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# Soft supersymmetry breaking terms from $A_4$ lepton flavor symmetry

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Abstract: We study the supersymmetric model with the $A_4$ lepton flavor symmetry, in particular soft supersymmetry breaking terms, which are predicted from the $A_4$ lepton flavor symmetry. We evaluate soft slepton masses and A-terms within the framework of supergravity theory. Constraints due to experiments of flavor changing neutral current processes are examined.

Keywords: Supersymmetry Breaking, Discrete and Finite Symmetries.



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## Contents



## 1. Introduction

Recent experiments of the neutrino oscillation go into the new phase of precise determination of mixing angles and mass squared differences [1, 2]. Those indicate the tri-bimaximal mixing for three flavors in the lepton sector [3]. Indeed, various types of models leading to the tri-bimaximal mixing have been proposed, e.g. by assuming several types of non-Abelian flavor symmetries.

One of natural models realizing the tri-bimaximal mixing has been proposed based on the non-Abelian finite group $A_4$. Since the original papers [4] on the application of the non-Abelian discrete symmetry $A_4$ to quark and lepton families, much progress has been made in understanding the tri-bimaximal mixing for neutrinos in a number of specific models [5−8]. Therefore, it is important to clarify the physical implication of the $A_4$ model carefully.

The supersymmetric extension of the standard model is one of interesting candidates for physics beyond the weak scale. Within the framework of supersymmetric models, flavor symmetries constrain not only quark and lepton mass matrices, but also mass matrices of their superpartners, i.e., squarks and sleptons. That is, flavor symmetries realizing realistic quark/lepton mass matrices would lead to specific patterns of squark and slepton mass matrices as their predictions, which could be tested in future experiments. For example, $D_4$ flavor models [9−12] would also lead to the lepton tri-bimaximal mixing. Their supersymmetric models have been studied in ref. [13] and it is shown that the $D_4$ flavor models predict the degeneracy between the second and third families of slepton masses.[1] The $A_4$ model would lead to a different prediction in slepton masses.

[1] The $D_4$ flavor symmetry can be realized in superstring models [14, 15]. From this viewpoint, the $D_4$ flavor symmetry as well as certain flavor symmetries are interesting, too.



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| | $(L_e, L_\mu, L_\tau)$ | $R_e$ | $R_\mu$ | $R_\tau$ | $H_u$ | $H_d$ | $(\chi_1, \chi_2, \chi_3)$ | $(\chi'_1, \chi'_2, \chi'_3)$ | $\chi$ | $\Phi$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $A_4$ | $\mathbf{3}$ | $\mathbf{1}$ | $\mathbf{1'}$ | $\mathbf{1''}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{3}$ | $\mathbf{3}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $Z_3$ | $\omega$ | $\omega^2$ | $\omega^2$ | $\omega^2$ | 1 | 1 | 1 | $\omega$ | $\omega$ | 1 |
| $\mathrm{U}(1)_F$ | 0 | $2q$ | $q$ | 0 | 0 | 0 | 0 | 0 | 0 | $-1$ |

**Table 1:** $A_4$, $Z_3$ and $\mathrm{U}(1)_F$ charges for leptons and scalars

Although squarks and sleptons have not been detected yet, their mass matrices are strongly constrained by experiments of flavor changing neutral current (FCNC) processes [16].[2] When squark and slepton masses are of order of the weak scale, the FCNC experimental bounds, in particular the $\mu \to e\gamma$ decay, requires strong suppression of off-diagonal elements in squark and slepton mass squared matrices in the basis, where fermion mass matrices are diagonalized. Non-Abelian flavor symmetries and certain types of their breaking patterns are useful to suppress FCNCs. (See e.g. [18, 19, 13].) In addition to flavor symmetries, their breaking patterns are important to derive quark and lepton mass matrices and to predict squark and slepton mass matrices. Thus, it is important to study which pattern of slepton mass matrices is predicted from the $A_4$ model and to examine whether the predicted pattern of slepton mass matrices is consistent with the current FCNC experimental bounds. That is the purpose of this paper.

The paper is organized as follows. In section 2, we review the $A_4$ model [6], showing values of parameters consistent with neutrino oscillation experiments. In section 3, we evaluate soft supersymmetry (SUSY) breaking terms of sleptons, i.e. soft scalar mass matrices and A-terms. We examine FCNC constraints on those SUSY breaking terms as mass insertion parameters. Section 4 is devoted to conclusion and discussion. In appendix, we give a brief review on the $A_4$ group.

## 2. $A_4 \times Z_3$ model for leptons

Here, we discuss the $A_4$ model [6] leading to the tri-bimaximal mixing and show proper values of parameters. In the non-Abelian finite group $A_4$, there are twelve group elements and four irreducible representations: $\mathbf{1}$, $\mathbf{1}'$, $\mathbf{1}''$ and $\mathbf{3}$. We consider the supersymmetric $A_4$ model based on [6], with the $A_4$ and $Z_3$ charge assignments listed in table 1. Under the $A_4$ symmetry, the chiral superfields for three families of the left-handed lepton doublets $L_I$ ($I = e, \mu, \tau$) are assumed to transform as $\mathbf{3}$, while the right-handed ones of the charge lepton singlets $R_e$, $R_\mu$ and $R_\tau$ are assigned with $\mathbf{1}$, $\mathbf{1}'$, $\mathbf{1}''$, respectively. The third row of table 1 shows how each chiral multiplet transforms under $Z_3$, where $\omega = e^{2\pi i/3}$. The flavor symmetry is spontaneously broken by vacuum expectation values (VEV) of two $\mathbf{3}'s$, $\chi_i$, $\chi'_i$, and by one singlet, $\chi(\mathbf{1})$, which are $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ singlets. Their $Z_3$ charges are also shown in table 1. Here and hereafter, we follow the convention that the chiral superfield and its lowest component are denoted by the same letter.

[2] See also e.g ref. [17] and references therein.



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The allowed terms in the superpotential including charged leptons are written by

$$W_L = \frac{y_e}{\Lambda}(L_e\chi_1 + L_\mu\chi_3 + L_\tau\chi_2)R_e H_d + \frac{y_\mu}{\Lambda}(L_e\chi_2 + L_\mu\chi_1 + L_\tau\chi_3)R_\mu H_d \\ + \frac{y_\tau}{\Lambda}(L_e\chi_3 + L_\mu\chi_2 + L_\tau\chi_1)R_\tau H_d\,, \tag{2.1}$$

where $y_e$, $y_\mu$ and $y_\tau$ are couplings, and $\Lambda$ is the cut-off scale of the effective superpotential. In order to obtain the natural hierarchy among lepton masses $m_e$, $m_\mu$ and $m_\tau$, the Froggatt-Nielsen mechanism [20] is introduced as an additional $\mathrm{U}(1)_F$ flavor symmetry under which only the right-handed lepton sector is charged. The $\mathrm{U}(1)_F$ charge values are taken as 0, $q$ and $2q$ for $R_\tau$, $R_\mu$ and $R_e$, respectively. By assuming that the flavon $\Phi$, carrying a negative unit charge of $\mathrm{U}(1)_F$, acquires a VEV $\langle\Phi\rangle/\Lambda \equiv \lambda < 1$, the following magnitudes of couplings are realized through the Froggatt-Nielsen mechanism,

$$y_\tau \simeq \mathcal{O}(1), \qquad y_\mu \simeq \mathcal{O}(\lambda^q), \qquad y_e \simeq \mathcal{O}(\lambda^{2q})\ . \tag{2.2}$$

When $q = 1$, we estimate $\lambda \sim 0.02$. The $\mathrm{U}(1)_F$ charges are shown in the fourth row of table 1.

The superpotential of the neutrino sector is given as

$$W_\nu = \frac{y_1}{\Lambda^2}(L_eL_e + L_\mu L_\tau + L_\tau L_\mu)H_uH_u\chi \tag{2.3} \\ + \frac{y_2}{3\Lambda^2}[(2L_eL_e - L_\mu L_\tau - L_\tau L_\mu)\chi'_1 + (-L_eL_\tau + 2L_\mu L_\mu - L_\tau L_e)\chi'_2 \\ + (-L_eL_\mu - L_\mu L_e + 2L_\tau L_\tau)\chi'_3]H_uH_u\,,$$

where $y_1$ and $y_2$ are couplings of $\mathcal{O}(1)$. After the $A_4 \times Z_3$ symmetry is spontaneously broken by VEVs of $\chi_i$, $\chi'_i$ and $\chi$, the charged lepton mass matrix $M_l$ and the neutrino mass matrix $M_\nu$ are obtained as follows,

$$M_l = \frac{v_d}{\Lambda}\begin{pmatrix} y_e\langle\chi_1\rangle & y_e\langle\chi_3\rangle & y_e\langle\chi_2\rangle \\ y_\mu\langle\chi_2\rangle & y_\mu\langle\chi_1\rangle & y_\mu\langle\chi_3\rangle \\ y_\tau\langle\chi_3\rangle & y_\tau\langle\chi_2\rangle & y_\tau\langle\chi_1\rangle \end{pmatrix}\,, \tag{2.4}$$

and

$$M_\nu = \frac{v_u^2}{3\Lambda^2}\begin{pmatrix} 3y_1\langle\chi\rangle + 2y_2\langle\chi'_1\rangle & -y_2\langle\chi'_3\rangle & -y_2\langle\chi'_2\rangle \\ -y_2\langle\chi'_3\rangle & 2y_2\langle\chi'_2\rangle & 3y_1\langle\chi\rangle - y_2\langle\chi'_1\rangle \\ -y_2\langle\chi'_2\rangle & 3y_1\langle\chi\rangle - y_2\langle\chi'_1\rangle & 2y_2\langle\chi'_3\rangle \end{pmatrix}\,, \tag{2.5}$$

where $v_u$ and $v_d$ denote VEVs of Higgs doublets, i.e. $\langle H_u\rangle = v_u$ and $\langle H_d\rangle = v_d$. Furthermore, we define $\tan\beta = v_u/v_d$.

If one can take the VEVs of gauge singlet scalar fields $\chi$, $\chi_i$ and $\chi'_i$ as follows

$$\langle\chi\rangle = V, \quad \langle(\chi_1,\chi_2,\chi_3)\rangle = (V_l, 0, 0), \quad \langle(\chi'_1,\chi'_2,\chi'_3)\rangle = (V_\nu, V_\nu, V_\nu), \tag{2.6}$$



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which are actually one of minima in the scalar potential at the leading order as shown in ref. [6], the mass matrices of charged leptons and neutrinos are reduced to

$$M_l = \frac{v_d V_l}{\Lambda}\begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}, \tag{2.7}$$

$$M_\nu = \frac{v_u^2}{3\Lambda^2}\begin{pmatrix} 3y_1V+2y_2V_\nu & -y_2V_\nu & -y_2V_\nu \\ -y_2V_\nu & 2y_2V_\nu & 3y_1V-y_2V_\nu \\ -y_2V_\nu & 3y_1V-y_2V_\nu & 2y_2V_\nu \end{pmatrix}. \tag{2.8}$$

The charged lepton mass matrix is diagonal. The neutrino mass matrix can be simplified as

$$M_\nu = \frac{y_1 v_u^2 V}{\Lambda^2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + \frac{y_2 v_u^2 V_\nu}{3\Lambda^2}\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}. \tag{2.9}$$

Then, it is easy to find the tri-bimaximal mixing for the lepton flavor mixing matrix $U_{MNS}$ [21] as,

$$U_{MNS} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}. \tag{2.10}$$

On the other hand, neutrino masses are given by

$$m_\nu = \frac{v_u^2}{\Lambda^2}(y_1V + y_2V_\nu, y_1V, -y_1V + y_2V_\nu), \tag{2.11}$$

and using the parameter $r = y_2V_\nu/y_1V$ the neutrino masses are expressed as

$$m_\nu = \frac{y_1 V v_u^2}{\Lambda^2}(1+r, 1, -1+r). \tag{2.12}$$

Then, the differences between masses squared are evaluated as,

$$\Delta m^2_{\rm atm} = \left| -4r\frac{y_1^2V^2v_u^4}{\Lambda^4} \right|, \qquad \Delta m^2_{\rm sol} = r(r+2)\frac{y_1^2V^2v_u^4}{\Lambda^4}, \tag{2.13}$$

that is,

$$\frac{\Delta m^2_{\rm atm}}{\Delta m^2_{\rm sol}} = \left| \frac{-4}{r+2} \right|, \tag{2.14}$$

which is reconciled with the experimental data for $r \sim -1.9$ or $r \sim -2.1$.

Let us estimate numerical values of $\alpha_l$, $\alpha_\nu$ and $\alpha$, which are determined by putting the neutrino experimental data. By using eqs. (2.7), (2.13) and (2.14), we obtain the following relations,

$$\alpha_l = \frac{m_\tau}{y_\tau v_d}, \quad \alpha^2 = \left| -\frac{\Delta m^2_{\rm atm}\Lambda^2}{4ry_1^2v_u^4} \right|, \quad \frac{y_2\alpha_\nu}{y_1\alpha} = -2 \pm 4\frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}}, \tag{2.15}$$

where

$$\alpha_l = \frac{V_l}{\Lambda}, \qquad \alpha_\nu = \frac{V_\nu}{\Lambda}, \qquad \alpha = \frac{V}{\Lambda}. \tag{2.16}$$

We use the experimental data,

$$m_\tau \simeq 1.8\mathrm{GeV}, \qquad \Delta m^2_{\mathrm{atm}} \simeq 2.4\times 10^{-3}\mathrm{eV}^2, \qquad \Delta m^2_{\mathrm{sol}} \simeq 7.6\times 10^{-5}\mathrm{eV}^2. \tag{2.17}$$

For example, in the case with $\tan\beta = 3$ and $|y_\tau| \simeq |y_1| \simeq |y_2| \simeq 1$, we can estimate

$$\alpha_l \sim 0.03, \quad \alpha \sim 6\times 10^{-16}\times\frac{\Lambda}{1\mathrm{GeV}}, \quad \alpha_\nu \simeq 2\alpha\ . \tag{2.18}$$

Moreover, if the breaking scales are assumed to be approximately the same $V_l \sim V_\nu \sim V$, the scale $\Lambda \simeq 10^{14}$GeV is determined. As the value of $\tan\beta$ becomes larger, $\alpha_l$ increases. For example, for $\tan\beta = 30$ and $|y_\tau| \simeq 1$, we obtain $\alpha_l \sim 0.3$. Similarly, $\alpha_l$ increases as $|y_\tau|$ decreases. Thus, the above value $\alpha_l \sim 0.03$ is the smallest value for $|y_\tau| \leq \mathcal{O}(1)$. On the other hand, if we allow a large value of $|y_\tau|$ like $|y_\tau| \sim 4\pi$, the value of $\alpha_l$ would be estimated as $\alpha_l \sim 0.002$. Hereafter, we restrict ourselves to the case with $\alpha_l \sim \alpha_\nu \sim \alpha$ and we denote their magnitudes by $\tilde{\alpha}$.

In the above scenario, it is crucial to choose the proper VEVs, i.e., (2.6). Indeed, such VEVs can be realized by a certain form of superpotential at the leading order, as shown in ref. [6]. Also, in ref. [6] it has been shown that when the next leading terms are taken into account, VEVs shift in the order of $\langle\chi_i\rangle/\Lambda$ and $\langle\chi'_i\rangle/\Lambda$. Actually, one can obtain

$$\begin{aligned}
&\langle(\chi_1,\chi_2,\chi_3)\rangle = (1+g_{l_1}\tilde{\alpha},\ g_{l_2}\tilde{\alpha},\ g_{l_3}\tilde{\alpha})V_l,\\
&\langle(\chi'_1,\chi'_2,\chi'_3)\rangle = (V_\nu+g_{\nu_1}\tilde{\alpha},\ V_\nu+g_{\nu_2}\tilde{\alpha},\ V_\nu+g_{\nu_3}\tilde{\alpha}),\\
&\langle\chi\rangle = V(1+g\tilde{\alpha})\ .
\end{aligned} \tag{2.19}$$

Here, the parameters, $g_{l_i}$, $g_{\nu_i}$ and $g$, are of $\mathcal{O}(1)$ when $\alpha_l \sim \alpha_\nu \sim \alpha$ and couplings in the superpotential are of $\mathcal{O}(1)$. With these VEVs, the mass matrices are modified as,

$$M_l = v_d\alpha_l\begin{pmatrix} y_e(1+g_{l_1}\tilde{\alpha}) & y_e g_{l_3}\tilde{\alpha} & y_e g_{l_2}\tilde{\alpha}\\ y_\mu g_{l_2}\tilde{\alpha} & y_\mu(1+g_{l_1}\tilde{\alpha}) & y_\mu g_{l_3}\tilde{\alpha}\\ y_\tau g_{l_3}\tilde{\alpha} & y_\tau g_{l_2}\tilde{\alpha} & y_\tau(1+g_{l_1}\tilde{\alpha})\end{pmatrix}, \tag{2.20}$$

for the charged leptons, and

$$M_\nu = \frac{y_1 v_u^2\tilde{\alpha}(1+g\tilde{\alpha})}{\Lambda}\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix} + \frac{y_2 v_u^2\tilde{\alpha}}{3\Lambda}\begin{pmatrix}2+2g_{\nu_1}\tilde{\alpha} & -1-g_{\nu_3}\tilde{\alpha} & -1-g_{\nu_2}\tilde{\alpha}\\ -1-g_{\nu_3}\tilde{\alpha} & 2+2g_{\nu_2}\tilde{\alpha} & -1-g_{\nu_1}\tilde{\alpha}\\ -1-g_{\nu_2}\tilde{\alpha} & -1-g_{\nu_1}\tilde{\alpha} & 2+2g_{\nu_3}\tilde{\alpha}\end{pmatrix}, \tag{2.21}$$

for neutrinos. These modified mass matrices give the deviation from the tri-bimaximal mixing.[3] That changes the lepton mixing angles by $\mathcal{O}(\tilde{\alpha})$. That implies that a large value of $\tan\beta$, $\tan\beta \gg 1$ and/or a small value of coupling $|y_\tau|$, $|y_\tau| \ll 1$ are unfavored

[3] Numerical analyses of the deviation from the tri-bimaximal mixing were presented in ref. [22]

in this scenario. On the other hand, if $\tilde{\alpha} < \mathcal{O}(0.1)$, the above deviations in the lepton mass matrices would not be important to the current accuracy of neutrino oscillation experiments. However, such deviations, in particular the deviation from the diagonal form in the charged lepton mass matrix, are important from the viewpoint of supersymmetry breaking terms as shown in the next section. For the later convenience, here we show the diagonalizing matrix of the charged lepton mass matrix, that is, we diagonalize the charged lepton mass matrix $M_l$ in eq. (2.4) by the matrices $V_L$ and $V_R$,

$$M_l = V_R^T(\theta_{R12}, \theta_{R23}, \theta_{R13}) \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} V_L(\theta_{L12}, \theta_{L23}, \theta_{L13}). \tag{2.22}$$

In the usual convention of $\theta_{R(L)ij}$, we get

$$\begin{aligned} &\theta_{R12} \sim \frac{y_e\langle\chi_3\rangle}{y_\mu\langle\chi_1\rangle} \sim \frac{m_e}{m_\mu}\mathcal{O}(\tilde{\alpha}), \quad &&\theta_{R23} \sim \frac{y_\mu\langle\chi_3\rangle}{y_\tau\langle\chi_1\rangle} \sim \frac{m_\mu}{m_\tau}\mathcal{O}(\tilde{\alpha}), \quad &&\theta_{R13} \sim \frac{y_e\langle\chi_2\rangle}{y_\tau\langle\chi_1\rangle} \sim \frac{m_e}{m_\tau}\mathcal{O}(\tilde{\alpha}), \\ &\theta_{L12} \sim \frac{\langle\chi_2\rangle}{\langle\chi_1\rangle} \sim \mathcal{O}(\tilde{\alpha}), &&\theta_{L23} \sim \frac{\langle\chi_2\rangle}{\langle\chi_1\rangle} \sim \mathcal{O}(\tilde{\alpha}), &&\theta_{L13} \sim \frac{\langle\chi_3\rangle}{\langle\chi_1\rangle} \sim \mathcal{O}(\tilde{\alpha}), \end{aligned} \tag{2.23}$$

where we take $\langle\chi_2\rangle \sim \langle\chi_3\rangle \sim \tilde{\alpha}\langle\chi_1\rangle$ taking account of non-leading corrections as given in eq. (2.19). Here we have assumed $\tilde{\alpha} < \mathcal{O}(0.1)$. Values of $\theta_{Lij}$ are large compared with those of $\theta_{Rij}$.

## 3. Soft SUSY breaking terms

We study soft SUSY breaking terms, i.e. soft slepton masses and A-terms, which are predicted from the $A_4$ model discussed in the previous section. We consider SUSY breaking within the framework of supergravity theory, where some moduli fields $Z$ and $\chi$, $\chi_i$ and $\chi'_i$ would have non-vanishing F-terms.[4] F-terms are given as

$$F^{\Phi_k} = -e^{\frac{K}{2M_p^2}} K^{\Phi_k \bar{I}} \left(\partial_{\bar{I}}\bar{W} + \frac{K_{\bar{I}}}{M_p^2}\bar{W}\right), \tag{3.1}$$

where $K$ denotes the Kähler potential, $K_{\bar{I}J}$ denotes second derivatives by fields, i.e. $K_{\bar{I}J} = \partial_{\bar{I}}\partial_J K$ and $K^{\bar{I}J}$ is its inverse. In general, the fields $\Phi_k$ in our notation include $A_4 \times Z_3$-singlet moduli fields $Z$ and $\chi$, $\chi_i$, $\chi'_i$. Furthermore, VEVs of $F_{\Phi_k}/\Phi_k$ are estimated as $\langle F_{\Phi_k}/\Phi_k\rangle = \mathcal{O}(m_{3/2})$, where $m_{3/2}$ denotes the gravitino mass, which is obtained as $m_{3/2} = \langle e^{K/2M_p^2} W/M_p^2\rangle$.

### 3.1 Slepton mass matrices

First let us study soft scalar masses. Within the framework of supergravity theory, soft scalar mass squared is obtained as [23]

$$m^2_{\bar{I}J} K_{\bar{I}J} = m^2_{3/2} K_{\bar{I}J} + |F^{\Phi_k}|^2 \partial_{\Phi_k}\partial_{\bar{\Phi}_k} K_{\bar{I}J} - |F^{\Phi_k}|^2 \partial_{\bar{\Phi}_k} K_{\bar{I}L} \partial_{\Phi_k} K_{\bar{M}J} K^{L\bar{M}}. \tag{3.2}$$

[4]SUSY breaking might be mediated through the gauge mediation and anomaly mediation. They are flavor-blind. Only if the gravity mediation has a sizable contribution with and without other SUSY breaking mediations, we would have a prediction of sfermion spectra from each flavor mechanism.



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The flavor symmetry $A_4 \times Z_3$ requires the following form of Kähler potential for left-handed and right-handed leptons

$$\begin{aligned} K^{(0)}_{\rm matter} &= a(Z,Z^\dagger)(L_e^\dagger L_e + L_\mu^\dagger L_\mu + L_\tau^\dagger L_\tau) \\ &\quad + b_e(Z,Z^\dagger)R_e^\dagger R_e + b_\mu(Z,Z^\dagger)R_\mu^\dagger R_\mu + b_\tau(Z,Z^\dagger)R_\tau^\dagger R_\tau \ , \end{aligned} \tag{3.3}$$

at the leading order, where $a(Z,Z^\dagger)$ and $b_I(Z,Z^\dagger)$ for $I = e,\mu,\tau$ are generic functions of moduli fields $Z$. Then, using eq. (3.2), the slepton mass squared matrices of left-handed and right-handed sleptons can be found to be

$$m_L^2 = \begin{pmatrix} m_L^2 & 0 & 0 \\ 0 & m_L^2 & 0 \\ 0 & 0 & m_L^2 \end{pmatrix}, \qquad m_R^2 = \begin{pmatrix} m_{R_1}^2 & 0 & 0 \\ 0 & m_{R_2}^2 & 0 \\ 0 & 0 & m_{R_3}^2 \end{pmatrix}, \tag{3.4}$$

where all of $m_L$ and $m_{R_i}$ for $i = 1,2,3$ would be of $\mathcal{O}(m_{3/2})$. These forms would be obvious from the flavor symmetry $A_4$, that is, three families of left-handed leptons are the $A_4$ triplet, while right-handed leptons are $A_4$ singlets. At any rate, it is the prediction of the $A_4$ model that three families of left-handed slepton masses are degenerate.

However, the flavor symmetry $A_4 \times Z_3$ is broken to derive the realistic lepton mass matrices and such breaking introduces corrections in the Kähler potential and the form of slepton masses. Let us study such corrections in the Kähler potential. Because $\chi'_i$ and $\chi$ have nontrivial $Z_3$ charges, their linear terms do not appear in the Kähler potential of lepton multiplets. In addition, because of $\langle\chi_2\rangle, \langle\chi_3\rangle \sim \tilde\alpha\langle\chi_1\rangle$, the most important correction terms would be linear terms of $\chi_1$. That is, the correction terms in the matter Kähler potential are obtained

$$\Delta K_{\rm matter} = \frac{\chi_1}{\Lambda'}\left[a'_1(Z,Z^\dagger)(2L_e^\dagger L_e - L_\mu^\dagger L_\mu - L_\tau^\dagger L_\tau) + a'_2(Z,Z^\dagger)(L_\mu^\dagger L_\mu - L_\tau^\dagger L_\tau)\right] + h.c., \tag{3.5}$$

up to $\mathcal{O}(\tilde\alpha^2\Lambda/\Lambda')$, where $a'_1(Z,Z^\dagger)$ and $a'_2(Z,Z^\dagger)$ are generic functions of moduli fields. The cut-off scale $\Lambda'$ might be independent of $\Lambda$, which appears in the effective superpotential. For example, if $\Lambda'$ is the Planck scale, the above corrections would be negligible. Hereafter, we concentrate to the case with $\Lambda' \sim \Lambda$. Note that linear correction terms of $\chi_i$ do not appear for the Kähler potential of right-handed lepton multiplets. All of off-diagonal Kähler metric entries for both left-handed and right-handed leptons appear at $\mathcal{O}(\tilde\alpha^2)$,

$$\frac{\partial^2 K_{\rm matter}}{\partial L_I^\dagger \partial L_J} = \mathcal{O}(\tilde\alpha^2), \qquad \frac{\partial^2 K_{\rm matter}}{\partial R_I^\dagger \partial R_J} = \mathcal{O}(\tilde\alpha^2), \tag{3.6}$$

where $I, J = e,\mu,\tau$ and $I \neq J$. For example, the (1,2) and (2,1) entries for left-handed leptons are induced by $(\chi_3/\Lambda')L_e^\dagger L_\mu$, $(\chi_2/\Lambda')L_\mu^\dagger L_e$, etc. Similarly, other entries for both left-handed and right-handed leptons are induced. Furthermore, corrections including $\Phi$ do not violate the structure of $K^{(0)}_{\rm matter}$ because $\Phi$ has a trivial charge under $A_4 \times Z_3$.

Including these corrections, the slepton masses are written by

$$m_L^2 = \begin{pmatrix} m_L^2 & 0 & 0 \\ 0 & m_L^2 & 0 \\ 0 & 0 & m_L^2 \end{pmatrix} + m_{3/2}^2 \begin{pmatrix} 2\bar{a}_1'\tilde{\alpha} & 0 & 0 \\ 0 & (\bar{a}_2' - \bar{a}_1')\tilde{\alpha} & 0 \\ 0 & 0 & -(\bar{a}_1' + \bar{a}_2')\tilde{\alpha} \end{pmatrix} + \mathcal{O}(\tilde{\alpha}^2 m_{3/2}^2),$$

$$m_R^2 = \begin{pmatrix} m_{R_1}^2 & 0 & 0 \\ 0 & m_{R_2}^2 & 0 \\ 0 & 0 & m_{R_3}^2 \end{pmatrix} + \mathcal{O}(\tilde{\alpha}^2 m_{3/2}^2), \tag{3.7}$$

in the flavor basis, where $\bar{a}_1', \bar{a}_2' = \mathcal{O}(1)$.

The leptonic FCNC is induced by off diagonal elements of scalar mass squared matrices in the diagonal basis of the charged lepton mass matrix. The following discussion presents that the off diagonal elements are enough suppressed in the left-handed slepton and the right-handed slepton mass matrices in the diagonal basis of the charged lepton mass matrix, i.e., $\tilde{m}_R^2 = V_R m_R^2 V_R^T$ and $\tilde{m}_L^2 = V_L m_L^2 V_L^T$. In this basis, the slepton mass squared matrices are obtained as

$$\tilde{m}_L^2 = \begin{pmatrix} m_L^2 & 0 & 0 \\ 0 & m_L^2 & 0 \\ 0 & 0 & m_L^2 \end{pmatrix} + m_{3/2}^2 \begin{pmatrix} O(\tilde{\alpha}) & O(\tilde{\alpha}^2) & O(\tilde{\alpha}^2) \\ O(\tilde{\alpha}^2) & O(\tilde{\alpha}) & O(\tilde{\alpha}^2) \\ O(\tilde{\alpha}^2) & O(\tilde{\alpha}^2) & O(\tilde{\alpha}) \end{pmatrix},$$

$$\tilde{m}_R^2 = \begin{pmatrix} m_{R_1}^2 & 0 & 0 \\ 0 & m_{R_2}^2 & 0 \\ 0 & 0 & m_{R_3}^2 \end{pmatrix} + \mathcal{O}(\tilde{\alpha}^2 m_{3/2}^2), \tag{3.8}$$

when $\tilde{\alpha} > m_e/m_\mu$.

We have a strong constraint on $(\tilde{m}_L^2)_{12}$ and $(\tilde{m}_R^2)_{12}$ from FCNC experiments [16], i.e.

$$\frac{(\tilde{m}_L^2)_{12}}{m_{\rm SUSY}^2} \leq \mathcal{O}(10^{-3}), \qquad \frac{(\tilde{m}_R^2)_{12}}{m_{\rm SUSY}^2} \leq \mathcal{O}(10^{-3}), \tag{3.9}$$

for $m_{\rm SUSY} \sim 100\,\text{GeV}$, where $m_{\rm SUSY}$ denotes the average mass of slepton masses and it would be of $\mathcal{O}(m_{3/2})$. The above prediction (3.8) of the $A_4$ model leads to $(\tilde{m}_L^2)_{12}/m_{\rm SUSY}^2 = \mathcal{O}(\tilde{\alpha}^2)$. Because of $\tilde{\alpha} \sim 0.03$ for $y_\tau \simeq 1$, our prediction, $(\tilde{m}_L^2)_{12}/m_{\rm SUSY}^2 = \mathcal{O}(\tilde{\alpha}^2) = \mathcal{O}(10^{-3})$, would be consistent with the current experimental bound. When we consider a larger value of $y_\tau$, e.g. $y_\tau \sim 3$, the predicted value of $(\tilde{m}_L^2)_{12}/m_{\rm SUSY}^2$ would be suppressed like $(\tilde{m}_L^2)_{12}/m_{\rm SUSY}^2 = \mathcal{O}(10^{-4})$. On the other hand, a large value of $\tilde{\alpha}$ like $\tilde{\alpha} = \mathcal{O}(0.1)$, which is obtained for a large value of $\tan\beta$ and/or a small value of $y_\tau$ would be ruled out. Similarly, we can estimate $(\tilde{m}_R^2)_{12}/m_{\rm SUSY}^2$ by using eq. (3.8) and results are the same.

We have studied soft scalar masses induced by F-terms. If we gauge $\mathrm{U}(1)_F$, another contribution to scalar masses would be induced through $\mathrm{U}(1)_F$ breaking, that is, contributions due to the D-term of the $\mathrm{U}(1)_F$ vector multiplet. Such D-term contributions $m_D^2$ are proportional to $\mathrm{U}(1)_F$ charges $Q$ of matter fields,[5]

$$m_D^2 = Q\langle D\rangle. \tag{3.10}$$

[5] See e.g. [24].

In general, such D-term contributions may be dangerous from the viewpoint of FCNC. However, those $D$-term contributions in the $A_4$ model do not violate the above form of soft scalar masses, (3.4), (3.7) and (3.8), because $\mathrm{U}(1)_F$ charges in table 1 are consistent with the $A_4$ flavor symmetry, that is, $L_I$ $(I = e, \mu, \tau)$ have the same $\mathrm{U}(1)_F$ charge, while three of $R_I$ $(I = e, \mu, \tau)$ have different $\mathrm{U}(1)_F$ charges. Thus, the predictions on $(\tilde{m}_L^2)_{12}/m_{\mathrm{SUSY}}^2$ and $(\tilde{m}_R^2)_{12}/m_{\mathrm{SUSY}}^2$ do not change.

Here, we give a comment on radiative corrections. The slepton masses, which we have studied above, are induced at a high energy scale such as the Planck scale or the GUT scale. The slepton masses have radiative corrections between such a high energy scale and the weak scale, although those have been neglected in the above analyses. In those radiative corrections to slepton masses, the gaugino contributions are dominant. For example, slepton masses at the weak scale are related with ones at the GUT scale $M_X$ as

$$\begin{aligned} m_L^2(M_Z) &= m_L^2(M_X) + 0.5M_{\tilde{W}}^2 + 0.04M_{\tilde{B}}^2, \\ m_R^2(M_Z) &= m_R^2(M_X) + 0.2M_{\tilde{B}}^2, \end{aligned} \tag{3.11}$$

where $M_{\tilde{B}}$ and $M_{\tilde{W}}$ are bino and wino masses, respectively. These radiative corrections do not change drastically the above results when these gaugino masses are of $\mathcal{O}(m_{3/2})$. On the other hand, FCNC constraints would be improved when these gaugino masses are much larger than initial values of slepton masses.

### 3.2 A-terms

Now, let us examine the mass matrix between the left-handed and the right-handed sleptons, which is generated by the so-called A-terms. The A-terms are trilinear couplings of two sleptons and one Higgs field, and are obtained as [23]

$$h_{IJ} L_J R_I H_d = h_{IJ}^{(Y)} L_J R_I H_d + h_{IJ}^{(K)} L_J R_I H_d, \tag{3.12}$$

where

$$\begin{aligned} h_{IJ}^{(Y)} &= F^{\Phi_k} \langle \partial_{\Phi_k} \tilde{y}_{IJ} \rangle, \\ h_{IJ}^{(K)} L_J R_I H_d &= -\langle \tilde{y}_{LJ} \rangle L_J R_I H_d F^{\Phi_k} K^{L\bar{L}} \partial_{\Phi_k} K_{\bar{L}I} \\ &\quad -\langle \tilde{y}_{IM} \rangle L_J R_I H_d F^{\Phi_k} K^{M\bar{M}} \partial_{\Phi_k} K_{\bar{M}J} \\ &\quad -\langle \tilde{y}_{IJ} \rangle L_J R_I H_d F^{\Phi_k} K^{H_d} \partial_{\Phi_k} K_{H_d}, \end{aligned} \tag{3.13}$$

where $K_{H_d}$ denotes the Kähler metric of $H_d$, and $\tilde{y}_{IJ}$ denotes the effective Yukawa couplings given as

$$\tilde{y}_{IJ} = \frac{1}{\Lambda} \begin{pmatrix} y_e\chi_1 & y_e\chi_3 & y_e\chi_2 \\ y_\mu\chi_2 & y_\mu\chi_1 & y_\mu\chi_3 \\ y_\tau\chi_3 & y_\tau\chi_2 & y_\tau\chi_1 \end{pmatrix}. \tag{3.14}$$

Furthermore, when we use the $\mathrm{U}(1)_F$ Froggatt-Nielsen mechanism in order to obtain the lepton mass hierarchy, the couplings, $y_e, y_\mu$ and $y_\tau$, are expressed as

$$y_I = c_I \left(\frac{\Phi}{\Lambda}\right)^{Q_I} \quad (I = e, \mu, \tau), \tag{3.15}$$

where $Q_I$ is $\mathrm{U}(1)_F$ charges. Here we assume that couplings, $c_e, c_\mu$ and $c_\tau$, do not include the moduli $Z$, i.e. $\partial_Z c_I = 0$ for $I = e, \mu, \tau$. After the electroweak symmetry breaking, these $A$-terms provide us with the left-right mixing mass squared $(m^2_{LR})_{IJ} = h_{IJ} v_d$. Furthermore, we use the basis $(\tilde{m}^2_{LR})_{IJ} = (V_R m^2_{LR} V_L^T)_{IJ}$. The third term in the right hand side of eq. (3.14) is diagonalized in this basis. Thus, we do not take the third term into account in the following discussion.

When we consider the leading order of Kähler potential $K^{(0)}_{\rm matter}$, the second terms in the right hand side of eq. (3.14), $h^{(K)}_{IJ}$, is written by [25]

$$h^{(K)}_{IJ} = \langle \tilde{y}_{IJ} \rangle (A^R_I + A^L_J), \tag{3.16}$$

where $A^R_I = -F^Z \partial_Z \ln b_I(Z, Z^\dagger)$ and $A^L_J = -F^Z \partial_Z \ln a(Z, Z^\dagger)$, that is, $A^L_I$ are degenerate up to $\mathcal{O}(\tilde{\alpha})$. Thus, the (2,1) entry of $(\tilde{m}^2_{LR})_{IJ}$ vanishes at the leading order. However, such a behavior is violated at the next order, that is, $A^L_1 - A^L_2 = \mathcal{O}(\tilde{\alpha} m_{3/2})$, because the diagonal (1,1) and (2,2) entries of Kähler metric $\Delta K_{\rm matter}$ for the left-handed lepton multiplets (3.5) have non-degenerate corrections of $\mathcal{O}(\tilde{\alpha})$. Then, the $h^{(K)}_{IJ}$ contribution to the (2,1) entry of $(\tilde{m}^2_{LR})_{IJ}$ is estimated as

$$(\tilde{m}^2_{LR})_{21} \sim \langle \tilde{y}_\mu \rangle v^d (A^L_1 - A^L_2) \theta_{L12} = \mathcal{O}(\tilde{\alpha}^2 m_\mu m_{3/2}). \tag{3.17}$$

Furthermore, the off-diagonal elements of Kähler metric have $\mathcal{O}(\tilde{\alpha}^2)$ of corrections (3.6), and these corrections also induce the same order of $(\tilde{m}^2_{LR})_{21}$, i.e. $(\tilde{m}^2_{LR})_{21} = \mathcal{O}(\tilde{\alpha}^2 m_\mu m_{3/2})$. Similarly, we can estimate the (1,2) entry and obtain the same result, i.e., $(\tilde{m}^2_{LR})_{12} = \mathcal{O}(\tilde{\alpha}^2 m_\mu m_{3/2})$ when $\tilde{\alpha} > m_e/m_\mu$. These entries have the strong constraint from FCNC experiments as $(\tilde{m}^2_{LR})_{12}/m^2_{\rm SUSY} \leq \mathcal{O}(10^{-6})$ and the same for the (2,1) entry for $m_{\rm SUSY} = 100\,\mathrm{GeV}$. However, the above prediction of the $A_4$ model leads to $(\tilde{m}^2_{LR})_{12}/m^2_{\rm SUSY} = \mathcal{O}(10^{-7})$ for $m_{\rm SUSY} = 100\,\mathrm{GeV}$ and $\alpha \sim 0.03$ and that is consistent with the experimental bound.

Now, let us estimate the first term in the right hand side of (3.12), $h^{(Y)}_{IJ}$, which can be written by,

$$\begin{aligned}(h^{(Y)}_{IJ}) = &\left(\frac{1}{\Lambda}\right) \begin{pmatrix} y_e \langle \chi_1 \rangle A_1 & y_e \langle \chi_3 \rangle A_3 & y_e \langle \chi_2 \rangle A_2 \\ y_\mu \langle \chi_2 \rangle A_2 & y_\mu \langle \chi_1 \rangle A_1 & y_\mu \langle \chi_3 \rangle A_3 \\ y_\tau \langle \chi_3 \rangle A_3 & y_\tau \langle \chi_2 \rangle A_2 & y_\tau \langle \chi_1 \rangle A_1 \end{pmatrix} \\ &+ \left(\frac{A_0}{\Lambda}\right) \begin{pmatrix} Q_e & 0 & 0 \\ 0 & Q_\mu & 0 \\ 0 & 0 & Q_\tau \end{pmatrix} \begin{pmatrix} y_e \langle \chi_1 \rangle & y_e \langle \chi_3 \rangle & y_e \langle \chi_2 \rangle \\ y_\mu \langle \chi_2 \rangle & y_\mu \langle \chi_1 \rangle & y_\mu \langle \chi_3 \rangle \\ y_\tau \langle \chi_3 \rangle & y_\tau \langle \chi_2 \rangle & y_\tau \langle \chi_1 \rangle \end{pmatrix},\end{aligned} \tag{3.18}$$

where

$$A_0 \equiv \frac{F_\Phi}{\Phi}, \qquad A_i \equiv \frac{F_{\chi_i}}{\chi_i}, \quad (i = 1, 2, 3). \tag{3.19}$$

These would be of $\mathcal{O}(m_{3/2})$. Since the second term of (3.18) is exactly the form of eq. (3.16) with the degenerate $A^L_I$, the second term does not change the above estimation of $(\tilde{m}^2_{LR})_{12}$ and $(\tilde{m}^2_{LR})_{21}$.[6]

[6]Indeed, the Kähler metric $b_I(Z, Z^\dagger)$ and couplings $y_I = c_I(\Phi/\Lambda)^{Q_I}$ lead to the same A-terms as the Kähler metric $b_I(Z, Z^\dagger)(\Phi/\Lambda)^{-Q_I}$ and couplings $y_I = c_I$.



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The first term of (3.18) contributes to $(\tilde{m}^2_{LR})_{21}$ as

$$(\tilde{m}^2_{LR})_{21} = y_\mu v_d \chi_2 (A_2 - A_1)/\Lambda \sim m_\mu \tilde{\alpha}(A_2 - A_1). \tag{3.20}$$

That is, we estimate $(\tilde{m}^2_{LR})_{21}/m^2_{\rm SUSY} \sim 10^{-5} \times (A_2 - A_1)/m_{3/2}$ for $\tilde{\alpha} \sim 0.03$. Thus, if $A_2 \neq A_1$ and $A_i = \mathcal{O}(m_{3/2})$, this value of $(\tilde{m}^2_{LR})_{21}/m^2_{\rm SUSY}$ would not be consistent with the experimental bound for $m_{\rm SUSY} = 100\,{\rm GeV}$. Hence, a smaller value of $\tilde{\alpha}$ like $\tilde{\alpha} = \mathcal{O}(0.001)$ would be favorable to be consistent with the experimental bound and that implies $y_\tau \sim 4\pi$. Alternatively, for $\tilde{\alpha} \sim 0.03$ it is required that $A_1 = A_2$ up to $\mathcal{O}(0.1)$. If the non-trivial superpotential leading to SUSY breaking does not include $\chi_i$, i.e. $\langle \partial_{\chi_i} W \rangle = 0$, we can realize

$$A_i = -\left(K^{(\chi)}_{i\bar{i}}\right)^{-1} m_{3/2}, \tag{3.21}$$

where $K^{(\chi)}_{i\bar{i}}$ is the Kähler metric of the fields $\chi_i$. Note that the Kähler metric for $\chi_i$ are degenerate at the leading order, because $\chi_i$ are the $A_4$ triplet. Hence, we obtain the degeneracy between $A_i$, i.e., $A_1 = A_2 = A_3$ up to $\mathcal{O}(\tilde{\alpha} m_{3/2})$. In this case, $(\tilde{m}^2_{LR})_{21}$ is suppressed and we can estimate $(\tilde{m}^2_{LR})_{21}/m^2_{\rm SUSY} \sim \tilde{\alpha}^2 m_\mu / m_{3/2} = \mathcal{O}(10^{-6})$ for $\tilde{\alpha} \sim 0.03$. This value is consistent with the experimental bound. However, the parameter region with larger $\alpha$ like $\tilde{\alpha} = \mathcal{O}(0.1)$ is still ruled out. Obviously, the value of $(\tilde{m}^2_{LR})_{21}$ depends on the difference between $A_1$ and $A_2$. If the difference $A_1 - A_2$ is smaller than $\mathcal{O}(\tilde{\alpha} m_{3/2})$, $(\tilde{m}^2_{LR})_{21}$ would be suppressed more.

Here we give a comment on radiative corrections. Similarly to slepton masses, radiative corrections to A-terms do not change drastically the above results. Note that Yukawa couplings are small, in particular the first and second families.

### 3.3 Comparison with other models

Here we give briefly comments comparing the $A_4$ model and other models. First, let us compare with the $D_4$ models [9, 12, 13], which also lead to the tri-bimaximal mixing for the lepton sector by choosing proper values of parameters. In both of the $A_4$ model and the $D_4$ models, the charged lepton mass matrices, $M_l$, are diagonal at the leading order, but there are small corrections at the next order. That is, in the $A_4$ model, the (1,2) entries, $\theta_{L12}$ and $\theta_{R12}$, of diagonalizing matrices of $M_l$ are estimated as $\theta_{L12} = \mathcal{O}(\tilde{\alpha})$ and $\theta_{R12} = \mathcal{O}(\tilde{\alpha} m_e/m_\mu)$ with $\tilde{\alpha} \sim 0.03$ for the typical values, while in the $D_4$ models, both of those entries are estimated as $\theta_{L12} = \theta_{R12} = \mathcal{O}(10^{-2})$ - $\mathcal{O}(10^{-3})$. Thus, these angles would be smaller in the $D_4$ models.

In the $D_4$ models, the second and third families of left-handed and right-handed charged leptons correspond to $D_4$ doublets and the first families correspond to $D_4$ singlets. Then, the second and third families of left-handed and right-handed slepton masses are degenerate at the leading order, while the first families of sleptons masses are, in general, independent of the others. On the other hand, in the $A_4$ model, all the three families of left-handed slepton masses are degenerate at the leading order, while the three families of right-handed slepton masses are, in general, different from each other. That is, we have different predictions in slepton mass spectra between the $A_4$ model and the $D_4$ models.



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In the $A_4$ model, a large value of the entry $\theta_{L12} = \mathcal{O}(\tilde{\alpha})$ like $\theta_{L12} \sim \tilde{\alpha} \sim 0.03$ would not be favorable from the viewpoint of FCNC experimental bounds. However, the triplet structure of left-handed leptons is helpful to suppress $(\tilde{m}^2_{LL})_{12}$ and $(\tilde{m}^2_{LR})_{12}$ and the predicted values in a wide parameter space become consistent with the FCNC experimental bounds.

On the other hand, in the $D_4$ models, small values of mixing angles $\theta_{L12} = \theta_{R12} = \mathcal{O}(10^{-3})$ are helpful to suppress $(\tilde{m}^2_{LL})_{12}$, $(\tilde{m}^2_{RR})_{12}$ and $(\tilde{m}^2_{LR})_{12}$, although the first and second families of slepton masses are not degenerate. This situation is the same as one for the right-handed sleptons in the $A_4$ model. Then, both models are consistent with the current FCNC experimental bounds.

Next, we give a comment on other $A_4$ models. Indeed, several $A_4$ models have been proposed. For those SUSY $A_4$ models, we can evaluate soft SUSY breaking terms in a way similar to section 3.1 and 3.2. We would obtain similar results in the models that left-handed and right-handed leptons are assigned to a triplet and singlets, respectively, and the angles, $\theta_{L12}$ and $\theta_{R12}$, are similar to the above values. Alternatively, we could construct the supersymmetric model, where three families of right-handed leptons are assigned with an $A_4$ triplet, while three families of left-handed leptons are assigned with three singlets, $\mathbf{1}$, $\mathbf{1}'$ and $\mathbf{1}''$, that is the opposite assignment of the $A_4$ model of [6].[7] In such a model, three families of right-handed slepton masses would be degenerate at the leading order, since the right-handed leptons are a triplet. On the other hand, three families of left-handed slepton masses would be different from each other. Thus, the prediction on soft SUSY breaking terms depend on flavor symmetries and assignments of matter fields.

## 4. Conclusion

We have studied soft SUSY breaking terms, which are derived from the $A_4$ model. Three families of left-handed slepton masses are degenerate, while three families of right-handed slepton masses are, in general, different from each other. That is the pattern different from slepton masses in the $D_4$ model, where only the second and third families of both left-handed and right-handed slepton masses are degenerate [13].

In the wide parameter region, the FCNCs predicted in the SUSY $A_4$ model are consistent with the current experimental bounds. Thus, the non-Abelain flavor symmetry in the $A_4$ model is useful not only to derive realistic lepton mass matrices, but also to suppress FCNC processes. If the bound of $BR(\mu \to e\gamma)$ is improved in future, e.g. by the MEG experiment [26], the allowed parameter space would be reduced, that is, a smaller value of $\tilde{\alpha}$, i.e. a larger value of $y_\tau$ like $y_\tau \sim 4\pi$, would become favorable.

**Note to be added.** While this paper was being completed, ref. [27] appeared, where a similar issue was studied.

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[7] Indeed, such non-SUSY model has been studied in [8].

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## A. $A_4$ group

The group $A_4$ has two generators $S$ and $T$, which satisfies $S^2 = (ST)^3 = T^3 = 1$. In the representation, where $T$ is taken to be diagonal, the elements $S$ and $T$ are expressed as

$$S = \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}. \tag{A.1}$$

Then, twelve elements of the $A_4$ group are given by

$$\begin{array}{llllll} 1, & S, & T, & ST, & TS, & T^2, \\ ST^2, & STS, & TST, & T^2S, & TST^2, & T^2ST. \end{array} \tag{A.2}$$

The product of two triplets $\mathbf{3} \times \mathbf{3}$, where

$$a = (a_1, a_2, a_3), \quad b = (b_1, b_2, b_3). \tag{A.3}$$

is decomposed as

$$\mathbf{3} \times \mathbf{3} = \mathbf{1} + \mathbf{1}' + \mathbf{1}'' + \mathbf{3} + \mathbf{3}', \tag{A.4}$$

where

$$\begin{aligned} &\mathbf{1} : (a_1 b_1 + a_2 b_3 + a_3 b_2), \quad \mathbf{1}' : (a_3 b_3 + a_1 b_2 + a_2 b_1), \quad \mathbf{1}'' : (a_2 b_2 + a_1 b_3 + a_3 b_1), \\ &\mathbf{3} : (2a_1 b_1 - a_2 b_3 - a_3 b_2, 2a_3 b_3 - a_1 b_2 - a_2 b_1, 2a_2 b_2 - a_1 b_3 - a_3 b_1), \\ &\mathbf{3}' : (a_2 b_3 - a_3 b_2, a_1 b_2 - a_2 b_1, a_1 b_3 - a_3 b_1). \end{aligned} \tag{A.5}$$

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