances, the $\theta$ (and subsequently, axion) field is homogeneous at QCD confinement and we need not consider spatial derivatives of the field. Thus, only the zero momentum mode is occupied and the equation of motion reduces to

$$\ddot{\theta} + 3 H(t) \dot{\theta} + m_a^2(t) \theta = 0, \quad (32)$$

i.e. the field satisfies the equation for a damped harmonic oscillator with time-dependent parameters. At early times, the axion mass is insignificant and $\theta$ is approximately constant. When the universe cools to the critical temperature, $T_1$, which we define via

$$m_a(T_1) \tau_1 \equiv 1, \quad (33)$$
the field will begin to oscillate in its potential [44]. Given the definition of the critical time, \( t_1 \), in equation (33) [40],

\[
t_1 \simeq 2 \times 10^{-7} \text{s} \left( \frac{f_a}{10^{12} \text{ GeV}} \right)^{1/3}
\]

and

\[
T_1 \simeq 1 \text{ GeV} \left( \frac{10^{12} \text{ GeV}}{f_a} \right)^{1/6}.
\]

The axion field can realign only as fast as causality permits, thus the momentum of a quantum of the axion field is

\[
p_a(t_1) \sim \frac{1}{t_1} \sim 10^{-9} \text{ eV}
\]

for \( f_a \simeq 10^{12} \text{ GeV} \), which corresponds to \( m_a \simeq 6 \mu \text{eV} \), by equation (24). Thus, this population is non-relativistic or cold.

The energy density of the scalar field around its potential minimum is

\[
\rho = \frac{f_a^2}{2} \left[ \dot{\theta}^2 + m_a^2(t)\theta^2 \right].
\]

By the Virial theorem,

\[
\langle \dot{\theta}^2 \rangle = m_a^2 \langle \theta^2 \rangle = \rho \frac{f_a^2}{2}.
\]

As axions are non-relativistic and decoupled,

\[
\rho \propto \frac{m_a(t)}{S^3(t)}.
\]

Thus, the number of axions per comoving volume is conserved, provided the axion mass varies adiabatically.

The initial energy density of the coherent oscillations is

\[
\rho_1 = \frac{1}{2} f_a^2 m_a^2(t_1) \theta_1^2
\]

and \( \theta_1 \) is the initial, ‘misalignment’ angle. Given the relationship of equation (39), the energy density in axions today is

\[
\rho_0 \sim \rho_1 \frac{m_a(t_0)}{m_a(t_1)} \frac{S^3(t_1)}{S^3(t_0)}.
\]

Using equations (33) and (40),

\[
\rho_0 \sim \frac{1}{2} f_a^2 \frac{m_a(t_1)}{t_1} \left( \frac{S(t_1)}{S(t_0)} \right)^3 \theta_1^2,
\]

which implies the axion energy density,

\[
\Omega_a \sim 0.15 \left( \frac{f_a}{10^{12} \text{ GeV}} \right)^{7/6} \theta_1^2,
\]

using equations (24), (34) and (35).
When $T_{PQ} < T_R$, the zero mode and higher momentum modes are populated. In this case, the axion field varies by $f_a$ over horizon scales. The energy density in the zero-momentum mode can still be derived as above, except we must now average over the initial misalignment angle, $\langle \theta_i^2 \rangle \sim 1$ and the results of equations (42) and (43) become

$$\rho_0 \sim \frac{1}{2} f_a^2 m_a \left( \frac{S(t_1)}{S(t_0)} \right)^3$$

and

$$\Omega_a \sim 0.15 \left( \frac{f_a}{10^{12} \text{GeV}} \right)^{7/6}.$$  \hspace{1cm} (44)

To approximate the higher momentum modes, we follow the discussion of Sikivie [40]. The energy density in the higher momentum modes can be approximated by considering the variation in the axion field ($\sim f_a$) over horizon scales ($\sim t$) for $t \ll t_1$,

$$\rho_{k=0 \sim 1} \sim \omega \frac{dn_a}{d\omega} |_{\omega \sim \Delta \omega \sim 1/t} \sim \frac{1}{2} \left( \nabla f_a \right)^2 \sim \frac{1}{2} \frac{f_a^2}{t^2},$$

where $\frac{dn_a}{d\omega}$ is the number density of axions with wavenumber $k = \omega$ for $\omega > 1/t$. Thus, the number density is

$$\frac{dn_a}{d\omega}(\omega, t) \sim \frac{1}{2} \frac{f_a^2}{t^2 \omega^2}.$$ \hspace{1cm} (46)

At $t_1$,

$$n_a(t_1) \sim \int_{1/t}^{\infty} d\omega \frac{dn_a}{d\omega}(\omega, t_1) \sim \frac{1}{2} \frac{f_a^2}{t_1}.$$ \hspace{1cm} (47)

Given that the majority of these axions are non-relativistic after $t_1$, the energy density in these modes today would be

$$\rho_{k=0}(t_0) \sim \frac{1}{2} f_a^2 m_a \left( \frac{S(t_1)}{S(t_0)} \right)^3,$$ \hspace{1cm} (48)

i.e. the higher momentum modes contribute the same order of magnitude to the axion energy density.

As the axion couplings are very small, these coherent oscillations do not decay and make axions a good candidate for the dark matter of the universe.

While we have not estimated the densities of axions from string or wall decay in this review, we note that the energy densities in axions from these contributions have the same dependence on $m_a$, $f_a$ and $t_1$ as the vacuum realignment contributions and thus, $\Omega_a^\text{string}$ and $\Omega_a^\text{wall}$ are also proportional to $f_a^{7/6}$. We refer the reader to [40] for further details.

4. Constraints on the axion

4.1. Laboratory bounds

The original PQWW axion would have been of order 100 keV mass, and possessed couplings large enough to enable the axion to have been produced and detected in conventional laboratory experiments. These included searches for axions emitted from reactors, where axions would
compete with M1 gamma transitions in radioactive decay; nuclear deexcitation experiments; beam-dump experiments; and axion decay from $1^-$ heavy quarkonia states, i.e. $J/\psi$ and $\Upsilon$. All results were negative, thus excluding the original PQWW axion within a decade of its prediction. As these limits are much weaker than the current astrophysical upper bounds for both the mass and coupling to radiation, the reader is directed to earlier reviews [21, 22, 39], and the Particle Data Group Review of Particle Properties for discussion and annotated limits [45].

Searches for axions have also been performed exploiting the coherent mixing of axions with photons in experiments realized with lasers and large superconducting dipole magnets. These include both searches for vacuum dichroism and birefringence through the production of real or virtual axions in a magnetic field [46], and photon regeneration (shining light through walls) [47]–[49]. Axion–photon mixing will be discussed briefly further on, but all such experiments to date have set limits on the axion–photon coupling, $g_{a\gamma\gamma} \sim 2 \times 10^{-7}$ GeV$^{-1}$ (weakening significantly for $m_a > 0.5$ meV) orders of magnitude weaker than current astrophysical limits and direct searches for solar axions. While there are no prospects for the polarization experiments to compete with these latter ($g_{a\gamma\gamma} \sim 10^{-10}$ GeV$^{-1}$), a new strategy to resonantly enhance photon regeneration may enable them to improve on current best limits by up to an order of magnitude [50], and at least one such experiment is in preparation.

Torsion-balance techniques have enabled searches for axions through their coupling to the nucleon spin, and rigorous bounds have been set on their short-range interactions for masses below 1 meV. While these are impressive tour de force experiments, how to relate these limits to expectations from the PQ axion is not straightforward [51].

4.2. Cosmological bounds

Whether by the vacuum realignment mechanism, or by radiation from topological strings, the production of very light axions in the early universe implies an increasing energy density of the universe in axions for lower masses: $\Omega_a = \rho_a / \rho_c \propto m_a^{-7/6}$. As the relative importance of the mechanisms is still controverted, a reliable lower bound on the axion mass should obtain where the vacuum realignment contribution becomes of order unity, $\Omega_a = O(1)$. From equation (43), this corresponds to $m_a \sim 6 \mu$eV (see section 3.3), although there is uncertainty in this estimate itself, owing to lack of knowledge of the initial value of $\theta$ within our horizon; the estimate above is based on the assumption that this parameter is of order unity. An accidentally small value would drive the mass associated with closure density downwards, including arbitrarily small masses. On the other hand, within the concordance model, $\Omega_{DM} = 0.23$, implying that the axion mass should be roughly a factor of 4 higher than cited above. Thus the Axion Dark Matter eXperiment (ADMX) microwave cavity experiment conservatively began its search campaign at $m_a \sim 2 \mu$eV, and continues to work upwards. Recent discussions of the cosmological bound from vacuum realignment can be found in [37, 52].

The above vacuum realignment bound applies when inflation has homogenized the axion field over our horizon. When the PQ scale breaks after inflation, cold axions are additionally produced by string and domain wall decay. These extra contributions mean that we require inflation to have produced less axions to avoid overclosing the universe and thus, the axion mass lower bound increases (see section 3.3). As the spectrum of axions from string radiation is debated, we review the possible bounds. If axion strings decay rapidly, giving a spectrum inversely proportional to momentum, the lower bound for the axion mass is $\sim 15 \mu$eV [17]. If the decay is less rapid and strings go through many oscillations, an analysis based on local
strings [15] gives a lower bound of 100 µeV. A similar analysis based on global strings [16] gives a smaller lower bound of 30 µeV. Given that the PQ symmetry is global, it is likely that the lower bound of 30 µeV is applicable if the PQ symmetry breaks after inflation and axion strings decay slowly.

4.3. Astrophysical bounds

In general, introducing a channel for direct or free-streaming energy loss from a star’s core accelerates the star’s evolution. The core will contract and heat up under the influence of gravity when axions (or other exotica) compete with the production of strongly trapped photons, whose radiation pressure acts to counterbalance gravitational pressure. Furthermore, for each stellar system, axions are excluded only over a finite range of couplings. As the axion’s coupling is increased in the stellar-evolution simulations, the free-streaming lower limit of the axion’s excluded couplings is reached at a point where deviations from an axion-free model first become noticeable. However, as the coupling is further increased, the axions themselves eventually become strongly trapped; the upper limit corresponds to the regime where their influence on evolution diminishes below the threshold of observation. A comprehensive treatment of such constraints on properties of axions and other exotica has been published by Raffelt [53]. The two most relevant astrophysical limits in framing the region of interest where axions may be the dark matter are described below.

The most stringent constraint on the axion–photon coupling presently is due to horizontal branch (HB) stars, i.e. those that are in their helium-burning phase, within globular clusters. Globular clusters provide a cohort population of stars of all the same age; those that are seen today are have masses somewhat less than that of our sun. By the ratio of the number observed in the HB phase to those in the red giant phase, i.e. after exhaustion of core-hydrogen burning but before the helium flash, one statistically infers an average HB lifetime. The concordance (within 10%) between the calculated and inferred HB lifetime precludes Primakoff production of axions (γ + Ze → a + Ze, i.e. axions produced by the interaction of a real plus virtual photon) at a level corresponding to an upper bound of $g_{a\gamma\gamma} < 10^{-10} \text{GeV}^{-1}$. A definitive study of axion cooling in stars using numerical methods aims to extend this analysis [54].

The lowest-lying upper bound for the axions mass is due to SN1987a. Axions produced by nucleon–nucleon bremsstrahlung ($N + N \rightarrow N + N + a$) during the core-bounce of the proto-neutron star would have competed with neutrino emission. That the duration of the neutrino pulse observed between the IMB and Kamioka water Cherenkov detectors (19 events over 10 s) was in good accord with core-collapse models precludes axions in the mass range $10^{-3} \text{eV} < m_a < 2 \text{eV}$. This range corresponds to the free-streaming regime; as mentioned previously, for axions above this range axions themselves are strongly trapped and thus are not effective in quenching the neutrino signal.

The current state of the standard QCD axion is shown in figure 1 [45]. It should be noted that stellar evolution establishes limits on the coupling of axions to radiation, electrons, nucleons, etc. (i.e. $g_{a\ell}$), and thus only indirectly limits mass through specific models. For example, the uncertainties in the upper limit for the ‘axion window’ set by SN1987a, germane to the search for dark matter axions, is roughly an order of magnitude [24]; i.e. $10^{-(2-3)} \text{eV}$ and thus the grey bar in figure 1 is set conservatively at the upper end of that uncertainty band. As discussed above, the uncertainty of the lower limit of the ‘axion window’ is at least an order of magnitude, for multiple reasons, i.e. uncertainties regarding the various axion production
mechanisms, uncertainties regarding early universe chronology and lack of any knowledge of the initial misalignment angle.

### 5. Phase-space structure of halo dark matter

The local velocity distribution of dark matter axions is of vital importance for direct detection. The ADMX, detailed in section 6.2 and [55], in this volume, uses a microwave cavity to search directly for axions. The observed signal is the power output from the cavity due to axion conversion to photons, as a function of frequency. The frequency corresponds to the energy distribution of axions undergoing conversion. As axions are non-relativistic, the signal frequency is given by

\[
\nu(v) = \frac{m_a}{\hbar} \left( c^2 + \frac{1}{2} v^2 \right).\tag{50}
\]
The local velocity distribution thus determines the signal shape. The signal amplitude is
determined by the density of axions of a particular energy. Thus, the phase-space distribution
determines the signal observed.

We expect that the dark halo of the Milky Way consists of a number of components
which ADMX is capable of observing: (i) a thermalized component with a Maxwell–Boltzmann
velocity distribution, (ii) discrete flows, from tidal stripping of satellite halos or coherent dark
matter flows crossing the halo, and (iii) overdense regions that are not gravitationally bound,
known as caustics.

The ADMX search technique assumes that the rates of change of velocity, velocity
dispersion and flow density are slow compared to the timescale of the experiment. The ADMX
medium resolution (MR) channel searches for an isothermal component of the halo as dark
matter axions. The ADMX high resolution (HR) channel searches for signals with a narrow
velocity dispersion, such as discrete flows.

Numerical simulations produce large halos within which hundreds of smaller clumps, or
subhalos, exist [56, 57]. Tidal disruption of these subhalos leads to flows in the form of ‘tidal
tails’ or ‘streams’. The Earth may currently be in a stream of dark matter from the Sagittarius A
dwarf galaxy [58, 59]. This stream of subhalo debris satisfies both the requirements of small
velocity dispersion and a repeatable signal and thus may be detectable by the ADMX HR
channel.

Non-thermalized flows from late infall of dark matter onto the halo have also been shown
to be expected [60, 61]. The idea behind these flows is that dark matter that has only recently
fallen into the gravitational potential of the galaxy will have had insufficient time to thermalize
with the rest of the halo and will be present in the form of discrete flows. There will be one
flow of particles falling onto the galaxy for the first time, one due to particles falling out of the
galaxy’s gravitational potential for the first time, one from particles falling into the potential for
the second time, etc. Furthermore, where the gradient of the particle velocity diverges, particles
‘pile up’ and form caustics. In the limit of zero flow velocity dispersion, caustics have infinite
particle density. The velocity dispersion of cold axions at a time, $t$, prior to galaxy formation
is approximately $\delta v_a \sim 3 \times 10^{-17} (10^{-5} \text{ eV} \, m^{-1}) (t_0/t)^{2/3}$ [62], where $t_0$ is the present age of the
universe. A flow of dark matter axions will thus have a small velocity dispersion, leading to a
large, but finite density at a caustic.

The caustic ring model, under the assumptions of self-similarity and axial symmetry,
predicts that the Earth is located near a caustic feature [63]. This model, fitted to rises in the
Milky Way rotation curve and a triangular feature seen in the IRAS maps, predicts that the
flows falling in and out of the halo for the fifth time contain a significant fraction of the local
halo density. The predicted densities are $1.5 \times 10^{-24}$ and $1.5 \times 10^{-25}$ g cm$^{-3}$ [64], comparable
to the local dark matter density of $9.2 \times 10^{-25}$ g cm$^{-3}$ predicted by Gates et al [65]. The flow of
the greatest density is known as the ‘Big Flow’.

A general treatment of the phase-space structure of dark matter halos, which does not
require assumptions of self-similarity or symmetry, has recently been developed [66]. This
treatment studies the statistics of dark matter caustics in the tidal debris remaining from mergers
of smaller halos to form galaxies and from the primordial coldness of dark matter. While more
general than the approach of Duffy and Sikivie [64], this treatment only results in a statistical
distribution and does not give specific predictions for our galactic halo.

Additionally, numerical methods have been developed to study caustics and flows in dark
matter halos. To date, most numerical simulations are too coarse-grained to resolve caustic
structure, although its presence can be observed when special techniques are used [67]–[71]. The recent work of Vogelsberger et al [72] predicts at least $10^5$ discrete streams near our Sun, although specific predictions are not made for the stream densities and velocities.

It has also recently been shown that dark matter axions can exist in the form of a Bose–Einstein condensate (BEC) [73]. If this is the case, the formation of caustics is suppressed within the BEC. However, vortices are expected to form at the center of galactic halos, due to their net rotation. Within a vortex, axion dark matter will exist in the normal phase and flows and caustics will still be present.

The possible existence of discrete flows provides an opportunity to increase the discovery potential of ADMX. A discrete axion flow produces a narrow peak in the spectrum of microwave photons in the experiment and such a peak can be searched for with higher signal-to-noise than the signal from axions in an isothermal halo. If such a signal is found, it will provide detailed information on the structure of the Milky Way halo.

6. Experimental searches for dark matter axions

6.1. Axion–photon mixing

As pseudoscalars, axions can be produced by the interaction of two photons, one of which can be virtual, \( \gamma + \gamma^* \rightarrow a \), the process being known as the Primakoff effect [74]. This implies that photons and axions may mix in the presence of an external electromagnetic field, through the Lagrangian density of equation (21). In all axion searches based on the Primakoff effect to date, \( E \) represents the electric field of the real photon, and \( B \) is an external magnetic field. Although the opposite combination is possible, it is vastly easier to produce and support a static magnetic field than the equivalent electric field, a 10 T magnetic field being equal to an electric field of 30 MV cm\(^{-1}\) in Gaussian units. In fact, fields of order 10 T are readily achieved today with superconducting magnets. The coherent mixing of axions and photons within magnetic fields of large spatial extent enables searches of exceedingly high sensitivity, although there is yet no experimental strategy capable of reaching the standard PQ axion over the entire open range.

A general formulism for axion–photon mixing in external magnetic fields, including plasma effects, is found in [75].

6.2. The microwave cavity experiment for dark matter axions

In 1983, Sikivie proposed two independent schemes to detect the axion based on the Primakoff effect [76, 77]. The first was a search for axions constituting halo dark matter by their resonant conversion to RF photons in a microwave cavity permeated by a strong magnetic field. Tuning the cavity to fulfill the resonant condition, \( h \nu = m_a c^2 (1 + \mathcal{O}(\beta^2 \sim 10^{-6})) \), and assuming axions saturate the galactic halo, the conversion power from an optimized experiment is given by

\[
P = \frac{s_{\gamma \gamma}^2 V B^2 \rho_a Q}{m_a},
\]

where \( B \) is the strength of the magnetic field, \( V \) the cavity volume and \( Q \) is the cavity quality factor. The most sensitive microwave cavity experiment (and in fact the only one currently in operation) is ADMX at Lawrence Livermore National Laboratory. This search has excluded axions of KSVZ axion–photon coupling as the local halo dark matter, for a narrow range of masses \( 1.9 < m_a < 3.4 \, \mu \text{eV} \).

The anticipated conversion power is minuscule, even for the largest superconducting magnets feasible; for ADMX the signal expected is of order $10^{-22}$ watts. Furthermore, as the experiment will necessitate tuning orders of magnitude of frequency in small frequency steps, there are limits on how long one may integrate at each frequency to improve the signal-to-noise, as governed by the Dicke radiometer equation [78]:

$$\frac{s}{n} = \frac{P_s}{P_n} = \frac{P_s \sqrt{\Delta v t}}{k T_n}.$$  \hspace{1cm} (52)

Here $s/n$ is the signal-to-noise ratio, $\Delta v$ the bandwidth of the signal, $t$ the integration time, and $P_s$ and $P_n$ the signal and noise power, respectively. This puts a clear premium on reducing the total system noise temperature, which is the sum of the physical temperature and the equivalent electronic noise temperature of the amplifier, $T_n = T_{\text{phys}} + T_{\text{elec}}$. ADMX has recently completed an upgrade from conventional heterojunction field-effect transistors (HFETs or HEMTs) with a noise equivalent temperature of $T_{\text{elec}} \sim 2 \text{ K}$, to superconducting quantum interference device (SQUID) amplifiers, whose noise equivalent noise temperatures can reach the quantum limit, $T_{\text{elec}} \sim 50 \text{ mK}$ at 750 MHz when cooled to comparable physical temperatures. This strategy will enable them ultimately to reach the DFSZ model axions, as well as cover the open mass range much faster. The microwave cavity experiments are described by Carosi elsewhere in this Focus issue [55].

6.3. Other current axion searches

In the same report, Sikivie also outlined how to detect axions free-streaming from the Sun’s nuclear burning core [76, 77]. Axion production would be dominated by the Primakoff process $\gamma + Ze \rightarrow a + Ze$; for KSVZ axions, the integrated solar flux at the Earth would be given by $F_\gamma = 7.4 \times 10^{11} m_a^2 \text{eV} \text{cm}^{-2} \text{s}^{-1}$, emitted with a thermal spectrum of mean energy $\sim 4.2 \text{ keV}$. For relativistic axions, the conversion probability to photons of the same energy in a uniform magnetic field is given by $P(a \rightarrow \gamma) = \Pi = (1/4) (g_{\gamma a} B L)^2 |F(q)|^2$, where $B$ is the strength of the magnetic field, and $L$ its length$^4$. $F(q) \equiv \int dx e^{i qx} B(x)/B_0 L$ represents the form factor of the magnetic field with respect to the momentum mismatch between the massive axion and massless photon of the same energy, $q = k_a - k_{\gamma} = (m_a^2 - m_{\gamma}^2)^{1/2} - \omega \sim m_a^2/2\omega$. $|F(q)|$ is unity in the limit $qL \ll 2\pi$, but oscillates and falls off rapidly for $qL > 2\pi$, where the axions are no longer sufficiently relativistic to stay in phase with the photon for maximum mixing.

Utilizing a prototype LHC dipole magnet as the basis for its axion helioscope, the CAST collaboration have recently published the best limits on the solar axions, $g_{\gamma a} < 0.88 \times 10^{-10} \text{ GeV}^{-1}$, valid for $m_a < 10^{-2} \text{ eV}$ [79], slightly more stringent than those derived from HB stars. This collaboration has also pushed the sensitivity of the search upward in mass into the region of axion models, by introducing a gas (4He) of variable pressure into the magnet bore. In this case, the plasma frequency $\omega_p = (4\pi \alpha N_e/m_a)^{1/2} \equiv m_a$ endows the x-ray photon with an effective mass; thus full coherence of the axion and photon states can be restored, and the theoretical maximum conversion probability achieved for any axion mass, by the filling the magnet with a gas of the appropriate density [80]. The mass range can thereby be extended upwards in scanning mode, by tuning the gas pressure in small steps to as high as feasible. In this manner, axions have now been excluded part-way into the PQ model band,

$^4$ A useful mnemonic in rate estimates for experiments is that, within a few percent, the factor $(g_{\gamma a} B_0 L)^2 \approx 10^{-16}$, where $g_{\gamma a} \approx 10^{-10} \text{ GeV}^{-1}$, $B_0 = 10^7 \text{T}$, and $L = 10 \text{ m}$.

$g_{a\gamma\gamma} < 2.2 \times 10^{-10} \text{GeV}^{-1}$ (95% c.l.), valid for $m_a < 0.4 \text{eV}$ [81]. This phase of the experiment continues with $^3\text{He}$ gas which will permit probing of yet higher masses; for further details, see Zioutas elsewhere in this Focus issue [82].

Purely laboratory bounds on axions or generalized pseudoscalars have also been established without relying on either astrophysical or cosmological sources. In photon regeneration (‘shining light through walls’) axions are coherently produced by shining a laser beam through a transverse dipole magnet, and reconverted to real photons in a collinear dipole magnet on the other side of an optical barrier [83, 84]. The probability to detect a photon per laser photon is given by $P(a \to \gamma \to a) = \Pi^2$. While current limits from photon regeneration ($g_{a\gamma\gamma} < 2 \times 10^{-7} \text{GeV}^{-1}$, $m_a < 0.5 \times 10^{-3} \text{eV}$) [47]–[49] have not so far been competitive with solar searches, the scheme may be resonantly enhanced utilizing actively locked Fabry–Perot optical cavities to strengthen limits potentially by an order of magnitude beyond those of CAST and HB stars [50].

7. Summary

After three decades, the confluence of laboratory bounds, cosmological and astrophysical constraints now restrict the QCD axion within four decades in mass range; making allowances for uncertainties, this range is $10^{-6}–10^{-2} \text{eV}$. With regards to axions as dark matter, independent of the production mechanism (i.e. vacuum realignment versus topological), the lower the axion mass, the greater fraction they will claim of the total matter density of the universe. While there is no a priori physics argument favoring a lighter axion to a heavier one, given the inherent uncertainties in the cosmological production, it would be difficult for a light axion not to be cosmologically significant.

Experiments based on axion–photon mixing have made impressive strides, and in particular the Sikivie microwave cavity experiment is now probing well into the QCD–axion model band beginning at the very lowest relevant masses and scanning upwards; see the report of Carosi in this Focus issue [55]. In fact it may be said that the microwave cavity experiment is the one search among all candidates where sensitivity is a solved issue, although pushing the mass range upwards to $100 \mu\text{eV}$ will become unwieldy under any presently envisioned concepts for cavity resonators. Thus at best, this experiment may only cover two of the four open decades in mass range. While resonantly enhanced photon regeneration may improve significantly upon limits of the CAST solar search and HB stars, there is no credible extrapolation of that experiment that will probe the QCD–axion couplings in the open range. New concepts will be needed if we are to finish the job from an experimental point of view. This does not diminish the importance of any experiment that can probe new masses and couplings, as there may be an entire sector of fundamental pseudoscalars unrelated to the strong CP problem; one needs to be open to surprise.

Should the axion be discovered in the microwave cavity experiment, it would constitute a unique quantum system with a very large coherence length to be investigated. The signal could possibly also exhibit non-trivial structure reflective of the history of the formation of the galaxy.

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