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COMMENT

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Comment

Comment on 'Information hidden in the velocity distribution of ions and the exact kinetic Bohm criterion'

A S Mustafaev[1], **V S Sukhomlinov**[2] **and N A Timofeev**[2]

[1] Department of General and Technical Physics, St. Petersburg Mining University, St. Petersburg, 199106, Russia
[2] Department of Physics, St. Petersburg State University, St. Petersburg, 198504, Russia

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Abstract
This Comment is devoted to some mathematical inaccuracies made by the authors of the paper 'Information hidden in the velocity distribution of ions and the exact kinetic Bohm criterion' (*Plasma Sources Science and Technology* **26** 055003). In the Comment, we show that the diapason of plasma parameters for the validity of the theoretical results obtained by the authors was defined incorrectly; we made a more accurate definition of this diapason. As a result, we show that it is impossible to confirm or refute the feasibility of the Bohm kinetic criterion on the basis of the data of the cited paper.

Introduction

The cited article [1] is devoted to the development of methods for the determination of the ion velocity distribution function (IVDF) in low temperature discharge plasmas with the help of experimental data about the velocity distribution of ion flux that bombards the surface of the plasma border. Obtained theoretical and experimental IVDF in inductively coupled plasma, as well as results of probe measurements of electron energy distribution functions (EEDF), are used to investigate and make more precise the known Bohm criterion [2].

The undoubted advantage of the study can be attributed to trying to implement a beautiful idea: to determine IVDF in quasi-neutral balk plasma using IVDF measurements just on the wall. Implementation of this idea would make it possible to determine IVDF in plasma volume away from walls with the help of relatively simple measurements, in comparison with the usual probe technique [3, 4] that is not applicable under all plasma conditions. At present, as known, IVDF has not been studied well in low temperature non-equilibrium plasmas, unlike EEDF. This is understandable because IVDF can be either weakly or strongly anisotropic depending on the parameter E/N (E, N are the electric field strength and neutral particle concentration in a plasma, respectively) [5–7]. Besides, this anisotropy depends upon the ion energy [3, 4, 6]. In the production of ion beams, the velocity distribution of ions in a plane orthogonal to the direction of the accelerating field determines the beam divergence. Thus, the angular dependence of IVDF is one of its key characteristics. In this case, the experimental determination of IVDF in the volume of a quasi-neutral plasma is complicated by the fact that most of the known methods using an ion analyzer with respect to energies of ions of various types [8, 9] and the methods of laser induced fluorescence (LIF) do not measure the function in energy and directions of motion, but the functions along one selected direction. Therefore, the information about the angular distribution of ions is lost. Theoretical consideration of IVDF on the basis of kinetic equation is complicated by the fact that, as shown in [6, 7], elastic collisions with neutral particles play an important role in the formation of the ion distribution, even at high values of the reduced electric field $\frac{E}{N} > 10^3\,Td$. Only recently, there are works in which, for a wide range of plasma conditions, IVDF is calculated [3–7, 10, 11] and is measured by the probe method [3–5] as a function of the velocity and directions of motion.

Unfortunately, in the formulation of the task that was considered in [1], the results of [1] are applicable to a limited list of plasma objects and in case a number of requirements are satisfied. This significantly narrows the range of experimental conditions. Since the authors do not speak about these restrictions and, accordingly, do not formulate the conditions for the applicability of their results, it seems necessary to us to do this. In addition, in our opinion, the authors allowed a number of mathematical inaccuracies, which require correction. Finally, one has to note that in the introduction of [1], the authors, for some reason, refer to the discussion of the non-Maxwellian type of EEDF and IVDF just on papers [12–14], where only the EEDF was determined and do not mention any works on the study of non-Maxwellian IVDF in a non-equilibrium low-temperature plasma, including the above quoted articles. In the works on the determination of IVDF [15–19], which the authors of [1] mentioned on page 2, the distribution of ions along the directions of motion was not also determined, but either the distribution of ions bombarding the surface was investigated or a LIF-method was used that did not provide information about ions angular distribution.

Basic inaccuracies and limits of applicability of the results of [1]

Let us consider in detail the inaccuracies and conditions for the applicability of the results obtained by the authors of [1].

1. The condition for the applicability of equation (1) of [1] was formulated by the authors as
$$T_g \ll T_e, \tag{1C}$$
where T_g, T_e are the temperatures of neutrals and electrons, respectively. It is difficult to agree with this. Indeed, equation (1) is the Boltzmann kinetic equation where the frequency of the collisions between ions and their own atoms is determined, in particular, by their relative velocity. Since there is no velocity of atoms in equation (1), it is obvious that the condition for the applicability of such an approximation is as follows:
$$\sqrt{kT_g} \ll \sqrt{E_{i\lambda}}, \tag{2C}$$
where $E_{i\lambda}$ is the ion energy which they acquire at the mean free path λ with respect to the resonant charge exchange. It is quite obvious that conditions (1C) and (2C) are fundamentally different.
Inequality (2C) is equivalent to
$$\sqrt{\frac{E}{P}\left(\frac{\mathrm{V}}{\mathrm{cm}\cdot\mathrm{Torr}}\right)\frac{1}{\sigma_{ex}(10^{-15}\,\mathrm{cm}^2)}} \gg 1, \tag{3C}$$
where σ_{ex} is the cross-section of resonant charge exchange. From (3C), it follows that, for example, for argon, the parameter $\frac{E}{P}$ should lie in the range $\frac{E}{P} > 600\frac{\mathrm{V}}{\mathrm{cm}\cdot\mathrm{Torr}}$.
2. Equation (1) of [1] is true, including under the condition that the ion energy corresponding to the motion in the plane orthogonal to the direction of the electric field is much less than the energy of the motion along this direction (and then the velocity along the field determines the frequency of ion-atom collisions). In addition, under the same condition, scattering at a considerable angle, due to elastic collisions, will be negligibly small. Scattering at small angles due to collisions with a large impact parameter has little effect on IVDF. It should be kept in mind that this condition does not apply to average values, but to quantities $\varepsilon_z = \frac{M{v_z}^2}{2}$, $\varepsilon_\rho = \frac{M{v_\rho}^2}{2}$ where v_z and v_ρ are velocities along the field and across it, with respect to which the Boltzmann equation must be written. It is quite obvious that the influence of elastic collisions (and, accordingly, the ratio $\frac{\varepsilon_\rho}{\varepsilon_z}$) increases with decreasing ion energy ε. Thus, at low energies neglecting elastic collisions will lead to significant errors in the calculation of IVDF [6, 7].
Calculations by the method described in [1, 7] show that to satisfy inequality
$$\varepsilon_\rho \ll \varepsilon_z, \tag{4C}$$
for example, for rare gases Ar, Ne, He, it is necessary that the ion energy should exceed 0.2–0.4 eV. Hence, it follows that if the average energy of ions $\overline{E_i}$ for these gases exceeds
$$\overline{E_i} > 2 \div 4\,\mathrm{eV}, \tag{5C}$$
the number of ions with energies less than 0.2–0.4 eV is negligibly small and elastic collisions in the Boltzmann equation for ions in the intrinsic gas can be neglected.
In view of the foregoing, it can be stated that under the conditions of [1] and at some distance from the volume axis, where the plasma was investigated, elastic collisions can be neglected in the Boltzmann equation for IVDF assuming that the ions move along the radius of the cylindrical volume of the plasma. However, it should be kept in mind that in a large number of plasma objects (a positive column, a low-voltage beam discharge, etc.) the average ion energy is less than 1 eV and elastic collisions must be taken into account when calculating IVDF. In this case, the possibility of reconstructing IVDF in the plasma volume by measuring the energy distribution of the ion flux bombarding the wall surface bounding this plasma is lost.
Thus, one of the necessary conditions for the applicability of the results obtained by the authors (at least in rare gases) is the fulfillment of inequality (5C).
3. The next remark also concerns equation (1) of [1], in which the dependence of the resonant charge exchange cross-section on the relative energy of colliding particles is not taken into account (in the approximation used by the authors, it is equivalent to the dependence on the ion energy). We note that an analogous problem, but for an arbitrary dependence of the charge exchange cross-section on the ion energy, was solved in [20]. In the same place, a solution of the Poisson equation

self-consistent with the Boltzmann equation was found for the case when $n_i \gg n_e$, where n_i and n_e are the concentrations of ions and electrons correspondingly. In the same paper, it was also shown that, for example, under conditions of glow discharge in a hollow cathode in argon at a pressure of the order of 1 Torr and a cathode drop of about 400 V, ignoring the above dependence leads to errors in the IVDF by tens of percent. Obviously, the error in the calculated IVDF due to the constant charge exchange cross-section depends on the energy range in which the calculation is performed. Estimates show that, for example, for He^+ at average ion energies of the order of 10 eV, the IVDF error amounts to 25% or more for the ion energy range from 1 to 50 eV.

4. In the right-hand side of equation (1), one of the two terms is the source type—the term corresponding to the production of ions as a result of resonant charge exchange:

$$\delta(v)\int\frac{|v|}{\lambda}f_i\,\mathrm{d}v.$$

The delta-function appears in the 'cold gas' approximation. The exact expression must use $f_g(v)$—Maxwellian function with some atomic temperature T_g normalized to 1. In cylindrical geometry, when the field is directed along the radius in the XY plane and the ion velocity component v_z can be neglected, in the limit of 'cold gas' i.e. at $T_a \to 0$, one should believe that $f_g(v) \to \frac{1}{2\pi}\delta\left(\frac{v^2}{2}\right)$ or to $\frac{1}{2\pi v}\delta(v)$, but not to $\delta(v)$. (v is the ion radial velocity). Atomic function $f_g(v)$ tends to *a* limit $\delta(v)$ or to $v\delta\left(\frac{v^2}{2}\right)$ only in plane geometry in the presence of one component of the ion velocity (see, for example, [21]). In spherical geometry, $f_g(v) \to \frac{1}{4\pi v}\delta\left(\frac{v^2}{2}\right)$ or $f_g(v) \to \frac{1}{4\pi v^2}\delta(v)$. It is easy to see that these two delta-functions differ to a great degree: the total number of ions (in units of atomic concentration), which this term gives, for example, for cylindrical geometry, is as follows:

$$\int_0^{2\pi}\int\frac{1}{2\pi}\delta\left(\frac{v^2}{2}\right)v\mathrm{d}v\,\mathrm{d}\varphi = 1,$$

$$\text{and}\ \int_0^{2\pi}\int\delta(v)v\mathrm{d}v\,\mathrm{d}\varphi = 0.$$

Strictly speaking, the presence of $\delta(v)$ on the right-hand side of the Boltzmann equation for spherical and cylindrical geometry does not change the balance of the particles, and terms proportional to $\delta(v)$ where v is the modulus of ion velocity can be ignored altogether. These inaccuracies do not lead to errors in the results obtained by the authors [1], only because in the calculation of the moments from IVDF in any geometry, the authors integrate using the differential dv, which is permissible only for planar geometry.

5. It is interesting to note that, as follows from the data in figure 2(b), the error in determining IVDF at 'low' energies (less than 10 eV) reaches values from 100% to 10 000%. Apparently this can explain the fact that the relative error in determining the value $B_i = B_{ic} + B_{i\lambda}$ (this value is introduced by the authors when discussing the Bohm criterion; B_{ic}, $B_{i\lambda}$ are determined by the formulas (6) and (7) of [1], respectively) reaches, at low ion energies according to figure 5(d), the value of ±80%, and at the boundary of the near-wall layer, it is 25% as well as the relative error in determining the value of B_e (see figure 5(d)). As follows from figure 6(b), deviation from quasineutrality begins at a distance of (2.25–2.5) mm from the wall. This, apparently, can be considered the boundary of the wall layer. In this range of distances, as follows from the data of the same figure 6(b), the error in determining the electron concentration is of the order of 30%. But the fulfillment of the Bohm criterion is important precisely at the boundary of the near-wall (near the probe) layer, since if it is performed at the boundary, it is also satisfied at any point of this layer. This can be easily proved using the Poisson equation [22]. It is in this area that the results of the work under discussion make it impossible to verify the fulfillment of this criterion, since the determined quantities entering this criterion for ions, as mentioned above, are determined with an error of the order of 25% or more.

Conclusion

Thus, the main limitations of the applicability of the concept developed by the authors [1] are the necessity of fulfilling the inequalities (*3C*) and (*5C*), as well as the restriction on the energy range in the calculation of the IVDF (see point 3). In general, we can state that the results obtained by the authors are applicable at not too high pressures and electric fields. The meaning of these limitations is that the energy that ion collects at the mean free path relative to the process of resonant charge exchange must be large enough so that the velocity of atoms and elastic collisions with atoms can be ignored. In particular, the results obtained by the authors of [1] cannot be used in a low-temperature non-equilibrium plasma of pulsed discharges, DC discharges when diffusion losses are small, in low voltage discharges, etc.

ORCID iDs

V S Sukhomlinov https://orcid.org/0000-0002-9986-4209

References

[1] Tsankov T V and Czarnetzki U 2017 *Plasma Sources Sci. Technol.* **26** 055003

[2] Bohm D 1949 Minimum ionic kinetic energy for a stable sheath *The Characteristics of Electrical Discharges in Magnetic Fields* ed A Guthrie and R K Wakerling 1st edn (New York: McGraw-Hill) pp 77–86

[3] Mustafaev A S, Sukhomlinov V S and Ainov M A 2015 *Technical Physics* **60** 1778–89

[4] Mustafaev A, Sukhomlinov V and Ainov M 2017 *High Temperature* **55** 346–51

[5] Sukhomlinov V, Mustafaev A S, Grabovskii A and Ainov M 2015 Ion velocity distribution function in the plasma of its own gas *42nd EPS Conf. on Plasma Physics* vol P5, p 168

[6] Wang H, Sukhomlinov V S, Kaganovich I D and Mustafaev A S 2017 *Plasma Sources Sci. Technol.* **26** 024001

[7] Wang H, Sukhomlinov V S, Kaganovich I D and Mustafaev. A S 2017 *Plasma Sources Sci. Technol.* **26** 024002

[8] Ellmer K, Wendt R and Wiesemann K 2003 *Int. J. Mass Spectrom.* **223–224** 679–93

[9] Goeckner M J, Goree J and Sheridan T E 1991 *Phys. Fluids.* B **3** 2913

[10] Lampe M, Röcker T B, Joyce G, Zhdanov S K, Ivlev A V and Morfill G E 2012 *Phys. Plasmas.* **19** 113703

[11] Mustafaev A, Sukhomlinov V and Ainov M 2017 *High Temperature* **55** 481–90

[12] Godyak V and Alexandrovich B 2015 *Plasma Sources Sci. Technol.* **24** 052001

[13] Godyak V A and Demidov V I 2011 *J. Phys. D: Appl. Phys.* **44** 233001

[14] Hagelaar G J M 2016 *Plasma Sources Sci. Technol.* **25** 015015

[15] Severn G D, Wang X, Ko E and Hershkowitz N 2003 *Phys. Rev. Lett.* **90** 145001

[16] Luggenhölscher D, Celik Y, Pu Y K and Czarnetzki U 2010 *J. Phys.: Conf. Series* **227** 012035

[17] Davis W D and Vanderslice T A 1963 *Phys. Rev.* **131** 219

[18] Olthoff J K, Van Brunt R J, Radovanov S B, Rees J A and Surowiec R 1994 *J. Appl. Phys.* **75** 115

[19] Bachet G, Chérigier L and Doveil F 1995 *Phys. Plasmas* **2** 1782

[20] Kuchinskii V V, Sukhomlinov V S and Sheikin E G 1985 *Zh. Tekh. Fiz.* **55** 67–72

[21] Kos L, Tskhakaya D D and Jeli´c N 2015 *Phys. Plasmas* **22** 093503

[22] Yu P 1991 *Raizer. Gas Discharge Physics.* (Berlin: Springer)