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A dual geometry of the hadron in dense matter

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ABSTRACT: We identify the dual geometry of the hadron phase of dense nuclear matter and investigate the confinement/deconfinement phase transition. We suggest that the low temperature phase of the RN black hole with the full backreaction of the bulk gauge field is described by the zero mass limit of the RN black hole with hard wall. We calculated the density dependence of critical temperature and found that the phase diagram closes. We also study the density dependence of the ρ meson mass.

Contents

1 Introduction

Recently, related to the RHIC and LHC experiments, understanding strongly interacting QCD is requesting much attention. Although a powerful method for this subject, the lattice QCD, is being developed, when it comes to the dense matter problem, lattice calculation has difficulty and not much result is produced so far. The AdS/CFT correspondence [1] in the string theory, can shed many aspect of hadron theory [2–10] as well as in strongly interacting quark gluon plasma [11–16]. The theory can easily accommodate the dense matter problem at least for deconfined phase [4–6, 17]. However, for the the hadron phase, the status is not very clear since even the phase diagram is qualitatively different from that of the real QCD [4, 18]. This may be traced back to the probe approach of the dense matter and one expect that if one fully account the back reaction of the gravity to the dense matter, one might overcome the situation.

In fact, in the previous work [10], it was suggested that if one considers bulk filling branes, one can easily account the full back reaction and the phase diagram actually closes. However, in that work, the back reacted geometry of the hadronic phase was not fully identified but was approximately treated as the thermal ads with electric potential.

In this paper we consider the zero black hole mass limit of the charged black hole with hard wall installed as the low energy pair of the charged black hole. Although it has the naked singularity at the origin it is hidden in the wall and we do not find any physical difficulty. With this identification, density dependence of the physical quantities can be easily calculated. As a first example, we calculated how meson spectrum depends on the density of the baryonic medium.

The rest of paper follows: In the section 2, we will briefly review the dual geometry for the quark-gluon plasma and then propose what is the dual geometry for the hadronic phase. In the section 3, we will investigate the Hawking-Page transition. We will first study the fixed chemical potential case and then consider the fixed density problem. In the section 4, we will investigate the mass of the excited vector mesons in dense medium. In the section 5, we will summarize our results and discuss some future works.

2 Dual geometry for QCD with quark matters

In AdS/CFT correspondence, the boundary value of the bulk gauge field is coupled to the dual operator in the QCD side, which is the quark current. Furthermore the boundary value of the time-component gauge field is the chemical potential its dual operator is the quark number density. Our main interest here is to see how the critical temperature of the phase transition depends on the quark chemical potential. To describe the region for the high chemical potential, we need to consider the back reaction of the bulk gauge field in the dual geometry. Here, we will investigate the asymptotically AdS geometry dual to QCD with hadronic matters.

We first review the gravity theory in the Mikowskian signature with introducing our conventions. The gravity action describing the five-dimensional asymptotic AdS space with the gauge field is given by

$$S_M = \int d^5x \sqrt{-G} \left[\frac{1}{2\kappa^2} \left(\mathcal{R} - 2\Lambda \right) - \frac{1}{4g^2} F_{MN} F^{MN} \right], \tag{2.1}$$

where $2\kappa^2$ is proportional to the five-dimensional Newton constant and g^2 is a five-dimensional gauge coupling constant. In the five dimensional AdS space, the cosmological constant is given by $\Lambda = -6/R^2$, where R is the radius of the AdS space. The equations of motion of this system becomes

$$\mathcal{R}_{MN} - \frac{1}{2} G_{MN} \mathcal{R} + G_{MN} \Lambda = \frac{\kappa^2}{g^2} \left(F_{MP} F_N^P - \frac{1}{4} G_{MN} F_{PQ} F^{PQ} \right),$$
$$0 = \partial_M \sqrt{-G} G^{MP} G^{NQ} F_{PQ}, \tag{2.2}$$

where $M, N = 0, 1, \ldots, 4$, $x^0 = t$ and $x^4 = z$. Under the following ansatz

$$\begin{aligned} A_0 &= A_0(z), \\ A_i &= A_4 = 0 \quad (i = 1, \ldots, 3), \\ ds^2 &= \frac{R^2}{z^2} \left(-f(z)dt^2 + dx_i^2 + \frac{1}{f(z)} dz^2 \right), \end{aligned} \tag{2.3}$$

the most general solution of eq. (2.2) known as the Reissner-Nordstrom AdS black hole (RNAdS BH) is

$$\begin{aligned} f(z) &= 1 - mz^4 + q^2 z^6, \\ A_0 &= \mu - Q z^2, \end{aligned} \tag{2.4}$$

where the boundary space, denoted by $x^\mu = \{x^0, x^i\}$, is located at $z = 0$. So, μ is a boundary value of A_0 and Q is related to the black hole charge q through

$$q^2 = \frac{2\kappa^2}{3g^2 R^2} Q^2. \tag{2.5}$$

In the AdS/QCD context [10], the gravitation constant $2\kappa^2$ and the five-dimensional coupling constant g^2 are related to the rank of the gauge group N_c and the number of the flavor N_f in QCD

$$\frac{1}{2\kappa^2} = \frac{N_c^2}{8\pi^2 R^3} \quad \text{and} \quad \frac{1}{g^2} = \frac{N_c N_f}{4\pi^2 R}, \tag{2.6}$$

so that eq. (2.5) can be rewritten as

$$Q = \sqrt{\frac{3}{2} \frac{N_c}{N_f}} \; q. \tag{2.7}$$

2.1 Dual geometry of the quark-gluon plasma

For investigating the Hawking-Page transition dual to the C/D phase transition in QCD, it is more convenient to consider the Euclidean version. Here, we summarize the Euclidean RNAdS BH shortly, which corresponds to the deconfinement phase described by the quark-gluon plasma.

By the Wick rotation $t \to -i\tau$, the Euclidean version of the previous action in eq. (2.1) reads

$$S = \int d^5 x \sqrt{G} \left[\frac{1}{2\kappa^2} (-\mathcal{R} + 2\Lambda) + \frac{1}{4g^2} F_{MN} F^{MN} \right], \tag{2.8}$$

where G_{MN} is the Euclidean metric ansatz

$$ds^2 = \frac{R^2}{z^2} \left(f(z) d\tau^2 + d\vec{x}^2 + \frac{1}{f(z)} dz^2 \right). \tag{2.9}$$

The equations of motion for this system become

$$\mathcal{R}_{MN} - \frac{1}{2} G_{MN} \mathcal{R} + G_{MN} \Lambda = \frac{\kappa^2}{g^2} \left(F_{MP} F_N^P - \frac{1}{4} G_{MN} F_{PQ} F^{PQ} \right),$$
$$0 = \partial_M \sqrt{G} G^{MP} G^{NQ} F_{PQ}. \tag{2.10}$$

Under the following ansatz

$$A_\tau = A(z),$$
$$A_i = A_4 = 0, \tag{2.11}$$

the most general solution is nothing but the Euclidean RNAdS BH, so that the metric factor $f(z)$ is the same as one in the Minkowski version. Finally, the metric solution is

$$ds^2 = \frac{R^2}{z^2} \left((1 - m z^4 + q^2 z^6) d\tau^2 + d\vec{x}^2 + \frac{1}{1 - m z^4 + q^2 z^6} dz^2 \right). \tag{2.12}$$

– 3 –

From now on, we consider the Euclidean version only. Because $F_{zt} = -F_{tz} = \partial_z A_t$, the Maxwell equation can be reduced to the simple form

$$0 = \partial_z \left(\frac{R}{z} \, \partial_z A(z) \right), \tag{2.13}$$

and the solution is given by

$$A(z) = i \left(\mu - Q z^2 \right). \tag{2.14}$$

Note that the imaginary number, i, in the above is very important to satisfy the Einstein equation in eq. (2.10), which naturally appears due to the Wick rotation.

From the metric in eq. (2.12), the outer horizon denoted by r_+ should satisfy

$$0 = f(z_+) = 1 - m z_+^4 + q^2 z_+^6. \tag{2.15}$$

Using the above, we can replace the black hole mass m with a function of the outer horizon z_+ and the black hole charge q

$$m = \frac{1}{z_+^4} + q^2 z_+^2, \tag{2.16}$$

which is useful for the later convenience. The Hawking temperature of the RNAdS BH is given by

$$T_{RN} = \frac{1}{\pi z_+} \left(1 - \frac{1}{2} q^2 z_+^6 \right). \tag{2.17}$$

For the norm of $||A(z)|| \equiv g^{\tau\tau} A_\tau A_\tau$ at the black hole horizon to be regular, we should impose the Dirichlet boundary condition $A(z_+) = 0$ [4, 5, 10, 19, 20], which gives a relation between Q and μ

$$Q^2 = \frac{\mu^2}{z_+^4}. \tag{2.18}$$

Inserting this relation and eq. (2.5) into eq. (2.17), we can find z_+ as a function of μ and T_{RN}

$$z_+ = \frac{3g^2 R^2}{2\kappa^2 \mu^2} \left(\sqrt{\pi^2 T_{RN}^2 + \frac{4\kappa^2 \mu^2}{3g^2 R^2}} - \pi T_{RN} \right). \tag{2.19}$$

To describe a system having the fixed chemical potential, we should impose the Dirichlet boundary condition, $A(0) = i\mu$, at the UV cut-off $z = \epsilon$ where ϵ is very small. Then, the on-shell action becomes

$$S_{RN}^D = \frac{V_3 R^3}{\kappa^2} \frac{1}{T_{RN}} \left(\frac{1}{\epsilon^4} - \frac{1}{z_+^4} - \frac{2\kappa^2}{3g^2 R^2} \frac{\mu^2}{z_+^2} \right), \tag{2.20}$$

where V_3 corresponds to the spatial volume of the boundary space. In the above action, the superscript D and the subscript RN imply the Dirichlet boundary condition at the UV cut-off and the RNAdS BH, respectively. Since this action has a divergent term when $\epsilon \to 0$, we should renormalize it. For the renomalization, we use the background subtraction method,

in which the on-shell action for the AdS space is subtracted from S_{RN}^D. The on-shell action for the AdS space is

$$
\begin{aligned}
S_{AdS} &= \frac{V_3}{2\kappa^2} \int_0^\beta d\tau \int_\epsilon^\infty dz \ \sqrt{G} \left(-\mathcal{R} + 2\Lambda \right) \\
&= \frac{V_3 R^3}{\kappa^2} \frac{\beta}{\epsilon^4}.
\end{aligned} \tag{2.21}
$$

After identifying the circumference of the RNAdS BH and the AdS space at the boundary, we can rewrite β as

$$
\beta = \frac{1}{T_{RN}} \left(1 - \frac{1}{2} m\epsilon^4 + \mathcal{O}(\epsilon^6) \right). \tag{2.22}
$$

Therefore, the on-shell action of the AdS space becomes

$$
S_{AdS} = \frac{V_3 R^3}{\kappa^2} \frac{1}{T_{RN}} \left(\frac{1}{\epsilon^4} - \frac{1}{2z_+^4} - \frac{\kappa^2}{3g^2 R^2} \frac{\mu^2}{z_+^2} \right), \tag{2.23}
$$

where eq. (2.5), eq. (2.16) and eq. (2.18) are used. As a result, the renormalized action of the RNAdS BH is given by

$$
\begin{aligned}
\bar{S}_{RN}^D &= S_{RN}^D - S_{AdS} \\
&= -\frac{V_3 R^3}{\kappa^2} \frac{1}{T_{RN}} \left(\frac{1}{2z_+^4} + \frac{\kappa^2}{3g^2 R^2} \frac{\mu^2}{z_+^2} \right),
\end{aligned} \tag{2.24}
$$

and the grand potential becomes

$$
\begin{aligned}
\Omega_{RN} &= \bar{S}_{RN}^D T_{RN} \\
&= -\frac{V_3 R^3}{\kappa^2} \left(\frac{1}{2z_+^4} + \frac{\kappa^2}{3g^2 R^2} \frac{\mu^2}{z_+^2} \right).
\end{aligned} \tag{2.25}
$$

For describing the dependence of the particle number in this system, we should consider the free energy F obtained by the Legendre transformation of the grand potential. In the thermodynamics, it is given by

$$
F = \Omega + \mu N, \tag{2.26}
$$

where $N = -\frac{\partial \Omega}{\partial \mu}$. Using the fact that z_+ can be represented as a function of μ and T like eq. (2.19), N is given by

$$
N = \frac{2R}{g^2} Q V_3, \tag{2.27}
$$

where the particle number N is proportional to the quark number density Q. As a result, the free energy becomes

$$
F = \Omega + \frac{2R}{g^2} \mu Q V_3. \tag{2.28}
$$

Interestingly, this result can be reobtained from the action eq. (2.8) with the different boundary condition at the UV cut-off. For changing the Dirichlet boundary condition into the Neumann boundary condition, we should add a boundary term to fix Q. Then, the renormalized action is given by

$$
\bar{S}_{RN}^N = \bar{S}_{RN}^D + S_b, \tag{2.29}
$$

where the superscript N implies the Neumann boundary condition at the UV cut-off and the boundary action S_b is given by

$$S_b = \frac{1}{g^2} \int_{\partial \mathcal{M}} d^4x \sqrt{G^{(4)}} \, n^M A^N F_{MN}. \tag{2.30}$$

In the above, the unit normal vector is given by $n^M = \left\{ 0, 0, 0, 0, \frac{z}{R}\sqrt{f(z)} \right\}$ and $G^{(4)} = \frac{R^8}{z^8} f(z)$ is a determinant of the four-dimensional boundary metric. Using the solution for the bulk gauge field in eq. (2.14), the boundary term becomes

$$S_b = \frac{V_3}{T_{RN}} \frac{2R}{g^2} \mu Q. \tag{2.31}$$

So, the free energy reads

$$F = \bar{S}_{RN}^N T_{RN} = \Omega + \frac{2R}{g^2} \mu Q V_3, \tag{2.32}$$

which is the same as one obtained from the thermodynamics in eq. (2.28). From these results, we may conclude that the five-dimensional bulk action with the Dirichlet or Neumann boundary condition at the UV-cut off corresponds to the grand potential or free energy of the dual QCD, respectively.

2.2 Dual geometry of the hadronic phase

In QCD, it is well known that there exist the hadronic or confinement phase at the low temperature. In the absence of quark matters, the Schwarzschild AdS black hole (SAdS BH) corresponds to the deconfinement phase. At the low temperature, the dual geometry for the confinement phase is given by the thermal AdS with the IR cut-off, which is needed to explain the confining behavior [21]. What is the dual geometry describing the hadronic phase, in other words the confinement phase with quark matters? As shown in the previous section, we should include the bulk gauge field to explain the quark matters. Therefore, the geometry corresponding to the hadronic phase has to be a deformed AdS including the backreaction of the bulk gauge field, which is not a black hole. The metric we find out to answer the above question, is

$$ds^2 = \frac{R^2}{z^2} \left((1 + q^2 z^6) d\tau^2 + d\vec{x}^2 + \frac{1}{1 + q^2 z^6} dz^2 \right), \tag{2.33}$$

which together with eq. (2.14) satisfies the Einstein and Maxwell equation and becomes a AdS space asymptotically. This solution can be also easily obtained from the RNAdS BH by taking $m = 0$. For the convenience, we call this solution as a thermal charged AdS (tcAdS) solution. Especially, in the case of $q = 0$, the tcAdS and RNAdS BH are reduced to the thermal AdS (tAdS) and SAdS BH, respectively. Here, our proposition is that the tcAdS having the IR cut-off z_{IR} is the dual geometry corresponding to the hadronic phase.

Now, we study the thermodynamics of the tcAdS with the fixed chemical potential, for which the Dirichlet boundary condition is needed at the UV cut-off. At the IR cut-off we impose another Dirichlet boundary condition

$$A(z_{IR}) = i\alpha\mu, \tag{2.34}$$

where α is a arbitrary constant and will be determined later. This IR boundary condition together with eq. (2.14) gives a relation between μ and Q

$$Q = \frac{(1-\alpha)\mu}{z_{IR}^2}. \tag{2.35}$$

Using this, the on-shell action of the tcAdS is given by

$$S_{tc}^D = \frac{V_3 R^3}{\kappa^2} \frac{1}{T_{tc}} \left(\frac{1}{\epsilon^4} - \frac{1}{z_{IR}^4} - \frac{2\kappa^2}{3g^2 R^2} \frac{(1-\alpha)^2 \mu^2}{z_{IR}^2} \right), \tag{2.36}$$

where the subscript tc means the tcAdS. To renormalize this action, we subtract the on-shell action of the AdS space in eq. (2.21), with the identification between the circumferences of two backgrounds at the UV cut-off

$$\beta = \frac{1}{T_{tc}} \left(1 + \mathcal{O}(\epsilon^6) \right). \tag{2.37}$$

Then, in the limit of $\epsilon \to 0$ the renormalized action of the tcAdS becomes

$$\bar{S}_{tc}^D = -\frac{V_3 R^3}{\kappa^2} \frac{1}{T_{tc}} \left(\frac{1}{z_{IR}^4} + \frac{2\kappa^2}{3g^2 R^2} \frac{(1-\alpha)^2 \mu^2}{z_{IR}^2} \right). \tag{2.38}$$

From this, the particle number N is given by

$$N = \frac{2}{3}(1-\alpha) \frac{2R}{g^2} Q V_3. \tag{2.39}$$

Since the action with the Neumann condition should be a free energy as previously mentioned, $\mu N = S_b T_{tc}$ should be satisfied. Using the fact that the boundary action S_b of the tcAdS is given by

$$S_b = \frac{\mu}{T_{tc}} \frac{2R}{g^2} Q V_3, \tag{2.40}$$

α should becomes $-1/2$ for the consistency. As a result, the renormalized action becomes

$$\bar{S}_{tc}^D = -\frac{V_3 R^3}{\kappa^2} \frac{1}{T_{tc}} \left(\frac{1}{z_{IR}^4} + \frac{3\kappa^2}{2g^2 R^2} \frac{\mu^2}{z_{IR}^2} \right), \tag{2.41}$$

with the following relation

$$\mu = \frac{2}{3} Q z_{IR}^2. \tag{2.42}$$

3 Confinement/deconfinement phase transition

In QCD, there exists the C/D phase transition, which is dual to the Hawking-Page transition in the gravity theory side. So it is an interesting question to ask how the C/D phase transition depends on the quark matters. As mentioned previously, we should introduce the IR cut-off to describe the confining behavior of the hadronic phase, which is called the hard wall model. There were many interesting works to explain QCD depending on quark matters. In our previous work [9], we studied the dependence of the quark (number) density in the C/D phase transition by considering the gauge field fluctuation on the tAdS and

SAdS BH backgroud. Unfortunately, its result is valid only in the regime of the low quark number density so that it can not explain the dependence of the high chemical potential and quark density. Anyway, the crucial point of the tcAdS and RNAdS BH backgrounds is that since they include the full backreaction of the gauge field, we can investigate the dependence of quark matters even in the high chemical potential or quark density regime using this model.

3.1 Fixed quark chemical potential

For the fixed chemical potential, to describe the Hawking-Page transition we calculate the difference between the on-shell actions of two backgrounds, with the Dirichlet boundary condition at the UV cut-off,

$$\Delta S = S_{RN}^D - S_{tc}^D, \tag{3.1}$$

with

$$S_{RN}^D = \frac{V_3 R^3}{\kappa^2} \frac{1}{T_{RN}} \left(\frac{1}{\epsilon^4} - \frac{1}{z_+^4} - \frac{2\kappa^2}{3g^2 R^2} \frac{\mu^2}{z_+^2} \right),$$
$$S_{tc}^D = \frac{V_3 R^3}{\kappa^2} \frac{1}{T_{tc}} \left(\frac{1}{\epsilon^4} - \frac{1}{z_{IR}^4} - \frac{3\kappa^2}{2g^2 R^2} \frac{\mu^2}{z_{IR}^2} \right). \tag{3.2}$$

After requiring the same circumference of τ at the UV cut-off, the difference becomes

$$\Delta S = \frac{V_3 R^3}{\kappa^2} \frac{1}{T_{RN}} \left(\frac{1}{z_{IR}^4} - \frac{1}{2z_+^4} + \frac{3\kappa^2}{2g^2 R^2} \frac{\mu^2}{z_{IR}^2} - \frac{\kappa^2}{3g^2 R^2} \frac{\mu^2}{z_+^2} \right), \tag{3.3}$$

which is the same as one obtained from the renormalized actions in the limit of $\epsilon \to 0$. The Hawking-Page transition corresponding the C/D phase transition occurs at $\Delta S = 0$. Note that the C/D phase transition occurs only in the range of $z_+ \leq z_{IR}$. So we will consider the case of $z_+ > z_{IR}$ from now on. Suppose that ΔS is zero at a critical point $z_+ = z_c$. In the case of $z_+ < z_c$, ΔS becomes negative. So the RNAdS BH is stable, which implies that the dual boundary theory is described by quark-gluon plasma or the deconfinement phase. In the Schwarzschild black hole case, since there is no black hole charge, the dual theory is described by the gluon only, without including quark matters. Anyway, for $z_c < z_+ \leq z_{IR}$ the stable space is the tcAdS. Since the tcAdS corresponds to the confinement phase, the dual QCD describes the hadronic matters.

For the later convenience, we introduce dimensionless variables as the following

$$\tilde{z}_c \equiv \frac{z_c}{z_{IR}},$$
$$\tilde{\mu}_c \equiv \mu_c z_{IR},$$
$$\tilde{T}_c \equiv T_c z_{IR}, \tag{3.4}$$

where the subscript c means the critical values representing the C/D phase transition point. Then, the critical chemical potential $\tilde{\mu}_c$ and temperature $\tilde{T}_c$ can be represented as functions
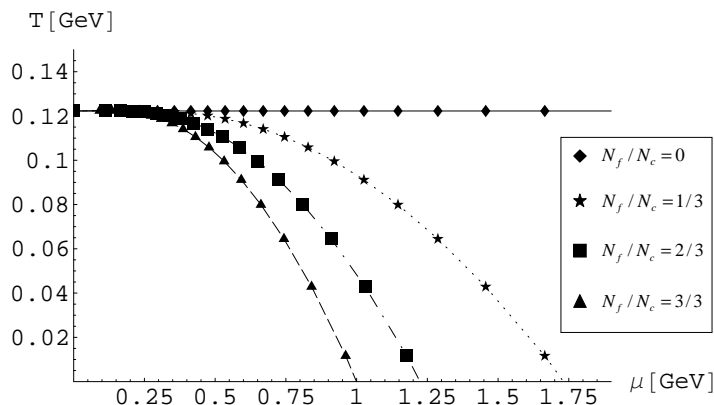
Figure 1. The deconfinement temperature depending on the chemical potential, which does not contain the quark mass.

of $\tilde{z}_c$

$$\tilde{\mu}_c = \sqrt{\frac{3N_c}{N_f} \frac{(1 - 2\tilde{z}_c^4)}{\tilde{z}_c^2 (9\tilde{z}_c^2 - 2)}},$$

$$\tilde{T}_c = \frac{1}{\pi \tilde{z}_c} \left(1 - \frac{1 - 2\tilde{z}_c^4}{9\tilde{z}_c^2 - 2}\right). \tag{3.5}$$

To obtain a well-defined chemical potential, the inside of the square root in the above should become positive. So the allowed range of $\tilde{z}_c$ is give by $0.4714 \lesssim \tilde{z}_c \lesssim 0.8409$. Though at $\tilde{\mu}_c = 0$ the deconfinement temperature $T_c = 122$ MeV is lower than the lattice estimation $T_c = 175 \sim 190$ MeV [22, 23] as noted by Herzog [21], we use $T_c = 122$ MeV and look at the qualitative behavior of the deconfinement temperature depending on the chemical potential. We obtain the expected phase diagram, which is given in the figure 1. Notice that the deconfinement temperature decreases more quickly as the ratio N_f/N_c goes up. In the holographic models with probe brane approach, the phase diagram does not close. However, in our case the phase diagram closes due to the backreaction of the gauge field as emphasized in ref. [10]. In the zero temperature the critical value of the phase transition, $\mu_c(T_c = 0)$ is given by 1002 MeV for $N_f/N_c = 1$. Note that our definition of the chemical potential does not include quark mass. From this, we can evaluate the the critical baryon chemical potential with mass,

$$\bar{\mu}_B = 3\mu + m_B \approx 4\text{GeV}, \tag{3.6}$$

where m_B is the baryon mass. For more realistic value, we should use different normalization or other models like the one suggested in ref. [24].

3.2 Fixed quark number density

For the fixed quark number density, we should add a boundary term for fixing Q, which corresponds to the Neumann boundary condition at the UV cut-off. The on-shell actions
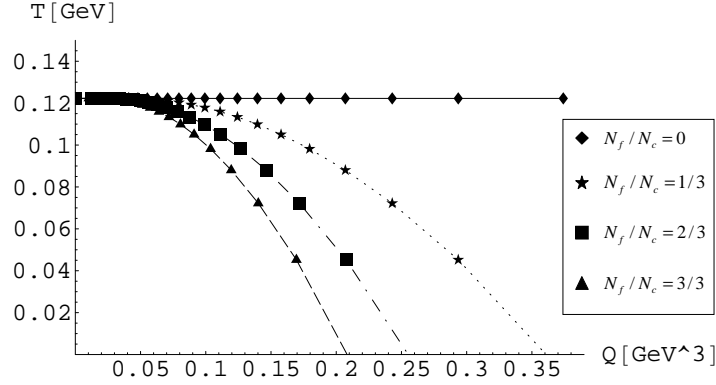
Figure 2. The Hawking-Page transition temperature depending on the quark number density.

of the RNAdS BH and the tcAdS are given by

$$
\begin{aligned}
S_{RN}^N &= \frac{V_3 R^3}{\kappa^2} \frac{1}{T_{RN}} \left(\frac{1}{\epsilon^4} - \frac{1}{z_+^4} + \frac{4\kappa^2 Q^2}{3g^2 R^2} z_+^2 \right), \\
S_{tc}^N &= \frac{V_3 R^3}{\kappa^2} \frac{1}{T_{tc}} \left(\frac{1}{\epsilon^4} - \frac{1}{z_{IR}^4} + \frac{2\kappa^2 Q^2}{3g^2 R^2} z_{IR}^2 \right).
\end{aligned}
\tag{3.7}
$$

Then, the difference between two on-shell actions becomes

$$
\begin{aligned}
\Delta S &\equiv S_{RN}^N - S_{tc}^N \\
&= \frac{V_3 R^3}{\kappa^2} \frac{1}{T_{RN}} \left[\frac{1}{z_{IR}^4} - \frac{1}{2z_+^4} + \frac{\kappa^2 Q^2}{3g^2 R^2} \left(5z_+^2 - 2z_{IR}^2 \right) \right].
\end{aligned}
\tag{3.8}
$$

Like the previous section, we assume that z_c is a critical value of z_+ where the C/D phase transition occurs. For $z_+ < z_c$, the RNAdS BH is more stable than the tcAdS. On the contrary, the tcAdS is stable for $z_+ > z_c$.

After introducing a new dimensionless variable, $\tilde{Q}_c = Q_c z_{IR}^3$, the dimensionless quark number density and critical temperature are given as functions of $\tilde{z}_c$

$$
\begin{aligned}
\tilde{Q}_c &= \sqrt{\frac{3N_c}{2N_f} \frac{(1 - 2\tilde{z}_c^4)}{\tilde{z}_c^4(5\tilde{z}_c^2 - 2)}}, \\
\tilde{T}_c &= \frac{1}{\pi \tilde{z}_c} \left[1 - \frac{\tilde{z}_c^2}{2} \frac{(1 - 2\tilde{z}_c^4)}{(5\tilde{z}_c^2 - 2)} \right].
\end{aligned}
\tag{3.9}
$$

In the first relation, $\tilde{Q}_c$ is well defined only in the range, $\sqrt{\frac{2}{5}} \leq \tilde{z}_c \leq \frac{1}{2^{1/4}}$, which is $0.6325 \lesssim \tilde{z}_c \lesssim 0.8409$. Using eq. (3.9), we numerically draw the deconfinement temperature depending on the quark density in the figure 2. As shown in the figure 2, the deconfinement temperature decreases as the quark density increases. Furthermore, when the number of flavor becomes large, the deconfinement temperature decreases more quickly than one having the smaller number of flavor.
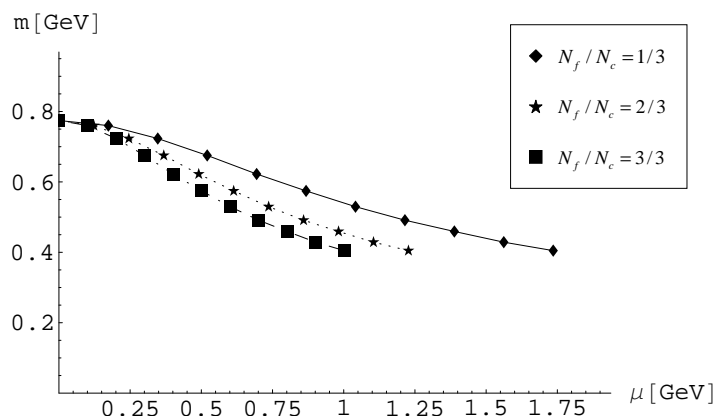
Figure 3. The ρ meson mass depending on the chemical potential.

4   The mass of the excited vector mesons

At first, we consider the ρ meson mass at zero temperature and finite baryon chemical potential μ_B, which is given by three times as much as the quark chemical potential, $\mu_B = 3\mu_q$. To describe this, we should consider the fluctuation of the vector field in the tcAdS. The ansatz for the vector field fluctuation is given by

$$\delta A_\mu = V_\mu(z, p)e^{ip \cdot x}. \tag{4.1}$$

Since the rotation symmetry in the τ-x_i plane is broken, we should distinguish the δA_0 with δA_i. The equation of motion for δA_0 is the exactly same as one obtained in tAdS. So, from now on we will concentrate on the δA_i fluctuation having the following equation of motion

$$0 = \partial_z^2 V_i - \frac{1}{z}\frac{(1 - 5q^2 z^6)}{(1 + q^2 z^6)} \partial_z V_i + m_m^2 V_i, \tag{4.2}$$

where the meson mass is given by $m_m^2 = -p^2$. After introducing the new dimensionless variables

$$\tilde{V}_i = V_i z_{IR} \ , \quad \tilde{z} = \frac{z}{z_{IR}} \quad \text{and} \quad \tilde{m}_m = m_m z_{IR}, \tag{4.3}$$

eq. (4.2) can be rewritten as

$$0 = \partial_{\tilde{z}}^2 \tilde{V}_i - \frac{1}{\tilde{z}}\frac{(1 - 5\tilde{q}^2 \tilde{z}^6)}{(1 + \tilde{q}^2 \tilde{z}^6)} \partial_{\tilde{z}} \tilde{V}_i + \tilde{m}_m^2 \tilde{V}_i. \tag{4.4}$$

For $\tilde{q} = 0$, the above equation reproduces the result in tAdS [21]. Due to the C/D phase transition, the range of the quark chemical potential $\tilde{\mu}$ at the zero temperature is limited from 0 to 5.373 for $\frac{N_f}{N_c} = \frac{1}{3}$. Similarly, $\tilde{\mu}$ runs from 0 to 3.799 for $\frac{N_f}{N_c} = \frac{2}{3}$ and from 0 to 3.10217 for $N_f/N_c = 1$. Through the numerical evaluation, we find the relation between the chemical potential and the ρ meson mass in the figure 3. As shown in the figure, the ρ meson mass decreases as the quark chemical potential increases. Note that in the figure 3,
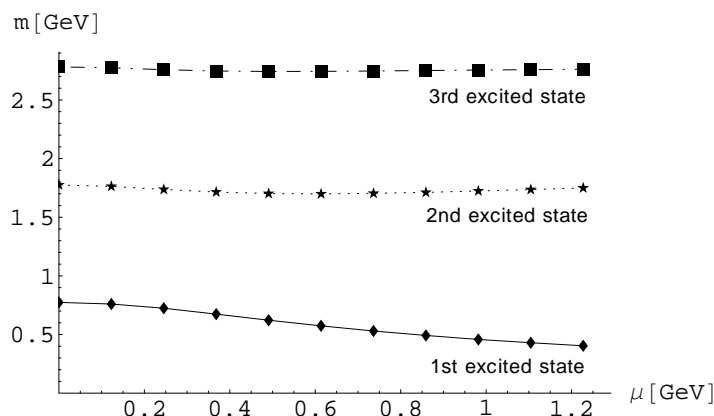
Figure 4. For $N_f/N_c = 2/3$, the chemical potential dependence of the excited meson masses.

	$\mu = 0$	$\mu = 0.245$	$\mu = 0.491$	$\mu = 0.736$	$\mu = 0.982$	$\mu = 1.227$
mass of the 1st	0.774	0.724	0.622	0.530	0.458	0.404
mass of the 2nd	1.775	1.737	1.702	1.704	1.724	1.750
mass of the 3rd	2.782	2.758	2.743	2.747	2.755	2.762

Table 1. The meson masses of the excited states depending on the chemical potential in the GeV unit.

the right ends of the curves implies the C/D phase transition point. In addition, the larger the number of flavor becomes, the more quickly the ρ meson mass decreases.

Finally, we further calculate the masses of the higher excited meson states. After the numerical evaluation, we draw several excited meson masses for $\frac{N_f}{N_c} = \frac{2}{3}$ in the figure 4 (see also table 1. for the numerical values of the masses in the several points of μ). For the ρ meson, its mass decreases as the chemical potential increases. In the higher mode cases, their masses decrease for some period of μ and then increase as μ increases (see table 1).

5 Discussion

In this paper, we proposed the thermal charged AdS space as the dual geometry corresponding on the hadronic phase of QCD, which is the zero mass limit of the Reissner-Nordstrom AdS black hole with the hard wall. This tcAdS was installed as the low temperature pair of the RNAdS black hole. By comparing the on-shell actions of two backgrounds, we investigated the confinement/deconfinement phase transition depending on the chemical potential or the quark number density. Interestingly, we found out that above the critical chemical potential there exists only the deconfinement phase even at zero temperature and evaluated the critical chemical potential depending on the flavor number. In addition, using the dual geometry of the hadronic phase, we calculated how the mass spectrum of the vector mesons depends on the chemical potential in the baryonic medium.

In this paper, we evaluated the baryonic chemical potential ($\approx 4\,\text{GeV}$), which is too high comparing with the known result (≈ 1 Gev). So it is important to know how to cure this discrepancy. Another interesting problem is to study the chemical potential dependence (or the quark density dependence) of other physical quantities. We will report those results elsewhere.

Acknowledgments

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