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Neutrino physics and spontaneous CP violation in the $\mu\nu$SSM

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Abstract: The $\mu\nu$SSM provides a solution to the μ problem of the MSSM and explains the origin of neutrino masses by simply using right-handed neutrino superfields. We have completed the analysis of the vacua in this model, studying the possibility of spontaneous CP violation through complex Higgs and sneutrino vacuum expectation values. As a consequence of this process, a complex MNS matrix can be present. Besides, we have discussed the neutrino physics and the associated electroweak seesaw mechanism in the $\mu\nu$SSM, including also phases. Current data on neutrino masses and mixing angles can easily be reproduced.

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1 Introduction

Although the minimal supersymmetric extension of the standard model (MSSM) reveals as a solution to the hierarchy problem, we still remain puzzled about the origin of the μ-term in the superpotential, known as the μ-problem [1]. On the other hand, the fact that neutrinos are not massless [2] suggests that the MSSM is incomplete. Motivated by these two facts, the "μ from ν" supersymmetric standard model ($\mu\nu$SSM) [3–5], which relies on the existence of right-handed neutrinos, arises as an alternative to the MSSM, providing a solution to the μ-problem and explaining the origin of neutrino masses.

In particular, the superpotential of the $\mu\nu$SSM contains, in addition to the usual Yukawa couplings for quarks and charged leptons, Yukawa couplings for neutrinos $\hat{H}_u\,\hat{L}\,\hat{\nu}^c$, terms of the type $\hat{\nu}^c\hat{H}_d\hat{H}_u$ producing an effective μ term through right-handed sneutrino vacuum expectation values (VEVs), and also terms of the type $\hat{\nu}^c\hat{\nu}^c\hat{\nu}^c$ avoiding the existence of a Goldstone boson and contributing to generate effective Majorana masses for neutrinos at the electroweak scale. Actually, the explicit breaking of R-parity in this model by the above terms produces the mixing of neutralinos with left- and right-handed neutrinos, and as a consequence a generalized matrix of the seesaw type that gives rise at tree level to three light eigenvalues corresponding to neutrino masses [3].

Following this proposal, several papers have studied different aspects of the $\mu\nu$SSM. In [4] the parameter space of the $\mu\nu$SSM was analyzed in detail, studying the viable regions which avoid false minima and tachyons, as well as fulfill the Landau pole constraint. The structure of the mass matrices, and the associated particle spectrum was also computed, paying special attention to the mass of the lightest Higgs. In [6] neutrino masses and mixing

angles were discussed, as well as the decays of the lightest neutralino to two body (W-lepton) final states. The correlations of the decay branching ratios with the neutrino mixing angles were studied as another possible test of the $\mu\nu$SSM at the LHC. The phenomenology of the $\mu\nu$SSM was also studied in [7], particularized for one and two generations of right-handed sneutrinos, and taking into account all possible final states when studying the decays of the lightest neutralino. Possible signatures that might allow to distinguish this model from other R-parity breaking models were discussed qualitatively in the last two papers. Let us finally mention that terms of the type $\hat{\nu}^c \hat{H}_d \hat{H}_u$ and $\hat{\nu}^c \hat{\nu}^c \hat{\nu}^c$ were also analysed as sources of the observed baryon asymmetry in the Universe [8] and of neutrino masses and bilarge mixing [9], respectively.

The goal of this work is twofold; first, we complete the analysis of the vacua of the $\mu\nu$SSM presented in [4], studying spontaneous CP violation (SCPV) of the tree-level neutral scalar potential. In particular, we explore CP violation in the lepton sector and show how phases for the tree-level Maki-Nakagawa-Sakata matrix (MNS) [10] may arise due to the fact that the minimum of the scalar potential with real parameters has complex VEV solutions. Second, we discuss neutrino physics and the seesaw mechanism in the $\mu\nu$SSM, including also phases.

Let us recall that, although there is evidence for CP violation in the quark sector of the standard model, there are not experimental traces of it in the leptonic part. CP can be explicitly broken through complex parameters in the Lagrangian or can arise spontaneously in a CP conserving Lagrangian (e.g. with all the parameters being real) through complex VEVs. Although the standard model as well as the MSSM do not allow for SCPV, in more complicated models both sources of CP violation, complex parameters and complex VEVs, could be present.

Concerning the quark sector, a recent study argues that the Cabibbo-Kobayashi-Maskawa (CKM) matrix is likely complex [11]. This conclusion is supported by the measurement of the unitarity triangle angle γ by BaBar and Belle collaborations [12, 13]. This evidence of a complex CKM matrix has ruled out Next-to-MSSM (NMSSM)-like models with SCPV (see e.g. [14]) for being the entire source of CP violation in the quark sector, since the CKM matrix in such models is real. Thus complex parameters are necessary in the quark sector. Given the structure of the $\mu\nu$SSM, this fact also holds for this model. On the other hand, as mentioned above, we will show that SCPV can be generated in the leptonic sector of the $\mu\nu$SSM, as well as phases for the MNS matrix.

One argument in favor of the presence of SCPV at the Lagrangian level is that, if the determinant of the quark mass matrix is real, it leads to a solution to the strong CP problem [15]. Extensions of the MSSM having this property, have been extensively studied in the literature (see e.g. [16]). In those scenarios, the quark sector of the model is extended in such a way that the effective 3×3 CKM matrix is complex whereas the determinant of the quark matrix is real.

Other authors have extended the Higgs sector of the models, leading to SCPV with a complex CKM matrix [17]. Last but not least, in supersymmetric (SUSY) models with both CP and Peccei-Quinn symmetries, SCPV can be used as a solution to the SUSY phase problem [18].

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Regarding extensions of the $\mu\nu$SSM, the SCPV scenario with a complex CKM matrix can be accomplished by adding two more families of Higgs doublets. In this case the model would contain three families of matter and Higgs fields. This possibility is well motivated phenomenologically, since the potential problem of flavor changing neutral currents can be avoided [19]. In addition, having three Higgs families is favored in some string scenarios [20]. Indeed, extensions of the quark sector of the model can also be studied, without altering the results here presented.

What we want to point out in this work is that SCPV is possible in the simplest version of the $\mu\nu$SSM, i.e. with only one family of Higgs doublets, and therefore it is worth studying its consequences. Following this philosophy, the paper is organized as follows. Section 2 is devoted to complete the analysis of the vacuum of the $\mu\nu$SSM started in [4], including SCPV solutions. In section 3 we examine the seesaw mechanism as the origin of neutrino masses and mixing angles in the model. In section 4 we carry out a detailed numerical analysis of the tree-level neutral scalar potential, showing explicitly that SCPV solutions are possible, and discussing their implications on the neutrino sector of the model. Finally, the conclusions are left for section 5. Minimization equations of the model and an approximate analytical formula for neutrino masses are given in the appendices.

2 Complex VEVs in the $\mu\nu$SSM

The superpotential of the $\mu\nu$SSM introduced in [3] is given by

$$W = \sum_{a,b}\sum_{i,j}\left[\epsilon_{ab}\left(Y_{u_{ij}}\,\hat{H}_u^b\,\hat{Q}_i^a\,\hat{u}_j^c + Y_{d_{ij}}\,\hat{H}_d^a\,\hat{Q}_i^b\,\hat{d}_j^c + Y_{e_{ij}}\,\hat{H}_d^a\,\hat{L}_i^b\,\hat{e}_j^c + Y_{\nu_{ij}}\,\hat{H}_u^b\,\hat{L}_i^a\,\hat{\nu}_j^c\right)\right]$$
$$-\sum_{a,b}\sum_{i}\epsilon_{ab}\lambda_i\,\hat{\nu}_i^c\,\hat{H}_d^a\hat{H}_u^b + \sum_{i,j,k}\frac{1}{3}\kappa_{ijk}\hat{\nu}_i^c\hat{\nu}_j^c\hat{\nu}_k^c\,, \tag{2.1}$$

where we take $\hat{H}_d^T = (\hat{H}_d^0, \hat{H}_d^-)$, $\hat{H}_u^T = (\hat{H}_u^+, \hat{H}_u^0)$, $\hat{Q}_i^T = (\hat{u}_i, \hat{d}_i)$, $\hat{L}_i^T = (\hat{\nu}_i, \hat{e}_{L_i})$, $i,j,k = 1,2,3$ are family indices, the 3×3 matrices Y are dimensionless Yukawa couplings, $a,b = 1,2$ are $\mathrm{SU}(2)_L$ indices and $\epsilon_{12} = 1$. As mentioned in the Introduction, in addition to the MSSM Yukawa couplings for quarks and charged leptons, the $\mu\nu$SSM superpotential contains Yukawa couplings for neutrinos, and two additional type of terms involving the Higgs doublet superfields, $\hat{H}_d$ and $\hat{H}_u$ and the three right-handed neutrino superfields, $\hat{\nu}_i^c$, with the dimensionless vector coupling λ and the totally symmetric tensor κ.

As discussed in [3], when the scalar components of the superfields $\hat{\nu}_i^c$, denoted by $\tilde{\nu}_i^c$, acquire VEVs of the order of the electroweak scale, an effective interaction $\mu\hat{H}_1\hat{H}_2$ is generated through the fifth term in eq. (2.1), with $\mu \equiv \lambda_i\langle\tilde{\nu}_i^c\rangle$. The last type of terms in eq. (2.1) is allowed by all symmetries, and avoids the presence of an unacceptable Goldstone boson associated to a global U(1) symmetry. In addition, it generates effective Majorana masses for neutrinos at the electroweak scale. These two type of terms break explicitly R-parity and lepton number.

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Working in the framework of gravity mediated supersymmetry breaking, the Lagrangian $\mathcal{L}_{\rm soft}$ is given by:

$$
\begin{aligned}
-\mathcal{L}_{\rm soft} = & \sum_{i,j}\Bigg[\sum_a m^2_{\tilde{Q}_{ij}}\,\tilde{Q}^{a\,*}_i\,\tilde{Q}^a_j + m^2_{\tilde{u}^c_{ij}}\,\tilde{u}^{c\,*}_i\,\tilde{u}^c_j + m^2_{\tilde{d}^c_{ij}}\,\tilde{d}^{c\,*}_i\,\tilde{d}^c_j + \sum_a m^2_{\tilde{L}_{ij}}\,\tilde{L}^{a\,*}_i\,\tilde{L}^a_j \\
& \qquad + m^2_{\tilde{e}^c_{ij}}\,\tilde{e}^{c\,*}_i\,\tilde{e}^c_j + m^2_{\tilde{\nu}^c_{ij}}\,\tilde{\nu}^{c\,*}_i\tilde{\nu}^c_j\Bigg] \\
& + \sum_a \left[m^2_{H_d}\,H^{a\,*}_d\,H^a_d + m^2_{H_u}\,H^{a\,*}_u H^a_u\right] \\
& + \sum_{a,b}\sum_{i,j}\epsilon_{ab}\Big[(A_uY_u)_{ij}\,H^b_u\,\tilde{Q}^a_i\,\tilde{u}^c_j + (A_dY_d)_{ij}\,H^a_d\,\tilde{Q}^b_i\,\tilde{d}^c_j + (A_eY_e)_{ij}\,H^a_d\,\tilde{L}^b_i\,\tilde{e}^c_j \\
& \qquad + (A_\nu Y_\nu)_{ij}\,H^b_u\,\tilde{L}^a_i\,\tilde{\nu}^c_j + \text{c.c.}\Big] \\
& + \left[-\sum_{a,b}\sum_i \epsilon_{ab}(A_\lambda\lambda)_i\,\tilde{\nu}^c_i\,H^a_d H^b_u + \sum_{ijk}\frac{1}{3}(A_\kappa\kappa)_{ijk}\,\tilde{\nu}^c_i\tilde{\nu}^c_j\tilde{\nu}^c_k \;+\text{c.c.}\right] \\
& - \frac{1}{2}\left(M_3\,\tilde{\lambda}_3\,\tilde{\lambda}_3 + M_2\,\tilde{\lambda}_2\,\tilde{\lambda}_2 + M_1\,\tilde{\lambda}_1\,\tilde{\lambda}_1 + \text{c.c.}\right).
\end{aligned}
\tag{2.2}
$$

In addition to terms from $\mathcal{L}_{\rm soft}$, the tree-level scalar potential receives the D and F term contributions also computed in [3]. In the following we will suppose that CP is a good symmetry of the model, taking all the parameters in the neutral scalar potential real and assuming that CP is only violated by the VEVs of the scalar fields

$$
\langle H^0_d\rangle = e^{i\varphi_{v_d}}\,v_d\,, \quad \langle H^0_u\rangle = e^{i\varphi_{v_u}}\,v_u\,, \quad \langle\tilde{\nu}_i\rangle = e^{\varphi_{\nu_i}}\,\nu_i\,, \quad \langle\tilde{\nu}^c_i\rangle = e^{\varphi_{\nu^c_i}}\,\nu^c_i\,. \tag{2.3}
$$

We then obtain for the tree-level neutral scalar potential,

$$
V^0 = V_{\rm soft} + V_D + V_F\,, \tag{2.4}
$$

where

$$
\begin{aligned}
V_{\rm soft} = & \; m^2_{H_d}v_dv_d + m^2_{H_u}v_uv_u + \sum_{i,j} m^2_{\tilde{L}_{ij}}\,\nu_i\,\nu_j\cos(\chi_i-\chi_j) + \sum_{i,j} m^2_{\tilde{\nu}^c_{ij}}\nu^c_i\nu^c_j\cos(\varphi_{\nu^c_i}-\varphi_{\nu^c_j}) \\
& -2\sum_i (A_\lambda\lambda)_i\nu^c_i v_d v_u\cos(\varphi_v+\varphi_{\nu^c_i}) + \sum_{i,j,k}\frac{2}{3}(A_\kappa\kappa)_{ijk}\nu^c_i\nu^c_j\nu^c_k cos(\varphi_{\nu^c_i}+\varphi_{\nu^c_j}+\varphi_{\nu^c_k}) \\
& +2\sum_{i,j}(A_\nu Y_\nu)_{ij}v_u\nu_i\nu^c_j\cos(\chi_i+\varphi_{\nu^c_j})\,,
\end{aligned}
\tag{2.5}
$$

$$
V_D = \frac{G^2}{8}\left(\sum_i \nu_i\nu_i + v_dv_d - v_uv_u\right)^2\,, \tag{2.6}
$$

with $G^2 \equiv g_1^2 + g_2^2$, and

$$
\begin{aligned}
V_F = & \sum_i (\lambda_i)^2 v_d^2 v_u^2 + \\
& + \sum_{i,j}\lambda_i\lambda_j v_d^2\nu^c_i\nu^c_j\cos(\varphi_{\nu^c_i}-\varphi_{\nu^c_j}) + \sum_{i,j}\lambda_i\lambda_j v_u^2\nu^c_i\nu^c_j\cos(\varphi_{\nu^c_i}-\varphi_{\nu^c_j})
\end{aligned}
$$

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$$
\begin{aligned}
&+\sum_{i,j,k,l}\sum_{m}\kappa_{imk}\kappa_{lmj}\nu_i^c\nu_j^c\nu_k^c\nu_l^c\cos(\varphi_{\nu_i^c}+\varphi_{\nu_j^c}-\varphi_{\nu_k^c}-\varphi_{\nu_l^c})\\
&+2\left[-\sum_{i,j}\sum_{k}\kappa_{ikj}\lambda_k v_d v_u\nu_i^c\nu_j^c\cos(\varphi_{\nu_i^c}+\varphi_{\nu_j^c}-\varphi_v)\right.\\
&\quad+\sum_{i,j,k}\sum_{l}Y_{\nu_{jl}}\kappa_{ilk}v_u\nu_j\nu_i^c\nu_k^c\cos(\varphi_{\nu_i^c}+\varphi_{\nu_k^c}-\chi_j)\\
&\quad-\sum_{i,j,k}Y_{\nu_{ij}}\lambda_k v_d\nu_i\nu_j^c\nu_k^c\cos(\chi_i+\varphi_{\nu_j^c}-\varphi_{\nu_k^c}-\varphi_v)\\
&\quad\left.-\sum_{i}\sum_{j}Y_{\nu_{ij}}\lambda_j v_d v_u^2\nu_i\cos(\varphi_v-\chi_i)\right]\\
&+\sum_{i,j,k,l}Y_{\nu_{ij}}Y_{\nu_{kl}}\nu_i\nu_j^c\nu_k\nu_l^c\cos(\chi_i-\chi_k+\varphi_{\nu_j^c}-\varphi_{\nu_l^c})\\
&+\sum_{i,j}\sum_{k}Y_{\nu_{ik}}Y_{\nu_{jk}}v_u^2\nu_i\nu_j\cos(\chi_i-\chi_j)\\
&+\sum_{i,j}\sum_{k}Y_{\nu_{ki}}Y_{\nu_{kj}}v_u^2\nu_i^c\nu_j^c\cos(\varphi_{\nu_i^c}-\varphi_{\nu_j^c})\ .
\end{aligned}
\tag{2.7}
$$

We observe that in the potential there are seven independent phases, and we have defined them as

$$
\varphi_v=\varphi_{v_u}+\varphi_{v_d}\ ,\quad \chi_i=\varphi_{\nu_i}+\varphi_{v_u}\ ,\quad \varphi_{\nu_i^c}.
\tag{2.8}
$$

Now one can derive the fifteen minimization conditions with respect to the moduli v_d, v_u, ν_i^c, ν_i, and phases φ_v, χ_i, and $\varphi_{\nu_i^c}$. These are written in appendix A. Finding minima requires the solutions of equations (A.1)–(A.7). A standard way to obtain this is to give the values of the cosines of the phases in terms of the moduli, using the triangle method [21–23] for the equation of the phases, and then substitute the expressions in the minimum equations for the moduli, solving them numerically. This method permits to demonstrate the existence at tree level of only real minima in several models. This is for example the case of the NMSSM [24], and the MSSM with extra doublets. The latter result has been proved for the MSSM with an extra pair of Higgs doublets [22] (the so called 4D model), the bilinear R-parity violation model (analogous to a 5D model because of the VEVs of the left-handed sneutrinos), and the MSSM with two extra pair of Higgs doublets (6D model) [23].

Another way of finding minima consists of using as input the phases and solve the fifteen equations to fix the variables that are linear in these equations, as it is the case of some of the soft terms. This is the procedure that we will follow in section 4.

A simple way to prove the existence of CP violating minima in the $\mu\nu$SSM is using the results of ref. [23], where the authors prove that SUSY scenarios for SCPV require singlets. In particular, they found that, if the singlets do not introduce dimensional parameters in the superpotential (i.e. no linear or bilinear terms), the MSSM extended with two gauge singlets would be the minimal SUSY model where CP violation can be generated spontaneously. Since that model is a limiting case of the $\mu\nu$SSM with vanishing neutrino Yukawa couplings $Y_{\nu_{ij}}=0$, $\lambda_3=0$, and $\kappa_{333}=\kappa_{322}=\kappa_{332}=\kappa_{311}=\kappa_{331}=\kappa_{123}=0$, this would prove

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that the $\mu\nu$SSM can break CP spontaneously. Let us remark that, since in the $\mu\nu$SSM one is using a seesaw at the electroweak scale, the $Y_{\nu_{ij}}$ have to be very small compared with the other parameters [3], and as a consequence the neutral scalar potential can be understood as a deformation of the MSSM extended with three gauge singlets. Although there is no literature about general solutions that break CP spontaneously in the latter, it is obvious that this model contains the MSSM extended with two singlets as a limiting case when $\kappa_{333} = \kappa_{322} = \kappa_{332} = \kappa_{311} = \kappa_{331} = \kappa_{123} = 0$, and $\lambda_3 = 0$. As already mentioned, SCPV solutions are well known in this case [23, 25]. Thus one could argue that a subset of solutions with neutrino masses different from zero could be obtained deforming the scalar potential of the MSSM extended with three singlets[1] through non-zero $Y_{\nu_{ij}}$.

In section 4 we will do a thorough numerical analysis showing explicitly how SCPV is realizable in the leptonic sector. Nevertheless, it is worth pointing out here that to find complex solutions is a non-trivial task compared to the search of real ones. As we will show, the key of SCPV is on the $(A_\kappa \kappa)_{ijk}$ terms used as inputs. In order to fulfill the minimization equations, the basic requirement is that entries different from $(A_\kappa \kappa)_{iii}$ must be allowed. In addition, these parameters have to be chosen carefully to obtain SCPV as a global minimum.

In the next section we will study the seesaw mechanism in the model as the origin of neutrino masses and mixing angles.

3 Neutrino masses and mixing angles

In the $\mu\nu$SSM the MSSM neutralinos mix with the left- and right-handed neutrinos as a consequence of R-parity violation. Therefore the right-handed neutrinos behave as singlino components of the neutralinos. In the basis $\chi^{0T} = (\tilde{B}^0, \tilde{W}^0, \tilde{H}_d, \tilde{H}_u, \nu_{R_i}, \nu_{L_i})$ the neutralino-neutrino mass matrix was given in [3, 4] for real VEVs. Considering now the possibility of complex VEVs the result is given by

$$\mathcal{M}_n = \begin{pmatrix} M & m \\ m^T & 0_{3\times 3} \end{pmatrix}, \tag{3.1}$$

where the neutralino mass matrix is

$$M = \begin{pmatrix} M_1 & 0 & -A\langle H_d^0\rangle^* & A\langle H_u^0\rangle^* & 0 & 0 & 0 \\ 0 & M_2 & B\langle H_d^0\rangle^* & -B\langle H_u^0\rangle^* & 0 & 0 & 0 \\ -A\langle H_d^0\rangle^* & B\langle H_d^0\rangle^* & 0 & -\lambda_i\langle\tilde{\nu}_i^c\rangle & -\lambda_1\langle H_u^0\rangle & -\lambda_2\langle H_u^0\rangle & -\lambda_3\langle H_u^0\rangle \\ A\langle H_u^0\rangle^* & -B\langle H_u^0\rangle^* & -\lambda_i\langle\tilde{\nu}_i^c\rangle & 0 & -\lambda_1\langle H_d^0\rangle + Y_{\nu_{i1}}\langle\tilde{\nu}_i\rangle & -\lambda_2\langle H_d^0\rangle + Y_{\nu_{i2}}\langle\tilde{\nu}_i\rangle & -\lambda_3\langle H_d^0\rangle + Y_{\nu_{i3}}\langle\tilde{\nu}_i\rangle \\ 0 & 0 & -\lambda_1\langle H_u^0\rangle & -\lambda_1\langle H_d^0\rangle + Y_{\nu_{i1}}\langle\tilde{\nu}_i\rangle & 2\kappa_{11j}\langle\tilde{\nu}_j^c\rangle & 2\kappa_{12j}\langle\tilde{\nu}_j^c\rangle & 2\kappa_{13j}\langle\tilde{\nu}_j^c\rangle \\ 0 & 0 & -\lambda_2\langle H_u^0\rangle & -\lambda_2\langle H_d^0\rangle + Y_{\nu_{i2}}\langle\tilde{\nu}_i\rangle & 2\kappa_{21j}\langle\tilde{\nu}_j^c\rangle & 2\kappa_{22j}\langle\tilde{\nu}_j^c\rangle & 2\kappa_{23j}\langle\tilde{\nu}_j^c\rangle \\ 0 & 0 & -\lambda_3\langle H_u^0\rangle & -\lambda_3\langle H_d^0\rangle + Y_{\nu_{i3}}\langle\tilde{\nu}_i\rangle & 2\kappa_{31j}\langle\tilde{\nu}_j^c\rangle & 2\kappa_{32j}\langle\tilde{\nu}_j^c\rangle & 2\kappa_{33j}\langle\tilde{\nu}_j^c\rangle \end{pmatrix}, \tag{3.2}$$

[1] Since only mass differences for neutrino masses have been measured, in principle two right-handed neutrino supermultiplets are enough to give two tree-level masses and also break CP spontaneously. Thus a version of the $\mu\nu$SSM with only two right-handed neutrinos instead of three could be formulated. Nevertheless, we will follow the philosophy that the existence of three generations of all kind of leptons is more natural.

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with $A=\frac{G}{\sqrt{2}}\sin\theta_W$, $B=\frac{G}{\sqrt{2}}\cos\theta_W$, and

$$m^T=\begin{pmatrix} -\frac{g_1}{\sqrt{2}}\langle\tilde{\nu}_1\rangle^* & \frac{g_2}{\sqrt{2}}\langle\tilde{\nu}_1\rangle^* & 0 & Y_{\nu_{1i}}\langle\tilde{\nu}_i^c\rangle & Y_{\nu_{11}}\langle H_u^0\rangle & Y_{\nu_{12}}\langle H_u^0\rangle & Y_{\nu_{13}}\langle H_u^0\rangle \\ -\frac{g_1}{\sqrt{2}}\langle\tilde{\nu}_2\rangle^* & \frac{g_2}{\sqrt{2}}\langle\tilde{\nu}_2\rangle^* & 0 & Y_{\nu_{2i}}\langle\tilde{\nu}_i^c\rangle & Y_{\nu_{21}}\langle H_u^0\rangle & Y_{\nu_{22}}\langle H_u^0\rangle & Y_{\nu_{23}}\langle H_u^0\rangle \\ -\frac{g_1}{\sqrt{2}}\langle\tilde{\nu}_3\rangle^* & \frac{g_2}{\sqrt{2}}\langle\tilde{\nu}_3\rangle^* & 0 & Y_{\nu_{3i}}\langle\tilde{\nu}_i^c\rangle & Y_{\nu_{31}}\langle H_u^0\rangle & Y_{\nu_{32}}\langle H_u^0\rangle & Y_{\nu_{33}}\langle H_u^0\rangle \end{pmatrix}. \tag{3.3}$$

For simplicity the summation convention on repeated indices was used in the above two equations. The above matrix (3.1) is of the seesaw type giving rise to the neutrino masses which have to be very small. This is the case since the entries of the matrix M are much larger than the ones in the matrix m. Notice in this respect that the entries of M are of the order of the electroweak scale while the ones in m are of the order of the Dirac masses for the neutrinos [3, 4]. Therefore in a first approximation the effective neutrino mixing mass matrix can be written as

$$m_{\rm eff}=-m^T\cdot M^{-1}\cdot m. \tag{3.4}$$

Because $m_{\rm eff}$ is symmetric and $m_{\rm eff}^\dagger m_{\rm eff}$ is Hermitian, one can diagonalize them by a unitary transformation

$$U_{\rm MNS}^T m_{\rm eff} U_{\rm MNS} = diag(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}), \tag{3.5}$$
$$U_{\rm MNS}^\dagger m_{\rm eff}^\dagger m_{\rm eff} U_{\rm MNS} = diag(m_{\nu_1}^2, m_{\nu_2}^2, m_{\nu_3}^2). \tag{3.6}$$

In appendix B, eq. (B.1), we present an approximate analytical expression for the effective neutrino mass matrix of the $\mu\nu$SSM with SCPV, neglecting all the terms containing $Y_\nu^2\nu^2$, $Y_\nu^3\nu$ and $Y_\nu\nu^3$ in eq. (3.4) due to the smallness of Y_ν and ν [3]. In the limit of vanishing phases $\varphi_{v_u}=\varphi_{v_d}=\varphi_{\nu_i^c}=\varphi_{\nu_i}=0$, eq. (B.1) is reduced to eq. (B.9). This is the formula that we will use in the following, in order to have a qualitative idea of how the seesaw mechanism works in this model.

Let us first rewrite the expression (B.9) in the following form:

$$(m_{eff|real})_{ij}\simeq\frac{v_u^2}{6\kappa\nu^c}Y_{\nu_i}Y_{\nu_j}\left(1-3\,\delta_{ij}\right)-\frac{1}{2M_{\rm eff}}\left[\nu_i\nu_j+\frac{v_d\left(Y_{\nu_i}\nu_j+Y_{\nu_j}\nu_i\right)}{3\lambda}+\frac{Y_{\nu_i}Y_{\nu_j}v_d^2}{9\lambda^2}\right], \tag{3.7}$$

with

$$M_{\rm eff}\equiv M\left[1-\frac{v^2}{2M\left(\kappa\nu^{c^2}+\lambda v_u v_d\right)\,3\lambda\nu^c}\left(2\kappa\nu^{c^2}\frac{v_u v_d}{v^2}+\frac{\lambda v^2}{2}\right)\right], \tag{3.8}$$

which coincides with the result in [6], where the possibility of obtaining an adequate seesaw with diagonal Yukawa couplings was also pointed out. Here $v^2=v_u^2+v_d^2+\sum_i\nu_i^2\approx v_u^2+v_d^2$ with $v\approx 174\,\text{GeV}$ has been used, since $\nu_i\ll v_u, v_d$ [3], and let us recall that we are also using couplings $\lambda_i\equiv\lambda$, a tensor κ with terms $\kappa_{iii}\equiv\kappa_i\equiv\kappa$ and vanishing otherwise, diagonal Yukawa couplings $Y_{\nu_{ii}}\equiv Y_{\nu_i}$, VEVs $\nu_i^c\equiv\nu^c$, and $\frac{1}{M}=\frac{g_1^2}{M_1}+\frac{g_2^2}{M_2}$.

In the limit where gauginos are very heavy and decouple (i.e. $M\to\infty$), eq. (3.7) reduces to

$$(m_{eff|real})_{ij}\simeq\frac{v_u^2}{6\,\kappa\nu^c}Y_{\nu_i}Y_{\nu_j}\left(1-3\,\delta_{ij}\right). \tag{3.9}$$

It is interesting to note that in contrast with the ordinary seesaw (i.e. generated only through the mixing between left- and right-handed neutrinos), where the case of diagonal Yukawas would give rise to a diagonal mass matrix of the form

$$(m_{eff|ordinary\ seesaw})_{ij} \simeq \frac{-v_u^2 Y_{\nu_i} Y_{\nu_j} \delta_{ij}}{2\,\kappa\nu^c}, \tag{3.10}$$

in this case we have an extra contribution given by the first term of eq. (3.9). This is due to the effective mixing of the right-handed neutrinos and Higgsinos in this limit, and produces off-diagonal entries in the mass matrix. Besides, when right-handed neutrinos are also decoupled (i.e. $\nu^c \to \infty$), the neutrino masses are zero as corresponds to the case of a seesaw with only Higgsinos.

Another observation is that, independently on the nature of the lightest neutralino, Higgsino-like or ν^c-like or even a mixture of them (recall that the ν^c can be interpreted also as the singlino component of the neutralino since R-parity is broken), the form of the effective neutrino mass matrix is the same when the gauginos are decoupled, as given by (3.9).

Another limit which is worth discussing is $\nu^c \to \infty$. Then, eq. (3.7) reduces to the form

$$(m_{eff|real})_{ij} \simeq -\frac{1}{2M}\left[\nu_i\nu_j + \frac{v_d(Y_{\nu_i}\nu_j + Y_{\nu_j}\nu_i)}{3\lambda} + \frac{Y_{\nu_i}Y_{\nu_j}v_d^2}{9\lambda^2}\right]. \tag{3.11}$$

We can also see that for $v_d \to 0$ (i.e. $\tan\beta = \frac{v_u}{v_d} \to \infty$) one obtains

$$(m_{eff|real})_{ij} \simeq -\frac{\nu_i\nu_j}{2M}. \tag{3.12}$$

Note that this result can actually be obtained if $\nu_i \gg \frac{Y_{\nu_i}v_d}{3\lambda}$, and that this relation can be fulfilled with $v_d \sim v_u \sim 174\,\text{GeV}$ for suitable values of λ. It means that decoupling right-handed neutrinos/singlinos and Higgsinos, the seesaw mechanism is generated through the mixing of left-handed neutrinos with gauginos. This is a characteristic feature of the seesaw in the well-known bilinear R-parity violation model (BRpV) [26].

The seesaw in the $\mu\nu$SSM comes, in general, from the interplay of the above two limits. Namely, the limit where we suppress only certain Higgsino and gaugino mixing. Hence, taking $v_d \to 0$ in eq. (3.7), which means quite pure gauginos but Higgsinos mixed with right-handed neutrinos, we obtain

$$(m_{eff|real})_{ij} \simeq \frac{v_u^2}{6\kappa\nu^c} Y_{\nu_i} Y_{\nu_j}(1-3\delta_{ij}) - \frac{1}{2\,M_{\text{eff}}}\nu_i\nu_j\,, \tag{3.13}$$

As above, we remark that actually this result can be obtained if $\nu_i \gg \frac{Y_{\nu_i}v_d}{3\lambda}$. The effective mass $M_{\text{eff}} = M\left(1 - \frac{v^4}{12\kappa M{\nu^c}^3}\right)$ represents the mixing between gauginos and Higgsinos-ν^c that is not completely suppressed in this limit. Expression (3.13) is more general than the other two limits studied above. On the other hand, for typical values of the parameters involved in the seesaw, $M_{\text{eff}} \approx M$, and therefore we get a simple formula that can be used to understand the seesaw mechanism in this model in an qualitative way, that is

$$(m_{eff|real})_{ij} \simeq \frac{v_u^2}{6\kappa\nu^c} Y_{\nu_i} Y_{\nu_j}(1-3\delta_{ij}) - \frac{1}{2M}\nu_i\nu_j. \tag{3.14}$$

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$\Delta m^2_{\rm sol}/10^{-5}$ eV^2	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	$\Delta m^2_{\rm atm}/10^{-3}$ eV^2
7.14-8.19	0.263-0.375	< 0.046	0.331-0.644	2.06-2.81

Table 1. Allowed 3σ ranges for the neutrino masses and mixings as discussed in [27].

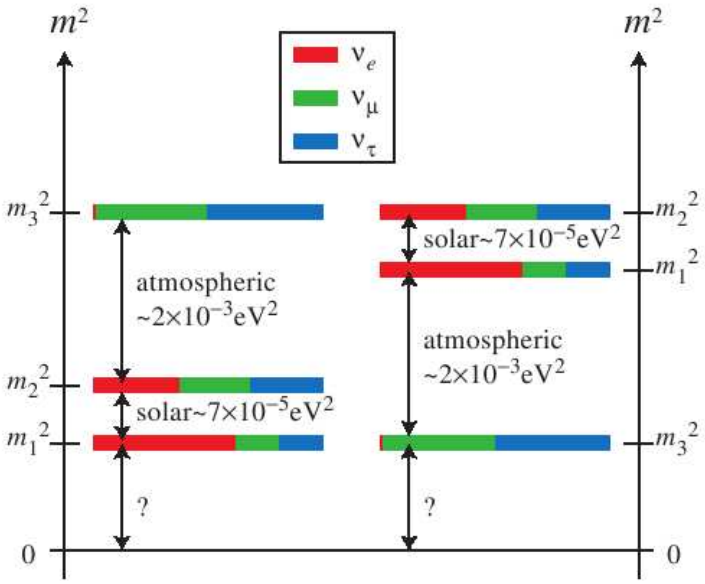

Figure 1. The two possible hierarchies of neutrino masses as shown in [28]. The pattern on the left side corresponds to the normal hierarchy and is characterized by one heavy state with a very little electron neutrino component, and two almost degenerate light states with a mass difference which is the solar mass difference. The pattern on the right side corresponds to the inverted hierarchy and is characterized by two almost degenerate heavy states with a mass difference that is the solar mass difference, and a light state which has very little electron neutrino component. In both cases the mass difference between the heaviest/lightest eigenstate and the almost degenerate eigenstates is the atmospheric scale.

The simplicity of eq. (3.14), in contrast with the full formula given by eq. (3.7), comes from the fact that the mixing between gauginos and Higgsinos-ν^c is neglected.

To continue the discussion of the seesaw in the $\mu\nu$SSM, let us remind that two mass differences and mixing angles have been measured experimentally in the neutrino sector. The allowed 3σ ranges for these parameters are shown in table 1. We also show the compositions of the mass eigenstates in figure 1 for the normal and inverted hierarchy cases. For the discussion, hereafter we will use indistinctly the subindices $(1,2,3) \equiv (e,\mu,\tau)$.

Due to the fact that the mass eigenstates have, in a good approximation, the same composition of ν_μ and ν_τ we start considering $Y_{\nu_2} = Y_{\nu_3}$ and $\nu_2 = \nu_3$, and therefore eq. (3.14) takes the form

$$m_{\rm eff} = \begin{pmatrix} d & c & c \\ c & A & B \\ c & B & A \end{pmatrix}, \tag{3.15}$$

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where

$$
\begin{aligned}
d &= -\frac{v_u^2}{3\kappa\nu^c}Y_{\nu_1}^2 - \frac{1}{2M}\nu_1^2, \\
c &= \frac{v_u^2}{6\kappa\nu^c}Y_{\nu_1}Y_{\nu_2} - \frac{1}{2M}\nu_1\nu_2, \\
A &= -\frac{v_u^2}{3\kappa\nu^c}Y_{\nu_2}^2 - \frac{1}{2M}\nu_2^2, \\
B &= \frac{v_u^2}{6\kappa\nu^c}Y_{\nu_2}^2 - \frac{1}{2M}\nu_2^2.
\end{aligned}
\tag{3.16}
$$

The eigenvalues of this matrix are the following:

$$
\frac{1}{2}\left(A+B-\sqrt{8c^2+(A+B-d)^2}+d\right), \frac{1}{2}\left(A+B+\sqrt{8c^2+(A+B-d)^2}+d\right), A-B\,, \tag{3.17}
$$

and the corresponding eigenvectors (for simplicity are not normalised) are

$$
\begin{gathered}
\left(-\frac{A+B+\sqrt{8c^2+(A+B-d)^2}-d}{2}, c, c\right), \\
\left(\frac{-A-B+\sqrt{8c^2+(A+B-d)^2}+d}{2c}, 1, 1\right), \\
(0,-1,1)\,.
\end{gathered}
\tag{3.18}
$$

We have ordered the eigenvalues in such a way that it is clear how to obtain the normal hierarchy for the ν_μ-ν_τ degenerate case. Then we see that $\sin^2\theta_{13}=0$ and $\sin^2\theta_{23}=\frac{1}{2}$, as in the tri-bimaximal mixing regime. Also we have enough freedom to fix the parameters in such a way that the experimental values for the mass differences and the remaining angle θ_{12} can be reproduced. It is important to mention that the above two values of the angles are a consequence of considering the example with ν_μ-ν_τ degeneration, and therefore valid even if we use the general formula (3.7) instead of the simplified expression (3.14). Notice that eqs. (3.15), (3.17) and (3.18) would be the same but with the corresponding values of A, B, c and d.

Let us remark that the fact that to obtain the correct neutrino angles is easy in this kind of seesaw is due to the following characteristics: R-parity is broken and the relevant scale is the electroweak one. In a sense we are giving an answer to the question why the mixing angles are so different in the quark and lepton sectors.

To show qualitatively how we can obtain an adequate seesaw with diagonal neutrino Yukawa couplings, let us first consider the limit[2] $c \to 0$. In this limit the electron neutrino is the lightest neutrino, and is completely decoupled from the rest. The second eigenvector has no ν_e composition ($\sin\theta_{12} \to 0$), and it is half ν_μ and half ν_τ. Understanding this case we can easily generalize the situation to the case $\sin\theta_{12} \neq 0$, switching on the parameter c. The eigenvalues in this limit are

$$
d\,,\ A+B,\ A-B, \tag{3.19}
$$

[2]Actually this limit can be obtained taking $Y_{\nu_1} \to 0$, $\nu_1 \to 0$, implying $c \to 0$, and also $d \to 0$, and leading to similar conclusions. This limit means that the electron neutrino is decoupled from the other two neutrinos, having a negligible mass.

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where

$$|d| = \left|\frac{v_u^2}{3\kappa\nu^c}Y_{\nu_1}^2 + \frac{1}{2M}\nu_1^2\right|,$$
$$|A+B| = \left|\frac{v_u^2}{6\kappa\nu^c}Y_{\nu_2}^2 + \frac{1}{M}\nu_2^2\right|,$$
$$|A-B| = \frac{v_u^2}{2\kappa\nu^c}Y_{\nu_2}^2. \tag{3.20}$$

We can see that $\Delta m^2_{\rm atm} \sim |4AB| = \left|4\left(\frac{v_u^4 Y_{\nu_2}^4}{18\kappa^2\nu^{c2}} - \frac{1}{4M^2}\nu_2^4 - \frac{v_u^2 Y_{\nu_2}^2 \nu_2^2}{12M\kappa\nu^c}\right)\right|$ and $\Delta m^2_{\rm sol} \sim |(A+B)^2 - d^2| = \left|\left(\frac{v_u^2}{6\kappa\nu^c}Y_{\nu_2}^2 + \frac{1}{M}\nu_2^2\right)^2 - \left(\frac{v_u^2}{3\kappa\nu^c}Y_{\nu_1}^2 + \frac{1}{2M}\nu_1^2\right)^2\right|$.

It is important to note that we need $|A-B| > |A+B|$ for the normal hierarchy case, otherwise the θ_{12} angle is zero even when c is not neglected. This is easy to obtain for $M \gg 2\kappa\nu^c$. If $M \sim 2\kappa\nu^c$, using different signs for the effective Majorana and gaugino masses helps to fulfill the above inequality. For this to hold with our convention, one must take $M < 0$.

In the inverted hierarchy scenario $|A-B| > |A+B|$ leads the angle θ_{12} to zero also with $c \neq 0$ which is not phenomenologically viable. Then we impose $|A-B| < |A+B|$. Note that when c is switched on, the parameter d has to be large enough for having the associated neutrino with an intermediate mass, as corresponds to the inverted hierarchy scenario. Therefore in this case we can also have easily the tri-bimaximal mixing regime for $M \ll 2\kappa\nu^c$. When $M \sim 2\kappa\nu^c$, having $M > 0$ helps to fulfill the above condition.

Let us finally remark that we can get the complete tri-bimaximal mixing regime $\sin^2\theta_{13} = 0$, $\sin^2\theta_{23} = 1/2$ and $\sin^2\theta_{12} = 1/3$ fixing in eq. (3.15) $c = A + B - d$. In this way we obtain the eigenvalues

$$-(A+B)+2d\,,\ 2(A+B)-d,\ A-B, \tag{3.21}$$

and from eq. (3.18), after normalization, we arrive to $\sin^2\theta_{12} = 1/3$.

Breaking the degeneracy between the Y_ν and ν of the muon and tau neutrinos, it is possible to find more general solutions in the normal and inverted hierarchy cases. We will show this with numerical examples in the next section, working always in the case $M \sim 2\kappa\nu^c$. Note also that in the case of degenerate ν_μ-ν_τ parameters, as the Dirac CP phase always appears in the MNS matrix in the form $\sin\theta_{13}e^{i\delta}$ (see eq. (4.2) below), the SCPV effect is suppressed since $\sin\theta_{13}$ is negligible. This is not the case if we break the degeneration between ν_μ and ν_τ.

When the vacuum is non CP-conserving the situation is more complicated since new relative phases are present, but the idea still holds. In the next section we will use the above results to find numerical examples in the general case where also phases are generated through complex vacua. Examples where changing the sign of M the second and third eigenvalue are interchanged and the behavior is similar to the one described in this section.

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4 Results

In section 2 we have given a simple argument to show that the $\mu\nu$SSM can violate CP spontaneously. In section 3 we have discussed how to obtain correct neutrino masses and mixing angles. In this section we sketch the numerical method used for the search of global minima of the $\mu\nu$SSM with SCPV, giving rise also to an effective neutrino mass matrix that reproduces correctly the phenomenology of the neutrino sector according to observations. We also give some examples.

For simplicity, we assume that all the parameters appearing in the potential are diagonal in flavor space at the electroweak scale, except the trilinear $(A_\kappa\kappa)_{ijk}$ terms whose entries different from $(A_\kappa\kappa)_{iii}$ are relevant to break CP spontaneously. We introduce the following notation for the flavor diagonal free parameters of the scalar potential: κ_i, Y_{ν_i}, $(A_\nu Y_\nu)_i$, $m^2_{\tilde{L}_i}$, $m^2_{\tilde{\nu}^c_i}$ with $i = 1, 2, 3$ being flavor indices. Under this assumption, the neutral scalar potential in (2.4) is obviously simplified, and as a consequence also the minimization conditions (A.1)–(A.7) are simplified. In addition to the complex VEVs, the potential depends on λ_i, κ_i, Y_{ν_i}, $(A_\kappa\kappa)_{ijk}$, $(A_\lambda\lambda)_i$, $(A_\nu Y_\nu)_i$, m_{H_d}, m_{H_u}, $m_{\tilde{\nu}^c_i}$ and $m_{\tilde{L}_i}$.

The strategy followed to find minima of the model consists of solving the minimization equations in terms of the soft parameters that are linear in those equations. More precisely, the three minimization equations (A.7), corresponding to $\frac{\partial V}{\partial \chi_i} = 0$, are used to solve the values of $(A_\nu Y_\nu)_i$. Using this result, eqs. (A.6) for $i = 2, 3$, corresponding to $\frac{\partial V}{\partial \varphi^c_{\nu_{2,3}}} = 0$, are then solved for $(A_\lambda\lambda)_{2,3}$. Repeating the procedure using the equation (A.5), $\frac{\partial V}{\partial \varphi_v} = 0$, one obtains $(A_\lambda\lambda)_1$. Finally, eq. (A.6) for $i = 1$ is used to get $(A_\kappa\kappa)_{111}$. The conditions with respect to the moduli of the VEVs (A.1)–(A.4) are used to get the squared soft masses. Once this is done, we ensure that the critical point found (i.e. with non-vanishing phases for the VEVs) is a global minimum through a numerical procedure. As discussed in [4], one has to check in particular that the minimum found is deeper than the local minima with some or all the VEVs vanishing.

To accomplish the numerical task of finding global minima we need as inputs the eight moduli and seven phases of the VEVs, the λ_i, κ_i and Y_{ν_i} couplings and the soft-trilinear terms $(A_\kappa\kappa)_{ijk}$ with $(i, j, k) \neq (1, 1, 1)$. For simplicity, we assume a special structure for the latter: $(A_\kappa\kappa)_{222} = (A_\kappa\kappa)_{333}$, a common value for $(A_\kappa\kappa)_{ijk}$ with $i, j, k \neq 1$, and another common value for $(A_\kappa\kappa)_{ijk}$ with one or two indices equal to 1. Moreover, let us recall that the modulus of the SUSY Higgs VEVs, can be determined from $v^2 = v_d^2 + v_u^2 + \sum_i \nu_i^2 \approx v_d^2 + v_u^2$ with $v \approx 174\ GeV$, and the value of $\tan\beta = \frac{v_u}{v_d}$.

Once we find global minima, the next step is to build the neutralino mass matrix and to diagonalize it perturbatively in order to extract the effective neutrino mass matrix. Diagonalizing the effective neutrino mass matrix, we can extract the mass differences and the mixing angles of the neutrino sector and compare them with the data. The key for obtaining a phenomenologically viable neutrino sector, once we are in a global minimum, consists of varying either the neutrino Yukawa couplings, the left-handed sneutrino VEVs or the soft gaugino masses. This approach does not alter the vacuum structure previously obtained.

Let us now describe the details on how we proceed with the phenomenological analysis of the neutrino sector of the model. First, we assume for simplicity the GUT inspired

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$\lambda_i = 0.13$	$\kappa_i = 0.55$	$\nu_i^c = 1000\ GeV$
$\tan\beta = 29$	$\varphi_v = -\pi$	$\varphi_{\nu_1^c} = \frac{\pi}{7}$
$\varphi_{\nu_2^c} = \varphi_{\nu_3^c} = -\frac{\pi}{7}$	$\chi_1 = -\frac{\pi}{6}$	$\chi_2 = \chi_3 = \frac{\pi}{6}$

Table 2. Numerical values of the relevant input parameters for a global minimum that breaks CP spontaneously.

relation between the gaugino masses M_1 and M_2, $M_1 = \frac{\alpha_1^2}{\alpha_2^2} M_2$, implying $M_2 \simeq 2M_1$ at low energy. As discussed in section 3, one has to diagonalize the neutrino effective mass matrix, $m_{\text{eff}} = -m^T \cdot M^{-1} \cdot m$. Since it is a complex symmetric matrix, it can be diagonalized with an unitary transformation, as it is shown in eqs. (3.5) and (3.6). For the MNS matrix we follow the standard parameterization

$$U_{\text{MNS}} = \text{diag}\left(e^{\text{i}\delta_e}, e^{\text{i}\delta_\mu}, e^{\text{i}\delta_\tau}\right) \cdot V \cdot \text{diag}\left(e^{-\text{i}\phi_1/2}, e^{-\text{i}\phi_2/2}, 1\right), \tag{4.1}$$

where ϕ_1 and ϕ_2 are the Majorana phases and V is given by

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}. \tag{4.2}$$

Here $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$ whereas δ is the Dirac CP violating phase. The conventions used for extracting the mixing angles and the Majorana and Dirac phases from eqs. (4.1) and (4.2) are outlined in ref. [29].

Taking all the above into account, we show in table 2 the parameters that characterize an example of a global minimum that breaks CP spontaneously. The values of the soft parameters not determined by the minimization equations have been chosen to be $(A_\kappa\kappa)_{iii} = 280\,\text{GeV}$ for $i \neq 1$, $(A_\kappa\kappa)_{ijk} = -40\,\text{GeV}$ for $i,j,k \neq 1$, and $(A_\kappa\kappa)_{ijk} = -120\,\text{GeV}$ for one or two indices equal to 1. In table 3 we show the neutrino/neutralino inputs used in order to obtain a ν_μ-ν_τ degenerated case with normal hierarchy, producing values of masses and angles within the ranges of table 1. In particular, we obtain $\sin^2\theta_{13} \sim 0$ and $\sin^2\theta_{23} = 0.5$, as expected from the discussion in section 3, $\sin^2\theta_{12} = 0.323$, and neutrino masses $m_1 = 0.00305\,\text{eV}$, $m_2 = 0.00949\,\text{eV}$ and $m_3 = 0.05091\,\text{eV}$, producing $\Delta m^2_{\text{solar}} = 8.08 \times 10^{-5}\,\text{eV}^2$ and $\Delta m^2_{\text{atm}} = 2.50 \times 10^{-3}\,\text{eV}^2$. The corresponding values of the soft terms calculated with the minimization equations are presented in table 4.

It is worth noticing that for this solution, the soft masses of the left-handed sneutrinos, $m_{\tilde{L}_i}$, do not need to be very different, and, actually, in this case they are almost degenerate $\sim 3700\,\text{GeV}$. This can be understood using the minimization equations (A.4), neglecting the terms with products of Yukawas. When $\frac{Y_{\nu_i}}{\nu_i} = \frac{Y_{\nu_j}}{\nu_j}$, $\forall\ i,j$, one obtains $m^2_{\tilde{L}_i} = m^2_{\tilde{L}_j}$. However, we have to point out that the values obtained for other soft parameters are not so natural in a SUSY framework. Notice for example that $A_\nu \sim -7\,\text{TeV}$, $A_{\lambda_1} \sim -11\,\text{TeV}$, whereas $A_{\kappa_{111}} \sim -0.5\,\text{GeV}$. Indeed, this is a consequence of the particular solution shown in table 2.

Although it is non-trivial to find realistic solutions, since many minima which apparently are acceptable, at the end of the day turn out to be false minima, we have been able to

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$Y_{\nu_1} = 4.25 \times 10^{-7}$	$Y_{\nu_2} = Y_{\nu_3} = 1.36 \times 10^{-6}$	$M_1 = -340$ GeV
$\nu_1 = 3.88 \times 10^{-5}$ GeV	$\nu_2 = \nu_3 = 1.24 \times 10^{-4}$ GeV	

Table 3. Numerical values of the neutrino/neutralino inputs that reproduce the neutrino experimental constraints, and correspond to the normal hierarchy scenario.

$(A_\nu Y_\nu)_1 \simeq -0.0031$ GeV	$(A_\nu Y_\nu)_2 \simeq -0.010\,\text{GeV}$	$(A_\nu Y_\nu)_3 \simeq -0.010$ GeV
$(A_\lambda \lambda)_1 \simeq -1487$ GeV	$(A_\lambda \lambda)_2 \simeq -679$ GeV	$(A_\lambda \lambda)_3 \simeq -679$ GeV
$(A_\kappa \kappa)_{111} \simeq -0.25$ GeV	$m^2_{H_d} \simeq 7.0325 \times 10^7$ GeV^2	$m^2_{H_u} \simeq -47200$ GeV^2
$m^2_{\tilde{\nu}^c_1} \simeq 260140$ GeV^2	$m^2_{\tilde{\nu}^c_2} \simeq -100820$ GeV^2	$m^2_{\tilde{\nu}^c_3} \simeq -100820$ GeV^2
$m^2_{\tilde{L}_1} \simeq m^2_{\tilde{L}_2} = m^2_{\tilde{L}_3} = 1.37 \times 10^7$ GeV^2		

Table 4. Values of the soft terms calculated with the minimization equations for the global minimum associated to the parameters shown in table 2.

$\lambda_i = 0.10$	$\kappa_i = 0.35$	$\nu^c_1 = 835\ GeV,\ \nu^c_2 = \nu^c_3 = 685\ GeV$
$\tan\beta = 29$	$\varphi_v = -\pi$	$\varphi_{\nu^c_1} = \frac{\pi}{7}$
$\varphi_{\nu^c_2} = \varphi_{\nu^c_3} = -\frac{\pi}{7}$	$\chi_1 = -\frac{\pi}{6}$	$\chi_2 = \chi_3 = \frac{\pi}{6}$

Table 5. Numerical values of the relevant inputs for the second global minimum discussed in the text, that breaks CP spontaneously.

$Y_{\nu_1} = 5.4 \times 10^{-7}$	$Y_{\nu_2} = Y_{\nu_3} = 9.2 \times 10^{-7}$	$M_1 = -340$ GeV
$\nu_1 = 3.7 \times 10^{-5}$ GeV	$\nu_2 = \nu_3 = 8.8 \times 10^{-5}$ GeV	

Table 6. Numerical values of the neutrino/neutralino inputs for the second global minimum discussed in the text, that reproduce the neutrino experimental constraints and correspond to the normal hierarchy scenario.

find more sensible solutions. This is the case of the one shown in table 5, with the values of the input soft parameters $(A_\kappa \kappa)_{iii} = -150\,\text{GeV}$ for $i \neq 1$, $(A_\kappa \kappa)_{ijk} = 75\,\text{GeV}$ for $i, j, k \neq 1$ and $(A_\kappa \kappa)_{ijk} = -50\,\text{GeV}$ for one or two indices equal to 1. For example, lowering the values of ν^c one is able to lower the trilinear terms $A_\nu \sim -3\,\text{TeV}$ in order to fulfill eqs. (A.7) (also lowering κ contributes to this result), and also the soft masses $m_{\tilde{L}_i} \sim 2.8\,\text{TeV}$, as shown in table 7. Lowering λ one is able to lower the trilinears $A_{\lambda_1} \sim -1.5\,\text{TeV}$, $A_{\lambda_{2,3}} \sim -840\,\text{GeV}$, in order to fulfill eqs. (A.5) and (A.6). Notice finally that the use of non-degenerate ν^c_i allows to increase the trilinear $A_{\kappa_{111}} \sim 36\,\text{GeV}$. In table 6 we show the corresponding neutrino/neutralino inputs producing values of masses and angles within the ranges of table 1.

Modifying the values of the angles we can also obtain other interesting solutions. See for example the one shown in tables 8, 9, and 10. In this case the values of the input soft parameters are chosen to be $(A_\kappa \kappa)_{iii} = -200\,\text{GeV}$ for $i \neq 1$, $(A_\kappa \kappa)_{ijk} = 125\,\text{GeV}$ for $i, j, k \neq 1$ and $(A_\kappa \kappa)_{ijk} = -75\,\text{GeV}$ for one or two indices equal to 1. Notice that now the

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$(A_\nu Y_\nu)_1 \simeq -0.00209$ GeV	$(A_\nu Y_\nu)_2 \simeq -0.00294$ GeV	$(A_\nu Y_\nu)_3 \simeq -0.00294$ GeV
$(A_\lambda \lambda)_1 \simeq -156$ GeV	$(A_\lambda \lambda)_2 \simeq -84$ GeV	$(A_\lambda \lambda)_3 \simeq -84$ GeV
$(A_\kappa \kappa)_{111} \simeq 12.7$ GeV	$m^2_{H_d} \simeq 5.36 \times 10^6$ GeV^2	$m^2_{H_u} \simeq -37910$ GeV^2
$m^2_{\tilde{\nu}^c_1} \simeq 51035$ GeV^2	$m^2_{\tilde{\nu}^c_2} \simeq 69155$ GeV^2	$m^2_{\tilde{\nu}^c_3} \simeq 69155$ GeV^2
$m^2_{\tilde{L}_1} = 8.07 \times 10^6$ GeV^2	$m^2_{\tilde{L}_2} = 3.92 \times 10^6$ GeV^2	$m^2_{\tilde{L}_3} = 3.92 \times 10^6$ GeV^2

Table 7. Values of the soft terms calculated with the minimization equations for the second global minimum discussed in the text, associated to the parameters shown in table 5.

$\lambda_i = 0.10$	$\kappa_i = 0.42$	$\nu^c_1 = 850\ GeV,\ \nu^c_2 = \nu^c_3 = 550\ GeV$
$\tan\beta = 29$	$\varphi_v = -\pi$	$\varphi_{\nu^c_1} = \frac{\pi}{5}$
$\varphi_{\nu^c_2} = \varphi_{\nu^c_3} = -\frac{\pi}{5}$	$\chi_1 = -\frac{\pi}{3}$	$\chi_2 = \chi_3 = \frac{\pi}{3}$

Table 8. Numerical values of the relevant inputs for the third global minimum discussed in the text, that breaks CP spontaneously.

$Y_{\nu_1} = 1.9 \times 10^{-7}$	$Y_{\nu_2} = Y_{\nu_3} = 8.5 \times 10^{-7}$	$M_1 = -100$ GeV
$\nu_1 = 6 \times 10^{-5}$ GeV	$\nu_2 = \nu_3 = 4.9 \times 10^{-5}$ GeV	

Table 9. Numerical values of the neutrino/neutralino inputs for the third global minimum discussed in the text, that reproduce the neutrino experimental constraints and correspond to the normal hierarchy scenario.

values obtained for the soft terms are also of this order. In particular, the trilinears are $A_{\nu_1} \sim -657\,\text{GeV}$, $A_{\nu_{2,3}} \sim -429\,\text{GeV}$, $A_{\lambda_1} \sim -990\,\text{GeV}$, $A_{\lambda_{2,3}} \sim -830\,\text{GeV}$, and $A_{\kappa_{111}} \sim 100\,\text{GeV}$. For the soft masses we obtain $m_{\tilde{L}_1} \sim 628\,\text{GeV}$, $m_{\tilde{L}_{2,3}} \sim 950\,\text{GeV}$.

A general analysis of the parameter space, finding other interesting complex vacua, is obviously extremely complicated given the large number of parameters involved, and beyond the scope of this paper. Nevertheless, we have checked that other sensible solutions can indeed be obtained modifying adequately the parameters. In the following we will work with the solution associated to the parameters of table 2, since the discussion below is essentially valid for other solutions. Our strategy will consist of varying the neutrino/neutralino inputs Y_{ν_i}, ν_i and M_1 in such a way that the derived neutrino mass differences and mixing angles are within the ranges of table 1. As mentioned above, this procedure will not alter the vacuum structure found. Notice in this respect that gaugino masses do not contribute to the minimization equations, and that the values of Y_{ν_i} and ν_i are very small. Let us also mention that this strategy can indeed be applied to the much more simple issue of analyzing real vacua. In particular, it was shown in [4] that many global minima with real VEVs can be found. For them neutrino/neutralino inputs Y_{ν_i}, ν_i, M_1, similar to those studied here are also valid.

As noted in section 3 we have chosen $M_1 < 0$ in order to guaranty a viable θ_{12} angle. It is worth pointing out here that a redefinition of the parameters leaving the Lagrangian invariant can be made, in such a way that M_1 becomes positive and other parameters such

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$(A_\nu Y_\nu)_1 \simeq -0.000125$ GeV	$(A_\nu Y_\nu)_2 \simeq -0.000365$ GeV	$(A_\nu Y_\nu)_3 \simeq -0.000365$ GeV
$(A_\lambda \lambda)_1 \simeq -99$ GeV	$(A_\lambda \lambda)_2 \simeq -83$ GeV	$(A_\lambda \lambda)_3 \simeq -83$ GeV
$(A_\kappa \kappa)_{111} \simeq 41.9$ GeV	$m^2_{H_d} \simeq 3.6 \times 10^6$ GeV^2	$m^2_{H_u} \simeq -25118$ GeV^2
$m^2_{\tilde{\nu}^c_1} \simeq -24393$ GeV^2	$m^2_{\tilde{\nu}^c_2} \simeq 208377$ GeV^2	$m^2_{\tilde{\nu}^c_3} \simeq 208377$ GeV^2
$m^2_{\tilde{L}_1} = 394777$ GeV^2	$m^2_{\tilde{L}_2} = 903528$ GeV^2	$m^2_{\tilde{L}_3} = 903528$ GeV^2

Table 10. Values of the soft terms calculated with the minimization equations for the third global minimum discussed in the text, associated to the parameters shown in table 8.

as the VEVs become negative, describing indeed the same physics. In our convention the VEVs, v_d, v_u, ν^c_i, ν_i, are always taken positive.

We would also like to stress that all the numerical results have been obtained without any approximation, that is, with the exact expression of the 10×10 neutralino mass matrix, calculating numerically the effective neutrino mass matrix and diagonalizing it.

Let us first study how the neutrino mass differences depend on the inputs. In section 3 we showed that in this scenario there are two different contributions to the seesaw mechanism; the one involving right-handed neutrinos (and Higgsinos) given by $\frac{(Y_{\nu_i} v_u)^2}{2\kappa\nu^c}$, where the Dirac and Majorana masses are parameterized by $Y_{\nu_i} v_u$ and $2\kappa\nu^c$, respectively, and the contribution coming from the gaugino seesaw given by $\frac{(g_1\nu_i)^2}{M_1} + \frac{(g_2\nu_i)^2}{M_2}$, where the Dirac and Majorana masses are parameterized by $g_\alpha \nu_i$ and M_α, respectively, with $\alpha = 1, 2$.

Figures 2a and 2b show that the heaviest eigenvalue (dashed line) has very little electron-neutrino component, as expected in the normal hierarchy scenario (see figure 1), and therefore it does not depend on $(Y_{\nu_1} v_u)^2/(2\kappa\nu^c)$, whereas the intermediate (solid line) and lightest (dotted line) eigenvalues, that have sizeable electron-neutrino components, grow with this term. As a consequence of the latter, the squared solar mass difference grows as well. On the other hand, following the arguments related to eq. (3.14), we can see in figures 2c and 2d that the heaviest eigenvalue is controlled by the contribution of the seesaw with right-handed neutrinos having an important muon/tau neutrino composition, thus we observe how the heaviest eigenvalue grows with $(Y_{\nu_2} v_u)^2/(2\kappa\nu^c)$ and, as a consequence, the squared atmospheric mass difference grows accordingly. The variation with $(Y_{\nu_3} v_u)^2/(2\kappa\nu^c)$ is analogue.

Figure 3 is analogous to figure 2 but showing the squared neutrino mass differences dependence on the gaugino seesaw component. In this case, because the heaviest eigenstate (dashed line) practically does not mix with the electron neutrino we can see that it does not vary with $((g_1\nu_i)^2/M_1 + (g_2\nu_i)^2/M_2)^2$ for $i = 1, 2, 3$. On the other hand, the intermediate eigenstate grows with the mixing with the gauginos, as explained in section 3 with $M_1 < 0$, therefore the squared solar mass difference also grows.

Let us now discuss the mixing angles. Note that in the ν_μ-ν_τ degenerate case with normal hierarchy and $M_1 < 0$ we have obtained $\sin^2\theta_{13} = 0$ and $\sin^2\theta_{23} = \frac{1}{2}$. In figure 4 we present the variation of $\sin^2\theta_{12}$ with the ratio of the parameters that control the gaugino seesaw, b_e^2/b_μ^2, where for the sake of simplicity we take $b_i = Y_{\nu_i} v_d + 3\lambda\nu_i$ and we do not consider the complicated factors containing phases in eqs. (B.5).

To obtain results different from $\sin^2\theta_{23} \sim \frac{1}{2}$ and $\sin^2\theta_{13} \sim 0$, in the following we

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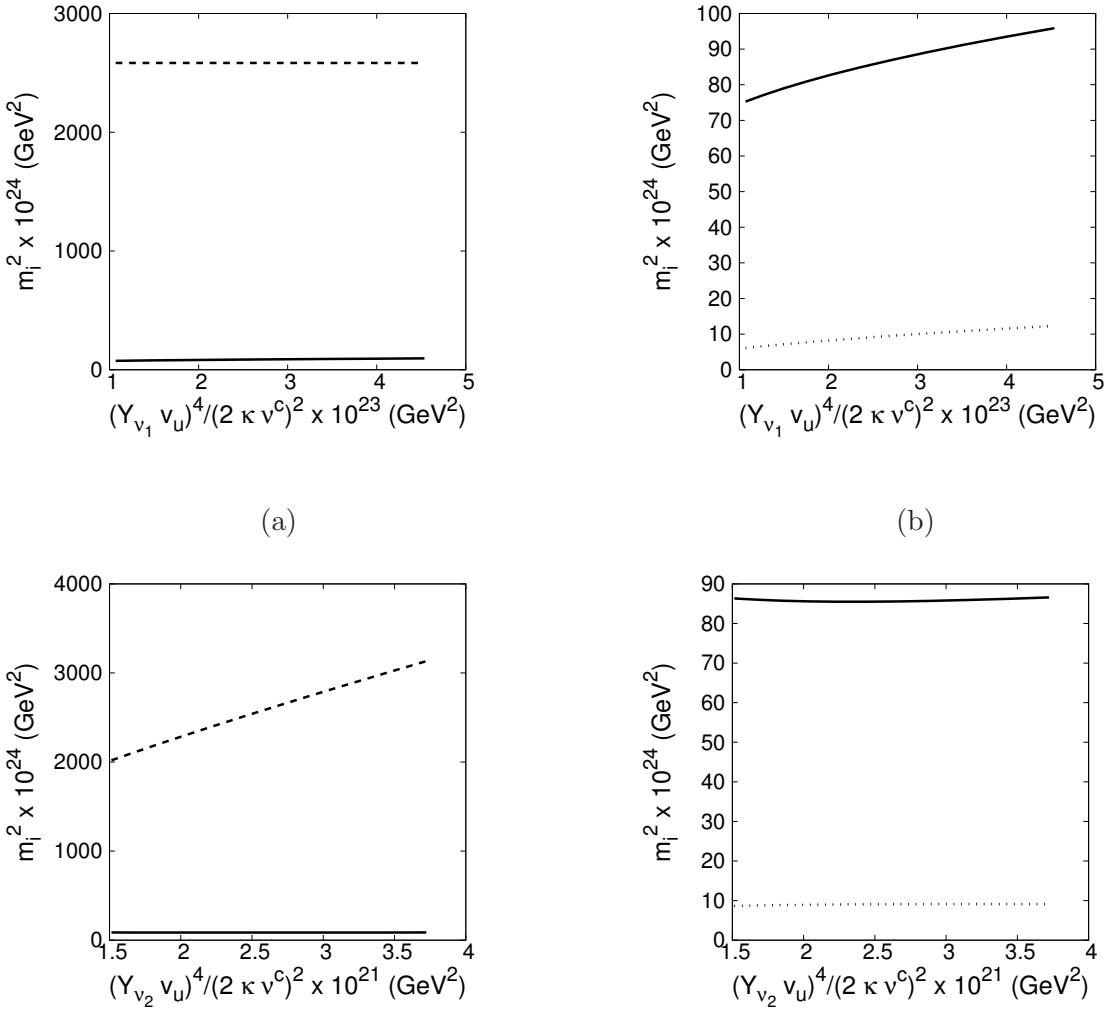

Figure 2. Squared neutrino masses versus $(Y_{\nu_i} v_u)^4/(2\kappa\nu^c)^2$. (a) and (b) show for $i = 1$ the two heaviest and lightest neutrinos, respectively. The same for (c) and (d) but for $i = 2$.

consider the possibility of having different values for the Y_ν and ν parameters for μ and τ neutrinos. We show in figure 5a $\sin^2\theta_{23}$ as a function of the ratio of the term that controls the Higgsino-ν^c seesaw, a_μ^2/a_τ^2. When a_μ/a_τ goes to 1, the ν_μ-ν_τ degeneracy is recovered and $\sin^2\theta_{23}$ goes to 1/2 as expected. In figure 5b we show $\sin^2\theta_{13}$ as a function of $\frac{4a_\mu a_\tau}{(a_\mu+a_\tau)^2}$ that is a good measure of the degeneration in this case. Note that when $4a_\mu a_\tau/(a_\mu + a_\tau)^2 \to 1$ the degeneracy is recovered and $\sin^2\theta_{13} \to 0$ as expected. The parameters a_i have been defined in eq. (B.2). Let us point out that $\sin^2\theta_{13} < 10^{-3}$ since we are breaking the degeneration between μ and τ neutrinos but the term that controls the higgsino-ν^c seesaw for the first family is very small compared to the other two families.

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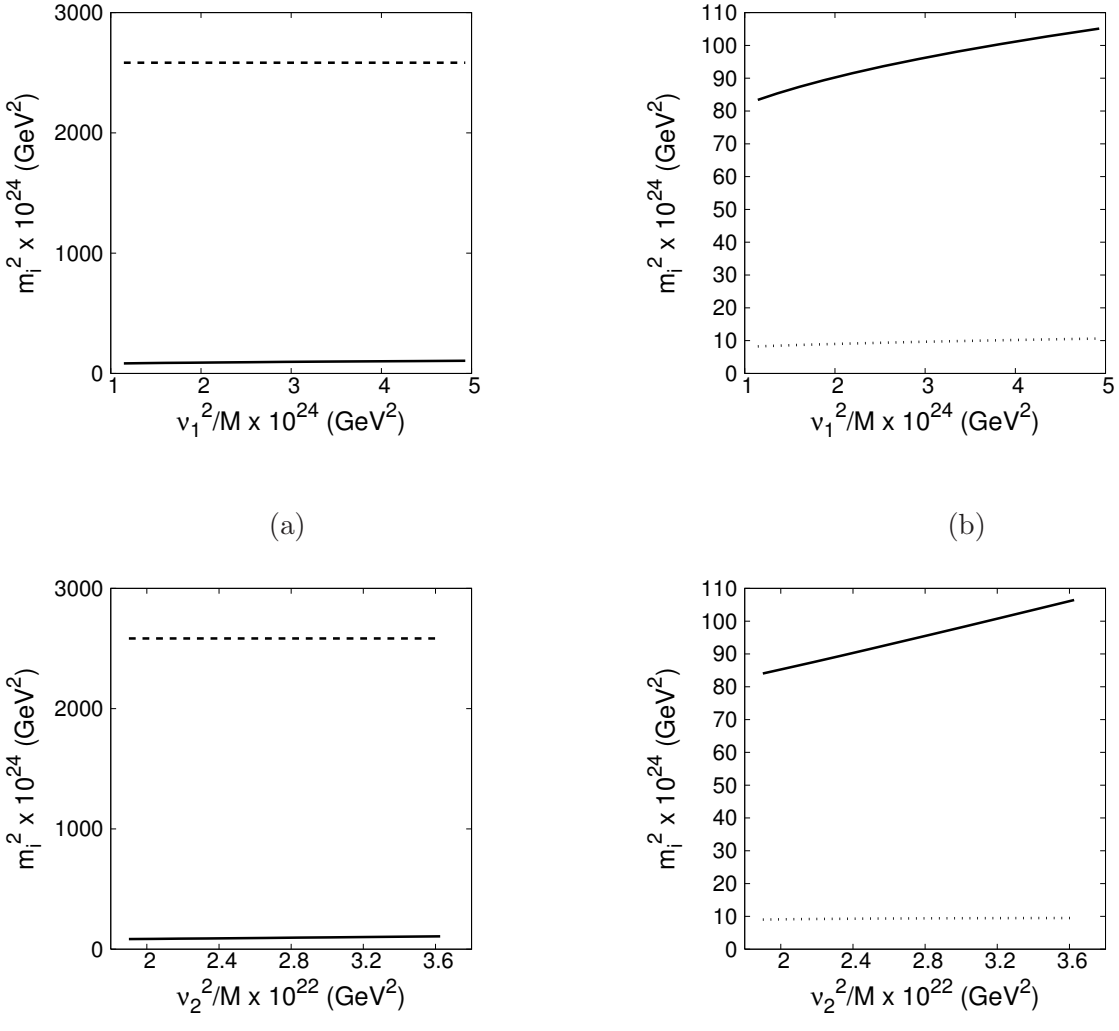

Figure 3. The same as in figure 2 but for the squared neutrino masses versus $[(g_1\nu_i)^2/M_1 + (g_2\nu_i)^2/M_2]^2$.

As mentioned previously, the $\mu\nu$SSM with SCPV also predicts non-zero CP phases in the MNS matrix. We have checked numerically that for each of the experimentally allowed regions found, the two Majorana CP phases and the Dirac CP phase are different from zero. This fact is reflected in figure 6 where we present two plots in the $\delta - \phi_1$ and $\delta - \phi_2$ planes (Dirac-Majorana CP phases) constructed varying all the inputs in the neutrino sector. However, it is fair to say that due to the smallness of $\sin^2\theta_{13} \sim 10^{-3}$ in this region, the CP violation effects of the phases of the VEVs turn out to be suppressed in the MNS matrix because the Dirac CP phase always appears in the form $\sin\theta_{13}e^{i\delta}$.

In order to complete the discussion about the neutrino sector in this scenario, we will consider the possibility $M_1 > 0$ instead of $M_1 < 0$. In section 3 we have seen that with

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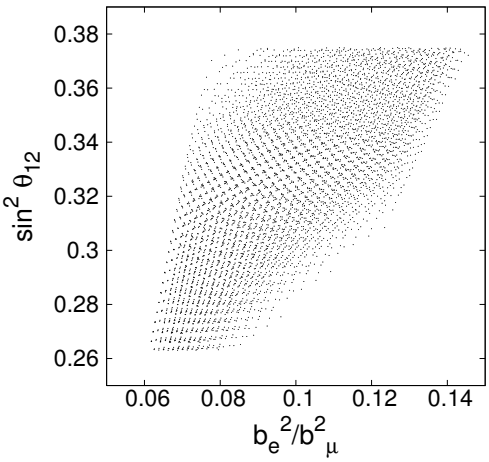
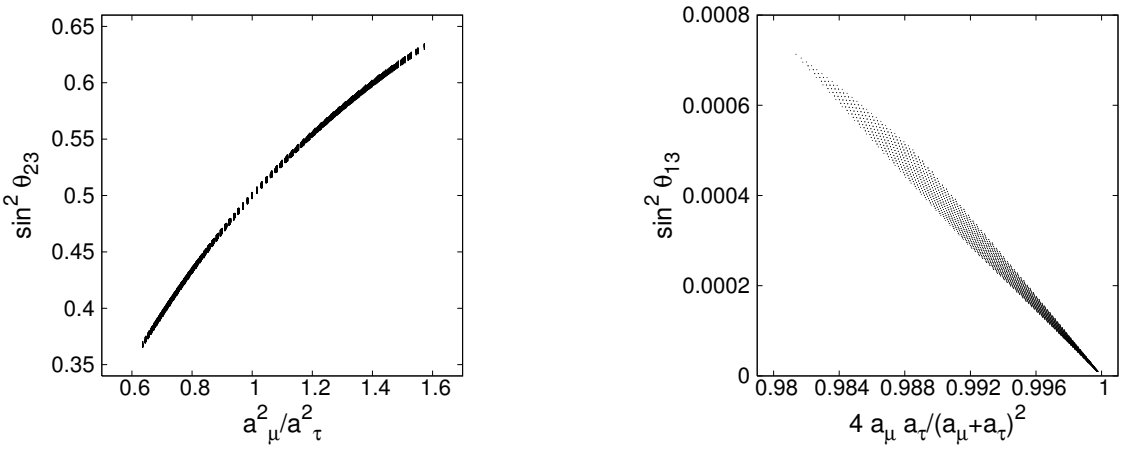


Figure 4. Variation of the solar mixing angle with respect to the relevant term that controls its evolution, b_e^2/b_μ^2.



(a) (b)

Figure 5. (a) The variation of $\sin^2\theta_{23}$ with respect to the relevant term that controls its evolution, a_μ^2/a_τ^2. (b) The variation of $\sin^2\theta_{13}$ with respect to the term that measures the ν_μ-ν_τ degeneracy.

$M_1 > 0$ it is more complicated to have a degeneracy between muon and tau neutrinos because it is easy to obtain $\sin^2\theta_{12} \sim 0$, in contradiction with the data (see table 1). Thus we will show a region where breaking the degeneracy ν_μ-ν_τ a normal hierarchy is obtained with $M_1 > 0$. This region is around the point of the parameter space shown in table 11. In this example the angle $\sin^2\theta_{13}$ can easily be made small as required by the data, but it is not necessarily negligible. Thus the CP violating effects would be present in the MNS matrix. Besides, we can roughly say that $\sin^2\theta_{13}$ and $\sin^2\theta_{12}$ are interchanged with respect to the case discussed above with $M_1 < 0$. For completeness, in figure 7a we show the variation of $\sin^2\theta_{13}$ with respect to the term that controls the gaugino seesaw

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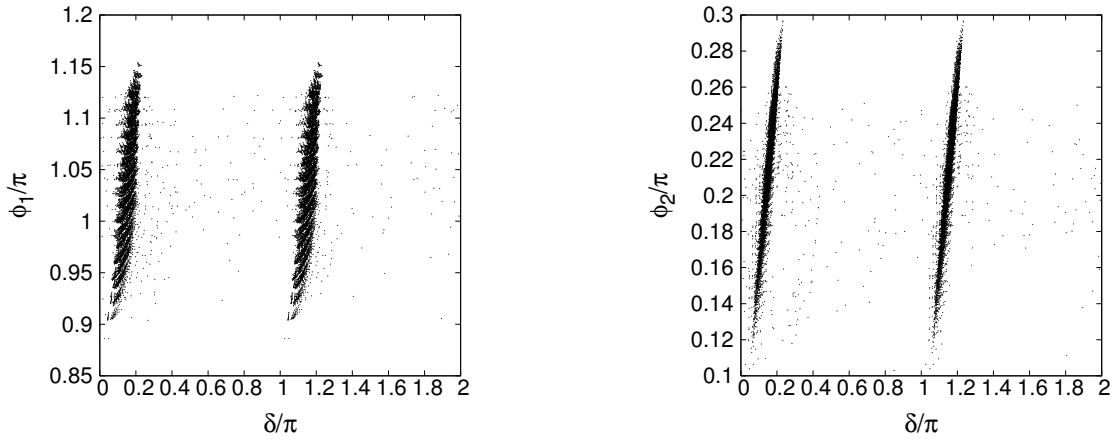
$Y_{\nu_1} = 9.54 \times 10^{-7}$	$Y_{\nu_2} = 9.47 \times 10^{-7}$	$Y_{\nu_3} = 2.31 \times 10^{-7}$	$M_1 = 350$ GeV
$\nu_1 = 8.59 \times 10^{-5}$ GeV	$\nu_2 = 2.25 \times 10^{-4}$ GeV	$\nu_3 = 2.29 \times 10^{-4}$ GeV	

Table 11. Numerical values of the relevant neutrino/neutralino-sector inputs that reproduce the neutrino experimental constraints, and correspond to the normal hierarchy scenario with $M_1 > 0$.



(a) (b)

Figure 6. $\delta - \phi_1$ plane (a) and $\delta - \phi_2$ plane (b) for the scenario with normal hierarchy and negative gaugino masses $M < 0$, varying simultaneously Y_{ν_i}, ν_i, M_1.

relevant in this case, namely $b_e^2/(b_\mu^2 + b_\tau^2)$. We also plot in figure 7b $\sin^2\theta_{12}$ as a function of the relevant term that controls the Higgsino-ν^c seesaw $\frac{4a_\mu a_\tau}{(a_\mu + a_\tau)^2}$. As mentioned above, an interesting feature of this region of the parameter space is that the effect of the Dirac CP phase in the MNS is not removed, since the value of $\sin\theta_{13}$ is not negligible. Figure 8 shows the derived CP phases of the MNS matrix.

For the sake of completeness, we show in table 12 an example where the inverse hierarchy scenario is achieved.

At this point it is clear that there are many regions with different characteristics, different compositions for the lightest neutralino or regions close to the tri-bimaximal mixing regime for normal or inverted hierarchy that can be found with different neutrino parameters. Furthermore, we have seen that the $\mu\nu$SSM with SCPV predicts non-zero CP-violating phases in the neutrino sector. If in the future a non-zero CP violating phase in the lepton sector is measured, SCPV as the one analyzed here could be a possible source.

Neutrino oscillations are sensitive only to the Dirac CP phase (insensitive to the Majorana phases). Let us briefly comment about the possible determination of δ in future neutrino experiments. The conservation of CP implies $P(\nu_\alpha \to \nu_\beta) = P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$. If CP

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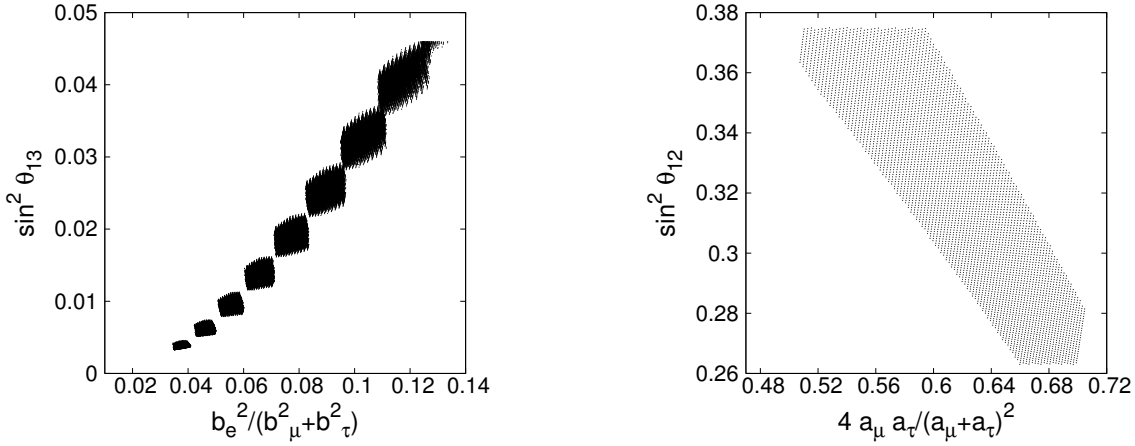
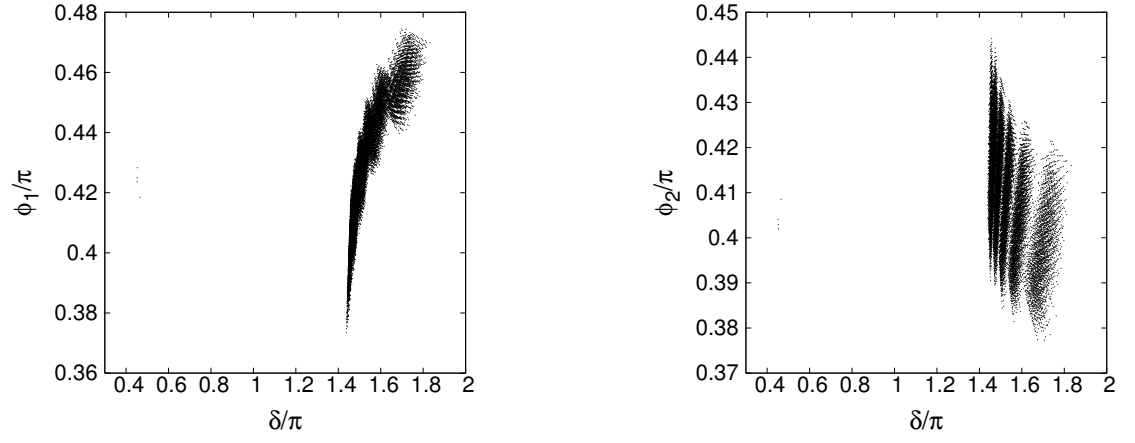


(a) (b)

Figure 7. (a) The variation of $\sin^2\theta_{13}$ with respect to the relevant term that controls its evolution. (b) The variation of $\sin^2\theta_{12}$ with respect to the relevant term $4a_\mu a_\tau/(a_\mu+a_\tau)^2$.



(a) (b)

Figure 8. $\delta-\phi_1$ plane (a) and $\delta-\phi_2$ plane (b) for the scenario with normal hierarchy and positive gaugino masses $M>0$, varying simultaneously Y_{ν_i}, ν_i, M_1.

is not conserved, we would have [30]

$$P(\nu_\mu\to\nu_e)-P(\bar\nu_\mu\to\bar\nu_e)=-16J\sin\left(\frac{\Delta m^2_{12}L}{4E}\right)\sin\left(\frac{\Delta m^2_{13}L}{4E}\right)\sin\left(\frac{\Delta m^2_{23}L}{4E}\right), \quad (4.3)$$

where L is the oscillation length, E is the neutrino beam energy and J is the Jarlskog invariant for the neutrino mass matrix which is given by $J=s_{12}c_{12}s_{23}c_{23}s_{13}c^2_{13}\sin\delta$. There is only an upper experimental limit for J, $J<0.04$. The reason is that J depends on θ_{13} and δ, which are currently unknown. If θ_{13} vanishes (recall the bound $\sin^2\theta_{13}<0.038$) J vanishes and the effect of CP violation via (4.3) would be unobservable. The same occurs if there was a degeneracy in the neutrino masses. In spite of these extreme situations the

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$Y_{\nu_1} = 5.98 \times 10^{-7}$	$Y_{\nu_2} = 1.32 \times 10^{-6}$	$Y_{\nu_3} = 1.40 \times 10^{-6}$	$M_1 = 340$ GeV
$\nu_1 = 3.276 \times 10^{-4}$ GeV	$\nu_2 = 6.20 \times 10^{-5}$ GeV	$\nu_3 = 6.56 \times 10^{-5}$ GeV	

Table 12. Numerical values of the relevant neutrino/neutralino inputs that reproduce the neutrino experimental constraints, and correspond to the inverted hierarchy scenario.

$\lambda_i = 0.13$	$\kappa_i = 0.55$	$\nu_1^c = 900\ GeV,\ \nu_2^c = \nu_3^c = 600\ GeV$
$\tan\beta = 29$	$\varphi_v = 0$	$\varphi_{\nu_1^c} = \frac{\pi}{100}$
$\varphi_{\nu_2^c} = \varphi_{\nu_3^c} = -\frac{\pi}{100}$	$\chi_1 = -\frac{\pi}{90}$	$\chi_2 = \chi_3 = \frac{\pi}{90}$

Table 13. Numerical values of the relevant inputs of a global minimum that breaks CP spontaneously with small phases.

process (4.3) implies that long baseline experiments allow the observation of CP violation due to the Dirac phase δ in the neutrino sector. Two experiments are designed for this purpose: NOνA [31] and the T2KK detector [32].

On the other hand, although Majorana phases affect neutrinoless double beta decay $0\nu\beta\beta$ [33], their determination turn out to be difficult.

Let us finally briefly discuss the implications of the CP-violating phases concerning the electric dipole moments (EDMs) in the $\mu\nu$SSM. As is well known, EDMs impose important constraints on supersymmetric theories. The MSSM (with explicit CP violation in the soft Lagrangian) predicts EDMs about three orders of magnitude larger than the experimental bounds for the EDM of the electron and neutron if the supersymmetric CP violating phases are $\mathcal{O}(1)$ and the supersymmetric particles have masses near their current experimental bounds $\mathcal{O}(100\ GeV)$ [34]. There are three kind of solutions to this problem in supersymmetric theories. First, if the supersymmetric CP violating phases are very small, of order $\mathcal{O}(10^{-2} - 10^{-3})$ the EDM bounds can be easily satisfied [34]. Second, if the supersymmetric scalar particles are decoupled with masses larger than about 3 TeV, and thus out of reach of the LHC, but not spoiling the solution of supersymmetry to the hierarchy problem, the EDM bounds could also be accomplished [35]. Third, there can be internal cancellations between the different contributions to the EDMs [36].

We would like to point out that the $\mu\nu$SSM with SCPV could implement these three kind of solutions. First of all, the possibility of small supersymmetric CP phases is present in our model. Let us show for example a global minimum that break CP spontaneously with $\mathcal{O}(10^{-2})$ CP phases (we have also found global minima with $\mathcal{O}(10^{-3})$ phases). The values of the soft parameters not determined by the minimization equations are chosen to be $(A_\kappa\kappa)_{iii} = -175\,\text{GeV}$ for $i \neq 1$, $(A_\kappa\kappa)_{ijk} = 100\,\text{GeV}$ for $i,j,k \neq 1$ and $(A_\kappa\kappa)_{ijk} = -100\,\text{GeV}$ for one or two indices equal to 1. The numerical values of the phases and the rest of input parameters are presented in tables 13 and 14.

It is worth remarking here that in models with SCPV small phases are not unnatural, since they arise as a consequence of the minimization conditions (notice that the use of phases as inputs in this work is just an artifact of the computation) for particular values of the soft terms.

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$Y_{\nu_1} = 1.9 \times 10^{-7}$	$Y_{\nu_2} = Y_{\nu_3} = 1.06 \times 10^{-6}$	$M_1 = 300$ GeV
$\nu_1 = 1.54 \times 10^{-4}$ GeV	$\nu_2 = \nu_3 = 2.4 \times 10^{-5}$ GeV	

Table 14. Numerical values of the neutrino/neutralino inputs that reproduce the neutrino experimental constraints for the global minimum with small phases.

The other two solutions, heavy scalars and internal cancellations, can also be implemented. Notice that the following soft parameters remain free in our model because they are not included in either the neutral scalar potential or the neutrino sector: $(A_u Y_u)_{ij}$, $m^2_{\tilde{u}^c_{ij}}$, M_3 , $(A_e Y_e)_{ij}$, $m^2_{\tilde{e}^c_{ij}}$. Thus, the solution with heavy scalars remains valid for scalar masses heavier than about 3 TeV. We also expect the internal cancellation solution to be valid in our model. This is because these free parameters enter in the calculation of the EDMs, and we will have enough freedom to find values where such cancellations can be accomplished, fulfilling the EDMs bounds.

5 Conclusions

In this work we have studied in detail the neutrino sector of the $\mu\nu$SSM. We have also shown that, even if all parameters in the scalar potential are real, SCPV is possible at tree level, and we have used these vacua to show how a complex MNS matrix can arise.

In particular, we have calculated first the scalar potential of the $\mu\nu$SSM with real parameters, assuming the most general situation where the VEVs of Higgses and sneutrinos can be complex. We have shown, using a simple argument, that CP can actually be spontaneously violated in this model.

Then we have discussed the neutralino-neutrino mass matrix of the $\mu\nu$SSM, and we have shown how to obtain from it the effective neutrino mass matrix. Although the discussion is general, we have applied it also to the particularly interesting case of real vacua. We have analyzed how the electroweak seesaw mechanism works in the $\mu\nu$SSM using approximate analytical equations, particularized for certain interesting limits that clarify the neutrino-sector behavior of the model. In addition, we have given the qualitative idea of how to find regions in the parameter space of the model that satisfy the neutrino experimental constraints. Let us remark that these constraints can be fulfilled even with a diagonal neutrino Yukawa matrix, since this seesaw does not involve only the right-handed neutrinos but also the MSSM neutralinos. Actually, to obtain the correct neutrino angles turns out to be easy due to the following characteristics of this seesaw: R-parity is broken and the relevant scale is the electroweak one. In a sense, this gives an answer to the question why the mixing angles are so different in the quark and lepton sectors.

Finally, we have presented our results describing the method to obtain numerically global minima with SCPV, and giving examples of such minima. Let us emphasize however that, unlike the case with real VEVs where many global minima can be found, for the case with complex VEVs such minima are not so easy to find. In particular, one has to choose carefully the parameters of the model. For the examples found we have shown the

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dependence of the neutrino mass differences (for both normal and inverted hierarchies), mixing angles, and CP phases of the MNS matrix, in terms of the relevant neutrino inputs. Last but not least, we have checked that different regions of the parameter space can reproduce the neutrino experimental constraints. In this context, future neutrino experiments could be able to measure a non-zero Dirac CP-violating phase, opening the possibility to SCPV in the $\mu\nu$SSM as the dominant source.

Acknowledgments

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A Minimization equations

Here we write first the eight minimization conditions with respect to the moduli v_d, v_u, ν_i^c, ν_i:

$$\begin{aligned}
&\frac{1}{4}G^2\left(\sum_i \nu_i\nu_i + v_d^2 - v_u^2\right)v_d + m_{H_d}^2 v_d + v_d v_u^2 \sum_i (\lambda_i)^2 - \sum_i (A_\lambda \lambda)_i \nu_i^c v_u \cos(\varphi_v + \varphi_{\nu_i^c}) \\
&\quad + \sum_{i,j} v_d \lambda_i \lambda_j \nu_i^c \nu_j^c \cos(\varphi_{\nu_i^c} - \varphi_{\nu_j^c}) - \sum_{i,j,k} \kappa_{ikj} \lambda_k v_u \nu_i^c \nu_j^c \cos(\varphi_{\nu_i^c} + \varphi_{\nu_j^c} - \varphi_v) \\
&\quad - \sum_{i,j,k} Y_{\nu_{ij}} \lambda_k \nu_i \nu_j^c \nu_k^c \cos(\chi_i + \varphi_{\nu_j^c} - \varphi_{\nu_k^c} - \varphi_v) - \sum_i \sum_j Y_{\nu_{ij}} \lambda_j v_u^2 \nu_i \cos(\varphi_v - \chi_i) = 0,
\end{aligned} \tag{A.1}$$

$$\begin{aligned}
&-\frac{1}{4}G^2\left(\sum_i \nu_i\nu_i + v_d^2 - v_u^2\right)v_u + m_{H_u}^2 v_u + v_u v_d^2 \sum_i (\lambda_i)^2 \\
&\quad + \sum_{i,j} (A_\nu Y_\nu)_{ij} \nu_i \nu_j^c \cos(\chi_i + \varphi_{\nu_j^c}) - \sum_i (A_\lambda \lambda)_i \nu_i^c v_d \cos(\varphi_v + \varphi_{\nu_i^c}) \\
&\quad + \sum_{i,j} \lambda_i \lambda_j v_u \nu_i^c \nu_j^c \cos(\varphi_{\nu_i^c} - \varphi_{\nu_j^c}) - \sum_{i,j} \sum_k \kappa_{ijk} \lambda_k v_d \nu_i^c \nu_j^c \cos(\varphi_{\nu_i^c} + \varphi_{\nu_j^c} - \varphi_v) \\
&\quad + \sum_{i,j,k} \sum_l Y_{\nu_{jl}} \kappa_{ilk} \nu_j \nu_i^c \nu_k^c \cos(\varphi_{\nu_i^c} + \varphi_{\nu_k^c} - \chi_j) - \sum_i \sum_j 2 Y_{\nu_{ij}} \lambda_j v_d v_u \nu_i \cos(\varphi_v - \chi_i) \\
&\quad + \sum_{i,j} \sum_k Y_{\nu_{ik}} Y_{\nu_{jk}} v_u \nu_i \nu_j \cos(\chi_i - \chi_j) + \sum_{i,j} \sum_k Y_{\nu_{ki}} Y_{\nu_{kj}} v_u \nu_i^c \nu_j^c \cos(\varphi_{\nu_i^c} - \varphi_{\nu_j^c}) = 0,
\end{aligned} \tag{A.2}$$

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$$\sum_j m^2_{\widetilde{\nu}^c_{ij}}\nu^c_j\cos(\varphi_{\nu^c_i}-\varphi_{\nu^c_j})-(A_\lambda\lambda)_i v_u v_d\cos(\varphi_v+\varphi_{\nu^c_i})+\sum_j(A_\nu Y_\nu)_{ji}\nu_j v_u\cos(\chi_j+\varphi_{\nu^c_i})$$
$$+\sum_{j,k}(A_\kappa\kappa)_{ijk}\nu^c_j\nu^c_k\cos(\varphi_{\nu^c_i}+\varphi_{\nu^c_j}+\varphi_{\nu^c_k})+\sum_j\lambda_i\lambda_j v_d^2\nu^c_j\cos(\varphi_{\nu^c_i}-\varphi_{\nu^c_j})$$
$$+\sum_j\lambda_i\lambda_j\nu^c_j v_u^2\cos(\varphi_{\nu^c_i}-\varphi_{\nu^c_j})+\sum_{j,k,l}\sum_m 2\kappa_{imk}\kappa_{lmj}\nu^c_j\nu^c_k\nu^c_l\cos(\varphi_{\nu^c_i}+\varphi_{\nu^c_j}-\varphi_{\nu^c_k}-\varphi_{\nu^c_l})$$
$$-\sum_j\sum_k 2\kappa_{ijk}\lambda_k v_d v_u\nu^c_j\cos(\varphi_{\nu^c_i}+\varphi_{\nu^c_j}-\varphi_v)+\sum_{j,k}\sum_l 2Y_{\nu_{jl}}\kappa_{ikl}v_u\nu_j\nu^c_k\cos(\varphi_{\nu^c_i}+\varphi_{\nu^c_k}-\chi_j)$$
$$-\sum_{j,k}Y_{\nu_{ji}}\lambda_k v_d\nu_j\nu^c_k\cos(\chi_j+\varphi_{\nu^c_i}-\varphi_{\nu^c_k}-\varphi_v)-\sum_{j,k}Y_{\nu_{kj}}\lambda_i v_d\nu_k\nu^c_j\cos(\chi_k+\varphi_{\nu^c_j}-\varphi_{\nu^c_i}-\varphi_v)$$
$$+\sum_{j,k,l}Y_{\nu_{ji}}Y_{\nu_{lk}}\nu_j\nu_l\nu^c_k\cos(\chi_j-\chi_k+\varphi_{\nu^c_i}-\varphi_{\nu^c_l})+\sum_j\sum_k Y_{\nu_{ki}}Y_{\nu_{kj}}v_u^2\nu^c_j\cos(\varphi_{\nu^c_i}-\varphi_{\nu^c_j})=0, \tag{A.3}$$

$$\frac{1}{4}G^2\left(\sum_j\nu_j\nu_j+v_d^2-v_u^2\right)\nu_i+\sum_j m^2_{\tilde{L}_{ij}}\nu_j\cos(\chi_i-\chi_j)+\sum_j(A_\nu Y_\nu)_{ij}\nu^c_j v_u\cos(\chi_i+\varphi_{\nu^c_j})$$
$$+\sum_{j,k}\sum_l Y_{\nu_{il}}\kappa_{jlk}v_u\nu^c_j\nu^c_k\cos(\varphi_{\nu^c_j}+\varphi_{\nu^c_k}-\chi_i)$$
$$-\sum_{j,k}Y_{\nu_{ij}}\lambda_k v_d\nu^c_j\nu^c_k\cos(\chi_i+\varphi_{\nu^c_j}-\varphi_{\nu^c_k}-\varphi_v)-\sum_j Y_{\nu_{ij}}\lambda_j v_d v_u^2\cos(\varphi_v-\chi_i)$$
$$+\sum_{j,k,l}Y_{ij}Y_{\nu_{kl}}\nu^c_j\nu_k\nu^c_l\cos(\chi_i-\chi_k+\varphi_{\nu^c_j}-\varphi_{\nu^c_l})+\sum_j\sum_k Y_{\nu_{ik}}Y_{\nu_{jk}}v_u^2\nu_j\cos(\chi_i-\chi_j)=0. \tag{A.4}$$

The seven minimization conditions with respect to the phases φ_v, $\varphi_{\nu^c_i}$ and χ_i are:

$$-\sum_{i,j}\sum_k 2\kappa_{ijk}\lambda_k v_d v_u\nu^c_i\nu^c_j\sin(\varphi_{\nu^c_i}+\varphi_{\nu^c_j}-\varphi_v)$$
$$-2\left[\sum_{i,j,k}Y_{\nu_{ij}}\lambda_k v_d\nu_i\nu^c_j\nu^c_k\sin(\chi_i+\varphi_{\nu^c_j}-\varphi_{\nu^c_k}-\varphi_v)-\sum_i\sum_j Y_{\nu_{ij}}\lambda_j v_d v_u^2\nu_i\sin(\varphi_v-\chi_i)\right]$$
$$+2\sum_i(A_\lambda\lambda)_i\nu^c_i v_d v_u\sin(\varphi_v+\varphi_{\nu^c_i})=0, \tag{A.5}$$

$$-\sum_j m^2_{\widetilde{\nu}^c_{ij}}\nu^c_i\nu^c_j\sin(\varphi_{\nu^c_i}-\varphi_{\nu^c_j})-\sum_j\lambda_i\lambda_j v_d^2\nu^c_i\nu^c_j\sin(\varphi_{\nu^c_i}-\varphi_{\nu^c_j})-\sum_j\lambda_i\lambda_j v_u^2\nu^c_i\nu^c_j\sin(\varphi_{\nu^c_i}-\varphi_{\nu^c_j})$$
$$-2\sum_{j,k,l}\sum_m\kappa_{imk}\kappa_{lmj}\nu^c_i\nu^c_j\nu^c_k\nu^c_l\sin(\varphi_{\nu^c_i}+\varphi_{\nu^c_j}-\varphi_{\nu^c_k}-\varphi_{\nu^c_l})$$
$$+2\sum_{j,k}\kappa_{ikj}\lambda_k v_d v_u\nu^c_i\nu^c_j\sin(\varphi_{\nu^c_i}+\varphi_{\nu^c_j}-\varphi_v)\ -2\sum_{j,k}\sum_l Y_{\nu_{jl}}\kappa_{ilk}v_u\nu_j\nu^c_i\nu^c_k\sin(\varphi_{\nu^c_i}+\varphi_{\nu^c_k}-\chi_j)$$
$$+\sum_{j,k}Y_{\nu_{ji}}\lambda_k v_d\nu_j\nu^c_i\nu^c_k\sin(\chi_j+\varphi_{\nu^c_i}-\varphi_{\nu^c_k}-\varphi_v)-\sum_{j,k}Y_{\nu_{kj}}\lambda_i v_d\nu_k\nu^c_j\nu^c_i\sin(\chi_k+\varphi_{\nu^c_j}-\varphi_{\nu^c_i}-\varphi_v)$$
$$-\sum_{j,k,l}Y_{\nu_{ji}}Y_{\nu_{kl}}\nu_j\nu^c_i\nu_k\nu^c_l\sin(\chi_j-\chi_k+\varphi_{\nu^c_i}-\varphi_{\nu^c_l})-\sum_j\sum_k Y_{\nu_{ki}}Y_{\nu_{kj}}v_u^2\nu^c_i\nu^c_j\sin(\varphi_{\nu^c_i}-\varphi_{\nu^c_j})$$

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$$+(A_\lambda\lambda)_i\nu_i^c v_d v_u\sin(\varphi_v+\varphi_{\nu_i^c})-\sum_{j,k}(A_\kappa\kappa)_{ijk}\nu_i^c\nu_j^c\nu_k^c\sin(\varphi_{\nu_i^c}+\varphi_{\nu_j^c}+\varphi_{\nu_k^c})$$
$$-\sum_j(A_\nu Y_\nu)_{ji}v_u\nu_j\nu_i^c\sin(\chi_j+\varphi_{\nu_i^c})=0, \tag{A.6}$$

$$-\sum_j m^2_{\tilde{L}_{ij}}\,\nu_i\,\nu_j\sin(\chi_i-\chi_j)+\sum_{j,k}\sum_l Y_{\nu_{il}}\kappa_{jlk}v_u\nu_i\nu_j^c\nu_k^c\sin(\varphi_{\nu_j^c}+\varphi_{\nu_k^c}-\chi_i)$$
$$+\sum_{j,k}Y_{\nu_{ij}}\lambda_k v_d\nu_i\nu_j^c\nu_k^c\sin(\chi_i+\varphi_{\nu_j^c}-\varphi_{\nu_k^c}-\varphi_v)-\sum_j Y_{\nu_{ij}}\lambda_j v_d v_u^2\nu_i\sin(\varphi_v-\chi_i)$$
$$-\sum_{j,k,l}Y_{\nu_{ij}}Y_{\nu_{kl}}\nu_i\nu_j^c\nu_k\nu_l^c\sin(\chi_i-\chi_k+\varphi_{\nu_j^c}-\varphi_{\nu_l^c})-\sum_j\sum_k Y_{\nu_{ik}}Y_{\nu_{jk}}v_u^2\nu_i\nu_j\sin(\chi_i-\chi_j)$$
$$-\sum_j(A_\nu Y_\nu)_{ij}v_u\nu_i\nu_j^c\sin(\chi_i+\varphi_{\nu_j^c})=0. \tag{A.7}$$

B Analitical formula for neutrino masses

The formula presented here is obtained from eq. (3.4) neglecting terms proportional to $Y_\nu^2\nu^2$, $Y_\nu^3\nu$ and $Y_\nu\nu^3$, and has been particularized for the simplified case discussed in section 4 of a common value of couplings $\lambda_i\equiv\lambda$, a tensor κ with terms $\kappa_{iii}\equiv\kappa_i\equiv\kappa$ and vanishing otherwise, diagonal Yukawa couplings $Y_{\nu_{ii}}\equiv Y_{\nu_i}$, and a common value of the VEVs $\nu_i^c\equiv\nu^c$. The phase structure of the global minimum discussed in section 4 for analyzing the neutrino sector, $\varphi_{\nu_1^c}=-\varphi_{\nu_2^c}=-\varphi_{\nu_3^c}\equiv\varphi_{\nu^c}$ and $\varphi_{\nu_1}=-\varphi_{\nu_2}=-\varphi_{\nu_3}\equiv\varphi_\nu$, has also been used in the computation. Then we arrive to the following formula:

$$(m_{\rm eff})_{ij}\simeq\frac{X_{ij}}{\Delta}+\frac{T_{ij}}{Z}\,\frac{a_i a_j}{2\kappa\nu^c}, \tag{B.1}$$

where the parameters have been defined as

$$\begin{aligned}
a_i &= Y_{\nu_i}v_u,\\
\Delta &= \left(e^{i\varphi_{\nu^c}}+2e^{i3\varphi_{\nu^c}}\right)\lambda^2\left(v_u^2+v_d^2\right)^2+\left(8e^{i\varphi_{\nu^c}}+4e^{i3\varphi_{\nu^c}}\right)\lambda\kappa\nu^{c2}v_d v_u e^{-i\varphi_v}\\
&\quad-\left(16+16e^{i2\varphi_{\nu^c}}+4e^{i4\varphi_{\nu^c}}\right)M\lambda^2\kappa\nu^{c3}-\left(8+20e^{i2\varphi_{\nu^c}}+8e^{i4\varphi_{\nu^c}}\right)M\lambda^3\nu^c v_d v_u e^{i\varphi_v},\\
Z &= e^{i\varphi_{\nu^c}}\Big[-4e^{i\varphi_{\nu^c}}\left(2+e^{i2\varphi_{\nu^c}}\right)\kappa\nu^{c2}v_d v_u+e^{i\varphi_v}\lambda\left(4M\left(2+e^{i2\varphi_{\nu^c}}\right)^2\kappa\nu^{c3}\right.\\
&\quad\left.-\,e^{i\varphi_{\nu^c}}\left(1+2e^{i2\varphi_{\nu^c}}\right)\left(v_d^2+v_u^2\right)^2\right)+4e^{i2(\varphi_{\nu^c}+\varphi_v)}\lambda^2 M\nu^c v_d v_u\left(5+4\cos 2\varphi_{\nu^c}\right)\Big],
\end{aligned} \tag{B.2}$$

with $\frac{1}{M}=\frac{g_1^2}{M_1}+\frac{g_2^2}{M_2}$,

$$\begin{aligned}
T_{11} &= 2e^{i2\varphi_v}[-4e^{i2(\varphi_{\nu^c}+\varphi_v)}(2+e^{i2\varphi_{\nu^c}})M\lambda^2\nu^c v_d v_u+4e^{i\varphi_{\nu^c}}\kappa\nu^{c^2}v_d v_u\\
&\quad+e^{i\varphi_v}\lambda(-4(2+e^{i6\varphi_{\nu^c}})M\kappa\nu^{c^3}+e^{i3\varphi_{\nu^c}}(v_u^2+v_d^2)^2)],\\
T_{22}=T_{33} &= 2e^{i(\varphi_{\nu^c}+2\varphi_v)}[-4e^{i2(\varphi_{\nu^c}+\varphi_v)}(2+e^{i2\varphi_{\nu^c}})M\lambda^2\nu^c v_d v_u+4e^{i3\varphi_{\nu^c}}\kappa\nu^{c^2}v_d v_u\\
&\quad+e^{i\varphi_v}\lambda(-4(1+e^{i2\varphi_{\nu^c}}+e^{i4\varphi_{\nu^c}})M\kappa\nu^{c^3}+e^{i3\varphi_{\nu^c}}(v_u^2+v_d^2)^2)],
\end{aligned}$$

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$$
\begin{aligned}
T_{12} = T_{13} = &-e^{i2\varphi_v}[-4e^{i2(\varphi_{\nu^c}+\varphi_v)}(2+e^{i2\varphi_{\nu^c}})M\lambda^2\nu^c v_d v_u + 4e^{i3\varphi_{\nu^c}}\kappa{\nu^c}^2 v_d v_u\cos(2\varphi_{\nu^c})\\
&+e^{i(3\varphi_{\nu^c}+\varphi_v)}\lambda(4(-3\cos(3\varphi_{\nu^c})+i\sin(3\varphi_{\nu^c}))M\kappa{\nu^c}^3+(v_u^2+v_d^2)^2)],\\
T_{23} = &-e^{i2\varphi_v}[-4e^{i2(2\varphi_{\nu^c}+\varphi_v)}(2+e^{i2\varphi_{\nu^c}})M\lambda^2\nu^c v_d v_u + 4e^{i3\varphi_{\nu^c}}\kappa{\nu^c}^2 v_d v_u\\
&+e^{i\varphi_v}\lambda(-4(-1+4e^{i3\varphi_{\nu^c}}\cos(\varphi_{\nu^c}))M\kappa{\nu^c}^3+e^{i5\varphi_{\nu^c}}(v_u^2+v_d^2)^2)],
\end{aligned}
\tag{B.3}
$$

and

$$
\begin{aligned}
X_{11} &= 2\kappa{\nu^c}^3(b_{11})^2 + 2\lambda\nu^c v_d v_u e^{i\varphi_v}(b'_{11})^2+\epsilon_{11},\\
X_{22} &= 2\kappa{\nu^c}^3(b_{22})^2 + 2\lambda\nu^c v_d v_u e^{i\varphi_v}(b'_{22})^2+\epsilon_{22},\\
X_{33} &= 2\kappa{\nu^c}^3(b_{33})^2 + 2\lambda\nu^c v_d v_u e^{i\varphi_v}(b'_{33})^2+\epsilon_{33},\\
X_{12} &= 2\kappa{\nu^c}^3(b_{11})(b_{22}) + 2\lambda\nu^c v_d v_u e^{i\varphi_v}(b'_{12})^2+\epsilon_{12},\\
X_{13} &= 2\kappa{\nu^c}^3(b_{11})(b_{33}) + 2\lambda\nu^c v_d v_u e^{i\varphi_v}(b'_{13})^2+\epsilon_{13},\\
X_{23} &= 2\kappa{\nu^c}^3(b_{22})(b_{33}) + 2\lambda\nu^c v_d v_u e^{i\varphi_v}(b'_{23})^2+\epsilon_{23},
\end{aligned}
\tag{B.4}
$$

with

$$
\begin{aligned}
(b_{11}) &= (2+e^{i2\varphi_{\nu^c}})\lambda e^{-i\varphi_\nu}\nu_1 + e^{i2\varphi_{\nu^c}}v_d Y_{\nu_1},\\
(b_{22}) &= (2+e^{i2\varphi_{\nu^c}})\lambda e^{i\varphi_\nu}\nu_2 + v_d Y_{\nu_2},\\
(b_{33}) &= (2+e^{i2\varphi_{\nu^c}})\lambda e^{i\varphi_\nu}\nu_3 + v_d Y_{\nu_3},\\
(b'_{11})^2 &= (2+5e^{i2\varphi_{\nu^c}}+2e^{i4\varphi_{\nu^c}})\lambda^2 e^{-i2\varphi_\nu}\nu_1^2\\
&\quad+(2+2e^{i2\varphi_{\nu^c}}+2e^{i4\varphi_{\nu^c}})\lambda v_d e^{-i\varphi_\nu}\nu_1 Y_{\nu_1}+e^{i2\varphi_{\nu^c}}v_d^2 Y_{\nu_1}^2,\\
(b'_{22})^2 &= (2+5e^{i2\varphi_{\nu^c}}+2e^{i4\varphi_{\nu^c}})\lambda^2 e^{i2\varphi_\nu}\nu_2^2\\
&\quad+(1+4e^{i2\varphi_{\nu^c}}+e^{i4\varphi_{\nu^c}})\lambda v_d e^{i\varphi_\nu}\nu_2 Y_{\nu_2}+e^{i2\varphi_{\nu^c}}v_d^2 Y_{\nu_2}^2,\\
(b'_{33})^2 &= (2+5e^{i2\varphi_{\nu^c}}+2e^{i4\varphi_{\nu^c}})\lambda^2 e^{i2\varphi_\nu}\nu_3^2\\
&\quad+(1+4e^{i2\varphi_{\nu^c}}+e^{i4\varphi_{\nu^c}})\lambda v_d e^{i\varphi_\nu}\nu_3 Y_{\nu_3}+e^{i2\varphi_{\nu^c}}v_d^2 Y_{\nu_3}^2,\\
(b'_{12})^2 &= (2+5e^{i2\varphi_{\nu^c}}+2e^{i4\varphi_{\nu^c}})\lambda^2\nu_1\nu_2+(1+e^{i2\varphi_{\nu^c}}+e^{i4\varphi_{\nu^c}})\lambda v_d e^{i\varphi_\nu}\nu_2 Y_{\nu_1}\\
&\quad+((1/2)+2e^{i2\varphi_{\nu^c}}+(1/2)e^{i4\varphi_{\nu^c}})\lambda v_d e^{-i\varphi_\nu}\nu_1 Y_{\nu_2}+(1/2)(1+e^{i4\varphi_{\nu^c}})v_d^2 Y_{\nu_1}Y_{\nu_2},\\
(b'_{13})^2 &= (2+5e^{i2\varphi_{\nu^c}}+2e^{i4\varphi_{\nu^c}})\lambda^2\nu_1\nu_3+(1+e^{i2\varphi_{\nu^c}}+e^{i4\varphi_{\nu^c}})\lambda v_d e^{i\varphi_\nu}\nu_3 Y_{\nu_1}\\
&\quad+((1/2)+2e^{i2\varphi_{\nu^c}}+(1/2)e^{i4\varphi_{\nu^c}})\lambda v_d e^{-i\varphi_\nu}\nu_1 Y_{\nu_3}+(1/2)(1+e^{i4\varphi_{\nu^c}})v_d^2 Y_{\nu_1}Y_{\nu_3},\\
(b'_{23})^2 &= (2+5e^{i2\varphi_{\nu^c}}+2e^{i4\varphi_{\nu^c}})\lambda^2 e^{i2\varphi_\nu}\nu_2\nu_3\\
&\quad+((1/2)+2e^{i2\varphi_{\nu^c}}+(1/2)e^{i4\varphi_{\nu^c}})\lambda v_d e^{i\varphi_\nu}(\nu_3 Y_{\nu_2}+\nu_2 Y_{\nu_3})+e^{i2\varphi_{\nu^c}}v_d^2 Y_{\nu_2}Y_{\nu_3}
\end{aligned}
\tag{B.5}
$$

and

$$
\begin{aligned}
\epsilon_{11} &= (4e^{i4\varphi_{\nu^c}}-4)\lambda^2\nu^c v_u^3 e^{i\varphi_v}e^{-i\varphi_\nu}\nu_1 Y_{\nu_1},\\
\epsilon_{22} &= (2-2e^{i4\varphi_{\nu^c}})\lambda^2\nu^c v_u^3 e^{i\varphi_v}e^{i\varphi_\nu}\nu_2 Y_{\nu_2},\\
\epsilon_{33} &= (2-2e^{i4\varphi_{\nu^c}})\lambda^2\nu^c v_u^3 e^{i\varphi_v}e^{i\varphi_\nu}\nu_3 Y_{\nu_3},\\
\epsilon_{12} &= (2e^{i4\varphi_{\nu^c}}-2)\lambda^2\nu^c v_u^3 e^{i\varphi_v}e^{i\varphi_\nu}\nu_2 Y_{\nu_1}+(1-e^{i4\varphi_{\nu^c}})\lambda^2\nu^c v_u^3 e^{i\varphi_v}e^{-i\varphi_\nu}\nu_1 Y_{\nu_2},\\
\epsilon_{13} &= (2e^{i4\varphi_{\nu^c}}-2)\lambda^2\nu^c v_u^3 e^{i\varphi_v}e^{i\varphi_\nu}\nu_3 Y_{\nu_1}+(1-e^{i4\varphi_{\nu^c}})\lambda^2\nu^c v_u^3 e^{i\varphi_v}e^{-i\varphi_\nu}\nu_1 Y_{\nu_3},\\
\epsilon_{23} &= (1-e^{i4\varphi_{\nu^c}})\lambda^2\nu^c v_u^3 e^{i\varphi_v}e^{i\varphi_\nu}(\nu_3 Y_{\nu_2}+\nu_2 Y_{\nu_3}).
\end{aligned}
\tag{B.6}
$$

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Let us discuss two particular limits where the formula becomes simple. In the limit $M \to \infty$ and $v_d \to 0$ we obtain

$$(m_{\rm eff})_{ij} \simeq F_{ij}\ \frac{a_i a_j}{2\kappa\nu^c}, \tag{B.7}$$

where

$$\begin{aligned}
F_{11} &= -2e^{i(2\varphi_v-\varphi_{\nu^c})}(2+e^{i6\varphi_{\nu^c}})\left(2+e^{2i\varphi_{\nu^c}}\right)^{-2},\\
F_{22} &= F_{33} = -2e^{i(2\varphi_v-\varphi_{\nu^c})}(1+e^{i2\varphi_{\nu^c}}+e^{i4\varphi_{\nu^c}})\left(2+e^{2i\varphi_{\nu^c}}\right)^{-2},\\
F_{12} &= F_{13} = e^{i2(\varphi_{\nu^c}+\varphi_v)}(3\cos(3\varphi_{\nu^c})-i\sin(3\varphi_{\nu^c})\left(2+e^{2i\varphi_{\nu^c}}\right)^{-2},\\
F_{23} &= e^{i(2\varphi_v-\varphi_{\nu^c})}(4e^{i3\varphi_{\nu^c}}\cos(\varphi_{\nu^c})-1)\left(2+e^{2i\varphi_{\nu^c}}\right)^{-2}.
\end{aligned} \tag{B.8}$$

In the limit of vanishing phases, i.e. real VEVs, we obtain

$$\begin{aligned}
(m_{eff|real})_{ij} \simeq\ & \frac{2}{3}\frac{(\kappa{\nu^c}^2+\lambda v_u v_d)\nu^c}{\lambda^2(v_u^2+v_d^2)^2+4\lambda\kappa{\nu^c}^2 v_u v_d-12M\lambda(\kappa{\nu^c}^2+\lambda v_u v_d)\lambda\nu^c}\ b_i b_j\\
&+\frac{1}{6\kappa\nu^c}(1-3\delta_{ij})a_i a_j,
\end{aligned} \tag{B.9}$$

where we have defined

$$b_i = Y_{\nu_i} v_d + 3\lambda\nu_i. \tag{B.10}$$

Regarding the previous parameters we note that for the real case

$$\begin{aligned}
b_i = b_{ii} &= b'_{ii},\\
b'^2_{ij} = b_{ii}\, b_{jj} &= b_i\, b_j,\\
\epsilon_{ij} &= 0.
\end{aligned} \tag{B.11}$$

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