

## A (running) bolt for new reasons

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## A (running) bolt for new reasons

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**ABSTRACT:** We construct a four-parameter family of smooth, horizonless, stationary solutions of ungauged five-dimensional supergravity by using the four-dimensional Euclidean Schwarzschild metric as a base space and “magnetizing” its bolt. We then generalize this to a five-parameter family based upon the Euclidean Kerr-Taub-Bolt. These “running Bolt” solutions are necessarily non-static. They also have the same charges and mass as a non-extremal black hole with a classically-large horizon area. Moreover, in a certain regime their mass can decrease as their charges increase. The existence of these solutions supports the idea that the singularities of non-extremal black holes are resolved by low-mass modes that correct the singularity of the classical black hole solution on large (horizon-sized) scales.

**KEYWORDS:** Black Holes in String Theory, Black Holes

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## 1 Introduction

Over the past few years there has been a significant shift in the discussion of the “fuzzball proposal” and its realization in terms of microstate geometries:<sup>1</sup> Met initially with considerable skepticism, there is now growing evidence (see [1] for reviews) that this proposal might well be realized for BPS black holes. In hind-sight, this may not appear so strange: extremal BPS black holes have a timelike singularity, and, as is fairly well known, string theory oftentimes resolves such singularities in terms of configurations that contain extra brane dipole moments, and that have a size that is parametrically much larger than the “size” of the original region of high curvature.<sup>2</sup>

The BPS microstate geometries constructed thus far [8] indicate that the timelike singularity of extremal BPS black holes is resolved in a similar manner and that the size of the configurations that resolve the singularity is of the same order as the size of the

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<sup>1</sup>Microstate geometries are defined to be smooth, horizonless solutions with the same asymptotic behavior at infinity as a black hole or black ring.

<sup>2</sup>A few of the better-known examples of such singularity-resolution mechanism can be found in Polchinski-Strassler [2], Klebanov-Strassler [3], the D1-D5 system [4], giant gravitons and LLM [5, 6] or the massive M2 supergravity dual [6, 7].

black-hole horizon. This means that one can no longer trust the “classical” space-time description of the region between the timelike singularity and the horizon of the extremal black hole, much as one does not trust the physical descriptions provided by the Klebanov-Tseytlin solution [10], the singular giant graviton, or the unpolarized Polchinski-Strassler solution [11] at scales smaller than the singularity-resolution scale. The resolution of these singularities involves low-mass degrees of freedom, that affect the physics at large distances; one cannot simply repair these solutions in the neighborhood of the singularity by some Planck-scale, or string-scale details.

The timelike singularity of BPS extremal black holes therefore appears to be resolved in the same way as its string theory cousins that do not sit behind a horizon, and this effectively implements the fuzzball proposal for this class of black holes. If this proposal also applies for *non-extremal* black holes, the would-be singularity resolution mechanism would be even more remarkable: The singularity of a non-extremal black hole is in the future of the horizon and if the classical black hole is to be replaced by a superposition of horizon-sized horizonless configurations this would imply that the resolution of non-extremal black hole singularities will affect the spacetime for a macroscopically-large distance in the past of the singularity! It is clearly important to understand this singularity-resolution mechanism, not only because there is, as yet, no rigorous example in string theory of how one might expect a space-like singularity to be resolved, but also because we live in a universe in which such singularities appear to be ubiquitous.

To establish that the singularity of non-extremal black holes is resolved by horizon-sized horizonless geometries that have the same mass and charges as the black hole, one first needs to construct such geometries, which is no easy task — only three such geometries are known at present [12–14] and some of their properties are studied in [15]. One then needs to see whether the physical properties of these geometries support thinking about them as microstates of the non-extremal black hole (and thus as examples of resolution of the black hole singularity). For example, the geometry constructed in [12], which has an ergosphere but no horizon was found to be unstable in [16] but the decay time was then computed in the dual CFT [17], and found to match exactly the decay time computed in gravity. This remarkable agreement strongly supports thinking about the geometries of [12] as microstates of non-extremal black holes, and brings hope that the fuzzball proposal will equally apply to such black holes.

Our purpose in this paper is to construct a new family of smooth, horizonless solutions that have the same charges and mass as non-extremal black holes, and that exist in the same regime of parameters where the black hole exists. The solutions we find can be thought of as asymptotically  $\mathbb{R}^{3,1} \times S^1$  solutions of  $U(1)^N$  supergravity in five dimensions, and from a four-dimensional “string theory on Calabi-Yau” perspective have D4, D2, D0 charges and angular momentum. Smoothness requires some of these charges to be related, and the solution depends on  $N + 2$  independent parameters.

Unlike the non-extremal solutions of [12–14], these solutions do have the same charges and mass as a black hole with a classically-large horizon area. Furthermore, they are asymptotically flat, and do not contain an AdS region. This is both an advantage – they can be microstates of more generic non-extremal black holes, and a disadvantage — one cannot use the power of the AdS-CFT correspondence to study them. Consequently, the

interpretation of these geometries as microstate geometries of a black hole will be somewhat less compelling than it has been for the BPS microstates.

The technical construction of these non-extremal geometries is not as complicated as one might expect, and is in fact very similar to the construction of BPS and non-BPS extremal multi-center solutions. To construct the latter one takes a four-dimensional hyper-Kähler manifold with self-dual curvature, turns on self-dual [18–20] two-forms (for BPS solutions) or anti-self dual [21] two-forms (for non-BPS solutions), and solves a linear system of equations [20] to determine the warp factors and the angular momentum. The nice observation of [21] that one can find “almost-BPS” non-supersymmetric solutions by simply flipping some relative orientations has been further exploited in [22, 23] where large classes of such solutions have been constructed. One way to think about these solutions is as coming from supersymmetric brane configurations that live in gravitational backgrounds that are incompatible with the supersymmetries of the branes.

One can, however, take this idea even further and ask whether the hyper-Kähler condition is really necessary if one only wishes to satisfy the supergravity equations of motion. Of course, having a hyper-Kähler base was needed for supersymmetry, but Einstein’s equations should only care about the Ricci tensor of the base and whether the base is Ricci-flat or not. Hence, if we consider the BPS equations of [18–20], or the non-BPS equations of [21], and replace the hyper-Kähler base by any Ricci-flat base we still expect to obtain a (non-BPS) solution to the equations of motion. In [24] we prove this result in detail, starting from the full equations of motion for five-dimensional supergravity coupled to three  $U(1)$  gauge fields.

Our purpose in this paper is to study some examples of this idea, and to find smooth, horizonless solutions in five dimensions using Ricci-flat, Euclidean four-dimensional base metrics. Perhaps the simplest, most interesting such metrics arise from the Euclideanization of the various pure-gravity black-hole metrics. Such solutions have a periodic “imaginary time” coordinate and are thus asymptotic to  $\mathbb{R}^3 \times S^1$ . These solutions also come with a “bolt,” that is, the center of these solutions is topologically  $\mathbb{R}^2 \times S^2$  where the  $S^2$  remains of finite size. This is because the Euclideanized geometry closes off smoothly where the (outer) horizon of the original Lorentzian black hole used to be and the  $S^2$  at the center is the same size as the original black-hole horizon. We therefore use these Euclidean metrics as base metrics for five-dimensional solutions and then add smooth (cohomological) self-dual or anti-self-dual magnetic fluxes to the bolt, and these fluxes also act as sources for the electric fields and angular momentum.

While the manner of obtaining these solutions is similar to that of BPS solutions, there are some fundamental differences. First, and most obviously, these solutions are not BPS. In addition, their mass depends on a combination of the electric charges and the mass parameter of the underlying Euclidean black hole. Furthermore, the smooth BPS bubbled geometries have an ambi-polar<sup>3</sup> base-space that by itself is rather pathological, and if one sets to zero some of the fluxes, the two-cycles of the base collapse. In contrast, the non-BPS

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<sup>3</sup>This means that the auxiliary, four-dimensional base metric can change signature from +4 to −4. The physical five-dimensional metric constructed from such a base will, however, be smooth and Lorentzian [31–33, 36].

solutions considered here have base spaces that can be either regular or ambi-polar, and that can give regular five-dimensional solutions even in the absence of fluxes. The two-cycle of the base space is the bolt. “Magnetizing” the bolt, by adding fluxes, distorts the size of this bolt and of the  $S^1$  at infinity but removing the flux does not collapse the bolt. Furthermore, when fluxes are added, the Euclidean time direction of the base (which is now the Kaluza-Klein direction) and the physical time direction mix. Hence these solutions are never static at infinity in the frame where the metric is static at the bolt, and viceversa. We therefore refer to them as “running Bolt” solutions.<sup>4</sup>

Before beginning we should also note that most, if not all the five-dimensional solutions that result from taking an Euclidean instanton and simply adding time<sup>5</sup> are unstable. This was demonstrated for the Euclidean Schwarzschild solution by Gross, Perry and Yaffe [26], and for the Taub-bolt instanton by Young [27]. The more general Kerr-Taub-bolt instanton has not yet, to our knowledge, been found to have negative modes. The solutions we construct also have magnetic and electric fluxes, as well as angular momentum, and their stability analysis is likely to be more complicated than that of [26, 27] (which is not simple either). By continuity, it is quite likely that the running Bolt solutions will still be unstable when the fluxes are small but when the fluxes and charges become larger the fate of the solutions is unknown — they may become stable or remain unstable. However, this need not hamper their interpretation as microstates of the corresponding black hole. On the contrary, as for the microstates of [12], the instability may be necessary from a microscopic perspective [17], and the variation of the decay time with the charges of our solutions might as well give a way to identify the black hole microstates that correspond to our solutions.

In section 2 we give our conventions and specify the class of solutions we are going to consider. In section 3 we generate new solutions from the Euclidean Schwarzschild metric. This solution is extremely simple since spherical symmetry is preserved throughout. We then go on, in section 4, to consider solutions generated from the the Euclidean Kerr-Taub-Bolt metric. While these solutions have some similar general features to the solution based upon the Schwarzschild metric, the Kerr-Taub-Bolt solution is richer and has more parameters. Indeed, *a priori*, this solution has three independent parameters but the combined effect of removing conical singularities and Dirac strings in the Euclidean base imposes a cubic constraint on the parameters and this implies that only two of them are independent. There are, however, potentially several branches in the solution space, but physical conditions, like the requirement of a positive definite Kerr-Taub-Bolt metric mean that one must discard some of the branches and parameter ranges [28]. This raises the interesting question of whether, by allowing an ambi-polar base metric, one can generate more solutions than those allowed by the naive positive-definite constraint on the base metric. We find that this is indeed possible, and that not only is there a broader range of admissible parameters when one allows ambi-polar base metrics, but that there are additional branches to the smooth five-dimensional solutions coming from allowable changes of sign in the cubic constraint. Finally, we conclude section 4 with a computation of the

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<sup>4</sup>Without, of course, any reference to the recent breaking of the 100 and 200 meters world records.

<sup>5</sup>Solutions of this type are usually referred to in the literature as static KK bubbles. For a general discussion of this class of solutions see, for example, [25].

asymptotic charges of running Bolt solutions, and section 5 contains some further remarks.

## 2 The family of solutions

### 2.1 Conventions

We consider  $\mathcal{N}=2$ , five-dimensional supergravity with three  $U(1)$  gauge fields and we use the conventions of [21]. The bosonic action may be written as:

$$S = \frac{1}{2\kappa_5} \int \sqrt{-g} d^5x \left( R - \frac{1}{2} Q_{IJ} F_{\mu\nu}^I F^{J\mu\nu} - Q_{IJ} \partial_\mu X^I \partial^\mu X^J - \frac{1}{24} C_{IJK} F_{\mu\nu}^I F_{\rho\sigma}^J A_\lambda^K \bar{\epsilon}^{\mu\nu\rho\sigma\lambda} \right), \quad (2.1)$$

with  $I, J = 1, 2, 3$ . One of the photons lies in the gravity multiplet and so there are only two vector multiplets and hence only two independent scalars. Thus the scalars,  $X^I$ , satisfy a constraint, and it is convenient to introduce three other scalar fields,  $Z_I$ , to parameterize these two scalars:

$$X^1 X^2 X^3 = 1, \quad X^1 = \left( \frac{Z_2 Z_3}{Z_1^2} \right)^{1/3}, \quad X^2 = \left( \frac{Z_1 Z_3}{Z_2^2} \right)^{1/3}, \quad X^3 = \left( \frac{Z_1 Z_2}{Z_3^2} \right)^{1/3}. \quad (2.2)$$

The metric for the kinetic terms can be written as:

$$Q_{IJ} = \frac{1}{2} \text{diag} \left( (X^1)^{-2}, (X^2)^{-2}, (X^3)^{-2} \right). \quad (2.3)$$

Note that the scalars,  $X^I$ , only depend upon the ratios  $Z_J/Z_K$  and it is convenient to parameterize a third independent scalar by:

$$Z \equiv (Z_1 Z_2 Z_3)^{1/3}. \quad (2.4)$$

We now use this scalar,  $Z$ , in the metric Ansatz:

$$ds_5^2 = -Z^{-2} (dt + k)^2 + Z ds_4^2, \quad (2.5)$$

where the powers guarantee that  $Z$  becomes an independent scalar from the four-dimensional perspective. We will denote the frames for (2.5) by  $e^A$ ,  $A = 0, 1, \dots, 4$  and let  $\hat{e}^a$ ,  $a = 1, \dots, 4$  denote frames for  $ds_4^2$ . That is, we take:

$$e^0 \equiv Z^{-1} (dt + k), \quad e^a \equiv Z^{1/2} \hat{e}^a. \quad (2.6)$$

The Maxwell Ansatz is:

$$A^I = -\varepsilon Z_I^{-1} (dt + k) + B^{(I)}, \quad (2.7)$$

where  $B^{(I)}$  is a one-form on the base (with metric  $ds_4^2$ ). The parameter,  $\varepsilon$ , will be related to the self-duality or anti-self-duality of the fields in the solution and is fixed to have  $\varepsilon^2 = 1$ . It is convenient to define the field strengths:

$$\Theta^{(I)} \equiv dB^{(I)} = \frac{1}{2} Z^{-1} \Theta_{ab}^{(I)} e^a \wedge e^b = \frac{1}{2} \Theta_{ab}^{(I)} \hat{e}^a \wedge \hat{e}^b. \quad (2.8)$$

Note that the frame components are defined relative to the frames on  $ds_4^2$ .

## 2.2 BPS and simple non-BPS solutions

Both the BPS and the almost-BPS solutions are given by taking the base to be hyper-Kähler with a self-dual curvature and then solving the linear system [20, 21]:

$$\Theta^{(I)} = \varepsilon *_4 \Theta^{(I)}, \quad (2.9)$$

$$\hat{\nabla}^2 Z_I = \frac{1}{2} \varepsilon C_{IJK} *_4 [\Theta^{(J)} \wedge \Theta^{(K)}], \quad (2.10)$$

$$dk + \varepsilon *_4 dk = \varepsilon Z_I \Theta_I. \quad (2.11)$$

As we explain in [24], one can obtain solutions to the equations of motion simply by solving the BPS system (2.9)–(2.10) with any Ricci-flat base metric on  $ds_4^2$ :

$$\hat{R}_{ab} = 0. \quad (2.12)$$

The most obvious such base is the Euclidean Schwarzschild metric, which we use in the next section to generate new solutions. In section 4 we will extend this to the more general Kerr-Taub-bolt solution.

## 3 Adding fluxes to euclidean Schwarzschild

### 3.1 The solution

The Euclidean Schwarzschild metric is given by:

$$ds_4^2 = \left(1 - \frac{2m}{r}\right) d\tau^2 + \left(1 - \frac{2m}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2. \quad (3.1)$$

It is, of course, Ricci flat, and if one restricts to the region  $r \geq 2m$  then the metric is globally regular provided one periodically identifies the Euclidean time by:

$$\tau \equiv \tau + 8\pi m. \quad (3.2)$$

Near  $r = 2m$  the manifold is then locally  $\mathbb{R}^2 \times S^2$  and at infinity it is asymptotic to  $\mathbb{R}^3 \times S^1$ . The “bolt” at the origin can be given a magnetic flux and we can take the  $\varepsilon$ -self-dual harmonic two-forms to be:

$$\Theta^{(I)} = q_I \left( \frac{1}{r^2} d\tau \wedge dr + \varepsilon \sin \theta d\theta \wedge d\phi \right), \quad (3.3)$$

for some magnetic charges,  $q_I$ .

With this flux it is trivial to solve the second equation (2.10) and one finds

$$Z_I = 1 - \frac{1}{2} C_{IJK} \frac{q_J q_K}{m} \frac{1}{r}. \quad (3.4)$$

We have chosen the homogeneous solution so as to exclude all other electric sources for  $Z_I$  and to arrange that  $Z_I \rightarrow 1$  as  $r \rightarrow \infty$ .

The last equation (2.11) is equally elementary, and setting

$$k = \mu d\tau + \omega d\phi \quad (3.5)$$



we find

$$\mu = (\varepsilon + \alpha)(q_1 + q_2 + q_3) \frac{1}{r} - \frac{3\varepsilon}{2m} q_1 q_2 q_3 \frac{1}{r^2} + \gamma, \quad (3.6)$$

$$\omega = \alpha(q_1 + q_2 + q_3)(\beta + \cos \theta), \quad (3.7)$$

where  $\alpha, \beta$  and  $\gamma$  parameterize homogeneous solutions. These parameters must be chosen to remove closed time-like curves (CTC's) in (2.5). First, to avoid CTC's on  $\phi$ -circles one must make sure that there are no Dirac strings in  $k$ , and hence  $\alpha = \beta = 0$ . Similarly, there are potential CTC's around the small  $\tau$  circles near  $r = 2m$  unless we choose  $\gamma$  so that  $\mu = 0$  at  $r = 2m$ . Thus we must take  $\omega = 0$  and

$$\mu = \varepsilon(q_1 + q_2 + q_3) \left( \frac{1}{r} - \frac{1}{2m} \right) - \frac{3\varepsilon}{2m} q_1 q_2 q_3 \left( \frac{1}{r^2} - \frac{1}{4m^2} \right). \quad (3.8)$$

The solution is now completely determined but it still remains to verify the absence of CTC's elsewhere. On the constant time slices the metric in the  $\tau$  direction is  $\mathcal{M}d\tau^2$  where  $\mathcal{M} \equiv Z^{-2}r^{-4}\mathcal{Q}$  and

$$\mathcal{Q} \equiv r^4 Z_1 Z_2 Z_3 \left( 1 - \frac{2m}{r} \right) - \mu^2 r^4. \quad (3.9)$$

This is a quartic function of  $r$  and must remain non-negative for  $2m \leq r < \infty$  and this places constraints on  $m$  and the  $q_I$ . In addition, the  $Z_I$  should remain positive definite for  $r > 2m$ .

To examine these conditions in more detail we simplify the analysis by taking  $q_I = q > 0$ ,  $I = 1, 2, 3$ . The positivity of the  $Z_I$  for  $r > 2m$  means that one must have:

$$q < \sqrt{2}m. \quad (3.10)$$

To analyze the positivity of  $\mathcal{M}$ , we first look at the behavior at infinity, where one has

$$\mathcal{M} \sim r^{-4} \mathcal{Q} \sim \left( 1 - \frac{3q}{8m^3}(q^2 - 4m^2) \right) \left( 1 + \frac{3q}{8m^3}(q^2 - 4m^2) \right). \quad (3.11)$$

For this to be positive, the two cubics in  $q$  must be positive and this implies the stronger condition:

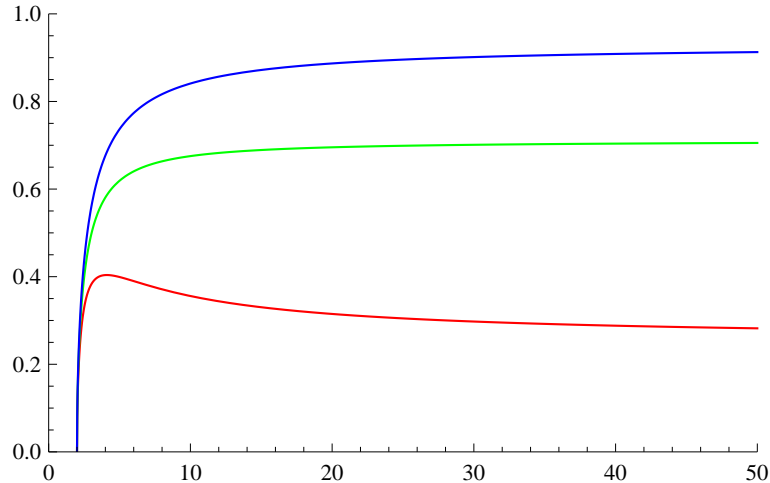
$$0 < \frac{q}{m} < \frac{4}{\sqrt{3}} \sin \frac{\pi}{9} \approx 0.78986. \quad (3.12)$$

Note that the function  $\mathcal{M}$ , and hence the condition above, do not depend on  $\varepsilon$ . More generally, the quartic that sets the scale of the  $\tau$ -circle is:

$$\mathcal{Q} \equiv (r - 2m) \left[ \left( r - \frac{q^2}{m} \right)^3 - \frac{9q^2}{64m^6} (r - 2m) ((q^2 - 4m^2)r + 2mq^2)^2 \right]. \quad (3.13)$$

One can verify that this is indeed positive definite for  $2m < r < \infty$  for  $q$  in the range (3.12).

It is interesting to note that (3.8) shows that  $\mu$  asymptotes to a finite value as  $r \rightarrow \infty$ . One can undo the rotation of this frame by shifting  $\tau \rightarrow \tau + at$  and the condition (3.12) simply reflects the fact that this rotation is sub-luminal.



**Figure 1.** Plot of the scale,  $\sqrt{\mathcal{M}}$ , of the compactification circle as a function of  $r/m$ . The three plots, from top to bottom, correspond to  $q/m$  of  $1/4$ ,  $1/2$  and  $3/4$ . Note that as one approaches the upper bound (3.12) the circle does not grow uniformly but attains a maximum scale before decreasing asymptotically.

We have thus created a “magnetized bolt” solution in which fluxes have been added to a pre-existing two-cycle. It is interesting to note that in the BPS “bubbled” solutions of [31–34] the fluxes were an essential part of blowing up the two-cycles and these bubbles would collapse without the fluxes. Another element of the BPS bubbled solutions was the presence of an ambi-polar base metric where the metric on the four-dimensional base changes sign but this sign change is canceled in the five-dimensional metric by a simultaneous sign change in warp factor,  $Z$ . In more physical terms, there is also a direct D-brane interpretation of the bubbling transition [31, 34].

The solution constructed here does not appear to have such a D-brane interpretation and does not involve an ambi-polar base. One could try to see whether the ranges of parameters or the range of  $r$  might be extended to give an ambi-polar four-dimensional base that still yields a smooth Lorentzian five-dimensional solution. There are obvious possibilities, like taking  $m < 0$  and trying to extend to  $r < 0$  but such extensions do not lead to an *overall* sign change in (3.1) and so cannot be canceled by the warp-factor  $Z$ . Thus, at least for this solution, the standard Euclidean Bolt is simply decorated by fluxes to give a running Bolt solution with electric and magnetic charges. We shall see later that there are richer possibilities once angular momentum and a NUT charge are included.

### 3.2 Asymptotic charges

The solution carries M5 charges, which are encoded in the magnetic part of the gauge field,  $B^{(I)}$ , and are equal to  $q_I$ . The solution also carries M2 charges. Note that the gauge field equations involve Chern-Simons terms:

$$d((X^I)^{-2} *_5 dA^I) = \frac{1}{2} C_{IJK} dA^J \wedge dA^K. \quad (3.14)$$

In the presence of such terms, the proper definition of the conserved electric charge associated with  $A^I$  is

$$Q^I = \int_{S^1 \times S^2} \left[ (X^I)^{-2} *_5 dA^I - \frac{1}{2} C_{IJK} A^J \wedge dA^K \right], \quad (3.15)$$

where the integral is computed over the  $S^1$  circle parameterized by  $\tau$  and the  $S^2$  sphere at spatial infinity. The Chern-Simons term gives a non-vanishing contribution to the charge, due to the fact that the one-form  $k$  goes to a constant non-zero value at infinity:

$$k \rightarrow \gamma d\tau, \quad \gamma = -\varepsilon \frac{q_1 + q_2 + q_3}{2m} + 3\varepsilon \frac{q_1 q_2 q_3}{8m^3}. \quad (3.16)$$

Using the identity

$$\begin{aligned} & (X^I)^{-2} *_5 dA^I - \frac{1}{2} C_{IJK} A^J \wedge dA^K \\ &= \varepsilon *_4 dZ_I - \frac{1}{2} C_{IJK} B^{(J)} \wedge \Theta^{(K)} + \frac{\varepsilon}{2} C_{IJK} d \left[ (dt + k) \wedge \frac{B^{(J)}}{Z_K} \right], \end{aligned} \quad (3.17)$$

one finds

$$Q^I = -(8\pi m)(4\pi) \frac{1}{2} C_{IJK} \left[ \varepsilon \frac{q_J q_K}{m} + \frac{\gamma}{2} (q_J + q_K) \right]. \quad (3.18)$$

To compute the mass and the KK electric charge of the solution one has to analyze the asymptotic form of the metric. The fact that the one-form,  $k$ , does not vanish at infinity implies that the coordinates  $(\tau, t)$  define a frame which is not asymptotically at rest, much like for the Black Ring in Taub-NUT constructed in [35]. One can go to an asymptotically static frame by re-writing the large  $r$  limit of the metric in the form

$$ds^2 \approx (1 - \gamma^2) \left( d\tau - \frac{\gamma}{1 - \gamma^2} dt \right)^2 - dt^2 (1 - \gamma^2)^{-1} + dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (3.19)$$

and redefining the coordinates as

$$\hat{\tau} = (1 - \gamma^2)^{1/2} \left( \tau - \frac{\gamma}{1 - \gamma^2} t \right), \quad \hat{t} = (1 - \gamma^2)^{-1/2} t. \quad (3.20)$$

The condition (3.12) reflects the fact that the rotation is sub-luminal ( $\gamma < 1$ ) and hence this change of coordinates is well-defined.

Dimensional reduction of the five-dimensional metric along the direction  $\hat{\tau}$  yields

$$ds_5^2 = \left( 1 - \frac{2m}{r} \right)^2 Z^{-2} \hat{I}_4 \left[ d\hat{\tau} - \left( 1 - \frac{2m}{r} \right)^{-2} \mu \hat{I}_4^{-1} d\hat{t} + \gamma d\hat{t} \right]^2 + \left( 1 - \frac{2m}{r} \right)^{-1} Z \hat{I}_4^{-1/2} ds_E^2, \quad (3.21)$$

where

$$\hat{I}_4 = (1 - \gamma^2)^{-1} \left[ Z^3 \left( 1 - \frac{2m}{r} \right)^{-1} - \mu^2 \left( 1 - \frac{2m}{r} \right)^{-2} \right], \quad (3.22)$$

and

$$ds_E^2 = -\hat{I}_4^{-1/2} d\hat{t}^2 + \hat{I}_4^{1/2} \left[ dr^2 + \left( 1 - \frac{2m}{r} \right) (r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2) \right] \quad (3.23)$$

is the four-dimensional Einstein metric. From the coefficient of  $d\hat{t}^2$  in the Einstein metric one can read off the mass of the solution:

$$G_4 M = \frac{1}{(1-\gamma^2)} \left( \frac{m}{2} - \frac{q_1 q_2 + q_1 q_3 + q_2 q_3}{4m} - \frac{3}{8} \frac{\gamma}{\varepsilon} \frac{q_1 q_2 q_3}{m^2} \right). \quad (3.24)$$

Here  $G_4$  is the four-dimensional Newton's constant, whose relation with the five-dimensional Newton's constant  $G_5$  is

$$G_4 = \frac{G_5}{(1-\gamma^2)^{1/2} (8\pi m)}. \quad (3.25)$$

The KK electric charge  $Q_e$  is encoded in the KK gauge field

$$A_{KK} = \left( \gamma - \left( 1 - \frac{2m}{r} \right)^{-2} \mu \hat{I}_4^{-1} \right) d\hat{t} \quad (3.26)$$

and is given by

$$G_4 Q_e = -\frac{1}{4(1-\gamma^2)} \left( \frac{3\varepsilon}{4} \frac{q_1 q_2 q_3}{m^2} + \gamma \frac{q_1 q_2 + q_1 q_3 + q_2 q_3}{m} + \varepsilon \gamma^2 (q_1 + q_2 + q_3) \right). \quad (3.27)$$

If one now computes the rest-mass,  $M_0$ , of the solution (i.e. the mass with respect to the  $(t, \tau)$  frame) one obtains:

$$M_0 \equiv (1-\gamma^2)^{-1/2} (M - \gamma Q_e) = \frac{\pi}{4G_5} \left( 16m^2 + \frac{\varepsilon}{4\pi^2} (Q^1 + Q^2 + Q^3) \right). \quad (3.28)$$

### 3.3 Some remarks on the mass of the running bolt

For  $\varepsilon = 1$ , Equation (3.28) indicates that the total rest mass is simply the sum of the mass of the uncharged bolt and the masses corresponding to the M2 branes. Hence, if one could ascribe a putative solitonic charge to the uncharged bolt, this formula would look very much like the mass of a BPS object. Furthermore, the fact that the M2 brane charge enters linearly in the total mass is also consistent with the fact that a probe M2 brane feels no force in this background.

For  $\varepsilon = -1$  the situation is even more interesting. The mass now decreases linearly with increasing the M2 charge. Hence, the mass formula is still linear, but the sign in front of the M2 charges is negative! We are not aware of any other such mass formula in the literature. One might object to this by noting that one can always flip the sign of M2 charges by reversing some orientations; however, by flipping the signs of some of the  $q_I$  one can change the sign of some of the M2 charges. Hence the total mass can either decrease or increase with increasing the mass corresponding to the M2 charges. Alternatively, if one absorbs  $\varepsilon$  by an orientation change, then the mass formula (3.28) is linear in the charges, and not in their absolute values as for BPS systems.

The fact that the mass of the solutions can decrease with increasing charge and the fact that M2 brane probes feel no force may lead one to believe naively that one could violate energy conservation: one can bring an M2 brane adiabatically from the infinity to the core of a solution, and the resulting solution will have a lower mass than the sum of the masses of the two pre-merger components. However, this does not happen. The charge

of the M2 brane probe that feels no force is oriented oppositely to the M2 brane charge of this solution! Bringing in this probe brane actually decreases the total M2 charge and therefore increases the mass of the solution to the mass of the soliton plus the mass of the probe M2 one brought in adiabatically, as expected.

Our analysis thus indicates that the uncharged bolt is the middle point of a family of magnetized solutions that can have both larger and smaller rest masses, and that moreover these masses can grow or decrease linearly with the M2 charges of the solution.

## 4 Adding fluxes to the Kerr-Taub-Bolt solution

The Euclidean Schwarzschild solution admits a very interesting generalization with additional quantum numbers: The Euclidean Kerr-Taub-Bolt solution [28]. In this section we add fluxes to the bolt and get a more general regular five-dimensional running Bolt solution.

### 4.1 The euclidean Kerr-Taub-Bolt solution

The four-dimensional metric is

$$ds_4^2 = \Xi \left( \frac{dr^2}{\Delta} + d\theta^2 \right) + \frac{\sin^2 \theta}{\Xi} (\alpha d\tau + P_r d\phi)^2 + \frac{\Delta}{\Xi} (d\tau + P_\theta d\phi)^2, \quad (4.1)$$

where

$$\begin{aligned} \Delta &\equiv r^2 - 2mr - \alpha^2 + N^2, & \Xi &\equiv P_r - \alpha P_\theta = r^2 - (N + \alpha \cos \theta)^2, \\ P_r &\equiv r^2 - \alpha^2 - \frac{N^4}{N^2 - \alpha^2}, & P_\theta &\equiv -\alpha \sin^2 \theta + 2N \cos \theta - \frac{\alpha N^2}{N^2 - \alpha^2}. \end{aligned} \quad (4.2)$$

This is a Ricci-flat metric where  $m$  is the mass,  $\alpha$  is the angular momentum and  $N$  is the NUT charge. If the metric is to be regular then these parameters are not independent, as we will see in the following. At infinity the metric behaves as

$$ds_4^2 \sim dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2) + (d\tilde{\tau} - (\alpha \sin^2 \theta + 2N(1 - \cos \theta)) d\phi)^2, \quad (4.3)$$

with

$$\tilde{\tau} \equiv \tau + 2N\phi - \frac{\alpha N^2}{N^2 - \alpha^2} \phi. \quad (4.4)$$

Thus the metric is asymptotic to  $\mathbb{R}^3 \times S^1$  provided that  $\phi$  has period  $2\pi$  and the fibration of  $\tau$  over the two-sphere in  $\mathbb{R}^3$  is regular if  $\tau$  is identified under shifts:

$$\tau \equiv \tau + 8N\pi. \quad (4.5)$$

We now need to examine regularity at the points where some of the metric coefficients vanish. First, at  $\theta = 0, \pi$ , the circle with  $d\tau = -P_\theta d\phi$  pinches off. Substituting  $d\tau = -P_\theta d\phi$  into the metric and ignoring the radial terms gives a metric:

$$\Xi \left[ d\theta^2 + \frac{1}{\Xi^2} \sin^2 \theta ((P_r - \alpha P_\theta) d\phi)^2 \right] = \Xi [d\theta^2 + \sin^2 \theta d\phi^2], \quad (4.6)$$

which is perfectly regular.

The second degeneracy appears at  $\Delta = 0$ . Define  $r_{\pm}$  by  $\Delta = (r - r_+)(r - r_-)$  with  $r_+ > r_-$ :

$$r_{\pm} = m \pm \sqrt{m^2 - N^2 + \alpha^2}. \quad (4.7)$$

Since we are interested in Euclidean black-hole solutions with non-trivial bolts, we will consider the situation where these roots are real:

$$m^2 + \alpha^2 \geq N^2. \quad (4.8)$$

We have to arrange that the metric is regular at  $r \rightarrow r_+$  and then restrict to  $r \geq r_+$ . As usual, this will lead to a periodic identification in the  $\tau$  coordinate. Before proceeding with the analysis here, it is worth noting that in the usual analysis of the Kerr-Taub-Bolt metric [28], one requires the metric to be positive definite and hence one requires  $\Xi > 0$  and hence  $m > |N|$ .

However, if one's purpose is to construct five-dimensional solutions, the four-dimensional base can be ambi-polar and hence  $\Xi$  can be allowed to change sign. We will indeed find that the warp factors also change sign to compensate for this and give a regular five-dimensional solution. Thus we will not impose that  $m > |N|$ , but only (4.8).

To explore regularity at  $r = r_+$  it is useful to define:

$$P_{r_+} \equiv P_r|_{r=r_+} = r_+^2 - \alpha^2 - \frac{N^4}{N^2 - \alpha^2}, \quad \kappa \equiv \left| \frac{r_+ - r_-}{2P_{r_+}} \right|. \quad (4.9)$$

The circle that pinches off at  $r = r_+$  has  $d\phi = -\alpha d\tau/P_{r_+}$ . Substituting this into the metric and expanding in  $x = (r - r_+)^{1/2}$  one obtains:

$$\frac{4\Xi}{r_+ - r_-} \left[ dx^2 + \kappa^2 x^2 d\tau^2 + \frac{1}{4}(r_+ - r_-) d\theta^2 \right]. \quad (4.10)$$

This is regular as  $x \rightarrow 0$  provided that  $\tau$  is periodically identified according to:

$$\tau \equiv \tau + \frac{2\pi}{\kappa}, \quad (4.11)$$

and hence the base space is smooth if

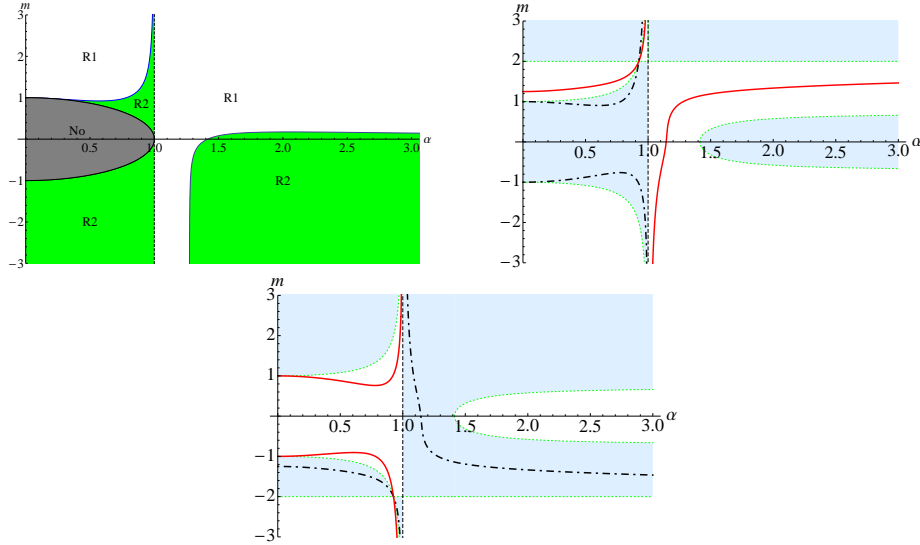
$$\kappa = \frac{1}{4|N|}. \quad (4.12)$$

This condition, together with (4.8) are the two necessary condition for absence of conical singularities in the base. Hence, the ambi-polar Kerr-Taub-Bolt metrics that we will use to generate running Bolt solutions depend on only two independent parameters.

We now explore the implications of equation (4.12) for the allowed range of  $N, m$  and  $\alpha$ . Since this equation only involves  $|N|$ , one can use the definition of  $\kappa$  to see that the sign of  $N$  is irrelevant; hence we will assume in the following that  $N$  is positive.

One can now square the square roots in (4.12), and obtain a constraint that is cubic in  $m$ . This constraint depends on the sign of  $P_{r_+}$ : if  $P_{r_+}$  is positive, we have

$$16N(N^2 - \alpha^2)^2 m^3 - 4(5N^6 - 8\alpha^2 N^4 + 2\alpha^4 N^2 + \alpha^6) m^2 - 16N(N^2 - \alpha^2)^3 m + 20N^8 - 52\alpha^2 N^6 + 49\alpha^4 N^4 - 16\alpha^6 N^2 = 0, \quad (4.13)$$



**Figure 2.** The three graphs are plots of  $m$  versus  $\alpha$ , in units in which  $N = 1$  (this choice can always be made because the equations are homogeneous). The first shows the regions where  $P_{r+}$  is either positive (R1, in white) or negative (R2, in green). The grey area, No, is forbidden by the reality condition, (4.8). The second and the third graph show the solutions of (4.13) and (4.14). In the shaded (blue) areas the square root in equation (4.12) is equal to a negative expression. Hence, the solutions that belong to these regions, or to the regions of the first graph where  $P_{r+}$  has the wrong sign, do not obey (4.12) and are “wrong branch” solutions. These solutions are represented using dotted lines, while the physical solutions, that obey (4.12), are represented using continuous lines and belong to the white areas.

while if  $P_{r+}$  is negative equation (4.12) implies:

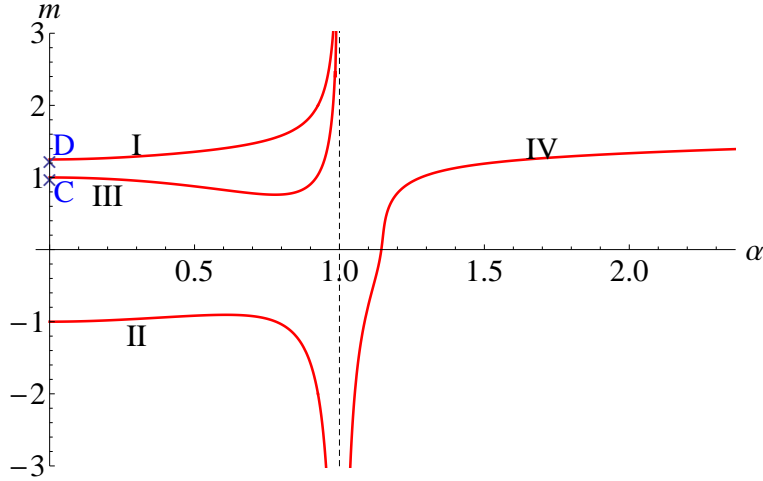
$$\begin{aligned}
 & -16N(N^2 - \alpha^2)^2 m^3 - 4(5N^6 - 8\alpha^2 N^4 + 2\alpha^4 N^2 + \alpha^6) m^2 \\
 & + 16N(N^2 - \alpha^2)^3 m + 20N^8 - 52\alpha^2 N^6 + 49\alpha^4 N^4 - 16\alpha^6 N^2 = 0, \quad (4.14)
 \end{aligned}$$

which is the same as (4.13) but with  $m \rightarrow -m$ . Note that a solution to (4.13) or (4.14) is not automatically a solution to (4.12). This only happens when two conditions are satisfied: first,  $P_{r+}$  must be respectively positive or negative; second, before squaring the square root in (4.12) one has to insure that the expression to which this square root is equal is positive. Hence, the cubic equations (4.13) and (4.14) contain “wrong branch” solutions, that do not solve (4.12).

The details of the parameter ranges are shown in figure 2. The first graph depicts the regions in which  $P_{r+}$  is positive or negative, and thus where we have to solve, respectively, (4.13) or (4.14). For  $\alpha < 1$ , each cubic has three real roots; for (4.13) two of them are solutions to (4.12), and one lies on the “wrong branch;” for (4.14) two of them lie on the “wrong branch.” For  $\alpha > 1$ , there is one real root to the cubics, and the only physical solution is the one with  $P_{r+} > 0$ .

The complete solution to (4.12) is shown on figure 3.

We would like to note that our analysis does not completely agree with the discussion in [28]. Indeed [28] only analyzes solutions with positive  $P_{r+}$ , and thus misses some of the



**Figure 3.** Plot of the values of  $m$  that give physical solutions of (4.12) for a given  $\alpha$ , in units in which  $N = 1$ . The solution has four disconnected branches: Branches I, II and III go from  $\alpha = 0$  to  $\alpha = 1$ , diverging as  $\alpha$  approaches 1. Branch IV starts from  $-\infty$  as  $\alpha \rightarrow 1_+$  and approaches  $m = 2$  as  $\alpha \rightarrow \infty$ . The intercepts, C and D, correspond, respectively, to the Taub-NUT and Taub-Bolt metrics.

ambi-polar branches of the five-dimensional solution. Furthermore, for  $\alpha < 1$  the solutions found and plotted in [28] do not have quite the same shape as the ones in figure 3.<sup>6</sup>

At  $\alpha = 0$  one obtains two interesting particular solutions: the Taub-NUT solution, for  $m = |N|$ , and the Taub-Bolt solution of [29] for  $m = 5/4N$ . It is worth noting that allowing the metric to be ambi-polar (see section 4.4) extends the range of parameters significantly. Indeed, forcing the four-dimensional metric to have a signature  $(+, +, +, +)$  imposes  $m > |N|$ , and thus would forbid the complete branch II, and part of branches III and IV (see figure 3).

## 4.2 Maxwell fields on the Kerr-Taub-Bolt

Introduce frames:

$$\begin{aligned} \hat{e}^1 &= \left( \frac{\Xi}{\Delta} \right)^{\frac{1}{2}} dr, & \hat{e}^2 &= \Xi^{\frac{1}{2}} d\theta, \\ \hat{e}^3 &= \frac{\sin \theta}{\Xi^{\frac{1}{2}}} (\alpha d\tau + P_r d\phi), & \hat{e}^4 &= \left( \frac{\Delta}{\Xi} \right)^{\frac{1}{2}} (d\tau + P_\theta d\phi), \end{aligned} \quad (4.15)$$

and define the self-dual and anti-self-dual two-forms by:

$$\Omega_{\pm} = \frac{1}{(r \mp (N + \alpha \cos \theta))^2} [\hat{e}^1 \wedge \hat{e}^4 \pm \hat{e}^2 \wedge \hat{e}^3]. \quad (4.16)$$

These forms are harmonic and have potentials satisfying  $dA_{\pm} = \Omega_{\pm}$ , of the form [30]:

$$A_{\pm} = \mp \cos \theta d\phi - \frac{1}{(r \mp (N + \alpha \cos \theta))} (d\tau + P_\theta d\phi). \quad (4.17)$$

<sup>6</sup>We believe this discrepancy can be most easily explained by evolving Moore's law backwards in time.



In the rest of this section we will focus on the self-dual Maxwell fields and take:

$$\Theta^{(I)} = q_I \Omega_+. \quad (4.18)$$

The extension to anti-self-dual Maxwell fields is rather straightforward.

Solving the second equation (2.10) yields:

$$Z_I = 1 - \frac{1}{2} C_{IJK} \frac{q_J q_K}{(m - N)} \frac{1}{(r - (N + \alpha \cos \theta))}. \quad (4.19)$$

We have, once again, chosen the homogeneous solution so as to exclude all singular electric sources for  $Z_I$  and to arrange that  $Z_I \rightarrow 1$  as  $r \rightarrow \infty$ . Notice that the denominator of  $Z_I$  is one of the factors of  $\Xi$  and, if  $N \geq 0$ , both this denominator and  $\Xi$  will change sign when  $r$  is small.<sup>7</sup> This suggests that the five-dimensional metric could be regular when the base space is ambi-polar.

### 4.3 The angular momentum vector

Solving the last equation (2.11) is a little non-trivial and we find it convenient to make the Ansatz:

$$k = \mu (d\tau + P_\theta d\phi) + \nu d\phi, \quad (4.20)$$

and solve the system for  $\mu$  and  $\nu$ . We find that this system may be recast in terms of a single function,  $F$ , for which:

$$\Xi \mu - \alpha \nu = \Delta \partial_r F, \quad \nu = \sin \theta \partial_\theta F. \quad (4.21)$$

The equation satisfied by  $F$  is:

$$\begin{aligned} \partial_r(\Delta \partial_r F) + \frac{1}{\sin \theta} \partial_\theta(\sin \theta \partial_\theta F) - \frac{2}{(r - (N + \alpha \cos \theta))} (\Delta \partial_r F + \alpha \sin \theta \partial_\theta F) \\ = (q_1 + q_2 + q_3) \frac{(r + N + \alpha \cos \theta)}{(r - (N + \alpha \cos \theta))} - \frac{3 q_1 q_2 q_3}{(m - N)} \frac{(r + N + \alpha \cos \theta)}{(r - (N + \alpha \cos \theta))^2}. \end{aligned} \quad (4.22)$$

Upon solving this equation we find the following solution for the angular momentum vector:

$$\begin{aligned} \mu = \gamma \left[ 1 - \frac{2N}{(r + N + \alpha \cos \theta)} \right] - (q_1 + q_2 + q_3) \frac{r}{\Xi} \\ + \frac{q_1 q_2 q_3}{2(m - N)^2} \left[ \frac{m - N - 2\alpha \cos \theta}{\Xi} + \frac{2(m - N)}{(r - (N + \alpha \cos \theta))^2} \right], \end{aligned} \quad (4.23)$$

$$\nu = \gamma \alpha \sin^2 \theta - \frac{\alpha q_1 q_2 q_3}{(m - N)^2} \frac{\sin^2 \theta}{(r - (N + \alpha \cos \theta))}, \quad (4.24)$$

where  $\gamma$  is an arbitrary constant that multiplies terms coming from the homogeneous solution of (4.22). As with the Schwarzschild solution, we suppress homogeneous solutions that lead to Dirac strings (and hence CTC's) in the  $\phi$  direction.

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<sup>7</sup>We will see later that the other factor in  $\Xi$  never vanishes for  $N > 0$ . If  $N < 0$  one can produce a similar result by starting with an anti-self-dual flux.

The parameter,  $\gamma$ , is now fixed by making sure that there are no CTC's near  $r = r_+$ . Since  $\Delta$  vanishes at  $r = r_+$ , one can make the spatial part of the metric vanish by moving on the circle with  $d\phi = -\alpha P_r^{-1} d\tau$ . It follows that, to avoid CTC's, the angular momentum vector,  $k$ , must vanish on this circle at  $r = r_+$ . This means that we must impose  $\Xi\mu - \alpha\nu = 0$  at  $r = r_+$ , for any value of  $\theta$ . This would follow from the first equation in (4.21) provided that  $F$  has no singularity at  $\Delta = 0$ . However,  $F$  generically has terms proportional to  $\log \Delta$ . On the other hand, the homogeneous solution to (4.22) (that yields the terms proportional to  $\gamma$  in (4.24)) also contains such terms:

$$F_{\text{hom}} = \gamma(r - \alpha \cos \theta) + \frac{(m - N)}{\sqrt{m^2 - N^2 + \alpha^2}} \left( r_+ \log(r - r_+) - r_- \log(r - r_-) \right). \quad (4.25)$$

Hence, we can cancel the singular behavior at  $r = r_+$  by choosing the coefficient of the homogeneous solution:

$$\gamma = \frac{(q_1 + q_2 + q_3)}{2(m - N)} + \frac{q_1 q_2 q_3}{4(m - N)^3} \left[ \frac{m + N}{r_+} - 2 \right]. \quad (4.26)$$

The full non-singular solution then has:

$$\begin{aligned} \mu = (\Delta + \alpha^2 \sin^2 \theta) & \left[ \frac{(q_1 + q_2 + q_3)}{2(m - N)\Xi} + \frac{3q_1 q_2 q_3}{2(m - N)^3 \Xi} \left( \frac{N}{r_+} - \frac{m - N}{2r} - 1 \right) \right. \\ & \left. - \frac{q_1 q_2 q_3}{2(m - N)^2} \frac{1}{r(r - (N + \alpha \cos \theta))^2} \right] \\ & - \frac{3q_1 q_2 q_3}{4(m - N)^2} \left( 1 + \frac{2N}{(r - (N + \alpha \cos \theta))} \right) \left( \frac{1}{r} - \frac{1}{r_+} \right) \end{aligned} \quad (4.27)$$

$$\begin{aligned} \nu = \alpha & \left[ \frac{(q_1 + q_2 + q_3)}{2(m - N)} - \frac{q_1 q_2 q_3}{(m - N)} \left( \frac{1}{(r - (N + \alpha \cos \theta))} \right. \right. \\ & \left. \left. + \frac{1}{4(m - N)^2} \left( \frac{m + N}{r_+} - 2 \right) \right) \right] \sin^2 \theta. \end{aligned} \quad (4.28)$$

Note that at  $r = r_+$ , this solution is proportional to  $\sin^2 \theta$ .

#### 4.4 Ambi-polar Kerr-Taub-Bolt

The Kerr-Taub-Bolt metric (4.1) can be recast in the form

$$ds_4^2 = V^{-1} (d\tau + P'_\theta d\phi)^2 + V \left( \frac{\Delta_\theta}{\Delta} dr^2 + \Delta_\theta d\theta^2 + \Delta \sin^2 \theta d\phi^2 \right), \quad (4.29)$$

with

$$\Delta_\theta = \Delta + \alpha^2 \sin^2 \theta, \quad V = \frac{\Xi}{\Delta_\theta}, \quad P'_\theta = P_\theta + \alpha \frac{\Xi}{\Delta_\theta} \sin^2 \theta. \quad (4.30)$$

In this form, we see that if  $\Xi$  becomes negative, the signature changes from  $(+, +, +, +)$  to  $(-, -, -, -)$ . If one is interested in four-dimensional Euclidean metrics, one must require  $\Xi > 0$  and thus impose  $m > |N|$ . However, we are interested in regular solutions of five-dimensional supergravity, and as it is well-established, such solutions can be obtained from four-dimensional base spaces that have such signature changes [31–33, 36]. We now investigate this possibility in more detail, and for simplicity we will assume  $N > 0$ .

The five-dimensional metric is

$$ds^2 = -Z^{-2}(dt + k)^2 + ZV^{-1}(d\tau + P'_\theta d\phi)^2 + ZVds_3^2, \quad (4.31)$$

with  $ds_3^2 = \frac{\Delta_\theta}{\Delta} dr^2 + \Delta_\theta d\theta^2 + \Delta \sin^2 \theta d\phi^2$ . The only factor in  $V$  that can change sign is  $\Xi$ :

$$\Xi \equiv r^2 - (N + \alpha \cos \theta)^2 = (r - (N + \alpha \cos \theta))(r + (N + \alpha \cos \theta)). \quad (4.32)$$

It is easy to see that because  $N > 0$ , the inequality (4.8) implies that the second factor,  $(r + (N + \alpha \cos \theta))$ , is always positive<sup>8</sup> and so  $\Xi$  changes sign when  $(r - (N + \alpha \cos \theta))$  changes sign. We therefore define

$$\eta \equiv (r - (N + \alpha \cos \theta)). \quad (4.33)$$

As  $\eta \rightarrow 0$ , we have

$$\begin{aligned} \Xi &= 2(N + \alpha \cos \theta) \eta \left( 1 + \frac{1}{2(N + \alpha \cos \theta)} \eta \right) + O(\eta^3), \\ \Delta_\theta &= 2(N - m)(N + \alpha \cos \theta) \left( 1 + \frac{N + \alpha \cos \theta - m}{(N - m)(N + \alpha \cos \theta)} \eta \right) + O(\eta^2), \\ Z_I &= \frac{C_{IJK}}{2} \frac{q^J q^K}{N - m} \frac{1}{\eta} \left( 1 - \frac{C_{IJK}}{2} \frac{N - m}{q^J q^K} \eta \right) + O(\eta), \\ \mu &= -\frac{q^1 q^2 q^3}{N - m} \frac{1}{\eta^2} \left( 1 + \left( -\frac{m - N - 2\alpha \cos \theta}{4(N - m)(N + \alpha \cos \theta)} + \frac{(q^1 + q^2 + q^3)(N - m)}{2q^1 q^2 q^3} \right) \eta \right) + O(1). \end{aligned} \quad (4.34)$$

The first possible divergences can come from the coefficient in front of the three-dimensional metric,  $ZV$ . But as  $\eta \rightarrow 0$ ,

$$ZV = \frac{(q^1 q^2 q^3)^{2/3}}{(N - m)^2} + O(\eta) \quad (4.35)$$

which is perfectly regular. The factor of  $Z/V \sim \eta^{-2}$  in front of the fiber metric is potentially more troublesome:

$$\begin{aligned} \frac{Z}{V} &= \frac{(q^1 q^2 q^3)^{2/3}}{\eta^2} \\ &+ \left( \frac{(N - m)(q^1 + q^2 + q^3)}{3(q^1 q^2 q^3)^{1/3}} - \frac{(q^1 q^2 q^3)^{2/3}}{2(N + \alpha \cos \theta)} \left( 1 + \frac{2(N - m + \alpha \cos \theta)}{(N - m)} \right) \right) \frac{1}{\eta} + O(1), \end{aligned} \quad (4.36)$$

and thus  $g_{\tau\tau}$  appears to blow up at  $\eta = 0$ . However, there is a similar set of terms coming from  $-Z^{-2}(dt + k)^2$  and we find that these cancel both the leading and the subleading divergences, and  $g_{\tau\tau}$  has a finite value as  $\eta = 0$ . Finally, one can also verify that the off-diagonal terms  $g_{t\tau}$  and  $g_{t\phi}$  are finite at  $\eta = 0$ .

This cancelation of singular terms and the ultimate regularity of the metric exactly parallels the story for the bubbled BPS solutions [31–33, 36]. Thus, when  $\Xi$  changes sign, the ambi-polar base metric leads to a regular five-dimensional metric and therefore, as described earlier, one can allow a wider range of parameters than merely  $m > |N|$  and still get a regular, Lorentzian metric in five dimensions.

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<sup>8</sup>It might appear that when  $\alpha > N$  this term can also become negative. However, the range of  $r$  is  $r \geq r_+$ , and one can check straightforwardly using the regularity constraints that this implies  $(r + (N + \alpha \cos \theta)) > 0$ .

#### 4.5 The running Taub-Bolt solution

If one sets the angular momentum parameter,  $\alpha$ , to zero then (4.13) requires  $m = 5N/4$  and the base becomes the Taub-Bolt space. The full solution then simplifies dramatically and has similar general features to the Schwarzschild solution described in section 3. Indeed, one finds that  $\nu = 0$  and

$$\mu = \frac{2(q_1 + q_2 + q_3)(r - 2N)(r - \frac{N}{2})}{N(r^2 - N^2)} + \frac{2q_1q_2q_3}{N^3} \frac{(r - 2N)(7r^2 - 2Nr + 4N^2)}{(r - N)^2(r + N)}. \quad (4.37)$$

We can, once again, easily check the conditions that there are no CTC's. The only danger comes from the  $\tau$ -circles, whose size is given by  $\mathcal{M}d\tau^2$ , where  $\mathcal{M} \equiv Z^{-2}\Xi^{-2}\mathcal{Q}$  and

$$\mathcal{Q} \equiv \Xi(\Delta Z_1 Z_2 Z_3 - \Xi\mu^2). \quad (4.38)$$

This is a quartic in  $r$  and must remain non-negative for  $r \geq r_+ = 2N$ .

To analyze this in more detail we consider equal fluxes  $q_I = q > 0$ ,  $I = 1, 2, 3$ . The positivity of the  $Z_I$  for  $r > 2N$  implies  $q \leq \frac{1}{2}N$ ; furthermore, the positivity of the quartic (4.38) imposes even stronger conditions. At infinity one has

$$\mathcal{M} \sim r^{-4}\mathcal{Q} \sim \left(1 - 6\left(\frac{q}{N}\right) + 14\left(\frac{q}{N}\right)^3\right)\left(1 + 6\left(\frac{q}{N}\right) - 14\left(\frac{q}{N}\right)^3\right). \quad (4.39)$$

For this to be positive (with  $0 < q < \frac{1}{2}N$ ), the two cubics in  $q$  must be positive and this implies:

$$0 < \frac{q}{N} < \frac{2}{\sqrt{7}} \cos\left(\frac{\pi}{3} + \frac{1}{3} \arccos\left(\frac{\sqrt{7}}{4}\right)\right) \approx 0.180355. \quad (4.40)$$

One can then verify that (4.39) is indeed positive definite for  $2N < r < \infty$  for  $q$  in the range (4.40).

Since the angular momentum parameter can be viewed as a perturbation of the foregoing solution, we anticipate qualitatively-similar solutions at least for when  $\alpha$  is small.

#### 4.6 Asymptotic charges

The computation of asymptotic charges proceeds along lines analogous to those outlined in section 3.2.

The solution has M5 charges encoded in the self-dual Maxwell fields  $\Theta^{(I)}$  and equal to  $q_I$ . The M2 charges, defined as in (3.15), are

$$Q^I = -(8\pi N)(4\pi)\frac{1}{2}C_{IJK}\left[\frac{q_J q_K}{m - N} + \frac{\gamma}{2}(q_J + q_K)\right], \quad (4.41)$$

with the parameter  $\gamma$  given in (4.26).

As for the running Schwarzschild bolt in section 3,  $\mu$  goes to a finite non-zero value,  $\gamma$ , at infinity. To find the mass of the solution one must then introduce coordinates  $\hat{\tau}$  and  $\hat{t}$  as in (3.20). It is also convenient to use the form (4.29) for the Kerr-Taub-Bolt metric and to rewrite the one-form  $k$  as

$$k = \mu(d\tau + P'_\theta d\phi) + \nu' d\phi, \quad (4.42)$$

with

$$\nu' = \nu - \alpha \frac{\Xi}{\Delta_\theta} \sin^2 \theta \mu = \alpha \frac{q_1 q_2 q_3 (m + N)}{2(m - N)^2 \Delta_\theta} \left(1 - \frac{r}{r_+}\right) \sin^2 \theta. \quad (4.43)$$

One can then rewrite the five-dimensional metric in a form ready for Kaluza-Klein reduction along  $\hat{\tau}$ :

$$ds^2 = \frac{\hat{I}_4}{(ZV)^2} \left( d\hat{\tau} + \hat{P}_\theta d\phi - \frac{\mu V^2}{\hat{I}_4} (d\hat{\tau} + \hat{\nu} d\phi) \right)^2 + \frac{VZ}{\hat{I}_4^{1/2}} ds_E^2, \quad (4.44)$$

where

$$ds_E^2 = -\hat{I}_4^{-1/2} (d\hat{\tau} + \hat{\nu} d\phi)^2 + \hat{I}_4^{1/2} \left( \frac{\Delta_\theta}{\Delta} dr^2 + \Delta_\theta d\theta^2 + \Delta \sin^2 \theta d\phi^2 \right) \quad (4.45)$$

is the four-dimensional Einstein metric and

$$\hat{I}_4 = (1 - \gamma^2)^{-1} (Z_1 Z_2 Z_3 V - \mu^2 V^2), \quad \hat{P}_\theta = (1 - \gamma^2)^{1/2} P'_\theta, \quad \hat{\nu} = (1 - \gamma^2)^{-1/2} \nu'. \quad (4.46)$$

From this one can read off the mass,  $M$  and the four-dimensional angular momentum,  $J$ :

$$G_4 M = \frac{1}{4(1 - \gamma^2)} \left[ 2m - \frac{q_1 q_2 + q_1 q_3 + q_2 q_3}{m - N} \right. \\ \left. + \frac{q_1 q_2 q_3 (q_1 + q_2 + q_3)}{2(m - N)^3} \left( 2 - \frac{m + N}{r_+} \right) - \frac{(q_1 q_2 q_3)^2}{4(m - N)^5} \left( 2 - \frac{m + N}{r_+} \right)^2 \right], \quad (4.47)$$

$$G_4 J = -\frac{1}{(1 - \gamma^2)^{1/2}} \frac{\alpha q_1 q_2 q_3 (m + N)}{4(m - N)^2 r_+}. \quad (4.48)$$

The geometry also carries Kaluza-Klein electric charge,  $Q_e$ , and the Kaluza-Klein magnetic charge,  $Q_m$ , given by<sup>9</sup>

$$G_4 Q_e = -\frac{1}{8(1 - \gamma^2)(m - N)^3} \left[ q_1 q_2 q_3 \left( \frac{m}{2} - 2N + \frac{N^2 - m^2}{r_+} \right) \right. \\ + \frac{1}{2} (q_1 q_2 + q_1 q_3 + q_2 q_3) (q_1 + q_2 + q_3) (m + 2N) + \frac{1}{2} (q_1^3 + q_2^3 + q_3^3) m \\ - \frac{1}{2} q_1 q_2 q_3 (q_1 q_2 + q_1 q_3 + q_2 q_3) \frac{m + 2N}{(m - N)^2} \left( 2 - \frac{m + N}{r_+} \right) \\ - \frac{1}{4} q_1 q_2 q_3 (q_1^2 + q_2^2 + q_3^2) \frac{2m + N}{(m - N)^2} \left( 2 - \frac{m + N}{r_+} \right) \\ + \frac{1}{8} q_1^2 q_2^2 q_3^2 (q_1 + q_2 + q_3) \frac{m + 2N}{(m - N)^4} \left( 2 - \frac{m + N}{r_+} \right)^2 \\ \left. - \frac{1}{16} q_1^3 q_2^3 q_3^3 \frac{N}{(m - N)^6} \left( 2 - \frac{m + N}{r_+} \right)^3 \right], \quad (4.49)$$

$$G_4 Q_m = (1 - \gamma^2)^{1/2} \frac{N}{2}, \quad (4.50)$$

where  $G_4$  is the four-dimensional Newton's constant

$$G_4 = \frac{G_5}{(1 - \gamma^2)^{1/2} (8\pi N)}. \quad (4.51)$$

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<sup>9</sup>We use the conventions of [35].

## 5 Conclusions

We have constructed a five-parameter family of smooth horizonless solutions of five-dimensional  $U(1)^3$  ungauged supergravity (or of the STU model) that are asymptotically  $\mathbb{R}^{3,1} \times S^1$  and that can have the same charges as a five-dimensional black string with M5, M2 and KK momentum charges and a macroscopically-large horizon area. We find that in these solutions the KK fiber and the time always mix at infinity and, since they contain a bolt, we refer to them as “running Bolt” solutions.

As these solutions exist in the same regime of parameters as the classical black hole, perhaps the most important question raised by their existence is whether they should be thought of as microstates of this black hole, and consequently whether their physics can be used to support the extension of the fuzzball proposal (also known as Mathur’s conjecture) to non-extremal black holes, and in particular to the Schwarzschild black hole. As explained in the Introduction, this would be the first example of string theory resolving a spacelike singularity — and the resolution mechanism would be rather amazing, as it would imply that the resolved spacetime differs from the singular spacetime on a macroscopic scale that is much larger than the Planck or string scale.

For black holes that have an AdS throat it is, more or less, straightforward to argue that a certain smooth geometry is a black hole microstate. By the AdS-CFT correspondence such a geometry must be dual (up to  $1/N$  corrections) to a coherent pure state of the dual CFT. Since the entropy of the CFT is reproduced by the entropy of the black hole [37], the bulk dual of one of the CFT microstates can be properly thought of as one of the black hole microstates. On the other hand, if a certain black hole does not have an AdS throat, the only way one can “define” its microstates is by requiring that they be smooth, horizonless, and have the same charges, mass and angular momentum.

The running Bolt solutions we construct here satisfy this criterion. On the other hand, it is clear that, much like the other non-extremal solutions constructed so far, they are quite different from the geometries that should correspond to the typical microstate of a non-extremal black hole. For example a point particle placed in any “Euclidean instanton times time” solution does not feel any attractive force. Similarly, an M2 brane probe placed in the running Bolt geometries does not feel any force. On the other hand, both the non-extremal black hole solution, and any typical microstate thereof will attract such probes.

It is clearly important to explore in more detail the relation between our running Bolt geometries and the black hole solutions that have the same charges. The latter are rotating four-dimensional non-extremal black holes with D4, D2 and D0 charge, and to our knowledge their supergravity solution is not known. When uplifted to five dimensions these black holes will become black strings with M5, M2 and KK momentum charges, and, at least for small-enough charges, they might suffer from Gregory-Laflamme-type instabilities [38].<sup>10</sup> On the other hand, when the charges become large these instabilities will most likely disappear, as they do for charged black strings [39]. It would be quite

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<sup>10</sup>Although it is quite possible that M5 branes along the putative Gregory-Laflamme direction might prevent this instability to occur even for arbitrarily-small M5 charges.

important to establish this, and if both the black hole and the microstates are unstable to compare their decay times.

Another important calculation is that of the decay time of our solutions, and its (rather non-trivial) dependence on the electric and magnetic charges and the angular momentum. Again, when these charges are zero, the Euclidean Schwarzschild [26] and the Euclidean Taub-Bolt solutions [27] have a negative-mass mode, and hence the five-dimensional solution coming from adding a time direction to these solutions is unstable. It is unlikely that a small amount of flux on the bolt will change this, but as the D2 and D4 charges grow the running Bolt solution may or may not become stable. If one knew the dependence of the decay time on charges, and if one used the fact that the non-extremal rotating black holes with D4-D2-D0 charges have a microscopic description in terms of an MSW string [40] with both left- and right-movers excited [41], one could try looking for a microscopic state whose decay time depends in the same non-trivial way on the charges — this will give us the dual of these geometries, and indicate their degree of typicality.

An interesting aspect of the running Bolt solutions we construct is the relation between their mass and their charges. When no charges are present, the mass of the five-dimensional uncharged Bolt solutions obtained by adding time to a four-dimensional Euclidean instanton is proportional to  $\frac{1}{g^2}$ , where  $g$  is the four-dimensional coupling constant coming from the KK reduction from five to four dimensions. As such, the uncharged Bolt solutions have a solitonic character. When charges are added, the KK fiber and the time always mix by a non-zero amount at infinity, and the bolt starts to run. If one computes its rest mass, one finds that when the base space is Euclidean Schwarzschild and the fluxes are self-dual, this mass can be written as a sum of the mass of the uncharged Bolt and of the three M2 brane charges. Hence, although this solution is non-BPS, if one naively ascribed a “solitonic charge” to the stationary Bolt, the mass of the self-dual running Bolt could be written in a BPS fashion, as the sum of its M2 and “solitonic” charges.

On the other hand, we have found that the rest mass of anti-self-dual running Bolts is given by the solitonic mass *minus* the M2 charges. Hence, the mass of the anti-self-dual running Bolt decreases linearly with the increasing of its M2 charges! We could not find other examples of such an unusual behavior of a mass formula in the literature; this comes essentially from the fact that the constant and  $r$ -dependent part of the warp factors have opposite signs. We have furthermore shown in section 3.3 that this rather unusual mass formula, combined with the existence of M2 brane probes that feel no force does not violate energy conservation, as one might naively have expected.

One could worry that the decreasing of the energy with the M2 charges might violate the positive energy theorem. However, since the KK circle pinches off at the bolt, the fermions must be antiperiodic around this circle, and this is incompatible with supersymmetry. Therefore the standard positive-energy theorems do not apply [42, 43].<sup>11</sup> It is clearly interesting to extend our analysis of the rest mass to the running Kerr-Taub-Bolt solution, and see whether for anti-self-dual fluxes the rest mass of that solution also decreases with increasing charge. It would be even more interesting to find an explanation for this phenomenon.

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<sup>11</sup>We thank Harvey Reall for pointing this argument to us.



Finally, the solutions we construct, as well as the ones of [12–14] represent a very tiny portion of the expected smooth horizonless microstate geometries for non-extremal black holes. Constructing large families of such geometries in a systematic fashion is a challenging, yet very worthwhile goal. The nature of these geometries is not more complicated than that of multiple concentric black rings, and we believe it should be possible to generalize the rod-structure and inverse-scattering methods used in their construction [44] to (at least) minimal five-dimensional ungauged supergravity, use one of the known solutions as a seed, and construct the multiple-bubble equivalent of running Bolts or of the geometries of [12–14]. One could also try constructing multiple-bubble non-extremal solutions by turning on fluxes on multi-Schwarzschild solitons. As one finds with the BPS solutions, the presence of fluxes might compensate the mutual attraction of the bolts, and lead to a five-dimensional smooth solution despite the presence of struts on the four-dimensional base space. Needless to say, if such large families of multiple-bubble solutions were found, and if their physics equally supported the extension of the fuzzball proposal of non-extremal black holes, this will greatly advance our understanding of not only these black holes, but also of the way in which string theory resolves space-like singularities.

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