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A total rip-off—crack propagation in paper

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Abstract

The explanation of material properties starts at a young age identifying materials using words such as strong or brittle, but it is not until higher education that we teach how and why materials break along with what brittle really means. It is an important concept to understand, as a material that could be thought strong can be made to appear weak with the addition of a very small crack. As force is applied, these cracks, introduced through dents, scratches or even from the manufacturing process, can rapidly grow, leading to catastrophic failure. To help educators explain this concept in class without the need for specialised equipment or teaching complex theory, we present a set of accessible experiments on the fracture strength of paper strips. We show how the complexity of the experiment can be modified for various age groups, ranging from an engaging session for younger students pulling paper strips to a more involved extended practical using analytical solutions and fitting to determine the fracture strength of paper. These experiments have been delivered successfully to students of various ages and have led to stimulating discussions on the subject of materials science and engineering.

Keywords: mechanical testing, materials science, fracture strength, paper, outreach, STEM

1. Failure in materials

Humans have exploited materials for millennia, but the subject of materials science and engineering is not well known and poorly signposted. Students are typically unaware of its existence until they start to consider a university degree, however realise quickly they have covered many aspects already from atomic bonding to the measurement of mechanical properties. This failure in promoting materials science and engineering has led to the recruitment of the course challenging even though graduates are in high demand by industry [1].

The exploration of material properties starts in reception (KS1) with the introduction of words such as fragile and brittle, continuing through GCSE and A-level (KS5) with discussion and measurement of mechanical properties such as

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Young's modulus and strength. More complicated mechanical properties such as impact fracture toughness [2] and bending [3] have been shown to work well in classrooms. Educational articles have also discussed the atomic bonding [4], the elastic properties of nylon rope [5] and the glass fibre-reinforced plastic vaulting poles failure [6] but very little discussion is made on how and why the material finally failed. It is only in the first year of an engineering degree that the key concept of how cracks and defects lead to failure is introduced. Given its importance in today's engineering, this article provides a simple, exciting accessible, hands-on practical that allows students at all levels, to see and discover how the presence of cracks in materials can lead to failure at lower-than-expected stresses.

We start with a brief introduction to cracks in engineering, providing context to experiment.

2. Why an experiment on fracture toughness?

In 1983 the estimated cost to the US economy for materials catastrophically fracturing was over $119 B (nearly half a trillion dollars in today's money) [7]. Today, as materials and devices are pushed to their limits, catastrophic failure is still a critical concern in engineering applications and the cause of many engineering disasters.

Fracture occurs under the action of applied stress. While ductile materials undergo slow permanent (plastic) deformation known as yielding before finally breaking (such as metals, plastic bags, and jelly sweets), brittle failure occurs with little or no plastic deformation. This leads to it breaking catastrophically, without warning and with a bang. This can be associated with materials such as glass, and ceramics, or simply the snapping of a crisp. For these (previously overlooked) classes of materials, we have developed an experiment that allows a hands-on experiment and discussion of materials failure.

If we could make materials defect-free, such as a perfect crystal, the amount of stress they could withstand is related to the bonding of the constitute atoms, typically in the order of giga-Pascals [4]. There is no 'weak' point for it to break early. Most materials however fail at a fraction of this, usually with mega-Pascal [8]. The reasoning was not well understood before 1913 when Inglis published a paper on the strength of ships showing how holes, cracks, and sharp corners were crucial to understanding failure [9]. During manufacturing, most materials end up with atomic-scale defects and/or micro cracks. Although the effective stress is unchanged (applied force over the cross-sectional area), these features have a profound impact on local stresses in the material. Around these features, a 'stress concentration' is formed, many times higher than the stress in the rest of the material. It starts to move the voids together forming larger cracks which grow until the material effectively 'unzips' itself from the inside in a catastrophic manner.

Sometimes this is wanted, like notches in chocolate (if you want to share a portion of chocolate with friends and family) or perforated paper on notebooks (and toilet paper!) to tear the paper in a straight line. However, in many cases, voids moving to form larger cracks are unwanted. An illustrative example is that of the 1928 the 56 kilo-tonne White Star Liner Majestic was fitted with a lift installed for its passengers [10]. As part of the installation, engineers had to cut rectangular holes in the hull, but these cuts had an unwanted side effect, they introduced sharp corners. This led to stress concentrations around them. On its way to Southampton, a crack formed from one of those holes and started running down the side of the ship for many metres before it was luckily stopped by a porthole! Any further, and it would have been catastrophic. There have been many other examples of catastrophic events due to stress concentration, the most famous of which are the Liberty ships in the 1940s [11] and the de Havilland Comet in the late 1950s [12]. These stories provide historical relevance and context of why studying the formation and effect of cracks in materials is crucial in the design of any engineered device or structure and can be used in class to support the delivery of the practical.

3. Fracture theory

One method of determining how cracks impact materials is measuring the impact fracture toughness of a material using an impact test such as Charpy machine [2, 13]. While a great experiment, to run it as a hands-on experiment requires

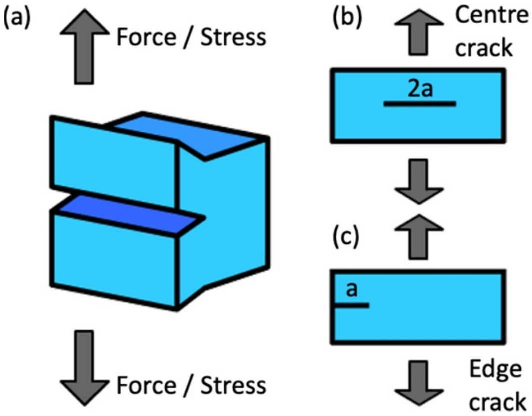

Figure 1. Highlighting the configuration of crack testing (a) general mode I crack testing using a tensile force or stress. (b) A centre crack of length 2a and (c) an edge crack of length a.

a collection of Charpy machines and identically notched samples, which can be expensive and challenging to obtain. An alternative method can be testing a pre-cracked sample using a tensile load, as seen in figure 1.

If a large enough stress is applied to a material incurred with cracks, the material will snap into two. This point is a measure of its fracture strength known as the critical stress intensity factor $K1c$.

Typical values of a range of materials are provided in table 1. Materials such as concrete and polystyrene have low values indicating small crack sizes are significant to failure, whereas metals with higher values show large-size cracks can still be supported by the material. Composites are dependent on their internal structure with cracks able to run faster in certain directions. This is why fibre alignment in carbon fibre reinforced composites is highly controlled, and why we cross-laminate wood (plywood) to provide resistance to facture and strength in all directions.

As Inglis stated, if a crack is present in the material, it generates a stress intensity factor, K_1 which is dependent on the size and shape of the crack along with the applied stress, s. If this value reaches the critical stress intensity factor, $K1c$, the crack will grow quickly resulting in a snap or bang! The fracture strength can be calculated using the following general equation:

$$K_1 = Y\sigma\sqrt{\pi a}. \quad (1)$$

Table 1. Values of the critical fracture strength of different materials extracted from various sources.

Material Type	Material	$K1c$ (MPa m$^{1/2}$)	Source
Metal	Aluminium	20–30	[8]
	Steel alloy	50–100	[8]
	Titanium	50–100	[8]
Ceramic	Glass	0.4–4.0	[14]
	Concrete	0.2–0.4	[14]
Polymer	Polystyrene	0.7–1.0	[8]
Composite	Multi-fibre composite	1.8–3.3	[16]
	Carbon fibre reinforced	20–100	[8]

The applied stress is given by σ, and the geometric factor, Y, is a phenomenological fitted geometric value. Here we focus on two common types, edge and centre cracks. Both geometric constants are given by Tada [16] and shown schematically in figures 1(b) and (c) respectively

$$Y_{\text{centre}} = 1.00 + 0.256\left(\frac{a}{w}\right)^1 - 1.152\left(\frac{a}{w}\right)^2 + 12.20\left(\frac{a}{w}\right)^3 \quad (2)$$

$$Y_{\text{edge}} = 1.12 - 0.231\left(\frac{a}{w}\right)^1 + 10.55\left(\frac{a}{w}\right)^2 - 21.71\left(\frac{a}{w}\right)^3 + 30.38\left(\frac{a}{w}\right)^4. \quad (3)$$

Note the value of 'a' changes with crack geometry. A centre crack is of length $2a$, figure 1(b), whereas an edge crack is length a as shown in figure 1(c). A centre crack can be considered as two edge cracks brought together. If the crack is small compared to the width of the plate, (an infinite case), the factors reduce to $Y = 1.00$ (centre) and $Y = 1.12$ (edge).

4. Material choice—paper

Providing a hands-on observation of key concepts is an essential component of physics education [15]. Although the topics can be covered in a lecture, we wanted to provide educators with a method of showing how cracks affect a material's strength. A key consideration was that no special equipment was required. As such we use strips of paper as it is safe (although for the odd paper cut) and easily shaped. Printer paper uses cellulose, extracted from trees, and made up of carbon, hydrogen, and oxygen atoms. The length of these molecules determines the quality of the paper made. The cellulose is mixed with water, chalk in some cases and starch. It is then dried, flattened, and heated to create the final product of the desired thickness, resulting in a multi-fibre composite.

This choice allows flexibility for educators to use it as a safe demonstration or prescribed task. It can also be extended to an open-ended project, allowing students to design and test their own cracks and record how they reduce the stress compared to a pristine, uncut sample. Depending on their level and ability, the learning goals range from the basic understanding that cracks are bad for materials' strength, to the ability of plotting, fitting, and linking experimental data to analytical solutions.

While we have selected printer paper, which is between 70 and 100 grams per square metre (gsm), cards and magazine covers are made from thicker 300 gsm. An interesting extension could be performing this on different types of paper from recycling to glossy photo paper. These would all lead to different results in the fracture strength and would be an interesting 'materials' experiment to explore. Links can be made too of the type of material use to engineers applications, such as laminated paper using a plastic surface to protect the paper and stop it tearing is similar the coatings for safety glass.

5. Experimental setup

A typical measurement of fracture toughness requires specialised equipment along with resource intensive pre-prepared samples. To mimic this for use in a classroom we have developed the experiment to use paper strips with a purposeful defect (cut) introduced. The force is applied by pulling the paper until it fails catastrophically (snaps).

To provide flexibility in the delivery of this, we present three different types of fracture strength of paper experiments, suitable for different levels of education. The first is an advanced experiment to determine the fracture strength of paper. It requires tensile testing apparatus and fitting of data and is well suitable for an open-ended physics project or a 1st-year undergraduate practical. The second determines the influences of cracks on a material's fracture strength, requiring access to a tensiometer, which could be as simple as a set of luggage scales. This is more suited to an intermediate-level student. The third requires only a pair of scissors and a piece of paper. This can be used as a demonstration for younger audiences to understand the concept of how materials break and how small cuts can make it a lot easier to break them.

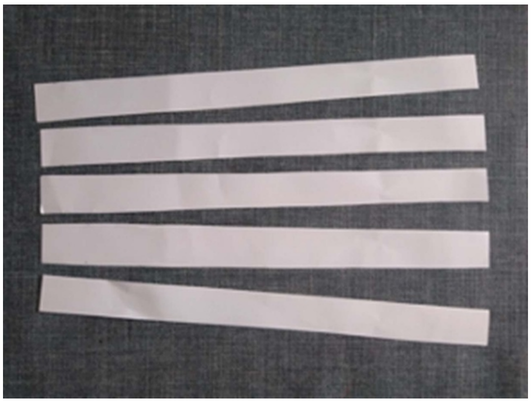

Figure 2. Showing a set of cut paper strips that can be cut out of an A4 piece of paper, each with dimensions 25 mm wide and 29 mm long.

To ensure we generate the force required to break the paper and any cuts introduced produce a varied response, we use an A4 piece of printer paper (70 gsm, thickness 0.095 mm) cut into strips of 25 mm wide work well, as shown in figure 2(a). This width provides a good balance between the force needed to break them, and the effect the cuts have on the paper's fracture strength. It is key when cutting these strips to ensure no small nicks or wobbles are made, which would introduce 'defects' further reducing its fracture strength.

6. Advanced level experiment—determining the fracture strength of paper

As there is a lack of literature on the fracture strength of paper, we use a Shimadzu tensile testing frame (Universal Testing Machine EZ-LX)

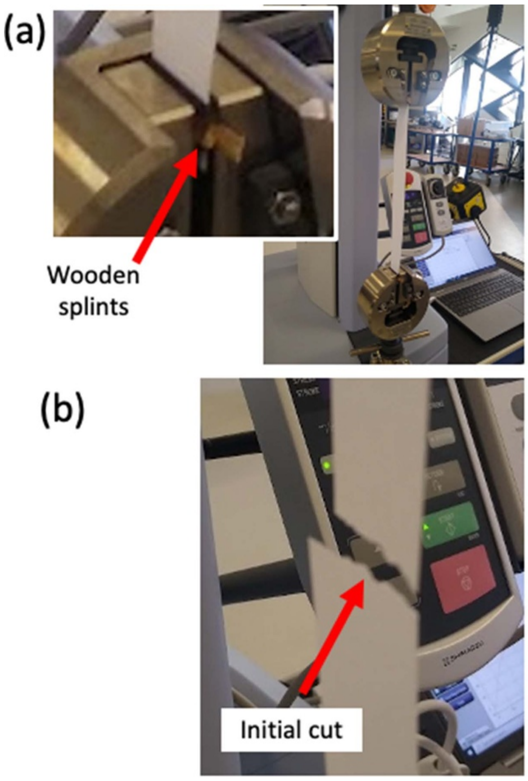

Figure 3. Showing the fracture of paper set up (a) the paper loaded in the tensile testing machine with wooden splints used to hold the paper firmly in the grips. (b) The paper after failure, showing the crack started from the initial centre cut in the material.

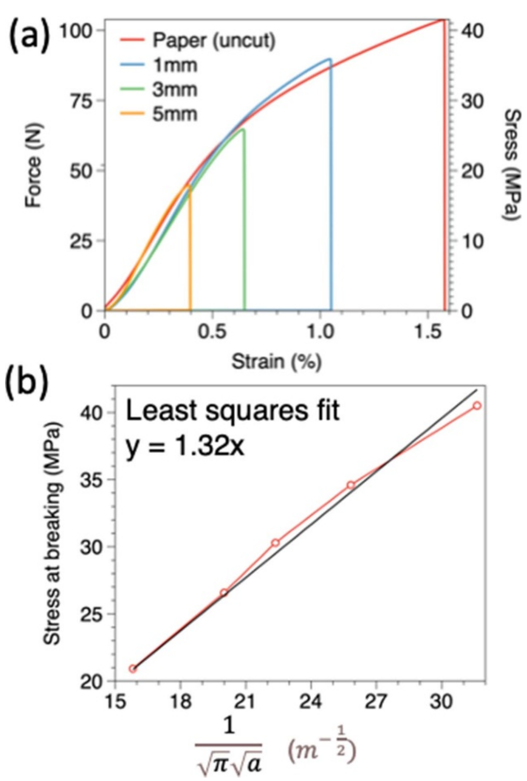

Figure 4. Results of tensile testing (a) the force–strain curves for centre cracked samples. The secondary axis shows the force converted into the associated stress. (b) The breaking strength using equation (4). A line of best fit is added to find the gradient, which corresponds to the fracture strength.

to determine this experimentally as accurately as possible. We note that while the method shown here provides a high level of control, it could be performed using a set of weights and clamps.

Each paper specimen was loaded into the Shimadzu tensile testing frame. We use small pieces of wood on each side to stop the paper from slipping in the grips, as shown in figure 3(a). The force–strain curves are shown in figure 4(a) with the maximum force at which it broke recorded.

To minimise error, we repeated each sample seven times, finding the average. We converted the maximum force into stress using the cross-sectional area of the paper strip (25 mm $\times$ 0.095 mm $=$ 2.38 $\times$ 10^{-6} m^2). This is shown as a secondary axis in figure 4(b) for comparison. This resulted in a failure strength of paper (uncut) as 40 $\pm$ 2.9 MPa (95 $\pm$ 6.9 N).

To find the fracture strength, $K1c$, we added pre-determined cuts into the centre of the strips using a sharp knife. A centre crack was used rather than an edge crack as we found edge cracks required a high degree of alignment to provide similar tension on either side of the paper and not cause it to buckle. Cuts ranged from 2 mm to 8 mm leading to geometrical factors between $Y = 1$ and $Y = 1.02$, equation (2).

As Y is very close to 1, we simplify and rearrange equation (1) to a linear form. This allows us to find the gradient using a least squares fit, which corresponds to the critical fracture strength

$$\sigma = K_{1c}\frac{1}{\sqrt{\pi}\sqrt{a}}. \tag{4}$$

As shown in figure 4(b), the result is near linear, although a slight curvature can be observed which is associated with Y changing as a function of the crack size. Using this linear method, we found the gradient to be 1.32 MPa $m^{1/2}$. Converting to the critical fracture strength using equation (4), we find $K1c = 2.34$ MPa $m^{1/2}$. It is worth noting that this value is in the range of multifibre composite and wood (table 1). We note we also used an advanced fit based on solving both equations (1) and (2). Only a small change in the fracture strength of the paper is found, leading to a value of $K1c = 2.40$ MPa $m^{1/2}$.

While this experiment was not been run with students at the time of publication, the development of an undergraduate practical based on this is underway. We believe this would also work well as an advanced practical for A-level physics in which students could test and compare different qualities of paper, cardboard and other materials.

7. Intermediate level experiment—how cracks influence the strength of paper

Using the fracture strength of $K1c = 2.40$ MPa $m^{1/2}$ and failure strength of 40 MPa, we formed table 2 to show a range of edge cracks, associated geometrical factors and intact cross-sectional areas.

As part of a $Y12$ session we delivered on engineering, we provided each student five paper strips and asked them to make cuts into them ranging between 1 mm and 10 mm. We asked them, using table 2, to predict the force required to break their paper strips assuming it was simply based on the intact cross-sectional area. To help in this, we gave students columns 1–3 of table 2, and the failure strength of the paper as 40 MP. We provide the estimate in column 4, table 2 if educators wish to provide this to their class instead of asking them to calculate it.

The final column of table 2 is the estimate of the force required due to the stress concentrate of the crack, which uses equations (1) and (3) in its calculation. As shown, the force required drops dramatically and is halved from the estimated 80 N to 40 N if a 4 mm crack is introduced.

As part of the Y12 session we obtained a set of inexpensive luggage scales to act as a tensiometer, figure 5(a). We reinforced the points where the paper strips were held with duct tape, figure 5(b) as well as the connection point to the luggage scales, figure 5(c). This increased the strength of these regions and avoided introducing any unwanted failure points due to it being held. The students were asked to pull their samples apart, measuring the weight (or more accurately the mass) that they failed at. This generated excitement due to the catastrophic nature of the breaking.

The scales we used, shown in figure 5(a), included a maximum needle. This helped the students record the maximum force after the paper snapped, however, this could work just as well pairing students together with one acting as a 'reader' and one as a 'puller'.

Only the strongest students (and the author) were able to obtain a measurement of the paper fracturing with a 4 mm cut with larger cuts broken by all. Using a pre-constructed Excel sheet, students entered their results, which automatically calculated the average failure stress of the paper strips. The class data is shown in figure 6. During the session, students mentioned how they were surprised to find only a few extra mm of a crack could make the paper much easier to break, with some commenting on how far off the initial prediction was.

To save time, we also created an Excel sheet to automatically plot the data using equation (4) and then provide a linear fit. Although the geometrical factor rises from $Y = 1.12$ to a 10 mm edge crack of $Y = 2.10$, the data still fits a straight line shown in figure 4(b), albeit with an intercept value. The gradient was found to be 1.25 MPa $m^{1/2}$ leading to a critical fracture strength of $K1c = 2.48$ MPa $m^{1/2}$, matching well the $K1c = 2.40$ MPa $m^{1/2}$ we calculated previously.

The session wrap was a discussion of what we saw and what it meant. One question was about the offset of the fit and what it might mean for the data. This allowed discussion on what errors could be arising in the practical, including changes in the geometric factor and accurately cutting the crack. One student asked, 'Is the paper uniform?' and another asked, 'Would different paper give different results?'. This led to a discussion of repeatability and manufacturing tolerances standards in engineering. We discussed why surfaces of

Table 2. The table shows the edge crack sizes used with their respective reduction in cross-section area due to the presence of the crack. The estimated force required to break the paper strip is compared to the force required to break the paper strip due to the stress intensity generated by the edge crack.

Crack size (mm)	Geom. factor Y	Cross sectional area ($\times 10^{-6}$ m^2)	Force required (area) (N)	Force required (crack) (N)
0	NA	2.38	95	95
1	1.13	2.28	91	90
2	1.16	2.19	87	62
3	1.21	2.09	84	48
4	1.26	2.00	80	40
5	1.37	1.90	76	33
6	1.47	1.81	72	28
7	1.59	1.71	68	24
8	1.73	1.62	64	20
9	1.90	1. 52	60	18
10	2.10	1.43	57	15

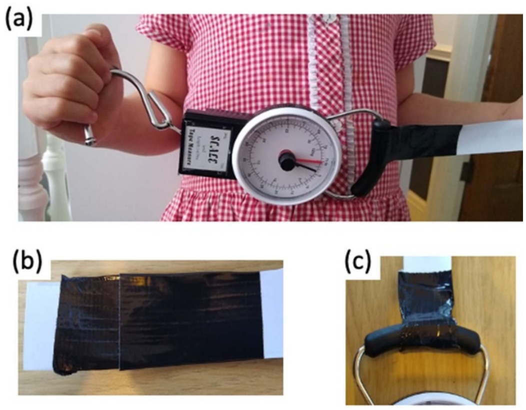

Figure 5. The pulling of the paper strips using a pair of luggage scales. (a) The luggage scales are used as a tensiometer, allowing the student to record the failure strength. (b) The end of the paper being held was reinforced with duct tape and (c) the point at which it connected to the luggage scales was also reinforced.

engineering materials are polished along with key components (e.g. aircraft fuselages, pressurised containers) being regularly inspected for dents or scratches providing real-world relevance.

8. Basic level experiment—paper snapping

We have developed this practical to be a quick engaging outreach session requiring little set-up and focused on observations. The key concept we wanted to highlight in the session is that cracks in a material make it much easier to break. We ran this as an interactive activity for Y12 students as part of an outreach lecture on materials failure but has also worked well as a more guided demonstration/practical for Y3 students.

To highlight the concept of 'strength' we first asked the students to find the force they can generate using the set of luggage scales, figure 7(a). Again, a set of scales which records the maximum weight was very useful here. The competitive element creates quite a bit of excitement and engagement. We found students could generate values between 25 and 35 N of force, well below the 95 N required to break a crack-free paper strip. It is however above that required to break a 5 mm edge cut if stress concentration is considered.

We explained the experiment we performed in our laboratory and explained the uncut paper strip needed nearly 100 N to break, more than double what they could produce. We then asked a few students to try and break an un-cut paper strip which allowed us to demonstrate how to hold and break them. A good 'pull' is achieved by holding each end carefully as shown in figure 7(b) and pulling the paper in a slow controlled manner. If failure occurred, we asked them to check this was in the middle near the crack and not near their fingers. If the paper broke near their fingers, we asked them to void it as a failed attempt and repeat it. This typically happened if students squeezed

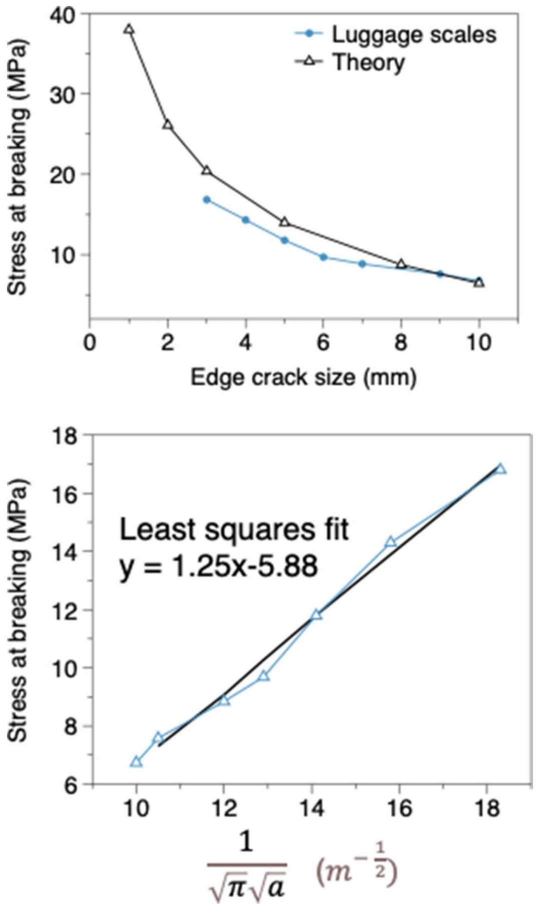

Figure 6. The results of the fracture strength experiment using a set of luggage scales. (a) The breaking stress as a function of the edge crack size, matches well with theory. (b) The linear fits using equation (4) with a line of best fit overlaid.

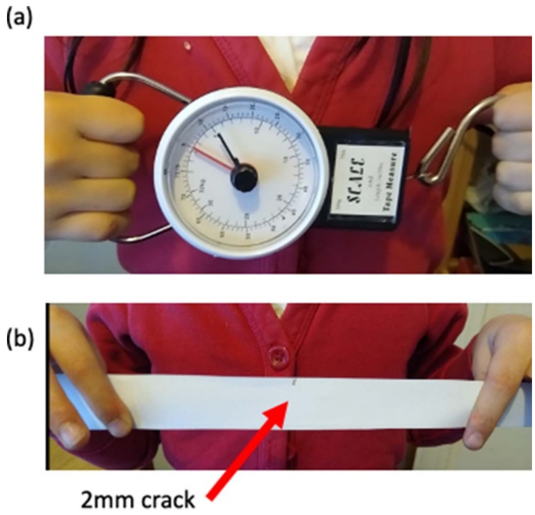

Figure 7. Breaking the paper by hand (a) the students estimate how much force they can generate using a luggage scale and (b) the technique of tensile testing the paper strip by hand.

or twisted the paper causing additional stresses at their 'clamped' points. Doing this led to no successful breaks.

We then provided students strips of paper with pre-marked 2, 3, 4 and 5 mm lines asking them to cut as accurately as they could to create an edge crack. All students easily broke the paper with a 5 mm cut, with comments about how easy it was even though the cut was small. Many could break a 4 mm, but none were able to break a 3 mm cut.

Summing up the session, we asked students what this means for materials with cracks. Students reacted with comments of *'it is bad news'* and *'it will break before you think it will'*. Some of our younger Y3 students told us it '*makes the paper weaker'* relating it to words they have learnt in class. We asked students how they might deal with cracks if they found them in the material, *'replace the material'* allowed us to talk about the lifetime of the component. One comment of *'fill them in with glue'* allowed us to talk about self-healing materials. Some of the Y3 students said gluing pieces of material over the top of the crack with one saying '*Can we cut it to get rid of it?'* is a great answer and related to polishing and shot peening surfaces—all real-world engineering solutions.

9. Summary

We show here a set of experiments that allow students of various ages to understand how materials fail through the introduction of cracks and their impact on apparent strength.

It has successfully allowed students across a diverse age range to gain the concept of crack propagation, stress intensity factors and fracture without understanding any complex background theory. The experiment also promoted discussions on why inspections of dents or cracks in aircraft and other pressurised structures are made along with why engineers like to polish and smooth fsurfaces along with removing sharp edges in their design.

These sessions can be used to support currently taught content in the syllabus as an interactive class experiment but furthermore stretch stronger physics students in exploring the more involved mathematics of fracture strength.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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