Inverse Problems

The first 100 years of the Radon transform

OPEN ACCESS
IOP Publishing

Inverse Problems

Inverse Problems **34** (2018) 090201 (4pp)

https://doi.org/10.1088/1361-6420/aacf27

Editorial

The first 100 years of the Radon transform

Abstract

This special issue is honoring the 100th anniversary of the publication of the famous paper by Johann Radon (1917 *Ber. über Verh. Königlich-Sächsischen Ges. Wiss. Leipzig* **69** 262–77).

Keywords: Radon transform, special issue, 100th anniversary

(Some figures may appear in colour only in the online journal)

1. Johann Radon and his fundamental paper on the Radon transform

Johann Radon (1887–1956) (see figure 1) had a remarkable career in mathematics: he was awarded a doctorate from the University of Vienna in Philosophy (for a thesis in the field of calculus of variations) in 1910. After professorships in Hamburg, Greifswald, Erlangen, Breslau, and Innsbruck, he returned to the University of Vienna, where he became dean and later president of the University of Vienna. Johann Radon is well-known for his ground-breaking achievements in mathematics, such as the Radon-transformation, the Radon-numbers, the theorem of Radon, the theorem of Radon–Nikodym and the Radon–Riesz theorem.

Interestingly, the paper [21], which we are honoring with this special issue, was not considered a break through publication for a long time. Johann Radon himself seems not to have reported about these results and in the obituary [9] of his colleague Paul Funk, also a distinguished mathematician of the University of Vienna, he outlined the above mentioned mathematical achievements but ignored the Radon transform.

From today's perspective this might seem curious but at this time his other achievements were mathematically outstanding, and had a much stronger impact in the field. After the realization of CT-scanners, which need the implementation of the inverse Radon transform, this needs to be reconsidered. It is an irony that Allan M Cormack and Godfrey Hounsfield, who received the Nobel-prize in Physiology or Medicine for the development of the first medical CT-scanner, developed their algorithms for image reconstruction not only independently from each other but also without knowledge of Radon's work. Allan M Cormack studied the propagation of x-rays through human tissue and Godfrey Hounsfield was an engineer who was designing scanners.

Moreover, today mathematics is established in many applications and permeates applications of biomedical engineering and of personalized medicine. The appreciation of the fundamental work of Johann Radon can also be seen in the newsletter article of the Austrian Chamber of Pharmacists. This article was published on the occasion of the 50th anniversary of the death of Johann Radon. The author explained the development of x-ray CTs and the importance of Radon's work. This article acknowledges for a broader community that Radon's work would have been worth a Nobel prize award.

Figure 1. The authors wish to express their gratitude to Brigitte Radon-Bukovics for providing the photo of her father.

2. On the special issue

This special issue collects 23 papers related to recent developments of the Radon transform and inverse problems.

Applications: reconstruction form limited x-ray CT data for underwater pipeline inspection has been presented by Riis *et al* [22]. Atmospheric turbulence profiling for reconstructing star images from blurred astronomical data has been considered by Helin *et al* [13]. Automatic alignment for 3D tomographic reconstruction of 2D sectional images data, which is an important practical problem in medical imaging, has been considered by Leeuwen *et al* [18]. A novel approach for solving practical inverse problems based on deep neural networks has been presented by Adler and Öktem [1]. Grathwohl *et al* [12] considered the numerical solution of a seismic imaging problem. An inverse problem of elastography has been studied by Stefanov *et al* [23].

From this list of exciting real world applications in this special issue one sees the enormous impact of mathematical tomography in today's applied sciences.

Stability: this is an important issue in limited angle tomography. Andersson and Boman [3] provide new results in this domain.

Discrete tomography: about recent developments in dynamic discrete tomography which is reported by Alpers and Gritzmann [2].

Developments in geodesic and weighted x-ray transforms have been considered in Holman *et al* [14]. Analysis of the Abel transforms and applications to x-ray tomography on spherically symmetric manifolds has been presented in Hoop and Ilmavirta [15]. An example of non-uniqueness for the weighted Radon transforms has been given by Goncharov and Novikov [11].

Tomography on non straight paths appear for instance in photoacoustics, where averages over spheres or circles are calculated. Photoacoustical inverse problems have been considered in Beigl *et al* [5] and Frikel and Haltmeier [8]. Gindikin [10] reports on curved version of Radon's inversion formula on the plane.

Compton camera imaging and the cone transforms are an emerging topic in inverse problems. The relation to generalized Radon transforms has been documented in an overview by Terzioglu *et al* [24]. A model for a Compton single scatter in PET has been presented by Kazantsev *et al* [17]. Reconstruction formulas and analysis of cone integral transforms have been reported in Palamodov [20].

Vector and tensor field tomography has been explored in the papers of Lehtonen *et al* [19]. Numerical solvers tensor field tomographical problems have been investigated in Derevtsov *et al* [6]. An exact inversion formula for solenoidal fields in cone beam vector tomography has been developed in Katsevich *et al* [16].

Numerical methods for obtaining higher contrast in tomographical images and to reconstruct patterns is a topic in the paper by Zibetti *et al* [25]. Electron paramagnetic resonance image reconstruction and pattern recognition with total variation and curvelets regularization is presented in the paper by Durand *et al* [7].

The treatment of nonlinear tomographical problems is considered in Bal *et al* [4]. In this paper a linearised hybrid data impedance tomography problem was analyzed.

Acknowledgments

RR and OS are supported by the Austrian Science Fund (FWF), in the frame of the Spezialforschungsbereich SFB F68, projects F6805-N36 and F6807-N36, respectively.

ORCID iDs

Ronny Ramlau https://orcid.org/0000-0002-0277-9615
Otmar Scherzer https://orcid.org/0000-0001-9378-7452

References

[1] Adler J and Öktem O 2017 Solving ill-posed inverse problems using iterative deep neural networks *Inverse Problems* **33** 124007
[2] Alpers A and Gritzmann P 2018 Dynamic discrete tomography *Inverse Problems* **34** 034003
[3] Andersson J and Boman J 2018 Stability estimates for the local radon transform *Inverse Problems* **34** 034004
[4] Bal G, Hoffmann K and Knudsen K 2017 Propagation of singularities for linearised hybrid data impedance tomography *Inverse Problems* **34** 024001
[5] Beigl A, Elbau P, Sadiq K and Scherzer O 2018 Quantitative photoacoustic imaging in the acoustic regime using SPIM *Inverse Problems* **35** 054003
[6] Derevtsov E Y, Louis A K, Maltseva S V, Polyakova A P and Svetov I E 2017 Numerical solvers based on the method of approximate inverse for 2D vector and 2-tensor tomography problems *Inverse Problems* **33** 124001
[7] Durand S, Frapart Y M and Kerebel M 2017 Electron paramagnetic resonance image reconstruction with total variation and curvelets regularization *Inverse Problems* **33** 114002
[8] Frikel J and Haltmeier M 2018 Efficient regularization with wavelet sparsity constraints in photoacoustic tomography *Inverse Problems* **34** 024006
[9] Funk P 1958 Nachruf auf Prof. Johann Radon *Math. Nachr.* **62** 191–9
[10] Gindikin S 2017 Curved version of Radon's inversion formula on the plane *Inverse Problems* **34** 014007

[11] Goncharov F O and Novikov R G 2018 An example of non-uniqueness for the weighted Radon transforms along hyperplanes in multidimensions *Inverse Problems* **34** 054001
[12] Grathwohl C, Kunstmann P, Quinto E T and Rieder A 2017 Approximate inverse for the common offset acquisition geometry in 2D seismic imaging *Inverse Problems* **34** 014002
[13] Helin T, Kindermann S, Lehtonen J and Ramlau R 2018 Atmospheric turbulence profiling with unknown power spectral density *Inverse Problems* **34** 044002
[14] Holman S, Monard F and Stefanov P 2018 The attenuated geodesic x-ray transform *Inverse Problems* **34** 064003
[15] de Hoop M V and Ilmavirta J 2017 Abel transforms with low regularity with applications to x-ray tomography on spherically symmetric manifolds *Inverse Problems* **33** 124003
[16] Katsevich A, Rothermel D and Schuster T 2017 An improved exact inversion formula for solenoidal fields in cone beam vector tomography *Inverse Problems* **33** 064001
[17] Kazantsev I G, Olsen U L, Poulsen H F and Hansen P C 2018 A spectral geometric model for compton single scatter in PET based on the single scatter simulation approximation *Inverse Problems* **34** 024002
[18] van Leeuwen T, Maretzke S and Batenburg K J 2018 Automatic alignment for three-dimensional tomographic reconstruction *Inverse Problems* **34** 024004
[19] Lehtonen J, Railo J and Salo M 2018 Tensor tomography on Cartan–Hadamard manifolds *Inverse Problems* **34** 044004
[20] Palamodov V 2017 Reconstruction from cone integral transforms *Inverse Problems* **33** 104001
[21] Radon J 1917 Über die Bestimmung von Funktionen durch ihre Integralwerte längs gewisser Mannigfaltigkeiten *Ber. über Verh. Königlich-Sächsischen Ges. Wiss. Leipzig* **69** 262–77
[22] Riis N A B, Frøsig J, Dong Y and Hansen P C 2018 Limited-data x-ray CT for underwater pipeline inspection *Inverse Problems* **34** 034002
[23] Stefanov P, Uhlmann G and Vasy A 2017 Local recovery of the compressional and shear speeds from the hyperbolic DN map *Inverse Problems* **34** 014003
[24] Terzioglu F, Kuchment P and Kunyansky L 2018 Compton camera imaging and the cone transform: a brief overview *Inverse Problems* **34** 054002
[25] Zibetti M V W, Lin C and Herman G T 2018 Total variation superiorized conjugate gradient method for image reconstruction *Inverse Problems* **34** 034001

Ronny Ramlau[2,3] **and Otmar Scherzer**[1,2]

[1] Faculty of Mathematics, University of Vienna, Oskar-Morgenstern-Platz 1, A-1090 Vienna, Austria
[2] Johann Radon Institute for Computational and Applied Mathematics (RICAM), Altenbergerstraße 69, A-4040 Linz, Austria
[3] Institute for Industrial Mathematics, Johannes Kepler University, Altenbergerstraße 69, A-4040 Linz, Austria

E-mail: ronny.ramlau@jku.at and otmar.scherzer@univie.ac.at