Spectroscopy of fermionic operators in AdS/CFT

Published by Institute of Physics Publishing for SISSA

Received: *August 9, 2006*
Accepted: *September 5, 2006*
Published: *September 21, 2006*

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Abstract: We compute the spectrum of color-singlet fermionic operators in the $\mathcal{N} = 2$ gauge theory on intersecting D3 and D7-branes using the AdS/CFT correspondence. The operator spectrum is found analytically by solving the equations for the dual D7-brane fluctuations. For the fermionic part of the D7-brane action, we use the Dirac-like form found by Martucci et al. ([26]). We also consider the baryon spectrum of a large class of supersymmetric gauge theories using a phenomenological approach to the gauge/gravity duality.

Keywords: Gauge-gravity correspondence, AdS-CFT Correspondence.

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Contents

1. Introduction

Within the AdS/CFT correspondence [1], there has been steady progress in the dual gravity description of strongly coupled Yang-Mills theories with matter in the fundamental representation of the gauge group.

One of the simplest brane configurations which realizes fundamental matter is given by the D3/D7 brane intersection. Strings stretching between the D3 and the D7-branes give rise to fundamental $\mathcal{N} = 2$ hypermultiplets ("quarks") which couple to the $\mathcal{N} = 4$ super Yang-Mills multiplet on the D3-branes. In the probe approximation, the dual supergravity background corresponds to probe D7-branes embedded into $AdS_5 \times S^5$ [2]. Here, open string fluctuations on the D7-branes are dual to meson operators in the $\mathcal{N} = 2$ gauge theory. This fluctuation-operator relation was used in [3] to derive the meson spectrum in dependence of the quark mass. A similar method was applied first in [4] to study meson spectra and chiral symmetry breaking in confining gauge theories. Further work on the string theory computation of meson spectra in strongly coupled gauge theories can be found in [5]–[24].

While the main focus has been on the holographic study of scalar and vector mesons, not much attention has been paid to the spectra of operators with half-integer spin. In

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particular, baryons have been widely neglected so far, although, in principle, a brane configuration for dynamical baryons is known [3] (see also [12] for a similar proposal): A baryon vertex is realized in string theory by N fundamental strings attached to a D5-brane which is wrapped on the five-sphere of $AdS_5 \times S^5$ [25]. By adding a stack of D7-branes at some finite distance in the AdS space, the N fundamental strings would end on the D7-branes rather than on the boundary of the AdS space. This corresponds to N dynamical quarks with a finite mass.

There are several difficulties which impede immediate progress towards such a holographic description of dynamical baryons: i) In large-N field theories baryons consist of N quarks. It is not clear whether one can neglect the backreaction of the baryonic D5-brane on the AdS geometry. ii) The set-up consists of several different branes and the topology of the baryon vertex is quite complex. iii) Baryons have half-integer spin and the dual fluctuations will be described by Dirac equations, Rarita-Schwinger equations, etc., all on curved spacetimes.

In this paper we will not directly approach baryons. Instead, we will discuss the spectrum of certain fermionic operators in the D3/D7 system. As the baryons, these operators have half-integer spin, but do not require the introduction of an additional brane in the dual string theory set-up. We may therefore develop techniques for dealing with fermionic open string fluctuations, while avoiding technical difficulties connected with a baryonic D5-brane.

The operators we are interested in are the supersymmetric partners of the meson-like operators discussed by Kruczenski *et al.* [3]. There are two towers of spin-$\frac{1}{2}$ operators: one with the dimension-$\frac{5}{2}$ operator $\mathcal{F} \sim \tilde{\psi} q$, the other with the dimension-$\frac{9}{2}$ operator $\mathcal{G} \sim \tilde{\psi}\lambda\psi$ at the bottom of the tower. Here q, $\tilde{q}$ and ψ, $\tilde{\psi}$ denote fundamental scalars and spinors. The operator $\tilde{\psi}\lambda\psi$ contains an additional adjoint spinor λ.

Similarly to the case of the scalar and vector mesons, we obtain the fermionic operator spectrum by considering D7-brane fluctuations dual to the above operators. Since the spin of the fluctuations must be identical to that of the operators, we start from the *fermionic* part of the D7-brane action as constructed in [26]. The corresponding equation of motion effectively describes a Dirac spinor on $AdS_5 \times S^3$ which couples to the self-dual five-form flux of type IIB string theory. The Kaluza-Klein reduction on the three-sphere S^3 then provides two sets of modes $\Psi_\ell^\pm$ with masses $m_\ell^\pm$ ($\ell = 0, 1, 2, \ldots$) satisfying the mass-dimension relation for spin-$\frac{1}{2}$ fields [27]:

$$\Delta_\ell = |m_\ell^\pm| + 2\,, \tag{1.1}$$

where Δ_ℓ denotes the conformal dimensions of the operators $\mathcal{F}^\ell$ and $\mathcal{G}^\ell$, the higher-ℓ relatives of $\mathcal{F}$ and $\mathcal{G}$. Since also the quantum numbers of these operators match exactly those of $\Psi_\ell^\pm$, this ensures the correct fluctuation-operator map between $\Psi_\ell^\pm$ and $\mathcal{F}^\ell$, $\mathcal{G}^\ell$. After recasting the equations of motion into a second order form, we find the spectrum of $\mathcal{F}^\ell$ and $\mathcal{G}^\ell$ by assuming a plane-wave behavior of the fluctuations $\Psi_\ell^\pm$. The resulting spectrum is linear in the quark mass and agrees exactly with the spectrum expected from supersymmetry.

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We finally come back to baryons in the last section of the paper in which we construct a phenomenological supergravity model for baryonic operators in a large class of supersymmetric gauge theories. Instead of starting from a ten-dimensional brane configuration, we fix the background from the properties of the dual field theory. This is known as the "bottom-up" approach to AdS/CFT [28 – 30] and has first been applied to baryons by Brodsky and Teramond [30]. As in [30], we assume that fluctuations dual to spin-$\frac{1}{2}$ baryons are effectively described by a massive Dirac equation on AdS_5. The mass of the Dirac spinor is fixed by the conformal dimension of the baryon operator. Deviating from [30], we consider gauge theories which are superconformal in the UV, at least in some parameter regime. The baryon spectrum will therefore be a function of the quark mass rather than the infra-red cut-off Λ_{QCD}. We fill find that the large-N baryon spectrum is linear in the quark mass and scales with N as expected from field theory [31].

The paper is organized as follows. In section 2 we review both the low-energy effective field theory of the D3/D7 intersection and the holographic computation of meson spectra. In section 3 we adapt the holographic method to find the spectrum of operators with half-integer spin. In section 4 we discuss large N baryons in an effective approach to the gauge/gravity duality. In section 5 we briefly summarize our results and discuss open problems.

2. Spectroscopy of meson operators in AdS/CFT

In the following we briefly discuss the $\mathcal{N} = 2$ world-volume theory of the D3/D7 intersection and review the holographic computation of its meson spectrum [3]. Those who are familiar with the meson spectroscopy in the D3/D7 theory may wish to proceed immediately to section 3 in which we adapt the method to compute the spectrum of fermionic operators.

2.1 The D3/D7 brane intersection

The D3/D7 brane intersection in flat space consists of a stack of N_c coincident D3-branes (along 0123) which is embedded into the world volume of N_f D7-branes (along 01234567). This system preserves 1/4 of the total amount of supersymmetry in type IIB string theory and has an $SO(4) \times SO(2)$ isometry in the directions transverse to the D3-branes. The $SO(4)$ rotates in x^4, x^5, x^6, x^7, while the $SO(2)$ group acts on x^8, x^9. Note that separating the D3-branes from the D7-branes in the 89 direction by a distance L explicitly breaks the $SO(2)$ group.

The world-volume theory describes a four-dimensional $\mathcal{N} = 4$ super Yang-Mills multiplet which is coupled to N_f $\mathcal{N} = 2$ hypermultiplets in the fundamental representation of the $U(N_c)$ gauge group. Under $\mathcal{N} = 1$ supersymmetry the $\mathcal{N} = 4$ vector multiplet decomposes into the vector multiplet W_α and the three chiral superfields Φ_1, Φ_2, Φ_3. The $\mathcal{N} = 2$ fundamental hypermultiplets can be written in terms of the $\mathcal{N} = 1$ chiral multiplets

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	components	spin	$SU(2)_\Phi \times SU(2)_\mathcal{R}$	$U(1)_\mathcal{R}$	Δ	$U(N_f)$	$U(1)_B$
Φ_1, Φ_2	X^4, X^5, X^6, X^7	0	$(\frac{1}{2}, \frac{1}{2})$	0	1	1	0
	λ_1, λ_2	$\frac{1}{2}$	$(\frac{1}{2}, 0)$	-1	$\frac{3}{2}$	1	0
Φ_3, W_α	$X_V^A = (X^8, X^9)$	0	$(0,0)$	$+2$	1	1	0
	λ_3, λ_4	$\frac{1}{2}$	$(0, \frac{1}{2})$	$+1$	$\frac{3}{2}$	1	0
	v_μ	1	$(0,0)$	0	1	1	0
Q, $\tilde{Q}$	$q^m = (q, \bar{\tilde{q}})$	0	$(0, \frac{1}{2})$	0	1	N_f	$+1$
	$\psi_i = (\psi, \tilde{\psi}^\dagger)$	$\frac{1}{2}$	$(0,0)$	∓ 1	$\frac{3}{2}$	N_f	$+1$

Table 1: Fields of the D3/D7 low-energy effective field theory and their quantum numbers under the global symmetries. Note that $U(1)_B \subset U(N_f)$.

$Q^r, \tilde{Q}_r$ $(r = 1, \ldots, N_f)$. In $\mathcal{N} = 1$ superspace notation, the Lagrangian is thus given by

$$\mathcal{L} = \mathrm{Im}\left[\tau \int d^4\theta \left(\mathrm{tr}\,(\bar{\Phi}_I e^V \Phi_I e^{-V}) + Q_r^\dagger e^V Q^r + \tilde{Q}_r^\dagger e^{-V} \tilde{Q}^r\right) + \tau \int d^2\theta (\mathrm{tr}\,(W^\alpha W_\alpha) + W) + c.c.\right], \tag{2.1}$$

where the superpotential W is

$$W = \mathrm{tr}\,(\varepsilon_{IJK} \Phi_I \Phi_J \Phi_K) + \tilde{Q}_r (m + \Phi_3) Q^r\,. \tag{2.2}$$

The $SO(2) \simeq U(1)$ isometry corresponds to a $U(1)_R$ R-symmetry in the field theory which is explicitly broken by a quark mass $m_q \sim L$. The field theory has also a global $SO(4) \approx SU(2)_\Phi \times SU(2)_\mathcal{R}$ symmetry which consists of a $SU(2)_\Phi$ symmetry and a $\mathcal{N} = 2$ $SU(2)_\mathcal{R}$ R-symmetry. The global symmetry $SU(2)_\Phi$ rotates the scalars in the adjoint hypermultiplet. There is also a group $U(1)_B$ which is a subgroup of the $U(N_f)$ flavor group. The fundamental superfields Q^r $(\tilde{Q}_r)$ are charged $+1$ (-1) under $U(1)_B$, while the adjoint fields are inert. The components of the $\mathcal{N} = 1$ superfields and their quantum numbers are summarized in the table 1 (see also [6]) which we will need for the construction of operators.

The exact perturbative $\mathcal{N} = 2$ beta function for the 't Hooft coupling is proportional to N_f/N_c [6, 14]. In the probe approximation, the D7-branes do not backreact on the $AdS_5 \times S^5$ near-horizon geometry of the D3-branes and the field theory is conformal corresponding to the strict $N_f/N_c \to 0$ limit. Beyond the probe approximation, N_f/N_c is finite, and the beta function is positive, implying an UV Landau pole in the field theory. This is a pathology of the perturbative field theory and is reflected by a dilaton divergence in the fully localized supergravity solution of the D3/D7 setup [32 – 35, 14]. Despite the occurrence of logarithmic tadpoles, this background is still consistent as far as the embedding of a nonconformal field theory is concerned, see ref. [14, 35, 36] and references therein. Note that logarithmic tadpoles do not represent gauge anomalies, but instead provide the correct one-loop running of the gauge coupling. The supergravity background perfectly reflects the properties of the perturbative field theory.

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2.2 Review on meson spectroscopy in the D3/D7 set-up

The $\mathcal{N}=2$ world-volume field theory of the D3/D7 brane intersection contains the scalar meson operators

$$\mathcal{M}_s^{A\ell} = \bar{\psi}_i \sigma^A_{ij} X^\ell \psi_j + \bar{q}^m X^A_V X^\ell q^m \qquad (i,m=1,2) \tag{2.3}$$

which have conformal dimensions $\Delta = 3+\ell$. Here X^A_V denotes the vector (X^8, X^9) and $\sigma^A = (\sigma^1, \sigma^2)$ is a doublet of Pauli matrices. Both X^A_V and σ^A transform in the **2** of $U(1)_R$. The components q^m and ψ_i of the fundamental hypermultiplets are as in table 1. X^ℓ denotes the symmetric, traceless operator insertion $X^{\{i_1}\cdots X^{i_\ell\}}$ of ℓ adjoint scalars X^i $(i=4,5,6,7)$ which transform in the fundamental representation $(\frac{1}{2},\frac{1}{2})$ of $SO(4) \approx SU(2)_\Phi \times SU(2)_\mathcal{R}$. The operators $\mathcal{M}_s^{A\ell}$ thus transform in the $(\frac{\ell}{2},\frac{\ell}{2})$ of $SO(4)$ and are charged $+2$ under $U(1)_R$.

The spectrum of these operators (and that of the vector mesons) was found in [3] by evaluating the Dirac-Born-Infeld (DBI) action for a probe D7-brane. The D7-brane wraps an $AdS_5 \times S^3$ submanifold inside the $AdS_5 \times S^5$ near-horizon geometry of the D3-branes. In static gauge, where the world volume coordinates of the D7 brane are identified with the spacetime coordinates by $\xi^a \sim t, x_1, \ldots, x_7$, the DBI action is given by [3]

$$\begin{aligned} S^b_{D7} &= -T_7 \int d^8\xi \sqrt{-\det g^{PB}_{ab}} \\ &= -T_7 \int d^8\xi \; \epsilon_3 \; \rho^3 \sqrt{1 + \frac{g^{ab}}{\rho^2 + x_8^2 + x_9^2}(\partial_a x_8 \partial_b x_8 + \partial_a x_9 \partial_b x_9)}\,, \end{aligned} \tag{2.4}$$

where g^{PB}_{ab} is the pullback of the $AdS_5 \times S^5$ metric on the D7 world-volume. Moreover, g_{ab} denotes the induced metric on the D7 brane and ϵ_3 is the determinant factor from the three sphere. x_8 and x_9 are the coordinates transverse to the D7-brane, while $\rho^2 = x_4^2 + \ldots + x_7^2$.

One easily verifies [3] that e.g. $x_8 = 0$, $x_9 = L$ is a solution to the corresponding Euler-Lagrange equations. The constant L is the distance between the D3- and the D7-branes and is proportional to the mass of the fundamental hypermultiplets ("quarks"). The spectrum of the operators $\mathcal{M}_s^\ell$ can then be found by considering fluctuations of the plane-wave type around this ground state solution. More precisely, one makes the ansatz

$$x_8 = 0\,, \qquad x_9 = L + f_\ell(\rho) e^{ik\cdot x} \mathcal{Y}^\ell(S^3)\,, \tag{2.5}$$

where $M^2 = -k^2$ is interpreted as the meson mass. The functions $\mathcal{Y}^\ell(S^3)$ are the scalar spherical harmonics on S^3 which have eigenvalues $-\ell(\ell+2)$ and transform in the $(\frac{\ell}{2},\frac{\ell}{2})$ of $SO(4)$. Substituting the ansatz into the Euler-Lagrange equations obtained from the DBI action (2.4) expanded to quadratic order in the fields, one obtains the following equation for the fluctuations $f_\ell(\rho)$:

$$\partial_\rho^2 f_\ell(\rho) + \frac{3}{\rho}\partial_\rho f_\ell(\rho) + \left(\frac{M^2}{(\rho^2+L^2)^2} - \frac{\ell(\ell+2)}{\rho^2}\right) f_\ell(\rho) = 0\,. \tag{2.6}$$

This equation is solved in terms of the hypergeometric function by [3]

$$f_\ell(\rho) = \frac{\rho^\ell}{(\rho^2+L^2)^{n+\ell+1}} F(-(n+\ell+1), -n; \ell+2; -\rho^2/L^2)\,, \tag{2.7}$$

where the excitation number n is related to the scalar meson mass by

$$M_s^2 = \frac{4L^2}{R^4}(n+\ell+1)(n+\ell+2) \qquad (n,\ell \geqslant 0)\,. \tag{2.8}$$

The discreteness of the spectrum follows from the normalizability of the states. Note that the spectrum is a linear function of the quark mass $m_q \sim L$. This is a particular feature of the superconformal field theory. It has been shown in [4] (see also [5]) that in a nonsupersymmetric deformation of the D3/D7 world-volume theory, the meson mass satisfies the Gell-Mann-Oakes-Renner-relation for small quark masses.

3. Spectroscopy of fermionic operators

3.1 Spectroscopy of spin-$\frac{1}{2}$ operators

We now use the holographic method to derive the spectrum of certain fermionic operators in the D3/D7 theory. More specifically, the operators we are interested in are the supersymmetric partners of the meson-like operators studied in [3]. There are two classes of spin-$\frac{1}{2}$ operators:

$$\mathcal{F}_\alpha^\ell \sim \bar{q}X^\ell\tilde{\psi}_\alpha^\dagger + \tilde{\psi}_\alpha X^\ell q\,, \tag{3.1}$$

$$\mathcal{G}_\alpha^\ell \sim \bar{\psi}_i\sigma_{ij}^B\lambda_{\alpha C}X^\ell\psi_j + \bar{q}^m X_V^B\lambda_{\alpha C}X^\ell q^m\,, \qquad (A,B,C=1,2) \tag{3.2}$$

which have the conformal dimensions $\Delta = \frac{5}{2}+\ell$ and $\Delta = \frac{9}{2}+\ell$ ($\ell \geqslant 0$), respectively. Both types of operators have fundamental fields at their ends: scalars $q^m = (q,\tilde{\bar{q}})^T$ and spinors $\psi_i = (\psi,\tilde{\psi}^\dagger)^T$. As in eq. (2.3), the operator insertion X^ℓ generates operators with higher angular momentum ℓ. The spinors $\lambda_{\alpha A}$ ($A=1,2$) belong to the adjoint hypermultiplets (Φ_1,Φ_2). The quantum numbers of these fields are listed in table 1.

The operators $\mathcal{F}_\alpha^\ell$ and $\mathcal{G}_\alpha^\ell$ have the following quantum numbers under the global symmetries of the theory: Since q^m, ψ^i and X^ℓ have the $SO(4)$ quantum numbers $(0,\frac{1}{2})$, $(0,0)$ and $(\frac{\ell}{2},\frac{\ell}{2})$, respectively, $\mathcal{F}_\alpha^\ell$ transforms in the $(\frac{\ell}{2},\frac{\ell+1}{2})$ of $SO(4) \approx SU(2)_\Phi \times SU(2)_\mathcal{R}$. The $U(1)_R$ charge of $\mathcal{F}_\alpha^\ell$ is $+1$. Similar operators can be found in the world-volume theory of intersecting D3/D5 branes [37]. The operators $\mathcal{G}_\alpha^\ell$ are obtained by inserting a doublet of adjoint spinors $\lambda_{\alpha A}$ ($A=1,2$) into the operators $\mathcal{M}_s^\ell$. Since $\lambda_{\alpha A}$ has the $SO(4)$ quantum numbers $(\frac{1}{2},0)$, the operators $\mathcal{G}_\alpha^\ell$ transform in the $(\frac{\ell+1}{2},\frac{\ell}{2})$ representations of $SO(4)$. The $U(1)_R$ charge of $\mathcal{G}_\alpha^\ell$ $(+1)$ is the same as for the operators $\mathcal{F}_\alpha^\ell$.

Let us consider the properties of the bulk modes Ψ_ℓ^+ and Ψ_ℓ^- dual to $\mathcal{G}_\alpha^\ell$ and $\mathcal{F}_\alpha^\ell$. The spin of $\Psi_\ell^\pm$ must be identical to that of the boundary states, *i.e.* their spin must be $\frac{1}{2}$. For spin-$\frac{1}{2}$ modes, the relation between the conformal dimension of the field theory operator and the mass of the dual mode in AdS_5 is given by eq. (1.1). The AdS_5 modes $\Psi_\ell^\pm$ should therefore have the masses

$$|m_\ell^+| = \frac{5}{2}+\ell\,, \qquad |m_\ell^-| = \frac{1}{2}+\ell\,. \tag{3.3}$$

Moreover, $\mathcal{F}_\alpha^\ell$ and $\mathcal{G}_\alpha^\ell$ have fundamental fields at their ends, for which reason the dual bulk modes $\Psi_\ell^\pm$ must descend from open string fluctuations. Recall that, since the D7-brane

does not backreact on the geometry, closed strings are only dual to pure adjoint operators. In other words, the modes must again correspond to open string fluctuations on the D7-brane. However, since the operators have half-integer spin, the D7 fluctuations must be fermionic. We now show that the D3/D7 brane configuration indeed contains spin-$\frac{1}{2}$ open string modes with these properties.

For this, we consider the fermionic part of the D7-brane action which to quadratic order in the fermions is given by [26]

$$S^f_{D7} = \frac{\tau_{D7}}{2}\int d^8\xi\sqrt{-\det g}\,\hat{\bar{\Psi}}\mathcal{P}_-\Gamma^{\hat{A}}(D_{\hat{A}} + \frac{1}{8}\frac{i}{2\cdot 5!}F_{\hat{N}\hat{P}\hat{Q}\hat{R}\hat{S}}\Gamma^{\hat{N}\hat{P}\hat{Q}\hat{R}\hat{S}}\Gamma_{\hat{A}})\hat{\Psi}\,. \tag{3.4}$$

Here $\xi^{\hat{A}}$ are the world-volume coordinates ($\hat{A} = 0,\ldots,7$) which, in static gauge, will be identified with the spacetime coordinates $t, x^1, \ldots, x^7$. The field $\hat{\Psi}$ is the 10d positive chirality Majorana-Weyl spinor of type IIB string theory and $\Gamma_{\hat{A}}$ is the pullback of the 10d gamma matrix $\Gamma_{\hat{M}}$ ($\hat{M}, \hat{N}, \ldots = 0,\ldots,9$), $\Gamma_{\hat{A}} = \Gamma_{\hat{M}}\partial_{\hat{A}}x^{\hat{M}}$. The integration goes over the world-volume of the D7-brane which wraps an $AdS_5 \times S^3$ submanifold of $AdS_5 \times S^5$. The spinor $\hat{\Psi} = \hat{\Psi}(x^M, \theta^m)$ depends on the coordinates x^M of AdS_5 and the three angles $\theta^m = (\theta^1, \theta^2, \theta^3)$ of the three-sphere S^3. The D7-brane is located at $\theta^4 = \theta^5 = 0$ corresponding to massless quarks in the field theory. The operator $\mathcal{P}_-$ is a κ-symmetry projector ensuring κ-symmetry invariance of the action. The action is therefore invariant under supersymmetries corresponding to any bulk Killing spinor.

We now decompose every ten-dimensional field or gamma matrix into parts associated with AdS_5 and S^5, respectively. Choosing a local Lorentz frame, the 10d gamma matrices $\Gamma^{\hat{M}}$ decompose as

$$\begin{aligned}\Gamma^M &= \sigma_y \otimes \mathbf{1}_4 \otimes \gamma^M\,, \quad (M = 0,1,2,3,4)\,,\\ \Gamma^m &= \sigma_x \otimes \gamma^m \otimes \mathbf{1}_4\,, \quad (m = 5,6,7,8,9)\,,\end{aligned} \tag{3.5}$$

where $\mathbf{1}_4$ is the 4d unit matrix and $\sigma_x, \sigma_y, \sigma_z$ are the Pauli matrices. The five-dimensional Minkowski and Euclidean gamma matrices, γ^A and γ^a, satisfy the relations

$$\{\gamma^M, \gamma^N\} = 2\eta^{MN}\,, \qquad \{\gamma^m, \gamma^n\} = 2\delta^{mn}\,. \tag{3.6}$$

From this, one obtains

$$\Gamma^{11} = \sigma_z \otimes \mathbf{1}_4 \otimes \mathbf{1}_4\,, \qquad \Gamma^{01234} = i\sigma_y \otimes \mathbf{1}_4 \otimes \mathbf{1}_4\,, \qquad \Gamma^{56789} = \sigma_x \otimes \mathbf{1}_4 \otimes \mathbf{1}_4\,. \tag{3.7}$$

The 10d spinor $\hat{\Psi}$ has positive chirality, $\Gamma^{11}\hat{\Psi} = \hat{\Psi}$, and can be decomposed as

$$\hat{\Psi} = \uparrow \otimes \chi \otimes \Psi\,, \tag{3.8}$$

where $\uparrow$ denotes the two-component spinor $(1,0)^T$, and χ and Ψ are four-component spinors of $SO(5)$ and $SO(1,4)$, respectively. These groups act on the tangent spaces of S^5 and AdS_5, respectively. The spinor $\chi = \chi_{||} \otimes \chi_\perp$ splits into a 3d spinor $\chi_{||}$ associated with S^3 and a 2d spinor $\chi_\perp$ acting transverse to the S^3.

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The action (3.4) leads to the Dirac equation

$$\not{D}\hat{\Psi} + \frac{1}{8}\frac{i}{2\cdot 5!}\Gamma^{\hat{A}}F_{\hat{N}\hat{P}\hat{Q}\hat{R}\hat{S}}\Gamma^{\hat{N}\hat{P}\hat{Q}\hat{R}\hat{S}}\Gamma_{\hat{A}}\hat{\Psi} = 0\,, \tag{3.9}$$

where $\not{D}$ is the Dirac-operator on $AdS_5 \times S^3$. The second term describes the coupling of $\hat{\Psi}$ to the self-dual five-form field strength $F_{\hat{N}\hat{P}\hat{Q}\hat{R}\hat{S}}$. We parametrize the five-form as

$$\begin{aligned} F_{NPQRS} &= \frac{4}{R}\,\varepsilon_{NPQRS}\,, \quad (N,P,\ldots = 0,1,2,3,4)\,, \\ F_{npqrs} &= \frac{4}{R}\,\varepsilon_{npqrs}\,, \quad (n,p,\ldots = 5,6,7,8,9)\,, \end{aligned} \tag{3.10}$$

where R is the AdS radius.

Using the decompositions (3.5), (3.8) and (3.10), we find

$$\frac{1}{8}\frac{i}{2\cdot 5!}\Gamma^{\hat{A}}F_{\hat{N}\hat{P}\hat{Q}\hat{R}\hat{S}}\Gamma^{\hat{N}\hat{P}\hat{Q}\hat{R}\hat{S}}\Gamma_{\hat{A}}\hat{\Psi} = \frac{i}{16R}\Gamma^{\hat{A}}\left((\sigma_x + i\sigma_y)\otimes \mathbf{1}_4\otimes \mathbf{1}_4\right)\Gamma_{\hat{A}}\hat{\Psi} = \frac{-i}{R}(\downarrow\otimes\chi\otimes\Psi) \tag{3.11}$$

and ($A = 0,\ldots,4$, $a = 5,6,7$)

$$\begin{aligned} \not{D}\hat{\Psi} &= \Gamma^{\hat{A}}D_{\hat{A}}\hat{\Psi} = \Gamma^{A}D_{A}\hat{\Psi} + \Gamma^{a}D_{a}\hat{\Psi} \\ &= (\sigma_y\otimes\mathbf{1}_4\otimes\gamma^A D_A + \sigma_x\otimes\gamma^a D_a\otimes\mathbf{1}_4)(\uparrow\otimes\chi\otimes\Psi) \\ &= \left(i(\mathbf{1}_2\otimes\mathbf{1}_4\otimes\gamma^A D_A) + (\mathbf{1}_2\otimes\gamma^a D_a\otimes\mathbf{1}_4)\right)(\downarrow\otimes\chi\otimes\Psi) \\ &\equiv (i\not{D}_{AdS_5} + \not{D}_{S^3})(\downarrow\otimes\chi\otimes\Psi)\,, \end{aligned} \tag{3.12}$$

where $\not{D}_{AdS_5}$ and $\not{D}_{S^3}$ are the Dirac operators on AdS_5 and S^3, respectively.

The Dirac operator $\not{D}_{S^n}$ on a n-sphere S^n of radius R and its eigenvalues λ are well-known, see e.g. [38]. The spinor spherical harmonics $\chi^{\pm}_{\ell}$ satisfy

$$\not{D}_{S^n}\chi^{\pm}_{\ell} = \mp i\lambda_\ell\chi^{\pm}_{\ell} = \mp\tfrac{i}{R}(\ell+\tfrac{n}{2})\chi^{\pm}_{\ell}\,. \qquad (\ell \geqslant 0) \tag{3.13}$$

For $n = 3$, $\lambda_\ell = \frac{i}{R}(\ell+\frac{3}{2})$ and the spinors $\chi^{\pm}_{||\ell}$ transform in the $(\frac{\ell+1}{2},\frac{\ell}{2})$ and $(\frac{\ell}{2},\frac{\ell+1}{2})$ of $SO(4)$ which rotates the S^3. Recall that $\chi_{||}$ is that part of the spinor $\chi = \chi_{||}\otimes\chi_{\perp}$ which is parallel to the S^3.

Substituting everything back into (3.9), we obtain the Dirac equation

$$(\not{D}_{AdS_5} \mp \tfrac{1}{R}(\ell+\tfrac{3}{2}) - \tfrac{1}{R})\Psi^{\pm}_{\ell} = \left\{\begin{array}{l}(\not{D}_{AdS_5} - \frac{1}{R}(\ell+\frac{5}{2}))\Psi^{+}_{\ell} \\ (\not{D}_{AdS_5} + \frac{1}{R}(\ell+\frac{1}{2}))\Psi^{-}_{\ell}\end{array}\right\} = 0\,. \tag{3.14}$$

We note that the effect of the RR five-form field is to shift the Kaluza-Klein mass by one unit. The same shift has been observed in the dilatino spectrum on $AdS_5\times S^5$ [39]. eq. (3.14) is basically a Dirac equation on AdS_5 which describes the fluctuation modes Ψ^{+}_{ℓ} and Ψ^{-}_{ℓ} with masses

$$m^{+}_{\ell} = \tfrac{5}{2} + \ell\,, \qquad m^{-}_{\ell} = -(\tfrac{1}{2}+\ell)\,, \tag{3.15}$$

respectively. These masses are exactly the ones expected from the field theory for fermionic bulk fields dual to the operators $\mathcal{G}^{\ell}_{\alpha}$ and $\mathcal{F}^{\ell}_{\alpha}$, cf. eq. (3.3). Moreover, the $SO(4)$ and $U(1)_R$ quantum numbers of Ψ^{+}_{ℓ} and Ψ^{-}_{ℓ}, $(\frac{\ell+1}{2},\frac{\ell}{2})_1$ and $(\frac{\ell}{2},\frac{\ell+1}{2})_1$, agree with those of $\mathcal{G}^{\ell}_{\alpha}$ and $\mathcal{F}^{\ell}_{\alpha}$. This completes the dictionary between the fermionic operators $\mathcal{G}^{\ell}_{\alpha}$ and $\mathcal{F}^{\ell}_{\alpha}$ and their dual fluctuation modes $\Psi^{\pm}_{\ell}$.

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3.2 Dirac equation in AdS_{d+1} spaces

For the computation of the spectrum of these operators, it turns out to be convenient to transform the Dirac equation (3.14) into a second order differential equation. For this, we briefly review Dirac equations on $d+1$-dimensional AdS spaces.

Consider a $d+1$-dimensional AdS geometry with metric given by

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2)\,, \tag{3.16}$$

where R is the AdS radius. The Dirac equation for a massive spin-$\frac{1}{2}$ mode $\Psi(x^\mu, z)$ on AdS_{d+1} is then given by

$$(\not{D}_{AdS} - m)\Psi(x^\mu, z) = 0\,, \tag{3.17}$$

where $\not{D}_{AdS} = e^M_A \gamma^A D_M$ is the Dirac operator and γ^A are the Dirac matrices of $d+1$-dimensional Minkowski space, $\{\gamma^A, \gamma^B\} = 2\eta^{AB}$. The curved-space covariant derivative

$$D_M = \partial_M + \frac{1}{4}\omega_{MBC}[\gamma^B, \gamma^C] \tag{3.18}$$

is determined by the spin connection ω_{MBC} which in turn is given in terms of the $d+1$-bein e^A_M [40]. For an AdS_{d+1} space, $e^A_M = \frac{R}{z}$ and the Dirac operator simplifies to $\not{D} = \frac{z}{R}\gamma^A\partial_A - \frac{d}{2R}\gamma^z$ [27]. The matrix γ^z is the higher-dimensional analog of the chirality operator γ^5 in $d=4$ dimensions. Multiplying the Dirac equation with $(\not{D}_{AdS} + m)$, one obtains the following second order differential equation [41]:

$$(z^2\partial^M\partial_M - dz\partial_z - m^2R^2 + \frac{d^2}{4} + \frac{d}{2} + mR\gamma^z)\Psi(x^\mu, z) = 0\,. \tag{3.19}$$

Similarly to the case of mesons, we are interested in spin-$\frac{1}{2}$ fluctuations which are plane waves along the four-momentum P_μ. As before, $M^2 = P^\mu P_\mu$ is interpreted as the mass of the dual spin-$\frac{1}{2}$ operator. Using the ansatz $\tilde{\Psi}(x, z) = e^{iP^\mu x_\mu} f(z)$, we find

$$(z^2\partial_z^2 - dz\partial_z + z^2M^2 - m^2R^2 + \frac{d^2}{4} + \frac{d}{2} + mR\gamma^z)f(z) = 0\,. \tag{3.20}$$

The solution of this equation [41] is given by

$$f(z) = z^{\frac{d+1}{2}}\left(J_{m-\frac{1}{2}}(zM)\,a^+ + J_{m+\frac{1}{2}}(zM)\,a^-\right), \tag{3.21}$$

where the spinors satisfy $\gamma^z a^\pm = \pm a^\pm$ and $a^- = \frac{\gamma^\mu P_\mu}{P}\,a^+$. For small z, the Bessel function $J_{m-\frac{1}{2}}$ behaves as $J_{m-\frac{1}{2}}(z) \approx z^{m-\frac{1}{2}}$, and $f(z)$ scales like z^Δ.

Eq. (3.20) is all what we will need to compute the fluctuation spectrum in the next section. It may, however, be interesting to extend the analysis to backgrounds which are only *asymptotically* AdS. The Dirac equation on asymptotic AdS spaces is studied in appendix A. Such an equation could become important for nonsupersymmetric supergravity backgrounds.

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3.3 Spectrum of spin-$\frac{1}{2}$ fluctuations in AdS_5

We now move on to compute the spectrum of the spin-$\frac{1}{2}$ modes $\Psi_\ell^\pm$. As shown in section 3.1, these modes are described by a Dirac equation on AdS_5, eq. (3.14), which follows from the fermionic part of the D7-brane action. In section 3.2, this equation has been transformed into a second order differential equation, eq. (3.19). The assumption of plane-wave fluctuations, $\Psi_\ell(x,r) = e^{iP_\mu x^\mu} f_\ell(r)$, led then to the differential equation (3.20) which we recast into the form ($d = 4$)

$$\big(\partial_r^2 + \frac{6}{r}\partial_r + \frac{1}{r^2}(-|m_\ell|^2 R^2 + 6 + |m_\ell| R\gamma^r) + \frac{M^2R^4}{r^4}\big) f_\ell(r) = 0\,, \tag{3.22}$$

where $r = R^2/z$ and the masses $m_\ell = m_\ell^\pm$ as in eq. (3.15). Without loss of generality, we work with the absolute value of m_ℓ, since $\gamma^r = \pm 1$.

The above equation of motion has been derived for overlapping D3 and D7-branes corresponding to massless quarks. We have not found a completely satisfying way to introduce a quark mass $m_q \neq 0$ right from the beginning. Nevertheless, it is possible to compute the mass spectrum in dependence of the quark mass by making the following assumption: Note that the equation for the fermionic operators, eq. (3.22), has a similar structure as the equation for the scalar mesons, eq. (2.6). We may therefore assume that massive quarks can be introduced by replacing

$$\frac{M^2}{r^4} \to \frac{M^2}{(r^2+L^2)^2} \tag{3.23}$$

in (3.22), where L is proportional to the quark mass m_q, $m_q \sim L$. This assumption will be justified below.

The resulting equation of motion

$$\left(\partial_r^2 + \frac{6}{r}\partial_r + \frac{1}{r^2}(-|m_\ell|^2 R^2 + 6 + |m_\ell| R\gamma^r) + \frac{M^2R^4}{(r^2+L^2)^2}\right) f_\ell(r) = 0\,, \tag{3.24}$$

can then be solved in terms of the hypergeometric function ${}_2F_1(a,b;c;-\frac{r^2}{L^2})$, where a, b, and c are constants. More precisely, we find the solutions

$$\begin{aligned} f_\ell(r) = {} & r^{|m_\ell|-3}(L^2+r^2)^{\frac{1}{2}-|m_\ell|-n_+}\, {}_2F_1\left(\tfrac{1}{2}-|m_\ell|-n_+, -n_+; |m_\ell|+\tfrac{1}{2}; -\tfrac{r^2}{L^2}\right) a^+ \\ & + r^{|m_\ell|-2}(L^2+r^2)^{-\frac{1}{2}-|m_\ell|-n_-}\, {}_2F_1\left(-\tfrac{1}{2}-|m_\ell|-n_-, -n_-; |m_\ell|+\tfrac{3}{2}; -\tfrac{r^2}{L^2}\right) a^-\,, \end{aligned} \tag{3.25}$$

where, as in eq. (3.21), the spinors $a^\pm$ satisfy $\gamma^r a^\pm = \pm a^\pm$. Here we defined the excitation numbers $n_\pm = 0, 1, 2, \ldots$ as functions of the mass M by imposing the quantization conditions

$$-n_+ = |m_\ell| - \tfrac{1}{2}\sqrt{1 + M^2/L^2}\,, \tag{3.26}$$

$$-n_- = |m_\ell| + 1 - \tfrac{1}{2}\sqrt{1 + M^2/L^2}\,. \tag{3.27}$$

These conditions ensure that the hypergeometric functions behave like $(r^2)^{n_\pm}$ at $r \to \infty$, and thus asymptotically $f_\ell(r) \sim r^{-\Delta}$. Since the fluctuations $\Psi_\ell^\pm$ are canonically normalized, this is the expected asymptotic behavior for normalizable modes.

The quantization conditions (3.26) and (3.27) determine the mass spectra of the operators $\mathcal{G}_\alpha^\ell$ and $\mathcal{F}_\alpha^\ell$. The function f_ℓ is a superposition of two solutions which are proportional to a^+ and a^-. For $m_\ell = m_\ell^+ > 0$, the dominant solution at $r \to \infty$ is that proportional to a^+. Solving the definition of n_+ for M and substituting $m_\ell^+ = \frac{5}{2} + \ell$, we obtain

$$\begin{aligned} M_{\mathcal{G}}^2 &= \frac{4L^2}{R^4}(|m_\ell^+| + n_+ - \tfrac{1}{2})(|m_\ell^+| + n_+ + \tfrac{1}{2}) \\ &= \frac{4L^2}{R^4}(n_+ + \ell + 2)(n_+ + \ell + 3) \qquad (n_+ \geqslant 0, \ell \geqslant 0)\,. \end{aligned} \tag{3.28}$$

This is the mass spectrum of the operators $\mathcal{G}_\alpha^\ell$ which have the same $SO(4)$ quantum numbers as χ^+, $(\frac{\ell+1}{2}, \frac{\ell}{2})$.

For $m_\ell = m_\ell^- < 0$, the two solutions interchange their roles. Inverting now the definition of n_- and substituting $|m_\ell^-| = \frac{1}{2} + \ell$, we obtain the mass spectrum of $\mathcal{F}_\alpha^\ell$:

$$\begin{aligned} M_{\mathcal{F}}^2 &= \frac{4L^2}{R^4}(|m_\ell^-| + n_- + \tfrac{1}{2})(|m_\ell^-| + n_- + \tfrac{3}{2}) \\ &= \frac{4L^2}{R^4}(n_- + \ell + 1)(n_- + \ell + 2) \qquad (n_- \geqslant 0, \ell \geqslant 0)\,. \end{aligned} \tag{3.29}$$

The mass spectrum transforms as χ^- in the $(\frac{\ell}{2}, \frac{\ell+1}{2})$ of $SO(4)$.

3.4 Fluctuation-operator matching

In the following we summarize all fluctuations of the D7-brane and assign the corresponding operators to them. As was found in [3], the complete set of open string fluctuations fits into a series of massive supermultiplets of the $\mathcal{N} = 2$ supersymmetry algebra. These multiplets have the masses

$$M^2 = \frac{4L^2}{R^4}(n + \ell + 1)(n + \ell + 2) \qquad (n, \ell \geqslant 0) \tag{3.30}$$

and are labeled by the quantum number ℓ. Since the supercharges commute with the generators of the global group $SU(2)_\Phi$, the $SU(2)_\Phi$ quantum number, $\frac{\ell}{2}$ is the same for all fluctuations in a supermultiplet.

The bosonic fluctuations of a multiplet are listed in the upper part of table 2. The notation of the fluctuations and their mass spectra is the same as in [3]. The numbers $(j_1, j_2)_q$ label a representation of $SO(4) \approx SU(2)_\Phi \times SU(2)_\mathcal{R}$, and q is the $U(1)_R$ charge. The last column shows the conformal dimension of the lowest operator in a series.

Let us consider the bosonic components in more detail. First, there is a scalar in the $(\frac{\ell}{2}, \frac{\ell}{2} + 1)_0$ which corresponds to the chiral primaries [3]

$$\mathcal{C}^{I\ell} = \bar{q}^m \sigma^I_{mn} X^\ell q^n\,, \tag{3.31}$$

where the Pauli matrices σ^I_{mn} $(I = 1, 2, 3)$ transform in triplet representation of $SU(2)_R$. Then, there are 2 scalars in the $(\frac{\ell}{2}, \frac{\ell}{2})_2$ which we identified in section 2.2 as the scalar

meson operators $\mathcal{M}_s^{A\ell}$ $(A = 1, 2)$. Moreover, there is 1 scalar and 1 vector[1] in the $(\frac{\ell}{2}, \frac{\ell}{2})_0$ which we identify as the operators

$$\mathcal{J}_B^{\mu\ell} = \bar{\psi}_i^\alpha \gamma^\mu_{\alpha\beta} X^\ell \psi_i^\beta + i\bar{q}^m X^\ell D^\mu q^m + i\bar{D}^\mu \bar{q}^m X^\ell q^m \qquad (\mu = 0, 1, 2, 3)\,, \tag{3.32}$$

with X^ℓ as in eq. (2.3). The operator $\mathcal{J}_B^{\mu 0}$ at the bottom of the tower is associated with the global $U(1)_B$ current. Finally, there is a scalar in the $(\frac{\ell}{2}, \frac{\ell}{2} + 1)_0$ $(\ell \geq 2)$ which is a higher descendant of $\mathcal{C}^{I\ell}$.

There are analogous operators in the defect conformal field theories located on the T-dual D3/D5 and D3/D3 brane intersections [37, 42]. For instance, the operator $\mathcal{M}_s^{A\ell}$ $(A = 1, 2)$ corresponds to the scalar mesons $\mathcal{E}^{A\ell}$ $(A = 1, 2, 3)$ [37] and $\mathcal{C}^{\mu\ell}$ $(\mu = 1, 2, 3, 4)$ [42] in the D3/D5 and D3/D3 theories, respectively. Also, the $U(1)_B$ current operator $\mathcal{J}_B^{\mu 0}$ is part of the lowest multiplet in each of the D3/Dp $(p = 3, 5, 7)$ theories.

The fermionic content of the multiplets is shown in the lower part of table 2. This part of the multiplet matches precisely the spectra of the fermionic operators:[2]

$$M_{\mathcal{G}}(n, \ell - 1) = M_{\mathcal{F}}(n, \ell) \qquad (\ell \geqslant 1)\,. \tag{3.33}$$

The lowest multiplet with $\ell = 0$ contains only the operator $\mathcal{F}_\alpha^0$. The operator $\mathcal{G}_\alpha^\ell$ appears first for $\ell = 1$. We have already discussed the structure of the operators $\mathcal{F}_\alpha^\ell$ and $\mathcal{G}_\alpha^\ell$ in section 3.1.

	fluctuation	d.o.f.	$(j_1, j_2)_q$	spectrum		op.	Δ
bosons	1 scalar	1	$(\frac{\ell}{2}, \frac{\ell}{2} + 1)_0$	$M_{I,-}(n, \ell + 1)$	$(\ell \geq 0)$	$\mathcal{C}^{I\ell}$	2
	2 scalars	2	$(\frac{\ell}{2}, \frac{\ell}{2})_2$	$M_s(n, \ell)$	$(\ell \geq 0)$	$\mathcal{M}_s^{A\ell}$	3
	1 scalar	1	$(\frac{\ell}{2}, \frac{\ell}{2})_0$	$M_{III}(n, \ell)$	$(\ell \geq 1)$	$\mathcal{J}_B^{\mu\ell}$	3
	1 vector	3	$(\frac{\ell}{2}, \frac{\ell}{2})_0$	$M_{II}(n, \ell)$	$(\ell \geq 0)$		
	1 scalar	1	$(\frac{\ell}{2}, \frac{\ell}{2} - 1)_0$	$M_{I,+}(n, \ell - 1)$	$(\ell \geq 2)$	–	4
fermions	1 Dirac	4	$(\frac{\ell}{2}, \frac{\ell+1}{2})_1$	$M_{\mathcal{F}}(n, \ell)$	$(\ell \geq 0)$	$\mathcal{F}_\alpha^\ell$	$\frac{5}{2}$
	1 Dirac	4	$(\frac{\ell}{2}, \frac{\ell-1}{2})_1$	$M_{\mathcal{G}}(n, \ell - 1)$	$(\ell \geq 1)$	$\mathcal{G}_\alpha^\ell$	$\frac{9}{2}$

Table 2: Field content of supermultiplets in the D3/D7 theory.

As required by supersymmetry, the number of bosonic components in a multiplet,

$$1(2(\tfrac{\ell}{2} + 1) + 1) + (2 + 1 + 3)(2\tfrac{\ell}{2} + 1) + 1(2(\tfrac{\ell}{2} - 1) + 1) = 8(\ell + 1)\,, \tag{3.34}$$

agrees with the number of fermionic components,

$$4(2\tfrac{\ell+1}{2} + 1) + 4(2\tfrac{\ell-1}{2} + 1) = 8(\ell + 1)\,. \tag{3.35}$$

The masses of the fermionic fluctuations matches exactly the spectrum expected from supersymmetry. With hindsight, this justifies the introduction of the quark mass into the equations of motion via the replacement (3.23).

[1] These fields fit into a 5d vector field of AdS_5.

[2] The mass spectra $M_{\mathcal{G}}^2$ and $M_{\mathcal{F}}^2$ are identical to the spectra M_{F2}^2 and M_{F1}^2 found in [3], respectively.

4. Baryon spectroscopy in the "bottom-up" approach

So far we have discussed the spectrum of bosonic and fermionic operators with two fundamental fields at the ends. In contrast, a baryon in large N $SU(N)$ (super) Yang-Mills theory is a color singlet bound state of N fundamental quarks. As discussed in the introduction, the construction of a brane configuration for dynamical baryons is remarkably difficult. Nevertheless, it is possible to derive the baryon spectrum in the so-called "bottom-up" approach to the AdS/CFT correspondence. Starting from a phenomenological supergravity model, we will study the baryon spectrum of a broad class of supersymmetric field theories.

4.1 Mass spectra in the D3/D7 theory in the effective approach

Let us first demonstrate this technique by computing once again the scalar meson spectrum in the $\mathcal{N} = 2$ theory of the D3/D7 configuration. Assume we had no knowledge about the dual gravity theory apart from its existence. We may then construct the supergravity background from the properties of the field theory. According to the standard prescription of AdS/CFT, the conformal invariance of the field theory requires the dual supergravity background to be AdS_5. The $SO(4,2)$ isometry of AdS_5 corresponds to the conformal group of the field theory.

Next, in order to compute the spectrum of the operators $\mathcal{M}_s^\ell$ as defined in eq. (2.3), we introduce scalar modes ϕ_ℓ in this AdS_5 space which are dual to the operators $\mathcal{M}_s^\ell$. These scalars are described by the equation of motion

$$\partial_M \sqrt{g} g^{MN} \partial_N \phi_\ell - m_\ell^2 \phi_\ell = 0\,, \tag{4.1}$$

where g_{MN} is the AdS_5 metric in the parametrization (3.16), and m_ℓ is fixed by the mass-dimension relation

$$m_\ell^2 = \Delta(\Delta - 4) = -3 + \ell(\ell + 2)\,. \tag{4.2}$$

Using again the plane-wave ansatz $\phi_\ell = e^{-ik\cdot x} f_\ell$ with $M^2 = -k^2$, this leads to

$$(z^2\partial_z^2 - 3z\partial_z + z^2 M^2 - m_\ell^2 R^2) f_\ell(z) = 0\,, \tag{4.3}$$

or, equivalently ($r = R^2/z$),

$$\partial_r^2 f_\ell(r) + \frac{3}{r}\partial_r f_\ell(r) + \left(\frac{M^2}{r^4} - \frac{\ell(\ell+2)}{r^2}\right) f_\ell(r) = 0\,, \tag{4.4}$$

where we redefined $f_\ell \to f_\ell/r$. After the replacement (3.23), this differential equation becomes identical to eq. (2.6) which we obtained from the DBI action of the D7-branes. The result for the meson spectrum is therefore the same as in the full ten-dimensional string approach.

Several remarks are in order here: First, the DBI computation in section 2 has shown that the eigenvalues of the spherical harmonics on S^5 lead to the dependence of the scalar mass m_ℓ on the angular momentum ℓ. In the phenomenological approach, the internal space S^5 may be ignored in the effective approach. The dependence on ℓ enters the mass via the

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conformal dimension of the higher-ℓ operators, see eq. (4.2). Second, it is not obvious how a nonvanishing quark mass can be introduced in this approach. The introduction of the appropriate dual field in the background of the induced metric on the D7-brane does not lead to the desired result. Inspection of the DBI action shows that the metric elements transverse to the D7-brane are relevant for nonvanishing quark mass. These metric elements do not appear in an effective computation. We therefore use the replacement (3.23) for the introduction of a quark mass. Third, we may also compute the spectrum of the fermionic operators $\mathcal{F}^\ell_\alpha$ and $\mathcal{G}^\ell_\alpha$ in the "bottom-up" approach. In this case we would introduce a Dirac spinor Ψ in AdS_5. Fixing the masses as in eq. (3.3), we immediately get eq. (3.14) from which we obtain the spectrum for $\mathcal{F}^\ell_\alpha$ and $\mathcal{G}^\ell_\alpha$.

4.2 Baryons in superconformal field theories

We now turn to baryon operators in a broad class of large N $SU(N)$ super Yang-Mills theories with N_f flavors. The only further assumption we make is that the theory is conformal invariant, at least in some parameter regime, and that baryons do exist in the theory. An example of such a theory would be super QCD in the conformal window ($3/2 \leq N_f/N_c \leq 3$) or asymptotic free theories at high energies. Conformal invariance ensures that the dual supergravity background has the structure of an AdS_5 space.

In this class of theories, we are interested in the spectrum of the totally antisymmetric baryon operator

$$\mathcal{B}^0 = \frac{1}{\sqrt{N!}}\,\varepsilon_{i_1 i_2 \ldots i_N}\psi_{i_1}\ldots\psi_{i_N} \tag{4.5}$$

which has conformal dimension $\Delta = \frac{3}{2}N$ and spins $\frac{1}{2},\ldots,\frac{N}{2}$ (N odd). For simplicity, we only consider the state with spin $\frac{1}{2}$. We can also construct operators $\mathcal{B}^\ell$ of higher orbital excitation by the insertion of ℓ derivatives D_{i_k} into $\mathcal{B}^0$. Such operators would have the conformal dimensions

$$\Delta = \frac{3}{2}N + \ell\,. \tag{4.6}$$

Let us now specify the characteristics of supergravity fields Ψ_ℓ dual to the operators $\mathcal{B}^\ell$. Again, the spin of Ψ_ℓ must be identical to that of the boundary states which we have chosen to be $\frac{1}{2}$. Moreover, the conformal dimensions of the operators $\mathcal{B}^\ell$ as given by eq. (4.6) determine the masses of the fields Ψ_ℓ. Using eq. (1.1), we find the masses

$$m_\ell = \tfrac{3}{2}N - 2 + \ell\,. \tag{4.7}$$

In the phenomenological approach the spin-$\frac{1}{2}$ component of the baryons $\mathcal{B}^\ell$ are quite similar to the fermionic operators $\mathcal{G}^\ell_\alpha$ ($\mathcal{F}^\ell_\alpha$). The dual fields Ψ_ℓ are described by a Dirac equation on AdS_5,

$$(\not{D} - m_\ell)\Psi_\ell(x,z) = 0\,, \tag{4.8}$$

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but now the masses m_ℓ are given by eq. (4.7). Proceeding as in section 3, we obtain the baryon masses

$$M_{\mathcal{B}}^2 = \frac{4L^2}{R^4}(m_\ell + n - \tfrac{1}{2})(m_\ell + n + \tfrac{1}{2}) \qquad (n, \ell \geqslant 0)\,, \tag{4.9}$$

cf. eq. (3.28). Substituting the masses m_ℓ as given by eq. (4.7), we find the baryon spectrum

$$M_{\mathcal{B}}^2 = \frac{4L^2}{R^4}(n + \ell + \tfrac{3}{2}(N-1))(n + \ell + \tfrac{3}{2}(N-1) - 1)\,. \tag{4.10}$$

For large N, and constant n and ℓ, the baryon masses $M_{\mathcal{B}}$ scale with N, as expected from field theory [31].

5. Conclusions and open problems

We have derived the mass spectra of certain fermionic operators in the $\mathcal{N} = 2$ field theory located on the world-volume of intersecting D3 and D7-branes. This supplements the analysis of [3] in which the spectrum of scalar and vector meson operators was found in the same theory. We showed that both the bosonic as well as the fermionic operators fit into $\mathcal{N} = 2$ supermultiplets and constructed explicit expressions for these operators. We finally made a prediction for the mass of baryon operators in a class of supersymmetric field theories using an effective approach to the AdS/CFT correspondence.

In the derivation of the fermionic operator spectrum we made the assumption that massive quarks can be introduced via the replacement (3.23). The procedure led exactly to the operator spectrum expected from supersymmetry, which justifies the replacement with hindsight. Certainly, it would be more desirable to include a nonvanishing quark mass already on the level of the brane set-up. Another goal would be to extend the analysis to nonsupersymmetric and nonconformal backgrounds. In this case the masses of the bosonic fluctuations would differ from that of the fermionic ones.

Finally, one purpose of this paper was to develop techniques which will also be relevant for the holographic discussion of dynamical baryons. We mainly focused on open string fluctuations dual to operators with half-integer spin. As outlined in the introduction, it would be interesting to find a 10d baryon configuration and compute the corresponding operator spectrum. If the brane configuration turns out to be of the form of an AdS space, at least in some parameter regime, the baryon spectrum should agree with that found in the effective approach. It would be nice, if the spectrum (4.10) could be verified from the full 10d string theory point of view.

Acknowledgments

I would like to thank N. Arkani-Hamed, C. Beasley, J. Erdmenger, Z. Guralnik, and L. Motl for helpful discussions related to this work. The author was supported by a fellowship within the Postdoc-Program of the German Research Society (DFG), grant KI 1084/1.

A. Dirac equation in asymptotic AdS_{d+1} spaces

In this section we study the Dirac equation on asymptotic AdS_{d+1} spaces. The general class of backgrounds we wish to consider is given by the metric and dilaton

$$ds^2 = \frac{R^2}{z^2} e^{2A(z)} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2) \,, \qquad \Phi = \Phi(z) \,, \tag{A.1}$$

where $A(z), \Phi(z) \to 0$ as $z = R^2/r \to 0$, and R is the AdS radius.

The Dirac action for a spinor $\Psi(x^\mu, z)$ in this background is given by

$$S = \int d^d x dz \, e^{-\Phi} \sqrt{-\det g} \left[\bar{\Psi} e^M_A \gamma^A (D_M - \frac{1}{2} \partial_M \Phi) \Psi - m \bar{\Psi} \Psi \right] , \tag{A.2}$$

where $x^M = (x^\mu, z)$. The resulting Dirac equation is

$$(\not{D} - \frac{1}{2} e^M_A \gamma^A \partial_M \Phi - m) \Psi(x^\mu, z) = 0 \,, \tag{A.3}$$

again with $\not{D} = e^M_A \gamma^A D_M$. The dependence on the dilaton can be separated by the ansatz

$$\Psi(x^\mu, z) = e^{\frac{1}{2}\Phi(z)} \tilde{\Psi}(x^\mu, z) \,, \tag{A.4}$$

and we just have to solve $(\not{D} - m)\tilde{\Psi} = 0$. If $\Phi(z)$ is a regular function, i.e. $e^{\frac{1}{2}\Phi(z)} > 0$ for all z, then Ψ has the same zeros as $\tilde{\Psi}$.

For the background (A.1), the $d+1$-bein is $e^A_M = \delta^A_M a(z)$ with $a(z) \equiv \frac{R}{z} e^{A(z)}$. The only non-vanishing component of ω_{MAB} is $\omega_{\mu az} = \eta_{\mu a} \frac{\partial_z a(z)}{a(z)}$ [43] and the Dirac operator becomes

$$\not{D} = a(z)^{-1} \left[\gamma^A \partial_A + \frac{d}{2} b(z) \gamma^z \right] , \qquad b(z) \equiv \partial_z \ln a(z) \,. \tag{A.5}$$

It is then straight-forward to get

$$(\partial_z^2 + d\, b(z)\, \partial_z + M^2 - a(z)^2 m^2 + \frac{d^2}{4} b(z)^2 + \frac{d}{2} \partial_z b(z) - \partial_z a(z) \gamma^z m) f(z) = 0 \,. \tag{A.6}$$

This equation reduces to eq. (3.20) for $a(z) = \frac{R}{z}$ ($A(z) = 0$). It can be used for the computation of the spectrum of spin-$\frac{1}{2}$ fluctuations in confining backgrounds with non-trivial warp factor. We do not pursue this issue any further here.

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