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# The effects of fourth generation on the total branching ratio and the lepton polarization in $\Lambda_{b} \rightarrow \Lambda \ell^{+} \ell^{-}$decay 

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Abstract: This study investigates the influence of the fourth generation quarks on the total branching ratio and the single lepton polarizations in $\Lambda_{b} \rightarrow \Lambda \ell^{+} \ell^{-}$decay. Taking $\left|V_{t^{\prime} s} V_{t^{\prime} b}\right| \sim\{0.01-0.03\}$ with phase just below $90^{\circ}$, which is consistent with the $b \rightarrow s \ell^{+} \ell^{-}$ rate and the $B_{s}$ mixing parameter $\Delta m_{B_{s}}$, we obtain that the total branching ratio and the single lepton $(\mu, \tau)$ polarizations are quite sensitive to the existence of fourth generation. It can serve as a good tool to search for new physics effects, precisely, to search for the fourth generation quarks $\left(t^{\prime}, b^{\prime}\right)$.

Keywords: B-Physics, Beyond Standard Model.

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## 1. Introduction

Although the standard model (SM) of electroweak interaction has very successfully described all existing experimental data, it is believed that it is a low energy manifestation of some fundamental theory. Therefore, intensive search for physics beyond the SM is now being performed in various areas of particle physics. One possible extension is SM with more than three generations.

The mass and mixing patterns of the fundamental fermions are the most mysterious aspects of the particle physics. Even the number of fermion generations is not fixed by the Standard Model(SM). In this sense, SM may be treated as an effective theory of fundamental interactions rather than fundamental particles. The Democratic Mass Matrix approach [1], which is quite natural in the SM framework, may be considered as the interesting step in true direction. It is intriguing that Flavors Democracy favors the existence of the fourth SM family 2 -島. The main restrictions on the new SM families come from experimental data on the $\rho$ and $S$ parameters [\#]. However, the common mass of the fourth quark ( $m_{t^{\prime}}$ ) lies between 320 GeV and 730 GeV considering the experimental value of $\rho=1.0002_{-0.0004}^{+0.0007}$ [5]. The last value is close to upper limit on heavy quark masses, $m_{q} \leq 700 \mathrm{GeV} \approx 4 m_{t}$, which follows from partial-wave unitarity at high energies $[6]$. It should be noted that with preferable value $a \approx g_{w}$ Flavor Democracy predicts $m_{t^{\prime}} \approx 8 m_{w} \approx 640 \mathrm{GeV}$. The above mentioned values for mass of $m_{t^{\prime}}$ disfavors the fifth SM family both because in general we expect that $m_{t} \ll m_{t^{\prime}} \ll m_{t^{\prime \prime}}$ and the experimental values of the $\rho$ and $S$ parameters [4] restrict the quark mass up to 700 GeV .

Moreover, Democratic Mass Matrix approach provides, in principle, the possibility to get the small masses [7] for the first neutrino species without see-saw mechanism. The fourth family quarks, if exist, will be copiously produced at the LHC [8]. Then the fourth family leads to an essential increase in Higgs boson production cross section via gluon fusion at hadron colliders [9].

One of the efficient ways to establish the existence of four generation is via their indirect manifestations in loop diagrams. Rare decays, induced by flavor changing neutral current (FCNC) $b \rightarrow s(d)$ transitions is at the forefront of our quest to understand flavor and the origins of CPV, offering one of the best probes for New Physics (NP) beyond the Standard

Model (SM). Several hints for NP have emerged in the past few years. For example, a large difference is seen in direct CP asymmetries in $B \rightarrow K \pi$ decays [10],

$$
\begin{align*}
\mathcal{A}_{K \pi} \equiv A_{\mathrm{CP}}\left(B^{0} \rightarrow K^{+} \pi^{-}\right) & =-0.093 \pm 0.015, \\
\mathcal{A}_{K \pi^{0}} \equiv A_{\mathrm{CP}}\left(B^{+} \rightarrow K^{+} \pi^{0}\right) & =+0.047 \pm 0.026, \tag{1.1}
\end{align*}
$$

or $\Delta \mathcal{A}_{K \pi} \equiv \mathcal{A}_{K \pi^{0}}-\mathcal{A}_{K \pi}=(14 \pm 3) \%$ [11]. As this percentage was not predicted when first measured in 2004, it has stimulated discussion on the potential mechanisms that it may have been missed in the SM calculations (12- 14].

Better known is the mixing-induced CP asymmetry $\mathcal{S}_{f}$ measured in a multitude of CP eigenstates $f$. For penguin-dominated $b \rightarrow s q \bar{q}$ modes, within SM, $\mathcal{S}_{s q \bar{q}}$ should be close to that extracted from $b \rightarrow c \bar{c} s$ modes. The latter is now measured rather precisely, $\mathcal{S}_{c \bar{c} s}=\sin 2 \phi_{1}=0.674 \pm 0.026$ [15], where $\phi_{1}$ is the weak phase in $V_{t d}$. However, for the past few years, data seem to indicate, at $2.6 \sigma$ significance,

$$
\begin{equation*}
\Delta \mathcal{S} \equiv \mathcal{S}_{s q \bar{q}}-\mathcal{S}_{c \bar{c} s} \leq 0, \tag{1.2}
\end{equation*}
$$

which has stimulated even more discussions.
Flavor-changing neutral current (FCNC) $b \rightarrow s(d) \ell^{+} \ell^{-}$decays provide important tests for the gauge structure of the standard model (SM) at one-loop level. Moreover, $b \rightarrow s(d) \ell^{+} \ell^{-}$decay is also very sensitive to the new physics beyond SM. New physics effects manifest themselves in rare decays in two different ways, either through new combinations to the new Wilson coefficients or through the new operator structure in the effective Hamiltonian, which is absent in the SM. One of the efficient ways in establishing new physics beyond the SM is the measurement of the lepton polarization in the inclusive $b \rightarrow s(d) \ell^{+} \ell^{-}$transition [16] and the exclusive $B \rightarrow K\left(K^{*}, \rho, \gamma\right) \ell^{+} \ell^{-}$decays [17]-25].

In this paper we investigate the possibility of searching for new physics in the heavy baryon decays $\Lambda_{b} \rightarrow \Lambda \ell^{+} \ell^{-}$using the SM with four generations of quarks $\left(b^{\prime}, t^{\prime}\right)$. The fourth quark ( $t^{\prime}$ ), like $u, c, t$ quarks, contributes in the $b \rightarrow s(d)$ transition at loop level. Note that, fourth generation effects have been widely studied in baryonic and semileptonic B decays [26]-[39]. But, there are few works related to the exclusive decays $\Lambda_{b} \rightarrow \Lambda l^{+} l^{-}$.

The main problem for the description of exclusive decays is to evaluate the form factors, i.e., matrix elements of the effective Hamiltonian between initial and final hadron states. It is well known that in order to describe baryonic $\Lambda_{b} \rightarrow \Lambda \ell^{+} \ell^{-}$decay a number of form factors are needed (see for example 40]). However, when heavy quark effective theory (HQET) is applied, only two independent form factors appear [41].

It should be mentioned here that the exclusive decay $\Lambda_{b} \rightarrow \Lambda \ell^{+} \ell^{-}$decay rate, lepton polarization and heavy $\left(\Lambda_{b}\right)$ or $\operatorname{light}(\Lambda)$ baryon polarization(readily measurable) is studied in the SM, the two Higgs doublet model and using the general form of the effective Hamiltonian, in [40, (42] and [43]-46], respectively.

The sensitivity of the forward-backward asymmetry to the existence of fourth generation quarks in the $\Lambda_{b} \rightarrow \Lambda \ell^{+} \ell^{-}$decay is investigated in [39] and it is obtained that the forward-backward asymmetry is very sensitive to the fourth generation parameters ( $m_{t^{\prime}}$, $V_{t^{\prime} b} V_{t^{\prime} s}^{*}$ ). In this connection it is natural to ask whether the total branching ratio and the
lepton polarizations are sensitive to the fourth generation parameters, in the "heavy baryon $\rightarrow$ light baryon $\ell^{+} \ell^{-}$" decays. In the present work we try to answer to this question.

The paper is organized as follows. In section 2, using the effective Hamiltonian, the general expressions for the longitudinal, transversal and normal polarizations of leptons are derived. In section 3 we investigate the sensitivity of these polarizations to the fourth generation parameters $\left(m_{t^{\prime}}, V_{t^{\prime} b} V_{t^{\prime} s}^{*}\right)$.

## 2. Lepton polarizations

The matrix element of the $\Lambda_{b} \rightarrow \Lambda \ell^{+} \ell^{-}$decay at quark level is described by $b \rightarrow s \ell^{+} \ell^{-}$ transition for which the effective Hamiltonian at $O(\mu)$ scale can be written as

$$
\begin{equation*}
\mathcal{H}_{\mathrm{eff}}=\frac{4 G_{F}}{\sqrt{2}} V_{t b} V_{t s}^{*} \sum_{i=1}^{10} \mathcal{C}_{i}(\mu) \mathcal{O}_{i}(\mu) \tag{2.1}
\end{equation*}
$$

where the full set of the operators $\mathcal{O}_{i}(\mu)$ and the corresponding expressions for the Wilson coefficients $\mathcal{C}_{i}(\mu)$ in the SM are given in 47 49. As it has already been noted, the fourth generation is introduced in the same way as three generations in the SM, and so new operators do not appear and clearly the full operator set is exactly the same as in SM. The fourth generation changes the values of the Wilson coefficients $C_{7}(\mu), C_{9}(\mu)$ and $C_{10}(\mu)$, via virtual exchange of the fourth generation up type quark $t^{\prime}$. The above mentioned Wilson coefficients will modify as

$$
\begin{equation*}
\lambda_{t} C_{i} \rightarrow \lambda_{t} C_{i}^{\mathrm{SM}}+\lambda_{t^{\prime}} C_{i}^{\text {new }} \tag{2.2}
\end{equation*}
$$

where $\lambda_{f}=V_{f b}^{*} V_{f s}$. The unitarity of the $4 \times 4$ CKM matrix leads to

$$
\begin{equation*}
\lambda_{u}+\lambda_{c}+\lambda_{t}+\lambda_{t^{\prime}}=0 \tag{2.3}
\end{equation*}
$$

Since $\lambda_{u}=V_{u b}^{*} V_{u s}$ is very small in strength compared to the others. Then $\lambda_{t} \approx-\lambda_{c}-\lambda_{t^{\prime}}$ and $\lambda_{c}=V_{c b}^{*} V_{c s} \approx 0.04$ is real by convention. It follows that

$$
\begin{equation*}
\lambda_{t} C_{i}^{\mathrm{SM}}+\lambda_{t^{\prime}} C_{i}^{\text {new }}=\lambda_{c} C_{i}^{\mathrm{SM}}+\lambda_{t^{\prime}}\left(C_{i}^{\text {new }}-C_{i}^{\mathrm{SM}}\right) \tag{2.4}
\end{equation*}
$$

It is clear that, for the $m_{t^{\prime}} \rightarrow m_{t}$ or $\lambda_{t^{\prime}} \rightarrow 0, \lambda_{t^{\prime}}\left(C_{i}^{\text {new }}-C_{i}^{\mathrm{SM}}\right)$ term vanishes, as required by the GIM mechanism. One can also write $C_{i}$ 's in the following form

$$
\begin{align*}
C_{7}^{\mathrm{tot}}(\mu) & =C_{7}^{\mathrm{SM}}(\mu)+\frac{\lambda_{t^{\prime}}}{\lambda_{t}} C_{7}^{\mathrm{new}}(\mu) \\
C_{9}^{\mathrm{tot}}(\mu) & =C_{9}^{\mathrm{SM}}(\mu)+\frac{\lambda_{t^{\prime}}}{\lambda_{t}} C_{9}^{\mathrm{new}}(\mu) \\
C_{10}^{\mathrm{tot}}(\mu) & =C_{10}^{\mathrm{SM}}(\mu)+\frac{\lambda_{t^{\prime}}}{\lambda_{t}} C_{10}^{\mathrm{new}}(\mu) \tag{2.5}
\end{align*}
$$

where the last terms in these expressions describe the contributions of the $t^{\prime}$ quark to the Wilson coefficients. $\lambda_{t^{\prime}}$ can be parametrized as:

$$
\begin{equation*}
\lambda_{t^{\prime}}=V_{t^{\prime} b}^{*} V_{t^{\prime} s}=r_{s b} e^{i \phi_{s b}} \tag{2.6}
\end{equation*}
$$

| $C_{1}$ | $C_{2}$ | $C_{3}$ | $C_{4}$ | $C_{5}$ | $C_{6}$ | $C_{7}^{\mathrm{SM}}$ | $C_{9}^{\mathrm{SM}}$ | $C_{10}^{\mathrm{SM}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -0.248 | 1.107 | 0.011 | -0.026 | 0.007 | -0.031 | -0.313 | 4.344 | -4.669 |

Table 1: The numerical values of the Wilson coefficients at $\mu=m_{b}$ scale within the SM. The corresponding numerical value of $C^{0}$ is 0.362 .

In deriving eq. (2.5) we factored out the term $V_{t b}^{*} V_{t s}$ in the effective Hamiltonian given in eq. (2.1). The explicit forms of the $C_{i}^{\text {new }}$ can easily be obtained from the corresponding expression of the Wilson coefficients in SM by substituting $m_{t} \rightarrow m_{t^{\prime}}$ (see 47, 48]). If the $\hat{s}$ quark mass is neglected, the above effective Hamiltonian leads to following matrix element for the $b \rightarrow s \ell^{+} \ell^{-}$decay

$$
\begin{align*}
\mathcal{H}_{\mathrm{eff}}=\frac{G \alpha}{2 \sqrt{2} \pi} V_{t b} V_{t s}^{*}\left[C_{9}^{\mathrm{tot}} \bar{s} \gamma_{\mu}\left(1-\gamma_{5}\right) b \bar{\ell} \gamma_{\mu} \ell\right. & +C_{10}^{\mathrm{tot}} \bar{s} \gamma_{\mu}\left(1-\gamma_{5}\right) b \bar{\ell} \gamma_{\mu} \gamma_{5} \ell  \tag{2.7}\\
& \left.-2 C_{7}^{\mathrm{tot}} \frac{m_{b}}{q^{2}} \bar{s} \sigma_{\mu \nu} q^{\nu}\left(1+\gamma_{5}\right) b \bar{\ell} \gamma_{\mu} \ell\right]
\end{align*}
$$

where $q^{2}=\left(p_{1}+p_{2}\right)^{2}$ and $p_{1}$ and $p_{2}$ are the final leptons four-momenta. The effective coefficient $C_{9}^{\text {tot }}$ can be written in the following form

$$
\begin{equation*}
C_{9}^{\mathrm{tot}}=C_{9}+Y(s), \tag{2.8}
\end{equation*}
$$

where $s^{\prime}=q^{2} / m_{b}^{2}$ and the function $Y\left(s^{\prime}\right)$ contains the contributions from the one loop matrix element of the four quark operators. A perturbative calculation leads to the result 47, 49,

$$
\begin{align*}
Y_{\mathrm{per}}\left(s^{\prime}\right)= & g\left(\hat{m}_{c}, s^{\prime}\right)\left(3 C_{1}+C_{2}+3 C_{3}+C_{4}+3 C_{5}+C_{6}\right) \\
& -\frac{1}{2} g\left(1, s^{\prime}\right)\left(4 C_{3}+4 C_{4}+3 C_{5}+C_{6}\right) \\
& -\frac{1}{2} g\left(0, s^{\prime}\right)\left(C_{3}+3 C_{4}\right)+\frac{2}{9}\left(3 C_{3}+C_{4}+3 C_{5}+C_{6}\right), \tag{2.9}
\end{align*}
$$

where $\hat{m}_{c}=\frac{m_{c}}{m_{b}}$. The explicit expressions for $g\left(\hat{m}_{c}, s^{\prime}\right), g\left(0, s^{\prime}\right), g\left(1, s^{\prime}\right)$ and the values of $C_{i}$ in the SM can be found in 47, 49].

In addition to the short distance contribution, $Y_{\text {per }}\left(s^{\prime}\right)$ receives also long distance contributions, which have their origin in the real $c \bar{c}$ intermediate states, i.e., $J / \psi, \psi^{\prime}, \cdots$. The $J / \psi$ family is introduced by the Breit-Wigner distribution for the resonances through the replacement 50]-52]

$$
\begin{equation*}
Y\left(s^{\prime}\right)=Y_{\mathrm{per}}\left(s^{\prime}\right)+\frac{3 \pi}{\alpha^{2}} C^{(0)} \sum_{V_{i}=\psi_{i}} \kappa_{i} \frac{m_{V_{i}} \Gamma\left(V_{i} \rightarrow \ell^{+} \ell^{-}\right)}{m_{V_{i}}^{2}-s^{\prime} m_{b}^{2}-i m_{V_{i}} \Gamma_{V_{i}}} \tag{2.10}
\end{equation*}
$$

where $C^{(0)}=3 C_{1}+C_{2}+3 C_{3}+C_{4}+3 C_{5}+C_{6}$. The phenomenological parameters $\kappa_{i}$ can be fixed from $\mathcal{B}\left(B \rightarrow K^{*} V_{i} \rightarrow K^{*} \ell^{+} \ell^{-}\right)=\mathcal{B}\left(B \rightarrow K^{*} V_{i}\right) \mathcal{B}\left(V_{i} \rightarrow \ell^{+} \ell^{-}\right)$, where the data for the right hand side is given in [53]. For the lowest resonances $J / \psi$ and $\psi^{\prime}$ one can use $\kappa=1.65$ and $\kappa=2.36$, respectively (see 54).


Figure 1: The dependence of the branching ratio for the $\Lambda_{b} \rightarrow \Lambda \mu^{+} \mu^{-}$decay on the fourth generation quark mass $m_{t^{\prime}}$ for three different values of $r_{s b}$.


Figure 2: The same as in figure 1, but for the $\tau$ lepton.

The amplitude of the exclusive $\Lambda_{b} \rightarrow \Lambda \ell^{+} \ell^{-}$decay can be obtained by sandwiching $\mathcal{H}_{\text {eff }}$ for the $b \rightarrow s \ell^{+} \ell^{-}$transition between initial and final baryon states, i.e., $\langle\Lambda| \mathcal{H}_{\text {eff }}\left|\Lambda_{b}\right\rangle$. It follows from eq. (2.7) that in order to calculate the $\Lambda_{b} \rightarrow \Lambda \ell^{+} \ell^{-}$decay amplitude the following matrix elements are needed

$$
\begin{gathered}
\langle\Lambda| \bar{s} \gamma_{\mu}\left(1 \mp \gamma_{5}\right) b\left|\Lambda_{b}\right\rangle, \\
\langle\Lambda| \bar{s} \sigma_{\mu \nu}\left(1 \mp \gamma_{5}\right) b\left|\Lambda_{b}\right\rangle, \\
\langle\Lambda| \bar{s}\left(1 \mp \gamma_{5}\right) b\left|\Lambda_{b}\right\rangle .
\end{gathered}
$$

Explicit forms of these matrix elements in terms of the form factors are presented in 43]


Figure 3: The dependence of the longitudinal lepton polarization asymmetry for the $\Lambda_{b} \rightarrow \Lambda \mu^{+} \mu^{-}$ decay on the fourth generation quark mass $m_{t^{\prime}}$ for three different values of $r_{s b}$.


Figure 4: The same as in figure 3, but for the $\tau$ lepton.
(see also 40]). The matrix element of the $\Lambda_{b} \rightarrow \Lambda \ell^{+} \ell^{-}$can be written as

$$
\begin{align*}
\mathcal{M}=\frac{G \alpha}{4 \sqrt{2} \pi} V_{t b} V_{t s}^{*}\{ & \bar{\ell}^{\mu} \ell \bar{u}_{\Lambda}\left[A_{1} \gamma_{\mu}\left(1+\gamma_{5}\right)+B_{1} \gamma_{\mu}\left(1-\gamma_{5}\right)\right.  \tag{2.11}\\
& \left.+i \sigma_{\mu \nu} q^{\nu}\left[A_{2}\left(1+\gamma_{5}\right)+B_{2}\left(1-\gamma_{5}\right)\right]+q_{\mu}\left[A_{3}\left(1+\gamma_{5}\right)+B_{3}\left(1-\gamma_{5}\right)\right]\right] u_{\Lambda_{b}} \\
& \left.\left.+\bar{\ell} \gamma^{\mu} \gamma_{5} \ell \bar{u}_{\Lambda}\left[E_{1} \gamma_{\mu}\left(1-\gamma_{5}\right)+i \sigma_{\mu \nu} q^{\nu} E_{2}\left(1-\gamma_{5}\right)+E_{3} q^{\mu}\left(1-\gamma_{5}\right)\right]\right] u_{\Lambda_{b}}\right\}
\end{align*}
$$

where $P=p_{\Lambda_{b}}+p_{\Lambda}$. Explicit expressions of the functions $A_{i}, B_{i}$, and $E_{i}(i=1,2,3)$ are


Figure 5: The dependence of the transversal lepton polarization asymmetry for the $\Lambda_{b} \rightarrow \Lambda \mu^{+} \mu^{-}$ decay on the fourth generation quark mass $m_{t^{\prime}}$ for three different values of $r_{s b}$.


Figure 6: The same as in figure 5, but for the $\tau$ lepton.
given as follows 43]:

$$
\begin{aligned}
A_{1} & =-\frac{4 m_{b}}{m_{\Lambda_{b}}} F_{2} C_{7}^{\mathrm{tot}} \\
A_{2} & =-\frac{4 m_{b}}{q^{2}}\left(F_{1}+\sqrt{r} F_{2}\right) C_{7}^{\mathrm{tot}} \\
A_{3} & =-\frac{4 m_{b} m_{\Lambda}}{q^{2} m_{\Lambda_{b}}} F_{2} C_{7}^{\mathrm{tot}} \\
B_{1} & =2\left(F_{1}+\sqrt{r} F_{2}\right) C_{9}^{\mathrm{tot}} \\
B_{2} & =\frac{2 F_{2}}{m_{\Lambda_{b}}} C_{9}^{\mathrm{tot}}
\end{aligned}
$$



Figure 7: The dependence of the normal lepton polarization asymmetry for the $\Lambda_{b} \rightarrow \Lambda \tau^{+} \tau^{-}$ decay on the fourth generation quark mass $m_{t^{\prime}}$ for three different values of $r_{s b}$.


Figure 8: The dependence of the combined normal lepton polarization asymmetry for the $\Lambda_{b} \rightarrow$ $\Lambda \tau^{+} \tau^{-}$decay on the fourth generation quark mass $m_{t^{\prime}}$ for three different values of $r_{s b}$.

$$
\begin{align*}
B_{3} & =\frac{4 m_{b}}{q^{2}} F_{2} C_{7}^{\mathrm{tot}} \\
E_{1} & =2\left(F_{1}+\sqrt{r} F_{2}\right) C_{10}^{\mathrm{tot}} \\
E_{2} & =E_{3}=\frac{2 F_{2}}{m_{\Lambda_{b}}} C_{10}^{\mathrm{tot}} \tag{2.12}
\end{align*}
$$

From the expressions of the above-mentioned matrix elements eq. (2.11) we observe that $\Lambda_{b} \rightarrow \Lambda \ell^{+} \ell^{-}$decay is described in terms of many form factors. When HQET is applied to the number of independent form factors, as it has already been noted, reduces to two ( $F_{1}$ and $F_{2}$ ) irrelevant with the Dirac structure of the corresponding operators and it is


Figure 9: The dependence of the combined transversal lepton polarization asymmetry for the $\Lambda_{b} \rightarrow \Lambda \tau^{+} \tau^{-}$decay on the fourth generation quark mass $m_{t^{\prime}}$ for three different values of $r_{s b}$.
obtained in 41 that

$$
\begin{equation*}
\left\langle\Lambda\left(p_{\Lambda}\right)\right| \bar{s} \Gamma b\left|\Lambda\left(p_{\Lambda_{b}}\right)\right\rangle=\bar{u}_{\Lambda}\left[F_{1}\left(q^{2}\right)+\not \psi F_{2}\left(q^{2}\right)\right] \Gamma u_{\Lambda_{b}} \tag{2.13}
\end{equation*}
$$

where $\Gamma$ is an arbitrary Dirac structure, $v^{\mu}=p_{\Lambda_{b}}^{\mu} / m_{\Lambda_{b}}$ is the four-velocity of $\Lambda_{b}$, and $q=p_{\Lambda_{b}}-p_{\Lambda}$ is the momentum transfer. Comparing the general form of the form factors with (2.14), one can easily obtain the following relations among them (see also 40)

$$
\begin{align*}
g_{1} & =f_{1}=f_{2}^{T}=g_{2}^{T}=F_{1}+\sqrt{r} F_{2} \\
g_{2} & =f_{2}=g_{3}=f_{3}=g_{T}^{V}=f_{T}^{V}=\frac{F_{2}}{m_{\Lambda_{b}}} \\
g_{T}^{S} & =f_{T}^{S}=0 \\
g_{1}^{T} & =f_{1}^{T}=\frac{F_{2}}{m_{\Lambda_{b}}} q^{2} \\
g_{3}^{T} & =\frac{F_{2}}{m_{\Lambda_{b}}}\left(m_{\Lambda_{b}}+m_{\Lambda}\right) \\
f_{3}^{T} & =-\frac{F_{2}}{m_{\Lambda_{b}}}\left(m_{\Lambda_{b}}-m_{\Lambda}\right) \tag{2.14}
\end{align*}
$$

where $r=m_{\Lambda}^{2} / m_{\Lambda_{b}}^{2}$.
Having obtained the matrix element for the $\Lambda_{b} \rightarrow \Lambda \ell^{+} \ell^{-}$decay, we next aim to calculate lepton polarizations with the help of this matrix element. We write the $\ell^{\mp}$ spin four-vector in terms of a unit vector $\vec{\xi}_{\mp}$ along the $\ell^{\mp}$ momentum in its rest frame as

$$
\begin{equation*}
s_{\mu}^{\mp}=\left(\frac{\vec{p}^{\mp} \cdot \vec{\xi}^{\mp}}{m_{\ell}}, \vec{\xi}^{\mp}+\frac{\vec{p}^{\mp}\left(\vec{p}^{\mp} \cdot \vec{\xi}^{\mp}\right)}{E_{\ell}+m_{\ell}}\right) \tag{2.15}
\end{equation*}
$$

and choose the unit vectors along the longitudinal, normal and transversal components of the $\ell^{-}$polarization to be

$$
\begin{equation*}
\vec{e}_{L}^{\mp}=\frac{\vec{p}^{\mp}}{\left|\vec{p}^{-}\right|}, \quad \vec{e}_{N}^{\mp}=\frac{\vec{p}_{\Lambda} \times \vec{p}^{\mp}}{\left|\vec{p}_{\Lambda} \times \vec{p}^{-}\right|}, \quad \vec{e}_{T}^{\mp}=\vec{e}_{N}^{\mp} \times \vec{e}_{L}^{\mp}, \tag{2.16}
\end{equation*}
$$

respectively, where $\vec{p}^{\mp}$ and $\vec{p}_{\Lambda}$ are the three momenta of $\ell^{\mp}$ and $\Lambda$, in the center of mass frame of the $\ell^{+} \ell^{-}$system. Obviously, $\vec{p}^{+}=-\vec{p}^{-}$in this reference frame.

The differential decay rate of the $\Lambda_{b} \rightarrow \Lambda \ell^{+} \ell^{-}$decay for any spin direction $\vec{\xi}^{\mp}$ can be written as

$$
\begin{equation*}
\frac{d \Gamma(\vec{\xi} \mp)}{d s}=\frac{1}{2}\left(\frac{d \Gamma}{d s}\right)_{0}\left[1+\left(P_{L}^{\mp} \vec{e}_{L}^{\mp}+P_{N}^{\mp} \vec{e}_{N}^{\mp}+P_{T}^{\mp} \vec{e}_{T}^{\mp}\right) \cdot \vec{\xi}^{\mp}\right], \tag{2.17}
\end{equation*}
$$

where $(d \Gamma / d s)_{0}$ corresponds to the unpolarized differential decay rate, $s=q^{2} / m_{\Lambda_{b}}^{2}$ and $P_{L}^{\mp}, P_{N}^{\mp}$ and $P_{T}^{\mp}$ represent the longitudinal, normal and transversal polarizations of $\ell^{\mp}$, respectively. The unpolarized decay width in eq. (2.17) can be written as

$$
\begin{equation*}
\left(\frac{d \Gamma}{d s}\right)_{0}=\frac{G^{2} \alpha^{2}}{8192 \pi^{5}}\left|V_{t b} V_{t s}^{*}\right|^{2} \lambda^{1 / 2}(1, r, s) v\left[\mathcal{T}_{0}(s)+\frac{1}{3} \mathcal{T}_{2}(s)\right] \tag{2.18}
\end{equation*}
$$

where $\lambda(1, r, s)=1+r^{2}+s^{2}-2 r-2 s-2 r s$ is the triangle function and $v=\sqrt{1-4 m_{\ell}^{2} / q^{2}}$ is the lepton velocity. The explicit expressions for $\mathcal{T}_{0}$ and $\mathcal{T}_{2}$ are given by:

$$
\begin{align*}
& \mathcal{T}_{0}= 4 m_{\Lambda_{b}}^{2}\{ \\
&\left\{m_{\ell}^{2} m_{\Lambda_{b}}^{2} \hat{s}(1+r-\hat{s})\left|E_{3}\right|^{2}+16 m_{\ell}^{2} m_{\Lambda_{b}} \sqrt{r}(1-r+\hat{s}) \operatorname{Re}\left[E_{1}^{*} E_{3}\right]+\right. \\
& 8\left(2 m_{\ell}^{2}+m_{\Lambda_{b}}^{2} \hat{s}\right)\left\{(1-r+\hat{s}) m_{\Lambda_{b}} \sqrt{r} \operatorname{Re}\left[A_{1}^{*} A_{2}+B_{1}^{*} B_{2}\right]-\right. \\
&\left.m_{\Lambda_{b}}(1-r-\hat{s}) \operatorname{Re}\left[A_{1}^{*} B_{2}+A_{2}^{*} B_{1}\right]-2 \sqrt{r}\left(\operatorname{Re}\left[A_{1}^{*} B_{1}\right]+m_{\Lambda_{b}}^{2} \hat{s} \operatorname{Re}\left[A_{2}^{*} B_{2}\right]\right)\right\}+ \\
& 2\left(4 m_{\ell}^{2}(1+r-\hat{s})+m_{\Lambda_{b}}^{2}\left[(1-r)^{2}-\hat{s}^{2}\right]\right)\left(\left|A_{1}\right|^{2}+\left|B_{1}\right|^{2}\right)+ \\
& 2 m_{\Lambda_{b}}^{2}\left(4 m_{\ell}^{2}[\lambda+(1+r-\hat{s}) \hat{s}]+m_{\Lambda_{b}}^{2} \hat{s}\left[(1-r)^{2}-\hat{s}^{2}\right]\right)\left(\left|A_{2}\right|^{2}+\left|B_{2}\right|^{2}\right)- \\
& 2\left(4 m_{\ell}^{2}(1+r-\hat{s})-m_{\Lambda_{b}}^{2}\left[(1-r)^{2}-\hat{s}^{2}\right]\right)\left|E_{1}\right|^{2}+  \tag{2.19}\\
&\left.2 m_{\Lambda_{b}}^{3} \hat{s} v^{2}\left(4(1-r+\hat{s}) \sqrt{r} \operatorname{Re}\left[E_{1}^{*} E_{2}\right]-m_{\Lambda_{b}}\left[(1-r)^{2}-\hat{s}^{2}\right]\left|E_{2}\right|^{2}\right)\right\} \tag{2.20}
\end{align*}
$$

The polarizations $P_{L}, P_{N}$ and $P_{T}$ are defined as:

$$
P_{i}^{(\mp)}\left(q^{2}\right)=\frac{\frac{d \Gamma}{d s}\left(\vec{\xi}^{\mp}=\vec{e}_{i}^{\mp}\right)-\frac{d \Gamma}{d s}\left(\vec{\xi}^{\mp}=-\vec{e}_{i}^{\mp}\right)}{\frac{d \Gamma}{d s}\left(\vec{\xi}^{\mp}=\vec{e}_{i}^{\mp}\right)+\frac{d \Gamma}{d s}\left(\vec{\xi}^{\mp}=-\vec{e}_{i}^{\mp}\right)},
$$

where $i=L, N, T . P_{L}$ and $P_{T}$ are $P$-odd, $T$-even, while $P_{N}$ is $P$-even, $T$-odd and $C P$-odd. The explicit forms of the expressions for the longitudinal $P_{L}$, transversal $P_{T}$ and normal
$P_{N}$ lepton polarizations are as follows:

$$
\begin{align*}
& P_{L}^{\mp}=\{- 32 m_{\ell} m_{\Lambda_{b}}^{3} v(1-\sqrt{r})\left[\hat{s}-(1+\sqrt{r})^{2}\right] \operatorname{Re}\left[E_{1}^{*} F_{1}\right] \pm 128 m_{\Lambda_{b}}^{4} \hat{s} v \sqrt{r} \operatorname{Re}\left[A_{1}^{*} E_{1}\right]  \tag{2.21}\\
& \mp 64 m_{\Lambda_{b}}^{5} \sqrt{r}(1-r+\hat{s}) \hat{s} v \operatorname{Re}\left[B_{1}^{*} E_{2}+B_{2}^{*} E_{1}\right] \pm 128 m_{\Lambda_{b}}^{6} \hat{s}^{2} v \sqrt{r} \operatorname{Re}\left[A_{2}^{*} E_{2}\right] \\
&-16 m_{\Lambda_{b}}^{4} \hat{s} v\left[\hat{s}-(1+\sqrt{r})^{2}\right] \operatorname{Re}\left[F_{1}^{*} F_{2}+2 m_{\ell} E_{3}^{*} F_{1}\right] \\
& \pm 64 m_{\Lambda_{b}}^{5} \hat{s} v(1-r-\hat{s}) \operatorname{Re}\left[A_{1}^{*} E_{2}+A_{2}^{*} E_{1}\right] \\
& \mp \\
& \frac{64}{3} m_{\Lambda_{b}}^{4} v\left[1+r^{2}+r(\hat{s}-2)+\hat{s}(1-2 \hat{s})\right] \operatorname{Re}\left[B_{1}^{*} E_{1}\right] \\
&\left.\mp \frac{64}{3} m_{\Lambda_{b}}^{6} v \hat{s}[2+r(2 r-4-\hat{s})-\hat{s}(1+\hat{s})] \operatorname{Re}\left[B_{2}^{*} E_{2}\right]\right\} /\left(\mathcal{T}_{0}(\hat{s})+\frac{1}{3} \mathcal{T}_{2}(\hat{s})\right), \\
& P_{T}^{\mp}=\left\{-16 \pi m_{\ell} m_{\Lambda_{b}}^{3} \sqrt{\hat{s} \lambda}\left(\left|A_{1}\right|^{2}-\left|B_{1}\right|^{2}\right)+32 \pi m_{\ell} m_{\Lambda_{b}}^{4} \sqrt{\hat{s} \lambda} \operatorname{Re}\left[A_{1}^{*} B_{2}-A_{2}^{*} B_{1}\right]\right. \\
& \mp 16 \pi m_{\ell} m_{\Lambda_{b}}^{4} \sqrt{\hat{s} \lambda} \operatorname{Re}\left[A_{1}^{*} E_{3}-A_{2}^{*} E_{1}\right] \mp 4 \pi m_{\Lambda_{b}}^{4} \sqrt{\hat{s} \lambda}(1+\sqrt{r}) \operatorname{Re}\left[\left(A_{1}+B_{1}\right)^{*} F_{2}\right] \\
&+16 \pi m_{\ell} m_{\Lambda_{b}}^{5} \sqrt{\hat{s} \lambda}(1-r)\left(\left|A_{2}\right|^{2}-\left|B_{2}\right|^{2}\right) \mp 16 \pi m_{\ell} m_{\Lambda_{b}}^{3} \sqrt{\frac{\lambda}{\hat{s}}}(1-r) \operatorname{Re}\left[B_{1}^{*} E_{1}\right] \\
&+16 \pi m_{\ell} m_{\Lambda_{b}}^{4} \sqrt{r \hat{s} \lambda} \operatorname{Re}\left[2 A_{1}^{*} A_{2}-2 B_{1}^{*} B_{2} \mp B_{1}^{*} E_{3} \mp B_{2}^{*} E_{1}\right]  \tag{2.22}\\
& \pm 4 \pi m_{\Lambda_{b}}^{5} \hat{s} \sqrt{\hat{s} \lambda}\left\{\operatorname{Re}\left[\left(A_{2}+B_{2}\right)^{*} F_{2}\right]+4 m_{\ell} \operatorname{Re}\left[B_{2}^{*} E_{3}\right]\right\} \\
&+4 \pi\left.m_{\Lambda_{b}}^{5} \hat{s} \sqrt{\lambda \hat{s}} v^{2} \operatorname{Re}\left[E_{2}^{*} F_{1}\right]-4 \pi m_{\Lambda_{b}}^{4} \sqrt{\lambda \hat{s}} v^{2}(1+\sqrt{r}) \operatorname{Re}\left[E_{1}^{*} F_{1}\right]\right\} /\left(\mathcal{T}_{0}(\hat{s})+\frac{1}{3} \mathcal{T}_{2}(\hat{s})\right), \\
& P_{N}^{\mp} \mp 16 \pi m_{\ell} m_{\Lambda_{b}}^{3} v \sqrt{\hat{s} \lambda} \operatorname{Im}\left[B_{1}^{*} E_{1}\right] \pm 16 \pi m_{\ell} m_{\Lambda_{b}}^{4} v \sqrt{\hat{s} \lambda} \operatorname{Im}\left[A_{2}^{*} E_{1}-A_{1}^{*} E_{2}\right] \\
&-4 \pi m_{\Lambda_{b}}^{4} v \sqrt{\hat{s} \lambda}(1+\sqrt{r}) \operatorname{Im}\left[ \pm\left(A_{1}+B_{1}\right)^{*} F_{1}+E_{1}^{*} F_{2}\right] \\
&+4 m_{\ell} \operatorname{Im}\left[E_{2}^{*} E_{3}\right] \pm 16 \pi m_{\ell} m_{\Lambda_{b}}^{5} v \sqrt{\hat{s} \lambda}(1-r) \operatorname{Im}\left[B_{2}^{*} E_{2}\right] \\
&+4 \pi m_{\Lambda_{b}}^{5} v \hat{s} \sqrt{\hat{s} \lambda} \operatorname{Im}\left[ \pm\left(A_{2}+B_{2}\right)^{*} F_{1}+E_{2}^{*} F_{2}\right] \\
&\left.-16 \pi m_{\ell} m_{\Lambda_{b}}^{4} v \sqrt{r \hat{s} \lambda} \operatorname{Im}\left[E_{2}^{*}\left( \pm B_{1}+E_{1}\right)+E_{1}^{*}\left( \pm B_{2}+E_{3}\right)\right]\right\} \times  \tag{2.23}\\
& \times 1 /\left.\mathcal{T}_{0}(\hat{s})+\frac{1}{3} \mathcal{T}_{2}(\hat{s})\right) .
\end{align*}
$$

where the $-(+)$ sign in these formulas corresponds to the particle (antiparticle), respectively.
It follows from eq. (2.21) that the difference between $P_{L}^{-}$and $P_{L}^{+}$, in massless lepton case, is the same as SM with three generations because it depends on form factors $F_{1}$ and $F_{2}$. Again in the same way, in massless lepton case, the difference between $P_{N}^{-}$and $P_{N}^{+}$ depends on fourth generation CP violation phase $\left(\phi_{s b}\right)$ and $r_{s b}$. We get the result of SM with three generations if $\phi_{s b}$ is zero.

Combined analysis of the lepton and antilepton polarizations can give additional information about the existence of new physics, since in the SM, $P_{L}^{-}+P_{L}^{+}=0, P_{N}^{-}+P_{N}^{+}=0$ and $P_{T}^{-}-P_{T}^{+} \simeq 0$ (in $m_{\ell} \rightarrow 0$ limit). Therefore, if nonzero values for the above mentioned combined asymmetries are measured in the experiments, it can be considered as an unambiguous indication of the existence of new physics. But looking at eq. (2.21), we see that $P_{L}^{-}+P_{L}^{+}$is the same as SM result in $m_{\ell} \rightarrow 0$ limit. Therefore, in order to look for fourth generation effects we can look at $P_{N}^{-}+P_{N}^{+}$and $P_{T}^{-}-P_{T}^{+}$in $m_{\ell} \rightarrow 0$ limit. The nonzero values of above mentioned quantities will indicate the existence of fourth generation effects.

|  | $F(0)$ | $a_{F}$ | $b_{F}$ |
| :---: | ---: | ---: | ---: |
| $F_{1}$ | 0.462 | -0.0182 | -0.000176 |
| $F_{2}$ | -0.077 | -0.0685 | 0.00146 |

Table 2: Transition form factors for $\Lambda_{b} \rightarrow \Lambda \ell^{+} \ell^{-}$decay in the QCD sum rules method.

| $r_{s b}$ | 0.005 | 0.01 | 0.02 | 0.03 |
| :---: | :---: | :---: | :---: | :---: |
| $m_{t^{\prime}}(\mathrm{GeV})$ | 511 | 373 | 289 | 253 |

Table 3: The experimental limit on $m_{t^{\prime}}$ for $\phi_{s b}=\pi / 2$.

## 3. Numerical analysis

In this section we will study the dependence of the total branching ratio and lepton polarizations as well as combined lepton polarization to the fourth quark mass $\left(m_{t^{\prime}}\right)$ and the product of quark mixing matrix elements ( $\left.V_{t^{\prime} b}^{*} V_{t^{\prime} s}=r_{s b} e^{i \phi_{s b}}\right)$. The main input parameters in the calculations are the form factors. Since the literature lacks exact calculations for the form factors of the $\Lambda_{b} \rightarrow \Lambda$ transition, we will use the results from QCD sum rules approach in combination with HQET [41, 55], which reduces the number of quite many form factors into two. The $\hat{s}$ dependence of these form factors can be represented in the following way

$$
F\left(q^{2}\right)=\frac{F(0)}{1-a_{F} s+b_{F} s^{2}},
$$

where parameters $F_{i}(0), a$ and $b$ are listed in table 2 .
We use the next-to-leading order logarithmic approximation for the resulting values of the Wilson coefficients $C_{9}^{\text {eff }}, C_{7}$ and $C_{10}$ in the SM [56, 57] at the re-normalization point $\mu=m_{b}$. It should be noted that, in addition to short distance contribution, $C_{9}^{\text {eff }}$ receives also long distance contributions from the real $\bar{c} c$ resonant states of the $J / \psi$ family. In the present work we do not take the long distance effects into account. The input parameters we used in this analysis are as follows:
$m_{\Lambda_{b}}=5.624 \mathrm{GeV}, m_{\Lambda}=1.116 \mathrm{GeV}, m_{b}=4.8 \mathrm{GeV}, m_{c}=1.35 \mathrm{GeV}, m_{\tau}=1.778 \mathrm{GeV}$, $m_{\mu}=0.105 \mathrm{GeV}, \lambda_{c}=0.045, \alpha^{-1}=129, G_{F}=1.166 \times 10^{-5} \mathrm{GeV}^{-2}$
In order to perform quantitative analysis of the total branching ratio and the lepton polarizations the values of the new parameters $\left(m_{t^{\prime}}, r_{s b}, \phi_{s b}\right)$ are needed. Using the experimental values of $B \rightarrow X_{s} \gamma$ and $B \rightarrow X_{s} \ell^{+} \ell^{-}$, the bound on $r_{s b} \sim\{0.01-0.03\}$ has been obtained [31] for $\phi_{s b} \sim\{0-2 \pi\}$ and $m_{t^{\prime}} \sim\{300,400\}(\mathrm{GeV})$. We are doing complete analysis about the range of the new parameters considering the recent experimental value of the $\mathcal{B}_{r}\left(B \rightarrow X_{s} \ell^{+} \ell^{-}=(1.59 \pm 0.5) \times 10^{-6}\right)$ 10. Right now, we have obtained that in the case of the $1 \sigma$ level deviation from the measured branching ratio the maximum values of $m_{t^{\prime}}$ are below than the theoretical upper limits The results shown in table 3 [58].

In the foregoing numerical analysis we vary $m_{t^{\prime}}$ in the range $175 \leq m_{t^{\prime}} \leq 600 \mathrm{GeV}$. The lower range is because of the fact that the fourth generation up quark should be
heavier than the third ones $\left(m_{t} \leq m_{t^{\prime}}\right)$ [勻. The upper range comes from the experimental bounds on the $\rho$ and $S$ parameters of SM, which we mentioned above(see Introduction). We took $r_{s b} \sim\{0.01-0.03\}$ with phase around $90^{\circ}\left(\phi_{s b} \approx 90^{\circ}\right)$, which are consistent with the $b \rightarrow s \ell^{+} \ell^{-}$rate and the $B_{s}$ mixing parameter $\Delta m_{B_{s}}[26,59]$.

Before performing numerical analysis, few words about lepton polarizations are in order. From explicit expressions of the lepton polarizations one can easily see that they depend on both $\hat{s}$ and the new parameters $\left(m_{t^{\prime}}, r_{s b}\right)$. We should eliminate the dependence of the lepton polarization on one of the variables. We eliminate the variable $\hat{s}$ by performing integration over $\hat{s}$ in the allowed kinematical region. The total branching ratio and the averaged lepton polarizations are defined as

$$
\begin{align*}
\mathcal{B}_{r} & =\int_{4 m_{\ell}^{2} / m_{\Lambda_{b}}^{2}}^{(1-\sqrt{r})^{2}} \frac{d \mathcal{B}}{d \hat{s}} d \hat{s}, \\
\left\langle P_{i}\right\rangle & =\frac{\int_{4 m_{\ell}^{2} / m_{\Lambda_{b}}^{2}}^{(1-\sqrt{2})^{2}} P_{i} \frac{d \mathcal{B}}{d \hat{s}} d \hat{s}}{\mathcal{B}_{r}} . \tag{3.1}
\end{align*}
$$

The dependence of the total branching ratio and lepton polarizations $\left\langle P_{L}^{-}\right\rangle,\left\langle P_{T}^{-}\right\rangle$, $\left\langle P_{N}^{-}\right\rangle,\left\langle P_{T}^{-}-P_{T}^{+}\right\rangle$and $\left\langle P_{N}^{-}+P_{N}^{+}\right\rangle$on the new parameters $\left(m_{t^{\prime}}, r_{s b}\right)$ are shown in Figs (1)-(9). From these figures we obtain the following results.

- $\mathcal{B}_{r}$ strongly depends on the fourth quark mass $\left(m_{t^{\prime}}\right)$ and the product of quark mixing matrix elements $\left(r_{s b}\right)$ for both $\mu$ and $\tau$ channels. Furthermore, for both channels, $\mathcal{B}_{r}$ is an increasing function of both $m_{t^{\prime}}$ and $r_{s b}$.
- Although, $\left\langle P_{L}^{-}\right\rangle$and $\left\langle P_{T}^{-}\right\rangle$strongly depends on the fourth quark mass $\left(m_{t^{\prime}}\right)$ and the product of quark mixing matrix elements $\left(r_{s b}\right)$ for both $\mu$ and $\tau$ channels. But, its magnitude is a decreasing function of both $m_{t^{\prime}}$ and $r_{s b}$. So, the existence of fourth generation of quarks will suppress the magnitude of $\left\langle P_{L}^{-}\right\rangle$and $\left\langle P_{T}^{-}\right\rangle$.
- The normal polarization following from eq. (2.23) is proportional to the imaginary parts of the combination of the products of the Wilson coefficients, $m_{t^{\prime}}$ and $r_{s b}$. There are two different contributions to the non-zero value of $\left\langle P_{N}^{-}\right\rangle$. First, is due to the imaginary part of the $C_{9}^{\mathrm{eff}}$, while the second, is due to $\phi_{s b}$, which we assume to be $\approx 90^{\circ}$ in this work, and which therefore makes a purely imaginary contribution to eq. (2.5). Moreover, since $\left\langle P_{N}^{-}\right\rangle$is proportional to the lepton mass, for the $\mu$ channel it is negligible in the SM3 and the $\operatorname{SM} 4\left(\left\langle P_{N}^{-}\right\rangle_{\max } \sim 1 \%\right)$. For the $\tau$ case, where $\left\langle P_{N}^{-}\right\rangle \sim 1 \%$ in the SM, it shows stronger dependence on ( $m_{t^{\prime}}, r_{s b}$ ). It is interesting to note that $\left\langle P_{N}^{-}\right\rangle \sim 5 \%$ at $300 \leq m_{t^{\prime}} \leq 400(\mathrm{GeV})$. Therefore, measurement of the $\left\langle P_{N}^{-}\right\rangle$for $\tau$ channel can serve as good clue for existence of fourth generation of quarks.

Our numerical analysis for the combined lepton and antilepton polarizations leads to the following results.

- In the $m_{\ell} \rightarrow 0$ limit, $\left\langle P_{L}^{-}+P_{L}^{+}\right\rangle$practically coincides with SM result.

For $\tau$ case, $\left\langle P_{L}^{-}+P_{L}^{+}\right\rangle$exhibits strong dependence only on $m_{\tau}$, rather than the new parameters $\left(m_{t^{\prime}}, r s b\right)$ and practically there is no difference between the result of SM with three generations.

- Situation for the combined $\left\langle P_{T}^{-}-P_{T}^{+}\right\rangle$polarization is as follows.

For the $\mu$ channel, $\left\langle P_{T}^{-}-P_{T}^{+}\right\rangle$is approximately zero in the SM3 and the SM4.
In the $\tau$ case, $\left\langle P_{T}^{-}-P_{T}^{+}\right\rangle$is observed to be strongly dependent on new parameters $\left(m_{t^{\prime}}, r_{s b}\right) .\left\langle P_{T}^{-}-P_{T}^{+}\right\rangle$is decreasing for increasing values of both $\left(m_{t^{\prime}}, r_{s b}\right)$. The magnitude of $\left\langle P_{T}^{-}-P_{T}^{+}\right\rangle$for the $\tau$ channel lies in the region ( $0.25 \div 0.85$ ) depending on the variations of the ( $m_{t^{\prime}}, r_{s b}$ ).

- Numerical calculations show that the combined $\left\langle P_{N}^{-}+P_{N}^{+}\right\rangle$polarization, exhibits strong dependence on the $\left(m_{t^{\prime}}, r_{s b}\right)$.
$\left\langle P_{N}^{-}+P_{N}^{+}\right\rangle$is approximately zero for $\mu$ channels in the SM, but considering the SM with four generations, it receives the maximum value of around $1 \%$ at $300 \leq m_{t^{\prime}} \leq$ $400(\mathrm{GeV})$. It may be hard for it to be measured in future experiments i.e., at LHC, unless a large amount of $\Lambda_{b}$ (i.e., $\sim 10^{12}$ ) are created. But, measurement of non zero value( $\sim 1 \%$ ) of $\left\langle P_{N}^{-}+P_{N}^{+}\right\rangle$for $\mu$ case will be the direct indication of new physics effects.

The situation for $\tau$ case is more interesting. The considerable ( $\sim 6$ times) enhancement can be seen at $300 \leq m_{t^{\prime}} \leq 400(\mathrm{GeV})$ in the magnitude of $\left\langle P_{N}^{-}+P_{N}^{+}\right\rangle$. The measurement of $\left\langle P_{N}^{-}+P_{N}^{+}\right\rangle$for $\tau$ case can serve as a good tool when looking for the fourth generation of quarks.

From these analyzes we can conclude that the measurement of the magnitude of not only the total branching ratio but also $\left\langle P_{i}^{-}\right\rangle$and $\left\langle P_{i}^{-}+(-) P_{i}^{+}\right\rangle(-$sign is for the transversal polarization case) is an indication of the existence of new physics beyond the SM.

In conclusion, we present the analysis of the total branching ratio and the lepton polarizations in the exclusive $\Lambda_{b} \rightarrow \Lambda \ell^{-} \ell^{+}$decay, by using the SM with four generations of quarks. The sensitivity of the total branching ratio, longitudinal, transversal and normal polarizations of $\ell^{-}$, as well as lepton-antilepton combined asymmetries on the new parameters that come out of fourth generations, are studied. We find out that both the total branching ratio and the lepton polarizations show a strong dependence on the fourth quark $\left(m_{t^{\prime}}\right)$ and the product of quark mixing matrix elements $\left(V_{t^{\prime} b}^{*} V_{t^{\prime} s}=r_{s b} e^{i \phi_{s b}}\right)$. The results can serve as a good tool to look for physics beyond the SM.

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